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Pairwise soft separation axioms in soft bigeneralized topological spaces

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Abstract: In this paper, we define and study some soft separation axioms in soft bigeneralized topological spaces in terms of pairwise soft bigeneralized T_i-spaces $(i = 0, 1, 2, 3, 4)$. Characterizations and properties of these soft separation axioms have been obtained. Finally, we discuss the soft hereditary properties for these spaces.

Keywords: Soft set, Soft topology, Soft bigeneralized topology, Soft bigeneralized topological space, Soft separation axioms, Pairwise soft separation axioms.

1 Introduction

In the year 1999, D. Molodtsov [17] introduced the soft set theory as a new mathematical tools for dealing with uncertainties inherent in many of real world problems. Since a good number of papers is being published every year, soft set theory is getting popularity among the researchers [13,14].

In 1963, N. Levine [12] tried to generalize a topology by replacing open sets with semi-open sets. Later, A. Csaszar [3] introduced the concept of $\gamma-$open sets and generalized these open sets. The theory of generalized topological spaces (briefly GT), which was founded by A. Csaszar [2] is one of the most important developments of general topology. Later, W. K. Min and Y. K. Kim [16] initiated the concept of bigeneralized topological spaces and quasi generalized open sets. In addition some separation axioms in bigeneralized topological spaces studied by P.Torton et al. [23].

Recently M. Shabir and M. Naz [20] initiated the study of soft topological spaces. Also they defined soft topology as a collection of soft sets over X and they showed that soft topological spaces gives a parameterized family of topological spaces. After then many researchers studied on soft topological spaces [1,4,5,7,15,18,21,24]. J.C. Kelly [10], first initiated the concept of bitopological spaces. In 2014 B.M.Ittanagi [8] introduced the soft bitopological spaces which are defined over an initial universe with a fixed set of parameters. In later years, many researchers studied some of basic concept and properties on soft bitopological spaces [8,9].

In the year 2014, J.Thomas and S.J.John [22] initiated the concept of generalized topological spaces. They showed that a soft generalized topology gave a parameterized family of generalized topologies on the initial universe. Recently, In 2018, T. Y. Ozturk et. al. [19] introduced the soft bigeneralized topological spaces. Also they defined soft open set, soft closed set, soft closure set etc. on the soft bigeneralized topological spaces. Thus they showed that soft bigeneralized topological spaces are important generalized of bigeneralized topological spaces. In [6] paper, they studied soft

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generalized continuity, soft generalized open (closed) mapping and soft generalized homeomorphism on soft bigeneralized topological spaces.

In the present paper, we introduced some pairwise soft separation axioms in soft bigeneralized topological spaces. Characterizations and properties of these pairwise soft separation axioms have been obtained. Finally, we showed that soft hereditary properties of pairwise soft bigeneralized $T_i - spaces$ $(i = 0,1,2,3,4)$.

2 Preliminary

In this section we will introduce necessary definitions and theorems for soft sets. Throughout this paper X denotes initial universe, E denotes the set of all parameters, $P(X)$ denotes the power set of X.

Definition 1. [17]. *A pair (F,E) is called a soft set over X, where F is a mapping given by $F : E \to P(X)$.*

In other words, the soft set is a parameterized family of subsets of the set X. For $e \in E$, $F(e)$ may be considered as the set of $e-$elements of the soft set (F,E), or as the set of $e-$approximate elements of the soft set, i.e.,

$$(F,E) = \{(e,F(e)) : e \in E,\, F : E \to P(X)\}.$$

After this, $SS(X)_E$ denotes the family of all soft sets over X with a fixed set of parameters E.

Definition 2. [14]. *For two soft sets (F,E) and (G,E) over X, (F,E) is called a soft subset of (G,E) if $\forall e \in E$, $F(e) \subseteq G(e)$. This relationship is denoted by $(F,E)\widetilde{\subseteq}(G,E)$.*

Similarly, (F,E) is called a soft superset of (G,E) if (G,E) is a soft subset of (F,E). This relationship is denoted by $(F,E)\widetilde{\supseteq}(G,E)$. Two soft sets (F,E) and (G,E) over X are called soft equal if (F,E) is a soft subset of (G,E) and (G,E) is a soft subset of (F,E).

Definition 3. [14]. *The intersection of two soft sets (F,E) and (G,E) over X is the soft set (H,E), where $\forall e \in E$, $H(e) = F(e) \cap G(e)$. This is denoted by $(F,E)\widetilde{\cap}(G,E) = (H,E)$.*

Definition 4. [14]. *The union of two soft sets (F,E) and (G,E) over X is the soft set (H,E), where $\forall e \in E$, $H(e) = F(e) \cup G(e)$. This is denoted by $(F,E)\widetilde{\cup}(G,E) = (H,E)$.*

Definition 5. [14]. *A soft set (F,E) over X is said to be a null soft set denoted by $(\widetilde{\phi},E)$ if for all $e \in E$, $F(e) = \varnothing$.*

Definition 6. [14]. *A soft set (F,E) over X is said to be an absolute soft set denoted by $(\widetilde{X},E)$ if for all $e \in E$, $F(e) = X$.*

Definition 7. [20]. *The difference (H,E) of two soft sets (F,E) and (G,E) over X, denoted by $(F,E)\backslash(G,E)$, is defined as $H(e) = F(e)\backslash G(e)$ for all $e \in E$.*

Definition 8. [20]. *The complement of a soft set (F,E), denoted by $(F,E)^c$, is defined $(F,E)^c = (F^c,E)$, where $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X \backslash F(e)$, $\forall e \in E$ and F^c is called the soft complement function of F.*

Definition 9. [20]. *Let (F,E) be a soft set over X and Y be a non-empty subset of X. Then the sub soft set of (F,E) over Y denoted by $({}^YF,E)$, is defined as ${}^YF(\alpha) = Y \cap F(\alpha)$, for all $\alpha \in E$. In other words $({}^YF,E) = (Y,E) \cap (F,E)$.*

Definition 10. [1]. *Let (F,E) be a soft set over X. The soft set (F,E) is called a soft point, denoted by (x_e,E), if for the element $e \in E$, $F(e) = \{x\}$ and $F(e^c) = \varnothing$ for all $e^c \in E - \{e\}$ (briefly denoted by x_e).*

It is obvious that each soft set can be expressed as a union of soft points. For this reason, to give the family of all soft sets on X it is sufficient to give only soft points on X.

Definition 11. [1]. *Two soft points x_e and $y_{e'}$ over a common universe X, we say that the soft points are different if $x \neq y$ or $e \neq e'$.*

Definition 12. [1]. *The soft point x_e is said to be belonging to the soft set (F,E), denoted by $x_e \widetilde{\in} (F,E)$, if $x_e(e) \in F(e)$, i.e., $\{x\} \subseteq F(e)$.*

Definition 13. [1]. *Let $(X,\widetilde{\tau},E)$ be a soft topological space over X. A soft set $(F,E) \widetilde{\subseteq} (X,E)$ is called a soft neighborhood of the soft point $x_e \widetilde{\in} (F,E)$ if there exists a soft open set (G,E) such that $x_e \widetilde{\in} (G,E) \widetilde{\subset} (F,E)$.*

Definition 14. [22]. *Let $\widetilde{\mu}$ be the collection of soft set over X. Then $\widetilde{\mu}$ is said to be a soft generalized topology on X if*

(1) *$(\widetilde{\phi},E)$ belongs to $\widetilde{\mu}$;*
(2) *the union of any number of soft sets in $\widetilde{\mu}$ belongs to $\widetilde{\mu}$.*

The triplet $(X,\widetilde{\mu},E)$ is called a soft generalized topological space (briefly $SGT-$space) over X.

