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Almost P_p-continuous functions

Shadya M. Mershkhan

Department of Mathematics, Faculty of Science, University of Zakho, Iraq

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Abstract: This paper is aimed to introduce a new class of functions called almost P_p-continuous functions by using P_p-open sets in topological spaces. Also some properties and characterizations are studied.

Keywords: P_p-open, preopen, almost P_p-continuous, almost precontinuous.

1 Introduction and Preliminaries

In 1982, Mashhour et al [13] defined a new class of sets called preopen sets and almost precontinuous functions is defined in [16]. In [10] the concept of P_p-open sets is introduced. In the present paper, we introduce and investigate the concept of almost P_p-continuous functions. It will be shown that almost P_p-continuity is weaker than P_p-continuity mentioned in [19], but it is stronger than almost precontinuity.

Throughout the present paper, a space X always means a topological space on which no separation axiom is assumed unless explicitly stated. Let A be a subset of a space X. The closure and interior of A with respect to X are denoted by $Cl(A)$ and $Int(A)$ respectively. A subset A of a space X is said to be preopen [13] (resp., semi-open [11], α-open [17], β-open [1] and regular open [22]), if $A \subseteq Int(Cl(A))$ (resp., $A \subseteq Cl(Int(A))$, $A \subseteq Int(Cl(Int(A))$, $A \subseteq Cl(Int(Cl(A))$ and $A = Int(Cl(A))$). The complement of a preopen (resp., semi-open, α-open, β-open and regular open) set is said to be preclosed (resp., semi-closed, α-closed, β-closed and regular closed). The family of all preopen (resp., semi-open, α-open, β-open and regular open) subsets of X is denoted by $PO(X)$ (resp., $SO(X)$, $\alpha O(X)$, $\beta O(X)$ and $RO(X)$). A subset A of a space X is called δ-open (resp., θ-open) if for each $x \in A$, there exists an open set G such that $x \in G \subseteq Int(Cl(G)) \subseteq A$ (resp., $x \in G \subseteq Cl(G) \subseteq A$). In 1968,Velicko [23] defined the concepts of δ-open and θ-open sets in X (denoted by $\delta O(X)$ and $\theta O(X)$ respectively).

A function $f : X \to Y$ is said to be precontinuous [13] (resp., super continuous [15], strongly θ-continuous [12]) if $f^{-1}(V)$ is preopen (resp., δ-open, θ-open) in X for every open set V of Y. A function $f : X \to Y$ is said to be almost precontinuous [16] if the inverse image of each regular open subset of Y is preopen in X. A function $f : X \to Y$ is said to be θ-continuous [7] (resp., almost strongly θ-continuous [18]) if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists an open set U of X containing x such that $f(ClU) \subseteq ClV$ (resp., $f(ClU) \subseteq sClV$).

Definition 1.[10] *A subset A of a space X is called P_p-open, if for each $x \in A \in PO(X)$, there exists a preclosed set F such that $x \in F \subseteq A$.The complement of a P_p-open is P_p-closed. The family of all P_p-open subsets of a topological space (X,τ) is denoted by $P_pO(X,\tau)$ or $P_pO(X)$.*

* Corresponding author e-mail: mathshadya@gmail.com

The intersection of all P_p-closed (resp., preclosed, semi-closed, α-closed and δ-closed) sets of X containing A is called the P_P-closure (resp. preclosure, semi-closure, α-closure and δ-closure) of A and is denoted by $P_pCl(A)$ (resp. $pCl(A)$, $sCl(A)$, $\alpha Cl(A)$ and $Cl\delta(A)$). The union of all P_P-open (resp., preopen, semi-open, α-open and δ-open) sets of X contained in A is called the P_p-interior (resp., preinterior, semi-interior, α-interior and δ-interior) of A and is denoted by $P_pInt(A)$ (resp. $pInt(A)$, $sInt(A)$, $\alpha Int(A)$ and $\delta Int(A)$).

