

A Method for Controlling Series and Parallel Nonlinear Time-Varying LRC Circuits

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Abstract: Series and parallel *LRC* circuit systems are widely encountered in numerous electrical, electronics, control issues and differential equation applications. Accurate control of the current in series and voltage in parallel *LRC* circuit systems with nonlinear time-varying inductance, resistance and capacitance is a challenge. In this study, a novel approach proposed for control of current in nonlinear time-varying series *LRC* circuit and voltage in nonlinear and time-varying parallel *LRC* circuit. The proposed controller is characterized by a nonlinear algebraic equation and straining the tracking error converge to zero. Illustrative results confirm the proposed approach for forcing the current /voltage in series and parallel nonlinear time-varying *LRC* circuit to follow the targeted current /voltage trajectories efficiently. The proposed approach shows great novelty to determine the dynamics behavior of nonlinear time-varying systems. Therefore, the obtained results generalize and improve the existing conclusions. Simulations illustrate the feasibility and validity of the theoretical results.

Keywords: Nonlinear time-varying *LRC* circuit, Tracking error, Voltage control, Current control.

1 Introduction

The *LRC* circuit is an important type of electronic circuit comprising of an inductor (L), a resistor (R) and a capacitor (C) connected in series and parallel. *LRC* circuits have many different applications such as second-order differential equations [1, 2, 3], fractional differential equations [4], higher-order differential systems (elliptic filters) [5], control issues [6, 7, 8, 9, 10], electrical transmission line models [11], oscillators and dividing power [12], computers, medical devices, mobile phones, TVs, and FM radios [13] are just a few examples that use different forms of *LRC* circuits. *LRC* circuits are traditionally considered to have constant inductance, resistance and capacitance. However, in many scenarios, the value of elements of *LRC* circuits can change over time depending on the environment [12]. Therefore, we further assume the mass of a moving particle $m(t, x, \dot{x})$ where m , t , x and \dot{x} represent mass, time, position and velocity, respectively. With this analogy, the value of the elements of *LRC* circuits may depend on the time (t) and the charge (q) and current \dot{q} that following through the elements of the circuits. For simplicity, we define the inductance, resistance and capacitance of the *LRC* circuits such as $L(t, q, \dot{q})$, $R(t, q)$ and $C(t, q)$, respectively. Since most of the physical systems encountered in the application are nonlinear. This is the overriding reason we study nonlinear systems. The mathematical model of *LRC* resembles the equation of mass-spring-damper systems, which are frequently encountered in many conformable robotics systems as a basic architecture [14, 15]. Therefore, it may be more advantageous to examine the dynamic behavior of some physical systems by using the notion of *LRC* circuit models. Because the *LRC* circuits are more reliable than the mechanical systems. For example, the energy function of a *LRC* circuit and its derivation can be constructed from power-energy relationship of circuit theory. Hence, the dynamical behavior of such systems can be predictable. Since the energy and its derivation determine the behavior of the system [16]. The construction of the energy function of a *LRC* circuit and the variation of this energy along the trajectories of the system is one of the advantages. Therefore, the study of

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control issues for *LRC* circuits with nonlinear time-varying inductance, resistance and capacitance are indispensable. In control theory, there is a great need to investigate the controlling of such systems with nonlinear time-varying coefficients. Hence, we are motivated to analyze nonlinear and time-varying *LRC* circuit models and to propose a new nonlinear control strategy. In addition, despite the long history and wide use of *LRC* circuits in practice, differential equations of such circuits are still writing incorrectly. In this study, some existing equations and conclusions [12, 14] were discussed and refined. This paper proposes a novel nonlinear control strategy for series figure 1 and parallel figure 2 *LRC* circuits with nonlinear time-varying inductance, resistance and capacitance. The nonlinear error function is defined twice and used to ensure that the tracking control error of current or voltage is stable by the Lyapunov method, and with linear case (tracking control error) current or voltage converges to zero. The resultant nonlinear controller takes the benefit of time derivative information of the desired current/voltage trajectories and can be defined by an algebraic expression. One of the disadvantages of our approach with the defined nonlinear systems is that the convergence rate is slow. This idea will be clear with figures 3 and 4.

The remainder of the paper is arranged as follows. Section 2 forms the backbone of the study, models and methods, controller design and some remarks present the theoretical analysis of this section. The main results with concluding remark presented in section 3. Simulations are given in section 4.