Definition 15. [22]. *A soft generalized topology $\widetilde{\mu}$ on (X,E) is called strong if $\widetilde{X} \in \widetilde{\mu}$.*

Definition 16. [22]. *Let $(X,\widetilde{\mu},E)$ be a $SGT-$space over (X,E). Then every element of $\widetilde{\mu}$ is called a soft $g_{\widetilde{\mu}}-$open set.*

Definition 17. [22]. *$(X,\widetilde{\mu},E)$ be a $SGT-$space over (X,E) and $(F,E) \widetilde{\subseteq} (X,E)$. Then (F,E) is called a soft $g_{\widetilde{\mu}}-$closed set if its soft complement $(F,E)^c$ is a soft $g_{\widetilde{\mu}}-$open set.*

Definition 18. [19]. *Let $\widetilde{g}_1$ and $\widetilde{g}_2$ be two soft generalized topologies on X and E be a set of parameters. Then the quadruple system $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is called a soft bigeneralized topological space (briefly $SBGT-$space).*

Definition 19. [19]. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a $SBGT-$space. A soft set $(F,E) \widetilde{\subseteq} X$ is called soft $\widetilde{g}_{1,2}-open$ set if $(F,E) = (F_1,E) \widetilde{\cup} (F_2,E)$ where $(F_1,E) \in \widetilde{g}_1$ and $(F_2,E) \in \widetilde{g}_2$.*

The complement of soft $\widetilde{g}_{1,2}-open$ set is called soft $\widetilde{g}_{1,2}-closed$ set. Clearly, a soft set (G,E) over X is a soft $\widetilde{g}_{1,2}-closed$ set in $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ if $(G,E) = (G_1,E) \widetilde{\cap} (G_2,E)$ such that $(G_1,E) \in \widetilde{g}_1^c$ and $(G_2,E) \in \widetilde{g}_2^c$, where

$$\widetilde{g}_i^c = \{(G,E)^c \in SS(X)_E : (G,E) \in \widetilde{g}_i\},\ i = 1,2.$$

Definition 20. [19]. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a $SBGT-$space and $(F,E) \in SS(X)_E$. Then, soft $\widetilde{g}_{1,2}-closure$ set of (F,E), denoted by $scl_{\widetilde{g}_{1,2}}(F,E)$, defined by*

$$scl_{\widetilde{g}_{1,2}}(F,E) = \widetilde{\cap} \left\{(G,E) \in \widetilde{g}_{1,2}^c : (F,E) \widetilde{\subseteq} (G,E)\right\}.$$

Note that, $scl_{\widetilde{g}_{1,2}}(F,E)$ is the smallest soft $\widetilde{g}_{1,2}-closed$ set that containing (F,E).

Theorem 1. [19]. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a $SBGT-$space and (F,E), $(G,E) \in SS(X)_E$. Then,*

(1) *$scl_{\widetilde{g}_{1,2}}(\widetilde{X},E) = (\widetilde{X},E)$,*
(2) *$(F,E) \widetilde{\subseteq} scl_{\widetilde{g}_{1,2}}(F,E)$,*
(3) *(F,E) is a soft $\widetilde{g}_{1,2}-closed$ set if and only if $scl_{\widetilde{g}_{1,2}}(F,E) = (F,E)$,*
(4) *if $(F,E) \widetilde{\subseteq} (G,E)$, then $scl_{\widetilde{g}_{1,2}}(F,E) \widetilde{\subseteq} scl_{\widetilde{g}_{1,2}}(G,E)$,*
(5) *$scl_{\widetilde{g}_{1,2}}(F,E) \widetilde{\cup} scl_{\widetilde{g}_{1,2}}(G,E) \widetilde{\subseteq} scl_{\widetilde{g}_{1,2}}\left[(F,E) \widetilde{\cup} (G,E)\right]$ and $scl_{\widetilde{g}_{1,2}}\left[(F,E) \widetilde{\cap} (G,E)\right] \widetilde{\subseteq} scl_{\widetilde{g}_{1,2}}(F,E) \widetilde{\cap} scl_{\widetilde{g}_{1,2}}(G,E)$,*
(6) *$scl_{\widetilde{g}_{1,2}}\left[scl_{\widetilde{g}_{1,2}}(F,E)\right] = scl_{\widetilde{g}_{1,2}}(F,E)$, i.e., $scl_{\widetilde{g}_{1,2}}(F,E)$ is a soft $\widetilde{g}_{1,2}-closed$ set.*

Definition 21. [19]. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT−space and $(F,E) \in SS(X)_E$. Then, soft $\widetilde{g}_{1,2}-$interior set of (F,E), denoted by $sint_{\widetilde{g}_{1,2}}(F,E)$, defined by*

$$sint_{\widetilde{g}_{1,2}}(F,E) = \widetilde{\cup}\left\{(U,E) \in \widetilde{g}_{1,2} : (U,E)\widetilde{\subseteq}(F,E)\right\}.$$

Note that, $sint_{\widetilde{g}_{1,2}}(F,E)$ is the largest soft $\widetilde{g}_{1,2}-$open set contained in (F,E).

Definition 22. [6]. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ and $(Y,\widetilde{k}_1,\widetilde{k}_2,E)$ be two SBGT−spaces and $(f,1_E):(X,\widetilde{g}_1,\widetilde{g}_2,E) \to (Y,\widetilde{k}_1,\widetilde{k}_2,E)$ (briefly denoted by f) be a soft mapping. For each soft $\widetilde{k}_{1,2}-nbd$ (G,E) of $f(x_e)$, if there exists a soft $\widetilde{g}_{1,2}-nbd$ (F,E) of soft point $x_e\widetilde{\in}SS(X)_E$ such that $f((F,E))\widetilde{\subseteq}(G,E)$, then f is said to be soft $\widetilde{g}_{1,2}-continuous$ mapping at x_e.*

If f is a soft $\widetilde{g}_{1,2}-continuous$ mapping for all $x_e\widetilde{\in}SS(X)_E$, then f is called a soft $\widetilde{g}_{1,2}-continuous$ mapping on a soft bigeneralized topological spaces $(X,\widetilde{g}_1,\widetilde{g}_2,E)$.