Definition 2. *A space X is said to be:*

(1) *locally indiscrete [5] if every open subset of X is closed.*
(2) *Pre-R_0 [4], if U is a preopen set and $x \in U$, then $PCl(\{x\}) \subseteq U$.*
(3) *Pre-T_1[9] if for each pair of distinct points x, y of X, there exist two preopen sets one containing x but not y and other containing y but not x.*

Definition 3. [21] *A space X is said to be pre-regular if for each preclosed set F and each point $x \notin F$, there exist disjoint preopen sets U and V such that $x \in U$ and $F \subseteq V$*

Proposition 1. [10] *The following statements are true:*

(1) *If a space X is pre-T_1, then $PO(X) = P_pO(X)$.*
(2) *If a space X is pre-regular, then $\tau \subseteq P_pO(X)$.*
(3) *If a space (X, τ) is locally indiscrete, then $PO(X) = P_pO(X)$.*

Corollary 1. [14] *For any space X, if X is pre-R_0, then $PO(X) = P_pO(X)$.*

Lemma 1. *Let X be a space. The following statements are true:*

(1) *$R \in RO(X)$ and $P \in PO(X)$, then $R \cap P \in PO(P)$ [5].*
(2) *Let $A \subseteq X$. Then $A \in PO(X, \tau)$ if and only if $sCl(A) = IntCl(A)$ [8].*
(3) *A is β-open if and only if $Cl(A)$ is regular closed [3].*

Lemma 2. *Let A be a subset of X. Then:*

(1) *If $A \in SO(X)$, then $pCl(A) = Cl(A)$ [6].*
(2) *If $A \in \beta O(X)$, then $\alpha Cl(A) = Cl(A)$ [2].*
(3) *If $A \in \beta O(X)$, then $Cl_\delta(A) = Cl(A)$ [24].*

Definition 4. [10] *A function $f : X \to Y$ is called P_p-continuous at a point $x \in X$ if for each open set V of Y containing $f(x)$, there exists a P_p-open set U of X containing x such that $f(U) \subseteq V$. If f is P_p-continuous at every point x of X, then it is called P_p-continuous.*

Definition 5. [19] *A function $f : X \to Y$ is called quasi θ-continuous at a point $x \in X$ if for each θ-open set V of Y containing $f(x)$, there exists a θ-open set U of X containing x such that $f(U) \subseteq V$.*

Corollary 2. [10] Every quasi θ-continuous is P_p-continuous.

Definition 6. [20] *A space X is said to be semi-regular if for any open set U of X and each point $x \in U$, there exists a regular open set V of X such that $x \in V \subseteq U$.*

2 Almost P_p-Continuous Functions

In this section, we introduce the notions of almost P_p-Continuous functions by using P_p-open sets. Some properties and characterizations are given.

Definition 7. *A function $f : X \to Y$ is called almost P_p-continuous at a point $x \in X$ if for each open set V of Y containing $f(x)$, there exists a P_p-open set U of X containing x such that $f(U) \subseteq IntCl(V)$. If f is almost P_p-continuous at every point x of X, then it is called almost P_p-continuous.*

Lemma 3. *The following results follows directly from their definitions:*

(1) *Every P_p-continuous function is almost P_p-continuous.*
(2) *Every almost P_p-continuous function is almost precontinuous.*

Corollary 3. *labelcsh* Every quasi θ-continuous function is almost P_p-continuous.

Proof. Follows from Corollary 2 and Lemma 3.

From Lemma 3, Corollary 2 and Corollary 5.4 in [10], the following diagram is obtained:

In the sequel, we shall show that none of the implications that concerning almost P_p- continuity in Diagram 1 is reversible.

Example 1. Consider $X = \{a,b,c,d\}$ with the two topologies $\tau = \{\phi,\{a\},\{a,b\},$ $\{c,d\},\{a,c,d\},X\}$ and $\sigma = \{\phi,\{b\},\{b,d\},\{b,c,d\},X\}$, $P_pO(X) = \{\phi,\{c\},\{d\},$ $\{a,b\},\{c,d\},\{a,b,c\},\{a,b,d\},X\}$. Let $f : (X,\tau) \to (X,\sigma)$ be the identity function. Then f is almost P_p-continuous, but it is not P_p-continuous, because $\{b\}$ is an open set in (X,σ) containing $f(b) = b$, there exists no P_p-open U in (X,τ) containing b such that $b \in f(U) \subseteq \{b\}$.