The above discussions will concretize with the following theoretical analysis.

2 Models and methods

The goal of tracking error problems is to design a feedback controller such that the output $x(t)$ asymptotically tracks the desired target (reference) signal $x_d(t)$; that is, $\lim_{x \rightarrow \infty} [x(t) - x_d(t)] = 0$, where all the state variables are initially bounded in magnitude. A commonly used model for our discussion is a nonlinear and non-autonomous system that is

$$x(t) = f(t, x(t), u(t)), \quad x(0) = x_0, \quad \forall t \geq 0, \quad (1)$$

where $t \in \mathfrak{R}^+$ ($\mathfrak{R}^+ = [0, \infty)$) denotes time, $x \in \mathfrak{R}$ denotes the state of the system, while $u \in \mathfrak{R}$ is called the input or the control function. However, $f : \mathfrak{R}^+ \times \mathfrak{R} \times \mathfrak{R} \rightarrow \mathfrak{R}$ satisfies Lipschitz condition. The state vector $x(t) \in D$, in which $D \subseteq \mathfrak{R}$ is a domain that contains the origin $x = 0$. We assume that (1) is well posed, that is, there exists a unique solution $x : [0, \infty) \rightarrow \mathfrak{R}$ for every initial data $x(0) = x_0 \in \mathfrak{R}$ and x depends continuously on x_0 according to the normed topology on \mathfrak{R} . Let $f(t, 0, 0) = 0$, $f(t, x, 0) \neq 0$ for $x \neq 0$. First, we will construct the mathematical models of nonlinear and time-varying series and parallel *LRC* circuit systems in the standard form of second-order differential equations. A special form of (1) may be written as:

$$\ddot{x} + f(t, x, \dot{x}) + g(t, x) = u(t). \quad (2)$$

We may transform (2) into the following state space form

$$\begin{cases} \dot{x} = y, \\ \dot{y} = - \left[\frac{f(t, x, y)}{y} \right] y - \left[\frac{g(t, x)}{x} \right] x + u(t) \end{cases} \quad (3)$$

where $x \neq 0, y \neq 0$.

2.1 Nonlinear and time-varying series LRC circuit model

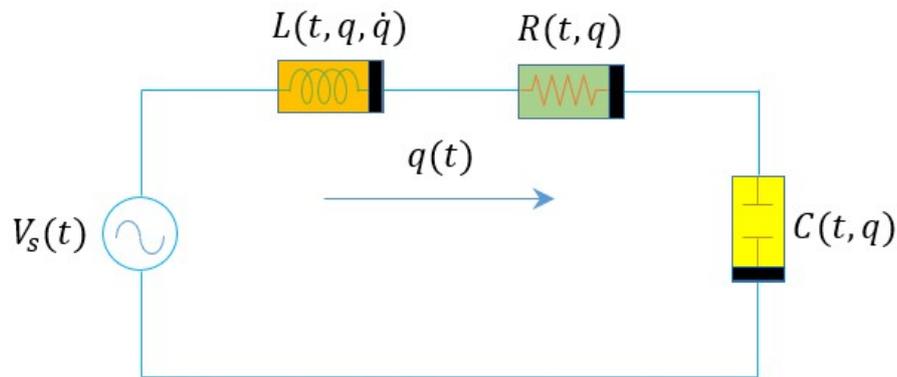


Fig. 1: Nonlinear and time-varying series LRC circuit.

The mathematical model of the above LRC circuit from Kirchoff's Law is described by the following equation:

$$L(t, q, \dot{q})\ddot{q} + R(t, q)\dot{q} + \frac{q}{C(t, q)} = V_s(t), \tag{4}$$

where $t \in \mathfrak{R}^+$ denotes the time, $q \in \mathfrak{R}$ represents the charge (the state variable), $L > 0$, $R \geq 0$ and $C > 0$, respectively, denote the nonlinear time-varying resistance, inductance and capacitance of the series LRC circuit, and $V_s(t)$ is the input voltage to be controlled. Assume that the existence and uniqueness solution to (4) satisfies the bounded initial data $q(0) = q_0$ and $\dot{q}(0) = q_0$. We may write (4) in the $x(= q)$ variable and in the standard form such as:

$$\ddot{x} + \frac{R(t, x)}{L(t, x, \dot{x})}\dot{x} + \frac{1}{L(t, x, \dot{x})C(t, x)}x = \frac{V(t)}{L(t, x, \dot{x})}. \tag{5}$$