Definition 23. [6]. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ and $(Y,\widetilde{k}_1,\widetilde{k}_2,E)$ be two SBGT−spaces, $f:(X,\widetilde{g}_1,\widetilde{g}_2,E) \to (Y,\widetilde{k}_1,\widetilde{k}_2,E)$ be a soft mapping. Then,*

(a) *f is called a soft $\widetilde{g}_{1,2}-open$ mapping if $f((F,E))$ is a soft $\widetilde{k}_{1,2}-open$ set in $(Y,\widetilde{k}_1,\widetilde{k}_2,E)$ for every soft $\widetilde{g}_{1,2}-open$ set (F,E) in $(X,\widetilde{g}_1,\widetilde{g}_2,E)$;*
(b) *f is called a soft $\widetilde{g}_{1,2}-closed$ mapping if $f((K,E))$ is a soft $\widetilde{k}_{1,2}-closed$ set in $(Y,\widetilde{k}_1,\widetilde{k}_2,E)$ for every soft $\widetilde{g}_{1,2}-$ closed set (K,E) in $(X,\widetilde{g}_1,\widetilde{g}_2,E)$.*

Theorem 2. [6]. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ and $(Y,\widetilde{k}_1,\widetilde{k}_2,E)$ be two SBGT−spaces and $f:(X,\widetilde{g}_1,\widetilde{g}_2,E) \to (Y,\widetilde{k}_1,\widetilde{k}_2,E)$ be a soft mapping. Then f is a soft $\widetilde{g}_{1,2}-continuous$ mapping on a soft bigeneralized topological spaces $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ iff $f^{-1}((G,E))$ is a soft $\widetilde{g}_{1,2}-open$ set, for every soft $\widetilde{k}_{1,2}-open$ set (G,E).*

Definition 24. [6]. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ and $(Y,\widetilde{k}_1,\widetilde{k}_2,E)$ be two SBGT−spaces, $f:(X,\widetilde{g}_1,\widetilde{g}_2,E) \to (Y,\widetilde{k}_1,\widetilde{k}_2,E)$ be a soft mapping. Then f is called a soft $\widetilde{g}_{1,2}-homeomorphism$, if*

(i) *f is a soft bijection,*
(ii) *f is a soft $\widetilde{g}_{1,2}-continuous$,*
(iii) *f^{-1} is a soft $\widetilde{g}_{1,2}-continuous$ mapping.*

3 Pairwise soft separation axioms

Definition 25. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT-space over X and each distinct soft points $x_\alpha, y_\beta\widetilde{\in}SS(X)_E$. If there exist a pairwise soft open $(F,E) \in \widetilde{g}_{12}$ such that $x_\alpha\widetilde{\in}(F,E)$, $y_\beta\widetilde{\notin}(F,E)$ or $x_\alpha\widetilde{\notin}(F,E)$, $y_\beta\widetilde{\in}(F,E)$, then $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is called a pairwise soft bigeneralized T_0-space.*

Example 1. Let $X=\{a,b\}$, $E=\{e_1,e_2\}$ and let

$$\widetilde{g}_1 = \{(\widetilde{\phi},E),(F_1,E),(F_2,E)\}$$
$$\widetilde{g}_2 = \{(\widetilde{\phi},E),(G_1,E),(G_2,E)\}$$

where

$$(F_1,E) = \{(e_1,\{a\}),(e_2,\phi)\}$$
$$(F_2,E) = \{(e_1,X),(e_2,\{a\})\}$$
$$(G_1,E) = \{(e_1,\{b\}),(e_2,\phi)\}$$
$$(G_2,E) = \{(e_1,X),(e_2,\{b\})\}$$

Then, $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a SBGT-space. Thus, the family of pairwise soft open sets over $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ becomes

$$\widetilde{g}_{12}=\{(\widetilde{\phi},E),(F_1,E),(F_2,E),(G_1,E),(G_2,E),(H_1,E),(H_2,E)\}$$

where

$$(H_1,E)=\{(e_1,X),(e_2,\phi)\}$$
$$(H_2,E)=\{(e_1,X),(e_2,X)\}$$

It is clear that $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a pairwise soft bigeneralized T_0-space.

Theorem 3. *A SBGT-space $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is pairwise soft bigeneralized T_0-space if and only if $scl_{\widetilde{g}_{12}}(x_\alpha,E)\neq scl_{\widetilde{g}_{12}}(y_\beta,E)$ where each distinct soft points $x_\alpha,y_\beta\widetilde{\in}SS(X)_E$.*

Proof. Suppose that $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is pairwise soft bigeneralized T_0-space and each distinct soft points $x_\alpha,y_\beta\widetilde{\in}SS(X)_E$. Then, there exist $(G,E)\in\widetilde{g}_{12}$ such that $x_\alpha\widetilde{\in}(G,E)$, $y_\beta\widetilde{\notin}(G,E)$. Hence $(G,E)^c$ is a soft $\widetilde{g}_{12}-$closed set which contain y_β but not x_α. By Definition-20, $scl_{\widetilde{g}_{12}}(y_\beta,E)$ is the smallest soft $\widetilde{g}_{12}-$closed set that containing (y_β,E). Therefore $scl_{\widetilde{g}_{12}}(y_\beta,E)\widetilde{\subseteq}(G,E)^c$. So $x_\alpha\widetilde{\notin}(G,E)^c$ implies that $x_\alpha\widetilde{\notin}scl_{\widetilde{g}_{12}}(y_\beta,E)$. Thus $x_\alpha\widetilde{\in}scl_{\widetilde{g}_{12}}(x_\alpha,E)$ but $x_\alpha\widetilde{\notin}scl_{\widetilde{g}_{12}}(y_\beta,E)$. Therefore $scl_{\widetilde{g}_{12}}(x_\alpha,E)\neq scl_{\widetilde{g}_{12}}(y_\beta,E)$.

Conversely, assume that each distinct soft points $x_\alpha,y_\beta\widetilde{\in}SS(X)_E$. and $scl_{\widetilde{g}_{12}}(x_\alpha,E)\neq scl_{\widetilde{g}_{12}}(y_\beta,E)$. Then by assumption, there exist at least one soft point $z_e\widetilde{\in}SS(X)_E$ such that $z_e\widetilde{\in}scl_{\widetilde{g}_{12}}(x_\alpha,E)$ but $z_e\widetilde{\notin}scl_{\widetilde{g}_{12}}(y_\beta,E)$. Now we show that $x_\alpha\widetilde{\notin}scl_{\widetilde{g}_{12}}(y_\beta,E)$. Suppose that $x_\alpha\widetilde{\in}scl_{\widetilde{g}_{12}}(y_\beta,E)$ then $\{x_\alpha\}\widetilde{\subseteq}scl_{\widetilde{g}_{12}}(y_\beta,E)$ which implies that $scl_{\widetilde{g}_{12}}(x_\alpha,E)\widetilde{\subseteq}scl_{\widetilde{g}_{12}}(y_\beta,E)$. Hence $z_e\widetilde{\in}scl_{\widetilde{g}_{12}}(x_\alpha,E)$ implies $z_e\widetilde{\in}scl_{\widetilde{g}_{12}}(y_\beta,E)$. The contradicts the fact that $z_e\widetilde{\notin}scl_{\widetilde{g}_{12}}(y_\beta,E)$. Therefore $x_\alpha\widetilde{\notin}scl_{\widetilde{g}_{12}}(y_\beta,E)$. So $x_\alpha\widetilde{\in}\left[scl_{\widetilde{g}_{12}}(y_\beta,E)\right]^c$ is a soft open set. Thus $\left[scl_{\widetilde{g}_{12}}(y_\beta,E)\right]^c$ is a soft open set containing x_α but not y_β. That is, $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is pairwise soft T_0-space.