Example 2. Consider $X = \{a,b,c\}$ with the topology $\tau = \sigma = \{\phi,\{a\},\{b\},$ $\{a,b\},X\}$. Let $f : (X,\tau) \to (X,\sigma)$ be the identity function. Then f is almost precontinuous, but it is not almost P_p-continuous, because $\{a\}$ is an open set in (X,σ) containing $f(a) = a$, there exists no P_p-open U in (X,τ) containing a such that $a \in f(U) \subseteq IntCl(\{a\})$.

Theorem 1. *For a function $f : (X,\tau) \to (Y,\sigma)$, the following statements are equivalent:*

(1) *f is almost P_p-continuous.*
(2) *For each $x \in X$ and each open set V of Y containing $f(x)$, there exists a P_p-open set U in X containing x such that $f(U) \subseteq sCl(V)$.*
(3) *For each $x \in X$ and each regular open set V of Y containing $f(x)$, there exists a P_p-open set U in X containing x such that $f(U) \subseteq V$.*
(4) *For each $x \in X$ and each δ-open set V of Y containing $f(x)$, there exists a P_p-open set U in X containing x such that $f(U) \subseteq V$.*

Proof. (1) $\Rightarrow$ (2). Let $x \in X$ and let V be any open set of Y containing $f(x)$. By (1), there exists a P_p-open set U of X containing x such that $f(U) \subseteq IntCl(V)$. Since V is open, hence V is preopen. Therefore, by Lemma 1 (2), $f(U) \subseteq sCl(V)$.
(2) $\Rightarrow$ (3). Follow directly from definition 7 and Lemma 1(2).
(3) $\Rightarrow$ (4). Let $x \in X$ and let V be any δ-open set of Y containing $f(x)$. Then for each $f(x) \in V$, there exists an open set G containing $f(x)$ such that $G \subseteq IntCl(G) \subseteq V$. Since $IntCl(G)$ is a regular open set of Y containing $f(x)$, by (3), there exists a P_p-open set U in X containing x such that $f(U) \subseteq IntCl(G) \subseteq V$. This completes the proof.
(4) $\Rightarrow$ (1). Let $x \in X$ and let V be any open set of Y containing $f(x)$. Then $IntCl(V)$ is δ-open of Y containing $f(x)$. By (4), there exists a P_p-open set U in X containing x such that $f(U) \subseteq IntCl(V)$. Therefore, f is almost P_p-continuous.

Theorem 2. *For a function $f : X \to Y$, the following statements are equivalent:*

(1) *f is almost P_p-continuous.*
(2) *$f^{-1}(IntCl(V))$ is a P_p-open set in X, for each open set V in Y.*

(3) $f^{-1}(ClInt(F))$ *is a* P_p*-closed set in* X*, for each closed set* F *in* Y.
(4) $f^{-1}(F)$ *is a* P_p*-closed set in* X*, for each regular closed set* F *of* Y.
(5) $f^{-1}(V)$ *is a* P_p*-open set in* X*, for each regular open set* V *of* Y.

Proof. $(1) \Rightarrow (2)$. Let V be any open set in Y. We have to show that $f^{-1}(IntCl(V))$ is P_p-open in X. Let $x \in f^{-1}(IntCl(V))$. Then $f(x) \in IntCl(V)$ and $IntCl(V)$ is a regular open set in Y. Since f is almost P_p-continuous, by Theorem 1, there exists a P_p-open set U of X containing x such that $f(U) \subseteq IntCl(V)$. Which implies that $x \in U \subseteq f^{-1}(IntClV)$. Therefore, $f^{-1}(IntCl(V))$ is P_p-open in X.
$(2) \Rightarrow (3)$. Let F be any closed set of Y. Then $Y - F$ is an open set of Y. By (2), $f^{-1}(IntCl(Y \setminus F))$ is P_p-open in X and $f^{-1}(IntCl(Y \setminus F)) = f^{-1}(Int(Y \setminus IntF)) = f^{-1}(Y \setminus ClIntF) = X \setminus f^{-1}(ClIntF)$ is P_p-open in X and hence $f^{-1}(ClInt(F))$ is P_p-closed in X.
$(3) \Rightarrow (4)$. Let F be any regular closed set of Y. Then F is a closed set of Y. By (3), $f^{-1}(ClInt(F))$ is P_p-closed in X. Since F is regular closed set, then $f^{-1}(ClInt(F)) = f^{-1}(F)$. Therefore, $f^{-1}(F)$ is P_p-closed set in X.
$(4) \Rightarrow (5)$. Let V be any regular open set of Y. Then $Y \setminus V$ is regular closed of Y and by (4), we have $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is P_p-closed in X and hence $f^{-1}(V)$ is P_p-open in X.
$(5) \Rightarrow (1)$. Let $x \in X$ and let V be any regular open set of Y containing $f(x)$. Then $x \in f^{-1}(V)$. By (5), we have $f^{-1}(V)$ is P_p-open in X. Therefore, we obtain $f(f^{-1}(V)) \subseteq V$. Hence, by Theorem 1, f is almost P_p-continuous.