With making an analogy between (3) and (5) for $\dot{x} = y$, we have the followings:

$$\frac{f(t, x, y)}{y} = \frac{R(t, x)}{L(t, x, y)}, \quad \frac{g(t, x)}{x} = \frac{1}{L(t, x, y)C(t, x)}, \quad u(t) = \frac{V(t)}{L(t, x, y)}.$$

The tracking error for the above series circuit is $e(t) = x(t) - x_d(t)$.

Remark 1 For linear time-varying case of (5), we have $u(t) = \frac{V(t)}{L(t)}$. But, for the same problem $u(t)$ stated in [12] such as $u(t) = \frac{\dot{V}_s(t)C(t) + V_s(t)\dot{C}(t) + I(0)}{C(t)L(t)}$. (5) also refines the coefficients of the differential system (4) in [12]. For convenience, see [12].

2.2 Nonlinear and time-varying parallel LRC circuit model

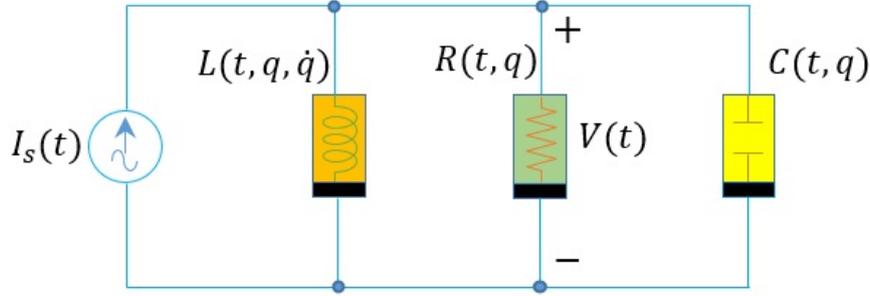


Fig. 2: Nonlinear and time-varying series LRC circuit.

The mathematical model of the above LRC circuit from Kirchoff's Law described by the following equation:

$$\frac{V(t)}{R(t, q)} + \int_0^t \frac{V_s(s)}{L(s, q(s), \dot{q}(s))} ds + I_L(0) + \dot{C}_t(t, q)V(t) + \dot{C}_q(t, q)\dot{q}V(t) + C(t, q)\ddot{V}(t) = I_s(t), \quad (6)$$

First, we differentiate (6) with respect to time t, then we have the followings:

$$\begin{aligned} & \frac{\dot{V}(\cdot)R(\cdot) - [\dot{R}(\cdot) + \dot{R}_q(\cdot)\dot{q}]V(\cdot)}{R^2(\cdot)} + \frac{V(\cdot)}{L(\cdot)} + \ddot{C}_t(t, q)V(t) + \ddot{C}_{tq}(t, q)\dot{q}V(t) + \\ & \dot{C}_t(t, q)\dot{V}(t) + \ddot{C}_{qt}(t, q)\dot{q}V(t) + \ddot{C}_q(t, q)\dot{q}^2V(t) + \dot{C}_q(t, q)\ddot{q}V(t) + \\ & \dot{C}_q(t, q)\dot{V}(t) + \dot{C}_t(t, q)\dot{V}(t) + \dot{C}_q(t, q)\dot{q}\dot{V}(t) + C(t, q)\ddot{V}(t) = \dot{I}_s(t), \end{aligned} \quad (7)$$

(7) can be rewritten in the standard form

$$\begin{aligned} & \ddot{V}(\cdot) + \frac{1}{C(\cdot)} \left[\frac{1}{R(\cdot)} + 2\dot{C}_t(\cdot) + 2\dot{C}_q(\cdot)\dot{q} \right] \dot{V}(\cdot) + \\ & \frac{1}{C(\cdot)} \left[-\frac{\dot{R}(\cdot) + \dot{R}_q(\cdot)\dot{q}}{R^2(\cdot)} + \frac{1}{L(\cdot)} + \ddot{C}_t(\cdot) + 2\ddot{C}_{tq}(\cdot)\dot{q} + \ddot{C}_q(\cdot)\dot{q}^2 + \dot{C}_q(t, q)\ddot{q} \right] V(\cdot) = \frac{\dot{I}_s(\cdot)}{C(\cdot)} \end{aligned} \quad (8)$$