Proposition 1. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT-space over X and $(X,\widetilde{g}_1,E)$ and $(X,\widetilde{g}_2,E)$ be two soft generalized topological spaces. If $(X,\widetilde{g}_1,E)$ or $(X,\widetilde{g}_2,E)$ is soft T_0-space, then $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a pairwise soft bigeneralized T_0-space.*

Proof. Suppose that each distinct soft points $x_\alpha,y_\beta\widetilde{\in}SS(X)_E$ and $(X,\widetilde{g}_1,E)$ is a soft T_0-space. Then there exist $(F,E)\in\widetilde{g}_1$ such that $x_\alpha\widetilde{\in}(F,E)$, $y_\beta\widetilde{\notin}(F,E)$ or $x_\alpha\widetilde{\notin}(F,E)$, $y_\beta\widetilde{\in}(F,E)$. Since $(F,E)\in\widetilde{g}_1\subseteq\widetilde{g}_{12}$, we obtain the requirement. Similarly it provide for $(X,\widetilde{g}_2,E)$ space. Thus, $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is pairwise soft bigeneralized T_0-space.

Remark. The converse of Proposition-1 is not true in general. It is shown in following example.

Example 2. According to Example–1, $(X,\widetilde{g}_1,E)$ is not soft T_0-space. For $b_{e_1}\neq a_{e_2}$, there is no soft open set in $\widetilde{g}_1$ which contains one of points but not contains the other. Similarly, $(X,\widetilde{g}_2,E)$ is not soft T_0-space. For $a_{e_1}\neq b_{e_2}$, there is no open soft set in $\widetilde{g}_2$ which contains one of points but not contains the other. Thus $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a pairwise soft bigeneralized T_0-space but $(X,\widetilde{g}_1,E)$ or $(X,\widetilde{g}_2,E)$ is not soft T_0-space.

Definition 26. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT-space over X and each distinct soft points $x_\alpha,y_\beta\widetilde{\in}SS(X)_E$. If there exist pairwise soft open sets $(F,E),(G,E)\in\widetilde{g}_{12}$ such that $x_\alpha\widetilde{\in}(F,E)$, $y_\beta\widetilde{\notin}(F,E)$ and $y_\beta\widetilde{\in}(G,E)$, $x_\alpha\widetilde{\notin}(G,E)$, then $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is called a pairwise soft bigeneralized T_1-space.*

Example 3. Let $X=\{a,b\}$, $E=\{e_1,e_2\}$ and let

$$\widetilde{g}_1=\{(\widetilde{\phi},E),(F_1,E),(F_2,E),(F_3,E)\}$$
$$\widetilde{g}_2=\{(\widetilde{\phi},E),(G_1,E),(G_2,E),(G_3,E)\}$$

where

$$\begin{aligned}(F_1,E) &= \{(e_1,X),(e_2,\phi)\}\\ (F_2,E) &= \{(e_1,\{a\}),(e_2,\{b\})\}\\ (F_3,E) &= \{(e_1,X),(e_2,\{b\}\}\\ (G_1,E) &= \{(e_1,\phi),(e_2,X)\}\\ (G_2,E) &= \{(e_1,\{b\}),(e_2,\{a\})\}\\ (G_3,E) &= \{(e_1,\{b\}),(e_2,X)\}\end{aligned}$$

Then, $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a SBGT-space. Thus, the family of pairwise soft open sets over $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ becomes

$$\widetilde{g}_{12} = \{(\widetilde{\phi},E),(F_1,E),(F_2,E),(F_3,E),(G_1,E),(G_2,E),(G_3,E),(H_1,E),(H_2,E),(H_3,E)\}$$

where

$$\begin{aligned}(H_1,E) &= \{(e_1,X),(e_2,X)\}\\ (H_2,E) &= \{(e_1,X),(e_2,\{a\})\}\\ (H_1,E) &= \{(e_1,\{a\}),(e_2,X)\}\end{aligned}$$

It is clear that $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a pairwise soft bigeneralized T_1-space.

Theorem 4. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT-space over X. Then $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a pairwise soft bigeneralized T_1-space if and only if every single soft point set over X is a pairwise soft closed set.*

Proof. $(\Rightarrow)$: Suppose that $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is pairwise soft bigeneralized T_0-space and each $x_\alpha\widetilde{\in}SS(X)_E$, $y_\beta\widetilde{\in}(x_\alpha,E)^c$. Then there exist $(G,E)\in\widetilde{g}_{12}$ such that $y_\beta\widetilde{\in}(G,E)$ and $x_\alpha\widetilde{\notin}(G,E)$. Hence we obtain $y_\beta\widetilde{\in}(G,E)\subseteq(x_\alpha,E)^c$. Therefore $(x_\alpha,E)^c$ is a pairwise soft closed set. Thus (x_α,E) is a pairwise soft closed set.

$(\Leftarrow)$: Suppose that for each distinct soft points $x_\alpha,y_\beta\widetilde{\in}SS(X)_E$, (x_α,E) is a pairwise soft closed set then $(x_\alpha,E)^c\in\widetilde{g}_{12}$. $(x_\alpha,E)^c$ is a pairwise soft open set such that $y_\beta\widetilde{\in}(x_\alpha,E)^c$ and $x_\alpha\widetilde{\notin}(x_\alpha,E)^c$. Similarly $(y_\beta,E)^c\in\widetilde{g}_{12}$ is such that $x_\alpha\widetilde{\in}(y_\beta,E)^c$ and $y_\beta\widetilde{\notin}(y_\beta,E)^c$. Thus $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a pairwise soft bigeneralized T_1-space.

Proposition 2. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT-space over X and $(X,\widetilde{g}_1,E)$ and $(X,\widetilde{g}_2,E)$ be two soft generalized topological spaces. If $(X,\widetilde{g}_1,E)$ or $(X,\widetilde{g}_2,E)$ is soft T_1-space, then $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a pairwise soft bigeneralized T_1-space.*

Proof. Suppose that each distinct soft points $x_\alpha,y_\beta\widetilde{\in}SS(X)_E$ and $(X,\widetilde{g}_1,E)$ is a soft T_1-space. Then there exist $(F_1,E),(F_2,E)\in\widetilde{g}_1$ such that $x_\alpha\widetilde{\in}(F_1,E)$, $y_\beta\widetilde{\notin}(F_1,E)$ and $x_\alpha\widetilde{\notin}(F_2,E)$, $y_\beta\widetilde{\in}(F_2,E)$. Since $(F_1,E),(F_2,E)\in\widetilde{g}_1\subseteq\widetilde{g}_{12}$, we obtain the requirement. Similarly it is provided for $(X,\widetilde{g}_2,E)$ space. Thus, $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is pairwise soft bigeneralized T_1-space.

Remark. The converse of Proposition-2 is not true in general. It is shown in following example.