Theorem 3. *For a function* $f : (X, \tau) \to (Y, \sigma)$*, the following statements are equivalent:*

(1) f *is almost* P_p*-continuous.*
(2) $f(P_pCl(A)) \subseteq Cl_\delta f(A)$ *, for each subset* A *of* X.
(3) $P_pCl(f^{-1}(B)) \subseteq f^{-1}(Cl_\delta(B))$*, for each subset* B *of* Y.
(4) $f^{-1}(F)$ *is a* P_p*-closed set in* X*, for each* δ*-closed set* F *of* Y.
(5) $f^{-1}(V)$ *is a* P_p*-open set in* X*, for each* δ*-open set* V *of* Y.
(6) $f^{-1}(Int_\delta(B)) \subseteq P_pInt(f^{-1}(B))$*, for each subset* B *of* Y.
(7) $Int_\delta(f(A)) \subseteq f(P_pInt(A))$*, for each subset* A *of* X.

Proof. $(1) \Rightarrow (2)$. Let A be a subset of X. Since $Cl_\delta f(A)$ is δ-closed in Y. By (1) and Theorem 2, $f^{-1}(Cl_\delta f(A))$ is P_p-closed set of X. Hence, $P_pClA \subseteq f^{-1}(Cl_\delta f(A))$. Therefore, we obtain that $f(P_pClA) \subseteq Cl_\delta f(A)$.
$(2) \Rightarrow (3)$. Let B be any subset of Y. Then $f^{-1}(B)$ is a subset of X. By (2), we have $f(P_pClf^{-1}(B)) \subseteq Cl_\delta f(f^{-1}(B)) = Cl_\delta B$. Hence, $P_pClf^{-1}(B) \subseteq f^{-1}(Cl_\delta B)$.
$(3) \Rightarrow (4)$. Let F be any δ-closed set of Y. By (3), we have $P_pClf^{-1}(F) \subseteq f^{-1}(Cl_\delta F) = f^{-1}(F)$ and hence $f^{-1}(F)$ is P_p-closed in X.
$(4) \Rightarrow (5)$. Let V be any δ-open set of Y. Then $Y \setminus V$ is δ-closed of Y and by (4), we have $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is P_p-closed in X. Hence $f^{-1}(V)$ is P_p-open in X.
$(5) \Rightarrow (6)$. For each subset B of Y. We have $Int_\delta B \subseteq B$. Then $f^{-1}(Int_\delta B) \subseteq f^{-1}(B)$. By (5), $f^{-1}(Int_\delta B)$ is P_p-open in X. Then $f^{-1}(Int_\delta B) \subseteq P_pIntf^{-1}(B)$.
$(6) \Rightarrow (7)$. Let A be any subset of X. Then $f(A)$ is a subset of Y. By (6), we have $f^{-1}(Int_\delta(f(A)) \subseteq P_pInt(f^{-1}(f(A)) \subseteq P_pInt(A)$. Therefore, $Int_\delta(f(A)) \subseteq f(P_pInt(A))$.
$(7) \Rightarrow (1)$. Let $x \in X$ and let V be any regular open set of Y containing $f(x)$. Then $x \in f^{-1}(V)$ and $f^{-1}(V)$ is a subset of X. By (7), we have $Int_\delta(f(f^{-1}(V)) \subseteq f(P_pInt(f^{-1}(V))$ implies that $Int_\delta(V) \subseteq f(P_pInt(f^{-1}(V))$. Since V is regular open and hence it is δ-open, then $V \subseteq f(P_pInt(f^{-1}(V))$. This implies that $f^{-1}(V) \subseteq P_pInt(f^{-1}(V))$. Therefore, $f^{-1}(V)$ is a P_p-open set in X which contains x and clearly $f(f^{-1}(V)) \subseteq V$. Hence, by Theorem 1, f is almost P_p-continuous.