To analyze the proposed control approach further, we may transform the above equation (7) to the state space form (3). Now, the coefficients of (3) are the followings:

$$\frac{f(t, x, y)}{y} = \frac{1}{C(\cdot)} \left[\frac{1}{R(\cdot)} + 2\dot{C}_t(\cdot) + 2\dot{C}_q(\cdot)\dot{q} \right]$$

and

$$\frac{g(t, x)}{x} = \frac{1}{C(\cdot)} \left[\frac{1}{L(\cdot)} + \ddot{C}_t(\cdot) + 2\ddot{C}_{tq}(\cdot)\dot{q} + \ddot{C}_q(\cdot)\dot{q}^2 \right]$$

and the control input is

$$u(t) = \frac{\dot{I}_s(\cdot)}{C(\cdot)}$$

We again assume that the existence and uniqueness solution to (8) satisfies any bounded initial data. For this case, we denote the state variables such as: $x(t) = V(t), y(t) = \dot{V}(t)$.

Remark 2 For linear time-varying case of (8) we have $u(t) = \frac{\dot{I}_s(t)}{C(t)}$. But, for the same problem $u(t)$ stated in [12] such as $u(t) = \frac{L(t)\dot{I}_s(t)+C(t)+\dot{L}(t)I_s(t)+V(0)}{C(t)L(t)}$. Further, for the same problem defined in [12], we have the following

$$\dot{V} + \frac{1}{C(t)} \left(\frac{1}{R(t)} + 2\dot{C}(t) \right) \dot{V} + \frac{1}{C(t)} \left(-\frac{\dot{R}}{R^2(t)} + \frac{1}{L(t)} + \ddot{C}(t) \right) V = \frac{\dot{I}_s(t)}{C(t)}.$$

Obviously, one can see the big differences between our equation above and (2) in [12]. The tracking error for the above parallel circuit is

Remark 3 When we carefully examine article [12], there are fundamental errors in equations (1),(3) and (8). Namely, in (1) and (3) $\frac{1}{L(t)}$ and $\frac{1}{C(t)}$ must be written in the associated integral (not outside of the integral) such as (6), and in (3) However, other coefficients are also wrong. In (8) of [12], and in (5) of [14] the two solutions must be linearly independent. That is, the second solution may be written in the form $c_2 t \exp(-rt)$. Furthermore, our Theorem 2 in section 3 improves the theoretical results in [12, 14]. In addition, the control functions $u(t)$ in [12] must not contain the initial values. They must disappear while taking the derivative of the associated integrals.

2.3 Controller Design

Our second goal in this work is to attain accurate control of the voltage/current of the nonlinear and time-varying series/parallel LRC circuit systems to the desired target value $x_d(t)$. To monitor the tracking control process of the circuit models, we consider the first error function and apply it as follows:

$$e_1(t) = x(t) - x_d(t), \quad (9)$$

where $x_d = q(t)$ or $x_d = V(t)$ coresspondignly. In addition to this, we may need to construct another error design formula forcing the tracking error to be zero:

$$\dot{e}_i(t) = -r\varphi(e_i(t)) = -re_i^{2n-1}(t), \quad i = 1, 2$$

where $n \in \mathbb{N}$, $\mathbb{N} = \{1, 2, 3, \dots\}$, and $r > 0$ is a design parameter that scales the convergence rate of the solution. For simplicity, let $n = 2$ in this work, the nonlinear activation function is $\varphi(e_i(t)) = e_i^3(t)$, $i = 1, 2$ to force the tracking error $e_1(t)$ to be zero as time t evolves. Thus, we have the tracking error formula characterized by

$$\dot{e}_1(t) = -re_1^3(t). \quad (10)$$

From (9) and (10), we obtain

$$\dot{x}(t) - \dot{x}_d(t) = -r(x(t) - x_d(t))^3.$$

Considering $\dot{x}(t) = y(t)$, we have

$$y(t) - \dot{x}_d(t) + r(x(t) - x_d(t))^3 = 0.$$

We may need to construct a second error function:

$$e_2(t) = y(t) - \dot{x}_d(t) + r(x(t) - x_d(t))^3 = 0.$$

By applying the design formula $\dot{e}_2(t) = -re_2^3(t)$ again, we obtain

$$\dot{y} - \ddot{x}_d + 3r(x - x_d)^2(\dot{x} - \dot{x}_d) + r \left[y + r\dot{x}_d + r^2(x - x_d)^3 \right]^3 = 0.$$