Example 4. According to Example–3, $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a pairwise soft bigeneralized T_1-space. But $(X,\widetilde{g}_1,E)$ and $(X,\widetilde{g}_2,E)$ is not soft T_1-space. We consider $(X,\widetilde{g}_1,E)$ space. For $a_{e_1}\neq b_{e_2}$, there is no soft open set in $\widetilde{g}_1$ which contains b_{e_2} but not contains a_{e_1}. Hence $(X,\widetilde{g}_1,E)$ is not soft T_1-space. Similarly, we consider $(X,\widetilde{g}_2,E)$ space. For $b_{e_1}\neq a_{e_2}$, there is no soft open set in $\widetilde{g}_2$ which contains b_{e_1} but not contains a_{e_2}. Hence $(X,\widetilde{g}_2,E)$ is not soft T_1-space. Thus $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a pairwise soft bigeneralized T_1-space but $(X,\widetilde{g}_1,E)$ or $(X,\widetilde{g}_2,E)$ is not soft T_0-space.

Theorem 5. *Every pairwise soft bigeneralized T_1-space is also a pairwise soft bigeneralized T_0-space.*

Proof. Suppose that $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is pairwise soft bigeneralized T_1-space. Then for each distinct soft points $x_\alpha, y_\beta \widetilde{\in} SS(X)_E$, there exist $(F,E),(G,E) \in \widetilde{g}_{12}$ such that $x_\alpha \widetilde{\in}(F,E)$, $y_\beta \widetilde{\notin}(F,E)$ and $y_\beta \widetilde{\in}(G,E)$, $x_\alpha \widetilde{\notin}(G,E)$. Hence we obtain required conditions for pairwise soft bigeneralized T_0-space. Thus $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a pairwise soft bigeneralized T_1-space.

Remark. The converse of Theorem-5 need not be true in general. It is shown in following example.

Example 5. According to Example–1, $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a pairwise soft bigeneralized T_0-space. For $a_{e_2} \neq b_{e_1}$, there is no soft open set in $\widetilde{g}_{12}$ which contains a_{e_2} but not contains b_{e_1}. Therefore $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is not pairwise soft bigeneralized T_1-space.

Definition 27. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT-space over X. If for each distinct soft points $x_\alpha, y_\beta \widetilde{\in} SS(X)_E$, there exist pairwise soft open sets $(U,E),(V,E) \in \widetilde{g}_{12}$ such that $(U,E)\widetilde{\cap}(V,E) = (\widetilde{\phi},E)$, where (U,E) and (V,E) means that $x_\alpha \widetilde{\in}(U,E)$ and $y_\beta \widetilde{\in}(V,E)$, respectively, then $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is called a pairwise soft bigeneralized T_2-space.*

Example 6. Consider a discrete SBGT-space $(X,\widetilde{g}_1,\widetilde{g}_2,E)$. Let $x_\alpha, y_\beta \widetilde{\in} SS(X)_E$ with $x_\alpha \neq y_\beta$. $\{x_\alpha\}$ and $\{y_\beta\}$ are soft open sets and $\{x_\alpha\} \cap \{y_\beta\} = (\widetilde{\phi}.E)$. Hence $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a pairwise soft bigeneralized T_2-space.

Theorem 6. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT-space over X. Then $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a pairwise soft bigeneralized T_2-space if and only if there exist a soft open set (U,E) containing x_α but not y_β such that $y_\beta \widetilde{\notin} scl_{\widetilde{g}_{12}}(V,E)$ for distinct point $x_\alpha, y_\beta \widetilde{\in} SS(X)_E$.*

Proof. $(\Rightarrow)$: Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a pairwise soft bigeneralized T_2-space and each distinct soft points $x_\alpha, y_\beta \widetilde{\in} SS(X)_E$. Then there exist disjoint pairwise soft open sets (U,E) and (V,E) such that $x_\alpha \widetilde{\in}(U,E)$ and $y_\beta \widetilde{\in}(V,E)$. This implies that $x_\alpha \widetilde{\in}(U,E)^c$. So $(U,E)^c = (V,E)$ is a pairwise soft closed set containing x_α but not y_β and $scl_{\widetilde{g}_{12}}(U,E) = (V,E)$. Hence $y_\beta \widetilde{\notin} scl_{\widetilde{g}_{12}}(U,E)$.

$(\Leftarrow)$: Let x_α and y_β be two distinct soft points in $(X,\widetilde{g}_1,\widetilde{g}_2,E)$. Then there exist a pairwise soft open set (U,E) containing x_α but not y_β such that $y_\beta \widetilde{\notin} scl_{\widetilde{g}_{12}}(V,E)$. This implies that $y_\beta \widetilde{\in} \left[scl_{\widetilde{g}_{12}}(V,E)\right]^c$. Hence (V,E) and $\left[scl_{\widetilde{g}_{12}}(V,E)\right]^c$ are two disjoint pairwise soft open sets containing x_α and y_β respectively and $(V,E)\widetilde{\cap}\left[scl_{\widetilde{g}_{12}}(V,E)\right]^c = (\widetilde{\phi},E)$. Thus $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is pairwise soft bigeneralized T_2-space.

Proposition 3. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT-space over X and $(X,\widetilde{g}_1,E)$ and $(X,\widetilde{g}_2,E)$ be two soft generalized topological spaces. If $(X,\widetilde{g}_1,E)$ or $(X,\widetilde{g}_2,E)$ is soft T_2-space, then $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a pairwise soft bigeneralized T_2-space.*

Proof. Suppose that each distinct soft points $x_\alpha, y_\beta \widetilde{\in} SS(X)_E$ and $(X,\widetilde{g}_1,E)$ is a soft T_2-space. Then there exist $(F_1,E),(F_2,E) \in \widetilde{g}_1$ such that $(F_1,E)\widetilde{\cap}(F_2,E) = (\widetilde{\phi},E)$, where (F_1,E) and (F_2,E) means that $x_\alpha \widetilde{\in}(F_1,E)$ and $y_\beta \widetilde{\in}(F_2,E)$, respectively. Since $(F_1,E),(F_2,E) \in \widetilde{g}_1 \subseteq \widetilde{g}_{12}$, we obtain the requirement. Similarly, it is provided for $(X,\widetilde{g}_2,E)$ space. Thus $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a pairwise soft bigeneralized T_2-space.

Theorem 7. *Every pairwise soft T_2-space is pairwise soft T_1-space.*

Proof. Suppose that $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT-space over X and each distinct soft points $x_\alpha, y_\beta \widetilde{\in} SS(X)_E$. If $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is pairwise soft T_2-space, then $(X,\widetilde{g}_1,E)$ or $(X,\widetilde{g}_2,E)$ is a soft T_2-space. If $(X,\widetilde{g}_1,E)$ is soft T_2-space, then there exist a $(F,E) \in \widetilde{g}_1$ and $(G,E) \in \widetilde{g}_1$ such that $x_\alpha \widetilde{\in}(F,E)$, $y_\beta \widetilde{\in}(G,E)$ and $(F,E)\widetilde{\cap}(G,E) = (\widetilde{\phi},E)$. Obviously $x_\alpha \widetilde{\in}(F,E)$, $y_\beta \widetilde{\notin}(F,E)$ and $x_\alpha \widetilde{\notin}(G,E)$, $y_\beta \widetilde{\in}(G,E)$. Therefore $(X,\widetilde{g}_1,E)$ is a soft T_1-space. According to Proposition-2, $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is pairwise soft bigeneralized T_1-space.

