Analytical investigation of thermal radiation effect on nanofluid forced convection in a semi porous channel

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Abstract: Nanofluid thermal radiation in a semi porous channel is investigated in existence of magnetic field. Influence of thermal radiation and Joule heating on energy equation is taken into account. Differential transform method is utilized to solve this problem. Effects of Reynolds number, Hartmann number, suction parameter, Eckert number, radiation parameter on flow and heat transfer are presented. Results show that temperature gradient augments with rise of Reynolds number, suction parameter, and Radiation parameter.

Keywords: Semi porous channel, nanofluid, magnetic field, single phase model, differential transform method.

\textbf{Nomenclature}

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_0$</td>
<td>Magnetic induction</td>
</tr>
<tr>
<td>$Rd$</td>
<td>Radiation parameter</td>
</tr>
<tr>
<td>$Ha$</td>
<td>Hartmann number</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Specific heat capacity</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>$q_r$</td>
<td>Thermal radiation</td>
</tr>
<tr>
<td>$T$</td>
<td>Fluid temperature</td>
</tr>
<tr>
<td>$v, u$</td>
<td>Vertical and horizontal velocities</td>
</tr>
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</table>

\textbf{Greek symbols}

<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>$\sigma$</td>
<td>Electrical conductivity</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Similarity independent variable</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Volume fraction of nanofluid</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>Stefan–Boltzmann constant</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Thermal diffusivity</td>
</tr>
<tr>
<td>$\beta_R$</td>
<td>mean absorption coefficient</td>
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<td>$\mu$</td>
<td>Dynamic viscosity</td>
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\textbf{Subscripts}

<table>
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<tr>
<th>Subscript</th>
<th>Meaning</th>
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<tr>
<td>$s$</td>
<td>Solid particles</td>
</tr>
<tr>
<td>$f$</td>
<td>Base fluid</td>
</tr>
<tr>
<td>$nf$</td>
<td>Nanofluid</td>
</tr>
<tr>
<td>$C$</td>
<td>Solutal quantity</td>
</tr>
<tr>
<td>$\infty$</td>
<td>for $\eta \rightarrow \infty$</td>
</tr>
<tr>
<td>0</td>
<td>at $\eta \rightarrow 0$</td>
</tr>
</tbody>
</table>

\textbf{1 Introduction}

Nanofluid was proposed as innovative way to enhance heat transfer. Khan et al. [1] investigated nanofluid flow with slip motion influence in existence of inclined magnetic field. Bhatti et al. [2] studied the electric double layer effect on two phase flow in existence of magnetic field. Sheikholeslami and Ganji [3] presented various application of nanofluid in their review paper. Sheremet et al. [4] simulated the unsteady MHD flow in an enclosure. They used FDM to simulate...

Selimefendigil and Oztop [10] examined nanofluid conjugate conduction-convection mechanism in a titled cavity. They proved that temperature gradient augments with enhance of Grashof number. Sheikholeslami and Ganji [11] investigated nanofluid transportation in presence of external magnetic source. Sheikholeslami et al. [12] investigated nanofluid flow in a porous media in existence of magnetic field. Sheikholeslami et al. [13] utilized AGM for nanofluid flow between two pipes. Influence of non-uniform Lorentz forces on nanofluid flow style has been studied by Sheikholeslami Kandelousi [14]. He concluded that improvement in heat transfer reduces with rise of Kelvin forces. Sheikholeslami and Ganji [15] studied the electric field effect on nanofluid forced convection. Sheikholeslami and Ganji [16] demonstrated the effect of magnetic field on nanofluid natural convection. Sheikholeslami and Ganji [17] presented the nanofluid transportation in a curved cavity in presence of Lorentz forces. Several kinds of nanoparticles have been used by several researchers [18-24].

The aim of this article is to investigate impact of magnetic field on CuO-water nanofluid in existence thermal radiation. Similarity transformation is utilized to find the ordinary differential equations. Differential transform method is chosen to simulate this paper.