From (3), by substituting $\dot{y} = -f(t, x, y) - g(t, x) + u(t)$, into the above equation, we can derive the control function and then write is as follows:

$$u(t) = f(t, x, y) + g(t, x) + \ddot{x}_d - 3r(x - x_d)^2(y - \dot{x}_d) - r \left[y + r\dot{x}_d + r^2(x - x_d)^3 \right]^3. \quad (11)$$

3 Main results

The applications of the nonlinear and linear controller will take place with the following theorems.

Theorem 1. *Starting from the bounded initial states, the solution of the tracking error $e_1(t)$ of the nonlinear time-varying series or parallel LRC circuit system equipped with controller (11) is stable. If the relation $\sqrt{h(t)} := \sup_{t \geq 0} \frac{1}{2}y^2(t)$ exists between the storage function $h(t)$ and the current $v(t)$ of the circuit then the solution $e_1(t)$ exponentially converges to zero.*

Proof 1 According to the associated design formula, we have the following equations:

$$e_2(t) = \dot{e}_1(t) + re_1^3(t), \quad \dot{e}_2(t) = -re_2^3(t). \quad (12)$$

From these two equations (12), we obtain the following second order differential equation:

$$\ddot{e}_1 + 3re_1^2\dot{e}_1 = -r(\dot{e}_1 + re_1^3)^3,$$

in $x = e_1$ variable, we have

$$\ddot{x} + r(\dot{x}^2 + 3x^2 + 3rx^3\dot{x} + 3r^2x^6)\dot{x} + r^4x^9 = 0 \quad (13)$$

Now, assume that the resistance component of the circuit (i.e., the coefficient of \dot{x}) must be a positive value. Therefore, for simplicity we may say that $x\dot{x} \geq 0$. Thus, the error function may represent a series nonlinear LRC circuit with zero input. (12) can be transformed into the equivalent system

$$\begin{cases} \dot{x} = y, \\ \dot{y} = -r(y^2 + 3rx^2 + 3rx^3y + 3r^2x^6)y - r^4x^9. \end{cases} \quad (14)$$

We may not have an explicit solution to (14), because of the nonlinearity of the equation. In such cases, the Lyapunov method is often an indispensable tool to make solution predictions about the behavior of the system [16]. Therefore, the Lyapunov (energy) function for the above system from the power-energy relationship of circuit theory may be written as:

$$h(t) = h(x, y) = \frac{1}{2}y^2 + \frac{1}{10}r^4x^{10}.$$

Then, we have $h(t) \geq \frac{1}{2}y^2(t)$, $h(0, 0) = 0$, $h(x, y) > 0$ with $x \neq 0$ or $y \neq 0$. Hence, h is a positive definite function. The derivative of the Lyapunov function $h(t)$ along the trajectories of system (14) gives

$$\dot{h}(t) = -r(y^2 + 3x^2 + 3rx^3y + 3r^2x^6)y^2$$

,

$$\dot{h}(t) \leq -ry^4 \leq 0.$$

Hence, the solution of the error equation is stable. From $\sqrt{h(t)} := \sup_{t \geq 0} \frac{1}{2}y^2(t)$, then it follows that

$$\dot{h}(t) \leq -4rh(t), \quad (15)$$

$$h(t) \leq ce^{-4rt}$$

for all $t \geq 0$, c is a constant value. (15) implies that both the solutions $x(t)$ and $y(t)$ converge to the equilibrium point $(0,0)$. The tracking error $x(t) - x_d(t)$ for series circuit and $V(t) - V_d(t)$ for parallel circuit exponentially converge to zero. Thus, the proof is complete.

Theorem 2. For the linear time-varying case of series or parallel LRC circuit equipped with linear controller form of (11), starting with bounded initial states $x(0)$, and $\dot{x}(0)$, the linear tracking error $e_1(t) = x(t) - x_d(t)$ of the whole system exponentially converges to zero.