Remark. The converse of Theorem-7 is not true in general. It is shown in following example.

Example 7. According to Example-3, $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ pairwise soft bigeneralized T_1-space but not pairwise soft bigeneralized T_2-space. For $a_{e_2} \neq b_{e_1}$, there is no soft open sets in $\widetilde{g}_{12}$ such that $a_{e_2}\widetilde{\in}(U,E)$, $b_{e_1}\widetilde{\in}(V,E)$ and $(U,E)\widetilde{\cap}(V,E) = (\widetilde{\phi},E)$. Thus $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is not pairwise soft bigeneralized T_2-space.

Theorem 8. *The property of being pairwise soft bigeneralized T_i-space$(i = 0,1,2)$ is preserved under a soft $\widetilde{g}_{12}-$homeomorphism mapping.*

Proof. We prove the theorem for $i = 2$, the other cases is similar. Let $f : (X,\widetilde{g}_1,\widetilde{g}_2,E) \to (Y,\widetilde{k}_1,\widetilde{k}_2,E)$ be a soft mapping such that:

(1) f is $1-1$, onto and soft open mapping.
(2) $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a pairwise soft bigeneralized T_2-space.

We want to show that $(Y,\widetilde{k}_1,\widetilde{k}_2,E)$ is a pairwise soft bigeneralized T_2-space. So, let $x_\alpha, y_\beta\widetilde{\in}SS(Y)_E$ such that $x_\alpha \neq y_\beta$. Since f is 1-1 and onto mapping, then there exist two soft points x'_α, y'_β in $\widetilde{X}$ such that $f(x'_\alpha) = x_\alpha$, $f(y'_\beta) = y_\beta$, and $x'_\alpha \neq y'_\beta$. But $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a pairwise soft T_2-space, so, there exist $(G,E),(H,E) \in \widetilde{g}_{12}$ such that $x'_\alpha\widetilde{\in}(G,E)$, $y'_\beta\widetilde{\in}(H,E)$ and $(G,E)\widetilde{\cap}(H,E) = (\widetilde{\phi},E)$. It follows that, $f(x'_\alpha) = x_\alpha\widetilde{\in}f((G,E))$, $f(y'_\beta) = y_\beta\widetilde{\in}f((H,E))$ and $f((G,E)\widetilde{\cap}(H,E)) = f((G,E))\widetilde{\cap}f((H,E)) = f((\widetilde{\phi},E)) = (\widetilde{\phi},E)$. Since $(G,E),(H,E) \in \widetilde{g}_{12}$ and f is an soft open mapping, $f((G,E)),f((H,E)) \in \widetilde{k}_{12}$. Then, there exist $f((G,E)),f((H,E)) \in \widetilde{k}_{12}$ such that $x_\alpha\widetilde{\in}f((G,E))$, $y_\beta\widetilde{\in}f((H,E))$ and $f((G,E))\widetilde{\cap}f((H,E)) = (\widetilde{\phi},E)$. Hence $(Y,\widetilde{k}_1,\widetilde{k}_2,E)$is a pairwise soft bigeneralized T_2-space

Definition 28. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT-space and pairwise soft bigeneralized T_1-space over X, (F,E) be a soft closed set in X and $x_\alpha\widetilde{\in}SS(X)_E$ such that $x_\alpha\widetilde{\notin}(F,E)$. If there exist $(G_1,E),(G_2,E) \in \widetilde{g}_{12}$ such that $x_\alpha\widetilde{\in}(G_1,E)$, $(F,E)\widetilde{\subset}(G_2,E)$ and $(G_1,E)\widetilde{\cap}(G_2,E) = (\widetilde{\phi},E)$, then $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is said to be a pairwise soft bigeneralized T_3-space.*

Definition 29. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT-space and pairwise soft bigeneralized T_1-space over X, (F,E) and (G,E) soft closed sets over X such that $(F,E)\widetilde{\cap}(G,E) = (\widetilde{\phi},E)$. If there exist $(F_1,E),(F_2,E) \in \widetilde{g}_{12}$ such that $(F,E)\widetilde{\subset}(F_1,E)$, $(G,E)\widetilde{\subset}(F_2,E)$ and $(F_1,E)\widetilde{\cap}(F_2,E) = \widetilde{\phi}$, then $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is called a pairwise soft bigeneralized T_4-space.*

Theorem 9. *The property of being pairwise soft bigeneralized T_i-space$(i = 3,4)$ is preserved under a soft $\widetilde{g}_{12}-$homeomorphism mapping.*

Proof. We prove the theorem for $i = 3$, the other cases are similar. So, let $f : (X,\widetilde{g}_1,\widetilde{g}_2,E) \to (Y,\widetilde{k}_1,\widetilde{k}_2,E)$ be a soft mapping such that:

(1) $f : (X,\widetilde{g}_1,\widetilde{g}_2,E) \to (Y,\widetilde{k}_1,\widetilde{k}_2,E)$ is soft $\widetilde{g}_{12}-$homeomorphism (i.e f is a soft bijection and f,f^{-1} are soft $\widetilde{g}_{12}-$continuous)
(2) $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a pairwise soft bigeneralized T_3-space.

Let (F,E) be a $\widetilde{k}_{12}-$closed soft subset of Y and let $y_\beta\widetilde{\in}SS(Y)_E$ such that $y_\beta\widetilde{\notin}(F,E)$. Since, f is an onto mapping, there exist $x_\alpha\widetilde{\in}SS(X)_E$ such that $f(x_\alpha) = y_\beta$.

Since f is soft continuous mapping and (F,E) is a $\widetilde{k}_{12}-$closed soft subset of Y, we have $f^{-1}((F,E))$ is a $\widetilde{g}_{12}-$closed soft subset of X. Since $y_\beta = f(x_\alpha)\widetilde{\notin}(F,E)$, we have $f^{-1}(f(x_\alpha)) = x_\alpha\widetilde{\notin}f^{-1}((F,E))$ (as, f is soft injective). Now, $f^{-1}((F,E))$ is a $\widetilde{g}_{12}-$soft closed subset of X, $x_\alpha\widetilde{\in}SS(X)_E$ such that $x_\alpha\widetilde{\notin}f^{-1}((F,E))$. Moreover $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a pairwise soft T_3-space, so, there exist $(G,E),(H,E) \in \widetilde{g}_{12}$ such that $x_\alpha\widetilde{\in}(G.E)$, $f^{-1}((F,E))\widetilde{\subseteq}(H,E)$ and $(G,E)\widetilde{\cap}(H,E) = (\widetilde{\phi},E)$ and therefore $f(x_\alpha) = y_\beta\widetilde{\in}f((G,E))$, $f(f^{-1}((F,E))) = (F,E)\widetilde{\subseteq}f((H;E))$ (as, f is soft surjective) and $f((G,E)\widetilde{\cap}(H,E)) = f((G,E))\widetilde{\cap}f((H,E)) = f((\widetilde{\phi},E)) = (\widetilde{\phi},E)$.