Theorem 4. *For a function* $f : X \to Y$*, the following statements are equivalent:*

(1) f *is almost* P_p*-continuous.*
(2) $P_pClf^{-1}(V) \subseteq f^{-1}(ClV)$ *, for each* β*-open set* V *of* Y.

(3) $f^{-1}(Int(F)) \subseteq P_pInt(f^{-1}(F))$, *for each* β*-closed set* F *of* Y.
(4) $f^{-1}(Int(F)) \subseteq P_pInt(f^{-1}(F))$, *for each semi-closed set* F *of* Y.
(5) $P_pClf^{-1}(V) \subseteq f^{-1}(ClV)$, *for each semi-open set* V *of* Y.

Proof. $(1) \Rightarrow (2)$. Let V be any β-open set of Y. By Lemma 1(3) that $Cl(V)$ is regular closed in Y. Since f is almost P_p-continuous, by Theorem 2, $f^{-1}(ClV)$ is P_p-closed set in X. Therefore, we obtain $P_pClf^{-1}(V) \subseteq f^{-1}(ClV)$.
$(2) \Rightarrow (3)$. Let F be any β-closed of Y. Then $Y \setminus F$ is β-open of Y and by (2), we have $P_pClf^{-1}(Y \setminus F) \subseteq f^{-1}(Cl(Y \setminus F))$ and $P_pCl(X \setminus f^{-1}(F)) \subseteq f^{-1}(Y \setminus IntF)$ and hence, $X \setminus P_pIntf^{-1}(F) \subseteq X \setminus f^{-1}(IntF)$. Therefore, $f^{-1}(IntF) \subseteq P_pIntf^{-1}(F)$.
$(3) \Rightarrow (4)$. Obvious since every semi-closed set is β-closed.
$(4) \Rightarrow (5)$. Let V be any semi-open set of Y. Then $Y \setminus V$ is semi-closed in Y and by (4), we have $f^{-1}(Int(Y \setminus V)) \subseteq P_pIntf^{-1}(Y \setminus V)$ and $f^{-1}(Y \setminus ClV) \subseteq P_pInt(X \setminus f^{-1}(V))$ and hence, $X \setminus f^{-1}(ClV) \subseteq X \setminus P_pClf^{-1}(V)$. Therefore, $P_pClf^{-1}(V) \subseteq f^{-1}(ClV)$.
$(5) \Rightarrow (1)$. Let F be any regular closed set of Y. Then F is a semi-open set of Y. By (5), we have $P_pClf^{-1}(F) \subseteq f^{-1}(ClF) = f^{-1}(F)$. This shows that $f^{-1}(F)$ is a P_p-closed set in X. Therefore, by Theorem 2, f is almost P_p-continuous.

Theorem 5. *For a function* $f : X \to Y$*, the following statements are equivalent:*

(1) f *is almost* P_p*-continuous.*
(2) $P_pClf^{-1}(V) \subseteq f^{-1}(\alpha ClV)$, *for each* β*-open set* V *of* Y.
(3) $P_pClf^{-1}(V) \subseteq f^{-1}(Cl_\delta V)$, *for each* β*-open set* V *of* Y.
(4) $P_pClf^{-1}(V) \subseteq f^{-1}(P_pClV)$, *for each semi-open set* V *of* Y.
(5) $P_pClf^{-1}(V) \subseteq f^{-1}(pCl(V)$, *for each semi-open set* V *of* Y.