Proof 2 The linear form of (12) is the following

$$e_2(t) = \dot{e}_1(t) + re_1(t), \quad \dot{e}_2(t) = -re_2(t).$$

The above equations yield the following differential equation

$$\ddot{e}_1(t) + 2r\dot{e}_1(t) + r^2e_1(t) = 0, \quad t > 0.$$

The solution of the above equation is

$$e_1(t) = A \exp(-rt) + Bt \exp(-rt), \quad t > 0,$$

where A and B are constant values. Obviously, $e_1(t)$ exponentially converges to zero as time t evolves. The solution $e_1(t)$ ((8)) in [12] and the solution $e_1(t)$ ((5)) in [14] may be not correct. They must be in the above solution form.

Remark 4 The proof of Theorem [2] is clearer for an analytic solution. Therefore, there is no need to give a numerical example.

4 Simulation results

Figure 3 shows the tracking control results for nonlinear time-varying LRC model (5) or (8) by the controller (11) with $r = 10$. One can observe that the model output $x(t)$ can achieve the reference as time t evolves. Therefore, the designed controller (11) can make the nonlinear time-varying LRC model (5) or (8) achieve the step response. Figure 4 demonstrates the corresponding tracking error of LRC models ((5),(8)) with the designed controller (11). The nonlinear steady-state tracking error $|x(t) - x_d(t)|$ diminished to zero gradually. The convergence rate of nonlinear systems is slow. This is obvious with figures 3, 4 and figure 5 shows the phase portrait of the system (14) with $r = 10$ attains to the equilibrium solution $(x(t), y(t)) = (0, 0)$. The last three figures illustrate the feasibility and validity of the theoretical results.

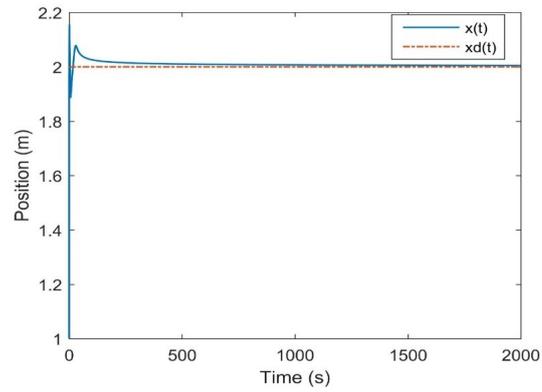


Fig. 3: Step response for the *LRC* model (5) or (8) by the controller

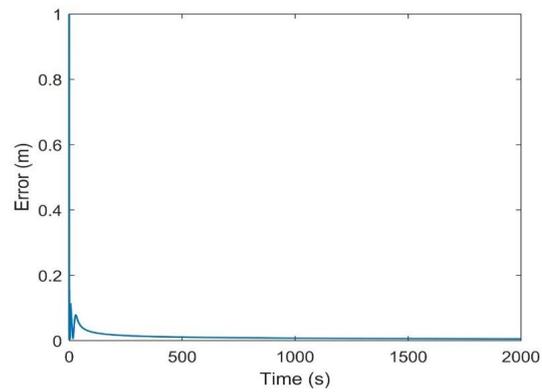


Fig. 4: Tracking error $|x(t) - x_d(t)|$ of *LRC* models

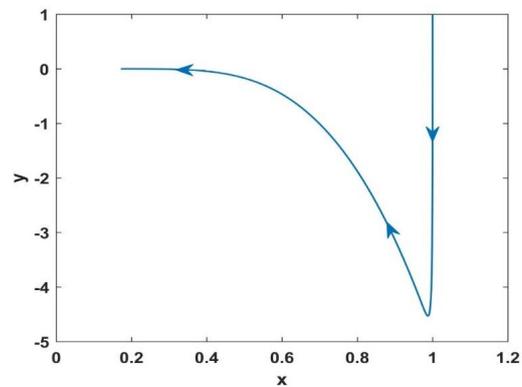


Fig. 5: Phase portrait of the system (14) with $r = 10$.

5 Conclusion

This paper proposes a novel approach for the control of series and parallels *LRC* circuits with nonlinear and time-varying inductance, resistance and capacitance. The proposed nonlinear controller is described by an algebraic equation and can provide the tracking error to slowly converge to zero. By the way, we realized that the convergence rate of the linear controller is faster than that of nonlinear systems. The simulations illustrate the feasibility and validity of the theoretical results.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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