Since f^{-1} is soft $\widetilde{g}_{12}-$continuous mapping, i.e. f is an soft open mapping (from Theorem-2). If $(G,E),(H,E) \in \widetilde{g}_{12}$ and f is a soft open mapping, then $f((G,E)),f((H,E)) \in \widetilde{k}_{12}$ (from Definition-23). Therefore, there exist $f((G,E)),f((H,E)) \in \widetilde{k}_{12}$ such that $y_\beta\widetilde{\in}f((G,E))$, $(F,E)\widetilde{\subseteq}f((H,E))$ and $f((G,E))\widetilde{\cap}f((H,E)) = (\widetilde{\phi},E)$. Thus, $(Y,\widetilde{k}_1,\widetilde{k}_2,E)$ is a pairwise soft bigeneralized T_3-space.

4 Soft hereditary property

Definition 30. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT-space over X and Y be a non-empty subset of X. Then $\widetilde{g}_{1Y}=\{({}^YF,E):(F,E)\in\widetilde{g}_1\}$ and $\widetilde{g}_{2Y}=\{({}^YG,E):(G,E)\in\widetilde{g}_2\}$ are said to be the relative soft topologies on $\widetilde{Y}$. Moreover $(Y,\widetilde{g}_{1Y},\widetilde{g}_{2Y},E)$ is called a soft bigeneralized subspace of $(X,\widetilde{g}_1,\widetilde{g}_2,E)$.*

Theorem 10. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT-space over X and Y be a non-empty subset of X. Then $(Y,\widetilde{g}_{1Y},\widetilde{g}_{2Y},E)$ is a SBGT-space on Y. Moreover $\widetilde{g}_{1Y2Y}=\widetilde{g}_{12Y}$, where $\widetilde{g}_{12Y}=\{(Y,E)\widetilde{\cap}(G,E):(G,E)\in\widetilde{g}_{12}\}$ and $\widetilde{g}_{1Y2Y}=\{(H,E)\in SS(Y)_E:(H,E)=(H_1,E)\widetilde{\cup}(H_2,E),\ (H_i,E)\in\widetilde{g}_{iY},\ i=1,2\}$.*

Proof. Since $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is a SBGT-space, $(X,\widetilde{g}_1,E)$ and $(X,\widetilde{g}_2,E)$ are soft generalized topologies on X. Since $Y\subseteq X$, $(X,\widetilde{g}_{1Y},E)$ and $(X,\widetilde{g}_{2Y},E)$ are soft generalized topologies on Y. Consequently, $(Y,\widetilde{g}_{1Y},\widetilde{g}_{2Y},E)$ is a SBGT-space on Y.

Let $(G,E)\in\widetilde{g}_{12Y}$. Then, there exist $(H,E)\in\widetilde{g}_{12}$ such that

$$\begin{aligned}(G,E)&=(Y,E)\widetilde{\cap}(H,E)\\&=(Y,E)\widetilde{\cap}[(H_1,E)\widetilde{\cup}(H_2,E)],(H_1,E)\in\widetilde{g}_1\quad\text{and}\quad(H_2,E)\in\widetilde{g}_2\\&=[(Y,E)\widetilde{\cap}(H_1,E)]\widetilde{\cup}[(Y,E)\widetilde{\cap}(H_2,E].\end{aligned}$$

Since $(Y,E)\widetilde{\cap}(H_1,E)\in\widetilde{g}_{1Y}$ and $(Y,E)\widetilde{\cap}(H_2,E)\in\widetilde{g}_{2Y}$, $[(Y,E)\widetilde{\cap}(H_1,E)]\widetilde{\cup}[(Y,E)\widetilde{\cap}(H_2,E]\in\widetilde{g}_{1Y2Y}$. Thus, $(G,E)\in\widetilde{g}_{1Y2Y}$. So, $\widetilde{g}_{12Y}\subseteq\widetilde{g}_{1Y2Y}$.

By similar way, we can prove that $\widetilde{g}_{1Y2Y}\subseteq\widetilde{g}_{12Y}$.

Proposition 4. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT-space over X and Y be a non-empty subset of X. If $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is pairwise soft bigeneralized T_0-space then $(Y,\widetilde{g}_{1Y},\widetilde{g}_{2Y},E)$ is pairwise soft bigeneralized T_0-space.*

Proof. Suppose that $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a pairwise soft bigeneralized T_0-space over X and $(Y,\widetilde{g}_{1Y},\widetilde{g}_{2Y},E)$ be a soft bigeneralized subspace of $(X,\widetilde{g}_1,\widetilde{g}_2,E)$. Assume that each distinct soft points $x_\alpha,y_\beta\widetilde{\in}SS(Y)_E$. Since $Y\subseteq X$, $x_\alpha,y_\beta\widetilde{\in}SS(X)_E$. So, there exist some pairwise soft set $(F,E)\in\widetilde{g}_{12}$ such that $x_\alpha\widetilde{\in}(F,E)$ and $y_\beta\widetilde{\notin}(F,E)$ or $x_\alpha\widetilde{\notin}(F,E)$ and $y_\beta\widetilde{\in}(F,E)$. Then $x_\alpha\widetilde{\in}(Y,E)\cap(F,E)=({}^YF,E)$ and $y_\beta\widetilde{\notin}(Y,E)\cap(F,E)=({}^YF,E)$ or $x_\alpha\widetilde{\notin}(Y,E)\cap(F,E)=({}^YF,E)$ and $y_\beta\widetilde{\in}(Y,E)\cap(F,E)=({}^YF,E)$. Since $({}^YF,E)\in\widetilde{g}_{12Y}$, $(Y,\widetilde{g}_{1Y},\widetilde{g}_{2Y},E)$ is a pairwise soft bigeneralized T_0-space.

Proposition 5. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT-space over X and Y be a non-empty subset of X. If $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is pairwise soft bigeneralized T_1-space then $(Y,\widetilde{g}_{1Y},\widetilde{g}_{2Y},E)$ is pairwise soft bigeneralized T_1-space.*

Proof. Suppose that $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a pairwise soft bigeneralized T_0-space over X and $(Y,\widetilde{g}_{1Y},\widetilde{g}_{2Y},E)$ be a soft bigeneralized subspace of $(X,\widetilde{g}_1,\widetilde{g}_2,E)$. Assume that each distinct soft points $x_\alpha,y_\beta\widetilde{\in}SS(Y)_E$. Since $Y\subseteq X$, $x_\alpha,y_\beta\widetilde{\in}SS(X)_E$. So, there exist some pairwise soft sets $(F,E),(G,E)\in\widetilde{g}_{12}$ such that $x_\alpha\widetilde{\in}(F,E)$, $y_\beta\widetilde{\notin}(F,E)$ and $x_\alpha\widetilde{\notin}(G,E)$, $y_\beta\widetilde{\in}(G,E)$. Then $x_\alpha\widetilde{\in}(Y,E)\cap(F,E)=({}^YF,E)$, $y_\beta\widetilde{\notin}(Y,E)\cap(F,E)=({}^YF,E)$ and $x_\alpha\widetilde{\notin}(Y,E)\cap(G,E)=({}^YG,E)$, $y_\beta\widetilde{\in}(Y,E)\cap(G,E)=({}^YG,E)$. Since $({}^YF,E),({}^YG,E)\in\widetilde{g}_{12Y}$, $(Y,\widetilde{g}_{1Y},\widetilde{g}_{2Y},E)$ is a pairwise soft bigeneralized T_1-space.