2 Governing equation

Steady nanofluid flow and heat transfer in a semi porous channel is considered. Fig. 1 shows the geometry and boundary conditions. Constant vertical magnetic field effect has been applied. Effects of thermal radiation and Joule heating on temperature distribution are taken into account. The lower plate is hot and the upper one is cold. Continuity, Navier stokes and energy equations are presented as follows:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

\[
\rho_n f \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu_n f \left( \frac{\partial^2 u}{\partial y^2} \right) - \sigma_n f B_0^2 u, \tag{2}
\]

\[
\rho_n f \left( \frac{\partial v}{\partial x} + u \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu_n f \left( \frac{\partial^2 v}{\partial x^2} \right) \tag{3}
\]

\[
(p C_p)_n f \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_n f \left( \frac{\partial^2 T}{\partial y^2} \right) + \sigma_n f B_0^2 u^2 - \frac{\partial q_r}{\partial y}, \tag{4}
\]

where the radiation heat flux \( q_r \) is considered according to Rosseland approximation such that \( q_r = -\frac{4\sigma e}{3} \frac{\partial T^4}{\partial y} \) where \( \sigma_e, \beta_R \) are the Stefan–Boltzmann constant and the mean absorption coefficient, respectively [27]. The fluid-phase temperature differences within the flow are assumed to be sufficiently small so that \( T^4 \) may be expressed as a linear function of temperature. This is done by expanding \( T^4 \) in a Taylor series about the temperature \( T_c \) and neglecting higher order terms to yield, \( T^4 \approx 4T_c^3 T - 3T_c^4 \).
The effective density, effective heat capacity and electrical conductivity of the nanofluid are defined as:

\[
\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_p,
\]

\[
(Cp)_f \rho_{nf} = (1 - \phi) (Cp)_f + \phi (Cp)_p,
\]

\[
\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3}{(\frac{\sigma_p}{\sigma_f} + 2) - (\frac{\sigma_p}{\sigma_f} - 1)} \phi.
\]

The KKL (Koo-Kleinstreuer-Li) correlation has been utilized for viscosity of nanofluid

\[
\mu_{eff} = \mu_{static} + \mu_{Brownian} = \mu_{static} + \frac{k_{Brownian}}{k_f} \times \frac{\mu_f}{Pr_f}
\]

\[
k_{Brownian} = 5 \times 10^4 \phi \rho_j c_p, f \sqrt{\frac{k_b T}{\rho_p d_p}} g'(T, \phi, d_p)
\]

\[
g'(T, \phi, d_p) = (a_1 + a_2 \ln(d_p) + a_3 \ln(\phi) + a_4 \ln(\phi) \ln(d_p) + a_5 \ln(d_p)^2) \ln(T)
\]

\[
+ (a_6 + a_7 \ln(d_p) + a_8 \ln(\phi) + a_9 \ln(\phi) \ln(d_p) + a_{10} \ln(d_p)^2).
\]

The related coefficient and properties of CuO-water nanofluid is presented in table 1 and 2. Maxwell model and Hamilton–Crosser model for irregular particle geometries by introducing a shape factor can be expressed as

\[
\frac{k_{nf}}{k_f} = \frac{k_p + (m + 1) k_f - (m + 1) \phi (k_f - k_p)}{k_p + (m + 1) k_f + \phi (k_f - k_p)}
\]

in which \(k_p\) and \(k_f\) are the conductivities of the particle material and the base fluid. In this equation ”m” is shaper factor. Table 3 shows the different values of shape factors for various shapes of nanoparticles.

Boundary conditions are:

\[
u = bx, \quad v = -v_0, \quad T = T_1 \quad \text{at} \quad y = -a
\]

\[
u = 0, \quad v = 0, \quad T = T_2 \quad \text{at} \quad y = +a
\]

where \(b < 0\) for shrinking walls channel and \(b > 0\) for stretching walls. Similarity transformation method has been utilized for obtain ordinary differential equations. The following non dimensional parameters are introduced:

\[
\eta = \frac{y}{a}, \quad u = bx f'(\eta), \quad v = -a b f(\eta), \quad \theta = \frac{T - T_1}{T_2 - T_1}
\]

By using the above transformation, the final equations are obtained as follows:

\[
f'' + \frac{Ha^2 A_5}{A_2} f''' - \frac{R A_1}{A_2} (f' f'' - f f''') = 0
\]

\[
(1 + \frac{4}{3A_4} Rd) \theta'' + Pr \frac{A_3}{A_4} f' \theta' + Ha^2 Ec \frac{Pr A_5}{R A_4} f''^2 = 0,
\]
where $R, Ha, Rd, Ec, Pr$ are stretching Reynolds number, Hartman number, Radiation parameter, Eckert number and Prandtl number. These constant parameters are defined as:

$$
R = \frac{a^2 b}{\nu_f}, Ha = B_0 a \sqrt{\frac{\sigma_f}{\mu_f}}, Rd = 4 \sigma_T T^3 / (\beta_k k_f),
$$

$$
Ec = \frac{\rho_f (\rho C_p)_f}{\rho C_p} \Delta T, \quad Pr = \frac{a^2 b}{\nu_f} \frac{\rho C_p}{k_f},
$$