Proof. $(1) \Rightarrow (2)$. Follows from Theorem 4 and Lemma 2(2).
$(2) \Rightarrow (3)$. Follows from the fact that $\alpha ClV \subseteq Cl_\delta V$.
$(3) \Rightarrow (4)$ and $(4) \Rightarrow (5)$. Follows from Theorem 4 and Lemma 2(1).
$(5) \Rightarrow (1)$. Follows from Theorem 4 and Lemma 2(1).

The following result also can be concluded directly.

Corollary 4. *For a function* $f : X \to Y$*, the following statements are equivalent:*

(1) f *is almost* P_p*-continuous.*
(2) $f^{-1}(\alpha IntF) \subseteq P_pIntf^{-1}(F)$, *for each* β*-closed set* F *of* Y.
(3) $f^{-1}(Int_\delta F) \subseteq P_pIntf^{-1}(F)$, *for each* β*-closed set* F *of* Y.
(4) $f^{-1}(P_pIntF) \subseteq P_pIntf^{-1}(F)$, *for each semi-closed set* F *of* Y.
(5) $f^{-1}(pIntF) \subseteq P_pIntf^{-1}(F)$, *for each semi-closed set* F *of* Y.

Theorem 6. *A function* $f : X \to Y$ *is almost* P_p*-continuous if and only if* $f^{-1}(V) \subseteq P_pIntf^{-1}(IntClV)$ *for each preopen set* V *of* Y.

Proof. **Necessity.** Let V be any preopen set of Y. Then $V \subseteq IntClV$ and $IntClV$ is a regular open set in Y. Since f is almost P_p-continuous, by Theorem 2, $f^{-1}(IntClV)$ is P_p-open in X and hence we obtain that $f^{-1}(V) \subseteq f^{-1}(IntClV) = P_pIntf^{-1}(IntClV)$.
Sufficiency. Let V be any regular open set of Y. Then V is a preopen set of Y. By hypothesis, we have $f^{-1}(V) \subseteq P_pIntf^{-1}(IntClV) = P_pIntf^{-1}(V)$. Therefore, $f^{-1}(V)$ is P_p-open in X and hence by Theorem 2, f is almost P_p-continuous.

Corollary 5. *The following statements are equivalent for a function* $f : X \to Y$*:*

(1) f *is almost* P_p*-continuous.*
(2) $f^{-1}(V) \subseteq P_pIntf^{-1}(sClV)$ *for each preopen set* V *of* Y.
(3) $P_pClf^{-1}(ClIntF) \subseteq f^{-1}(F)$ *for each preclosed set* F *of* Y.
(4) $P_pClf^{-1}(sIntF) \subseteq f^{-1}(F)$ *for each preclosed set* F *of* Y.

Corollary 6. *For a function* $f : X \to Y$*, the following statements are equivalent:*

(1) f *is almost* P_p*-continuous.*
(2) *For each neighborhood* V *of* $f(x)$, $x \in P_pIntf^{-1}(sClV)$.
(3) *For each neighborhood* V *of* $f(x)$, $x \in P_pInt(IntClV)$.

Proof. Follows from Theorem 6 and Corollary 5.

Theorem 7. *Let* $f : X \to Y$ *be an almost* P_p*-continuous function and let* V *be any open subset of* Y*. If* $x \in P_pClf^{-1}(V) \setminus f^{-1}(V)$*, then* $f(x) \in P_pClV$.

Proof. Let $x \in X$ be such that $x \in P_pClf^{-1}(V) \setminus f^{-1}(V)$ and suppose $f(x) \notin P_pClV$. Then there exists a P_p-open set H containing $f(x)$ such that $H \cap V = \phi$. Then $ClH \cap V = \phi$ implies $IntClH \cap V = \phi$ and $IntClH$ is a regular open set. Since f is almost P_p-continuous, by Theorem 1, there exists a P_p-open set U in X containing x such that $f(U) \subseteq IntClH$. Therefore, $f(U) \cap V = \phi$. However, since $x \in P_pClf^{-1}(V)$, $U \cap f^{-1}(V) \neq \phi$ for every P_p-open set U in X containing x, so that $f(U) \cap V \neq \phi$. We have a contradiction. It follows that $f(x) \in P_pClV$.