Proposition 6. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT-space over X and Y be a non-empty subset of X. If $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ is pairwise soft bigeneralized T_2-space then $(Y,\widetilde{g}_{1Y},\widetilde{g}_{2Y},E)$ is pairwise soft bigeneralized T_2-space.*

Proof. Suppose that $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a pairwise soft bigeneralized T_0-space over X and $(Y,\widetilde{g}_{1Y},\widetilde{g}_{2Y},E)$ be a soft bigeneralized subspace of $(X,\widetilde{g}_1,\widetilde{g}_2,E)$. Assume that each distinct soft points $x_\alpha,y_\beta\widetilde{\in}SS(Y)_E$. Since $Y\subseteq X$,

$x_\alpha, y_\beta \widetilde{\in} SS(X)_E$. So, there exist $(F,E),(G,E) \in \widetilde{g}_{12}$ such that $x_\alpha \widetilde{\in} (F,E)$, $y_\beta \widetilde{\in} (G,E)$ and $(F,E) \cap (G,E) = (\widetilde{\phi},E)$. Moreover there exists $(Y,E) \cap (G,E)$, $(Y,E) \cap (F,E) \in \widetilde{g}_{12Y}$ such that $x_\alpha \widetilde{\in} (Y,E) \cap (F,E)$, $y_\beta \widetilde{\in} (Y,E) \cap (G,E)$ and $[(Y,E) \cap (F,E)] \cap [(Y,E) \cap (G,E)] = (Y,E) \cap [(F,E) \cap (G,E)] = (Y,E) \cap (\widetilde{\phi},E) = (\widetilde{\phi},E)$. Hence $(Y, \widetilde{g}_{1Y}, \widetilde{g}_{2Y}, E)$ is a pairwise soft bigeneralized T_2-space.

Proposition 7. *Let $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ be a SBGT-space over X and Y be a non-empty subset of X. If $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ is pairwise soft bigeneralized T_3-space then $(Y, \widetilde{g}_{1Y}, \widetilde{g}_{2Y}, E)$ is pairwise soft bigeneralized T_3-space.*

Proof. Let $y_\beta \widetilde{\in} SS(Y)_E$ and (F,E) be a soft closed set in $\widetilde{Y}$ such that $y_\beta \widetilde{\notin} (F,E)$. Then $y_\beta \widetilde{\notin} ((Y,E) \widetilde{\cap} (G,E))$ where $(F,E) = ((Y,E) \widetilde{\cap} (G,E))$ for some soft closed set in $\widetilde{X}$. But $y_\beta \widetilde{\in} (Y,E)$, so $y_\beta \widetilde{\notin} (G,E)$. As $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ is a pairwise soft T_3-space, so there exist $(G_1,E),(G_2,E) \widetilde{\in} \widetilde{g}_{12}$ such that $y_\beta \widetilde{\in} (G_1,E)$, $(G,E) \widetilde{\subset} (G_2,E)$ and $(G_1,E) \widetilde{\cap} (G_2,E) = (\widetilde{\phi},E)$. Now if we take $(F_1,E) = (Y,E) \widetilde{\cap} (G_1,E)$ and $(F_2,E) = (Y,E) \widetilde{\cap} (G_2,E)$, then (F_1,E),$(F_2,E) \in \widetilde{g}_{12Y}$ such that $y_\beta \widetilde{\in} (F_1,E)$ and $(F,E) \widetilde{\subset} (Y,E) \widetilde{\cap}$
$(G_2,E) = (F_2,E)$ and $(F_1,E) \widetilde{\cap} (F_2,E) \widetilde{\subset} (G_1,E) \widetilde{\cap} (G_2,E) = (\widetilde{\phi},E)$. Thus $(Y, \widetilde{g}_{1Y}, \widetilde{g}_{2Y}, E)$ is a soft bigeneralized T_3-space.

Proposition 8. *Every closed soft subspace of a pairwise soft T_4-space is a pairwise soft bigeneralized T_4-space.*

Proof. Let $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ be a pairwise soft bigeneralized T_4-space and $(Y, \widetilde{g}_{1Y}, \widetilde{g}_{2Y}, E)$ be a soft closed subspace of $(X, \widetilde{g}_1, \widetilde{g}_2, E)$. We want to show that $(Y, \widetilde{g}_{1Y}, \widetilde{g}_{2Y}, E)$ is a pairwise soft bigeneralized T_4-space. So, let $(K_1,E),(K_2,E)$ be a non-empty disjoint closed soft subsets of $\widetilde{Y}$. Since $(K_1,E),(K_2,E) \in \widetilde{g}^c_{12Y}$, there exists $(F_1,E),(F_2,E) \in \widetilde{g}^c_{12}$ such that $(K_1,E) = (Y,E) \widetilde{\cap} (F_1,E)$ and $(K_2,E) = (Y,E) \widetilde{\cap} (F_2,E)$. Since $(Y,E) \in \widetilde{g}^c_{12}$ and $(F_1,E),(F_2,E) \in \widetilde{g}^c_{12}$, we have $(Y,E) \widetilde{\cap} (F_1,E), (Y,E) \widetilde{\cap} (F_2,E) \in \widetilde{g}^c_{12}$.

Now $(K_1,E),(K_2,E)$ are two non-empty disjoint soft closed subset of $\widetilde{X}$, but $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ is a pairwise soft T_4-space, so, $(G,E),(H,E) \in \widetilde{g}_{12}$ such that $(K_1,E) \widetilde{\subseteq} (G,E)$, $(K_2,E) \widetilde{\subseteq} (H,E)$ and $(G,E) \widetilde{\cap} (H,E) = (\widetilde{\phi},E)$. It follows that, $\exists (Y,E) \widetilde{\cap} (G,E)$, $(Y,E) \widetilde{\cap} (H,E) \in \widetilde{g}_{12Y}$ such that $(K_1,E) \widetilde{\subseteq} (Y,E) \widetilde{\cap} (G,E)$, $(K_2,E) \widetilde{\subseteq} (Y,E) \widetilde{\cap} (H,E)$ and
$((Y,E) \widetilde{\cap} (G,E)) \widetilde{\cap} ((Y,E) \widetilde{\cap} (H,E)) = (Y,E) \widetilde{\cap} ((G,E) \widetilde{\cap} (H,E)) = (Y,E) \widetilde{\cap} (\phi,E) = (\widetilde{\phi},E)$. Therefore $(Y, \widetilde{g}_{1Y}, \widetilde{g}_{2Y}, E)$ is a pairwise soft bigeneralized T_4-space.

5 Conclusion

Soft bigeneralized topological spaces was introduced by T.Y. Ozturk et. al. [19] in 2018. A soft bigeneralized topological spaces is an important generalization of the bigeneralized topological spaces. Hence in this paper we introduced pairwise soft bigeneralized $T_i - spaces$ $(i = 0,1,2,3,4)$. Characterizations of these spaces are obtained. Finally, we showed that soft hereditary properties of these pairwise soft separation axioms. We hope that the findings in this paper will help researcher enhance and promote the further study on SBGT-spaces.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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