Moreover, boundary conditions becomes

$$
f'(-1) = 1, \quad f'(1) = 0, \quad \theta (-1) = 1,
$$

$$
f(-1) = \lambda = \frac{\nu_0}{ab}, \quad f(1) = 0, \quad \theta (1) = 0.
$$

### 3 Differential transform method (DTM)

#### 3.1 Basic idea

Basic definitions and operations of differential transformation are introduced as follows. Differential transformation of the function $f(\eta)$ is defined as follows:

$$
F(k) = \frac{1}{k!} \left[ \frac{d^k f(\eta)}{d\eta^k} \right]_{\eta = \eta_0}.
$$

In (14), $f(\eta)$ is the original function and $F(k)$ is the transformed function which is called the T-function (it is also called the spectrum of the $f(\eta)$ at $\eta = \eta_0$, in the $k$ domain). The differential inverse transformation of $F(k)$ is defined as:

$$
f(\eta) = \sum_{k=0}^{\infty} F(k)(\eta - \eta_0)^k
$$

by combining (14) and (15) $f(\eta)$ can be obtained:

$$
f(\eta) = \sum_{k=0}^{\infty} \left[ \frac{d^k f(\eta)}{d\eta^k} \right]_{\eta = \eta_0} (\eta - \eta_0)^k.
$$

Equation (16) implies that the concept of the differential transformation is derived from Taylor’s series expansion, but the method does not evaluate the derivatives symbolically. However, relative derivatives are calculated by an iterative procedure that is described by the transformed equations of the original functions. From the definitions of (14) and (15), it is easily proven that the transformed functions comply with the basic mathematical operations shown in below. In real applications, the function $f(\eta)$ in (16) is expressed by a finite series and can be written as:

$$
f(\eta) = \sum_{k=0}^{N} F(k)(\eta - \eta_0)^k.
$$
Applying the differential transforms for Eqs. (10-11) gives:

\[
(k + 1)(k + 2)(k + 3)(k + 4)F[k + 4] - R \frac{A_1}{A_2} \sum_{m=0}^{k} ((m + 1)(m + 2)F[m + 2](k - m + 1)F[k - m + 1]) - R \frac{A_1}{A_2} \sum_{m=0}^{k} ((m + 1)(m + 2)(m + 3)F[m + 3]F[k - m]) = 0
\]

\[
\]

\[
\left(1 + \frac{4}{3A_4}Rd\right)(k + 2)(k + 1)\Theta[k + 2] + Pr \frac{A_3}{A_4} \sum_{m=0}^{k} ((m + 1)\Theta[m + 1]F[k - m]) + Ha^2Ec \frac{Pr A_5}{R A_4} \sum_{m=0}^{k} ((m + 1)F[m + 1](k - m + 1)F[k - m + 1]) = 0
\]

\[
\Theta[0] = a_5, \Theta[1] = a_6.
\]

According to previous equations:

\[
\]

\[
F[4] = \frac{1}{12}Ha^2 A_5 A_2 a_3 + \frac{1}{12}R A_1 A_2 a_5 - \frac{1}{4} R A_1 a_1 a_4,
\]

\[
F[5] = \frac{1}{20}Ha^2 A_5 A_2 a_4 + \frac{1}{30} R A_1 A_2 a_5 - \frac{1}{60} R A_1 Ha^2 A_5 A_2 a_1 a_3 + \frac{1}{20} \left(R \frac{A_1}{A_2}\right)^2 a_1^2 a_4, ...
\]

\[
\Theta[0] = a_5, \Theta[1] = a_6, \Theta[2] = -1.5 \frac{Pr}{R(3A_4 + 4Rd)} (RA_3 a_1 a_6 + Ha^2 A_5 Ec a_2^2),
\]

\[
\Theta[3] = 0.5 \frac{Pr}{R(3A_4 + 4Rd)} (-3RA_3 A_1 a_2 a_6 - 4RA_3 a_2 a_6 Rd + 3 a_1^2 Pr A_5^2 a_6 R + 3 a_1 Pr Ha^2 Ec A_3 A_5 a_2^2 - 12Ha^2 Ec a_2 a_3 A_5 A_4 - 16Ha^2 Ec a_2 a_3 A_5 Rd), ...
\]
Finally we have:

\[ F(\eta) = a_1 + a_2 \eta + a_3 \eta^2 + a_4 \eta^3 + \left( \frac{1}{12} Ha^2 A_5 A_2 a_3 + \frac{1}{12} R A_1 A_2 a_3 - \frac{1}{4} R A_1 a_4 \right) \eta^4 + \ldots \] (24)

\[ \Theta(\eta) = a_5 + a_6 \eta + \left( -1.5 \frac{Pr}{R (3A_4 + 4Rd)} (RA_3 a_1 a_6 + Ha^2 A_5 Ec a_2^2) \right) \eta^2 + \ldots \] (25)