Theorem 8. If $f : X \to Y$ is almost P_p-continuous and $g : Y \to Z$ is super continuous function, then the composition function $g \circ f : X \to Z$ is P_p-continuous.

Proof. Let W be any open subset of Z. Since g is super continuous, $g^{-1}(W)$ is δ-open of Y. Since f is almost P_p-continuous, by Theorem 3, $(g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W))$ is P_p-open in X. Therefore, by Definition 4, $g \circ f$ is P_p-continuous.

Theorem 9. If $f : X \to Y$ is almost P_p-continuous and $g : Y \to Z$ is continuous and open, then the composition function $g \circ f : X \to Z$ is almost P_p-continuous.

Proof. Let $x \in X$ and W be an open set of Z containing $g(f(x))$. Since g is continuous, $g^{-1}(W)$ is an open set of Y containing $f(x)$. Since f is almost P_p-continuous, there exists a P_p-open set U of X containing x such that $f(U) \subseteq Int(Cl(g^{-1}(W)))$. Also, since g is continuous, then we obtain $(g \circ f)(U) \subseteq g(Int(g^{-1}(Cl(W))))$. Since g is open, we obtain $(gof)(U) \subseteq Int(Cl(W))$. Therefore, $g \circ f$ is almost P_p-continuous.

Theorem 10. *If* $f : X \to Y$ *is an almost* P_p*-continuous function and* Y *is semi-regular, then* f *is* P_p*-continuous.*

Proof. Let $x \in X$ and let V be any open set of Y containing $f(x)$. By the semi-regularity of Y , there exists a regular open set G of Y such that $f(x) \in G \subseteq V$. Since f is almost P_p-continuous, by Theorem 1, there exists a P_p-open set U of X containing x such that $f(U) \subseteq G \subseteq V$. Therefore, f is P_p-continuous.

Proposition 2. If $f : X \to Y$ is an almost P_p-continuous function and $g : Y \to Z$ a strongly θ-continuous function, then $g \circ f : X \to Z$ is almost P_p-continuous.

Proof. Let W be an open subset of Z. In view of strong θ-continuity of g, $g^{-1}(W)$ is a θ-open subset of Y. Again, since f is almost P_p-continuous, $(g \circ f)^{-1}(W)$=$f^{-1}(g^{-1}(W))$ is P_p-open in X. Hence, $g \circ f$ is almost P_p-continuous.

Theorem 11. Let $f : X \to Y$ be almost P_p-continuous. If Y is a preopen subset of Z , then $f : X \to Z$ is almost P_p-continuous.

Proof. Let V be any regular open set of Z. Since Y is preopen, by Lemma 1(1), $V \cap Y$ is a regular open set in Y. Since $f : X \to Y$ is almost P_p-continuous, by Theorem 2, $f^{-1}(V \cap Y)$ is a P_p-open set in X. But $f(x) \in Y$ for each $x \in X$. Thus $f^{-1}(V) = f^{-1}(V \cap Y)$ is a P_p-open set of X. Therefore, by Theorem 2, $f : X \to Z$ is almost P_p-continuous.

Corollary 7. *Let $f : X \to Y$ be a function and let X be a pre-T_1 space. Then f is almost precontinuous if and only if f is almost P_p-continuous.*

Proof. Follows from Proposition 1(1).

Corollary 8. *Let $f : X \to Y$ be a function and let X be a pre-R_0 space. Then f is almost precontinuous if and only if f is almost P_p-continuous.*

Proof. Follows from Corollary 1.

Corollary 9. *Let $f : X \to Y$ be a function and let X be a pre-regular space. If f is almost continuous, then f is almost P_p-continuous.*

Proof. Follows from Proposition 1(2).

Corollary 10. *Let $f : X \to Y$ be a function and let X be a locally indiscrete space. Then f is almost precontinuous if and only if f is almost P_p-continuous.*

Proof. Follows from Proposition 1(3).

Theorem 12. *If a function $f : X \to Y$ is almost strongly θ-continuous, then f is almost P_p- continuous.*

Proof. Let V be any regular open set of Y. Since f is almost strongly θ-continuous, so $f^{-1}(V)$ is θ-open and hence it is P_p-open. Therefore, by Theorem 2, f is almost P_p-continuous.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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