According to following equations and boundary conditions, \( a_1, a_2, a_3, a_4, a_5, a_6 \) can be obtained. By inserting obtained \( a_1, a_2, a_3, a_4, a_5, a_6 \) into Equations (24-25), it can be obtained the expression of \( F(\eta) \) and \( \Theta(\eta) \) which are transform of \( f(\eta) \) and \( \theta(\eta) \), respectively.

### 4 Results and discussions

Influence of thermal radiation on Magnetohydrodynamic CuO-water nanofluid flow and heat transfer in a semi porous channel with stretching wall is investigated. Influence of Joule heating on energy equation is taken into account. The ordinary differential equations are obtained by means of similarity transformation. Differential transform method (DTM) is chosen for solving final ODEs. Fig. 2 illustrates the verifications of presented result with those of obtained in previous works ([28] and [29]). As depicted in this figure, the current MAPLE code has good accuracy.

Influences of nanofluid volume fraction, shape of nanoparticles, Hartmann number, suction parameter, Eckert number, radiation parameter on velocity profile, temperature profile are presented in graphs and tables. Influences of shape of the nanoparticles on Nusselt number is shown in Table 5. Selecting Platelet leads to find the maximum Nusselt number. Therefore, Platelet nanoparticle has been selected for further investigation. Influence of volume fraction of nanofluid on temperature profile is depicted in Fig. 3. Temperature profile decreases with rise nanofluid volume fraction. So thermal boundary layer thickness has reverse relationship with nanofluid volume fraction. Effect of Reynolds number on velocity and temperature profiles is shown in Fig. 4. Vertical velocity and temperature profiles decreases with increase of Reynolds number. Horizontal velocity near the bottom wall decreases with increase of Reynolds number while opposite trend is observed near the upper wall. Fig. 5 exhibits the effect of Hartmann number on velocity and temperature profiles. Temperature profile enhances with increase of Hartmann number. Also velocity reduces with increase of Lorentz forces. Fig. 6 shows the effect of suction parameter on velocity and temperature profiles. Temperature and vertical velocity profiles enhances with rise of suction parameter. Horizontal velocity reduces with augment of suction parameter. Also the minimum point of velocity shift to lower wall.

### 5 Conclusions

Effect of magnetic field on forced and thermal radiation heat transfer in a channel is examined considering Joule heating effect. Roles of nanofluid volume fraction, shape of nanoparticles, Hartmann number, suction parameter, Eckert number, radiation parameter are discussed. As Hartman number and Eckert number increases, Nusselt number decreases while opposite trend is observed for skin friction coefficient. Also it can be found that Nusselt number increases with augment of Reynolds number, suction parameter, Radiation parameter.
Competing interests

The authors declare that they have no competing interests.

Authors’ contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

References


Fig. 1: Geometry of the problem
Fig. 2: Comparison of the velocity and temperature profiles between the present work and (a) [28]; (b) [29] for different values of $Pr$ when $\lambda = 0.5$, $M = 1$, $R = 0.5$ and $Kr = 0.5$.

Fig. 3: Effect of nanofluid volume fraction on temperature profile when $R = 1, Ha = 1, Rd = 0.5, \lambda = 1, Ec = 0.5, m = 5.7, Pr = 6.2$. 

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Fig. 4: Effect of Reynolds number on velocity and temperature profiles.
Fig. 5: Effect of Hartmann number on velocity and temperature profiles.
Table 1: Thermo physical properties of water and nanoparticles.

<table>
<thead>
<tr>
<th>Material</th>
<th>( \rho ) (kg/m(^3))</th>
<th>( C_p ) (J/kgK)</th>
<th>( k ) (W/mK)</th>
<th>( d_p ) (nm)</th>
<th>( \sigma ) (( \Omega \cdot m ))(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure water</td>
<td>997.1</td>
<td>4179</td>
<td>0.613</td>
<td>-</td>
<td>0.05</td>
</tr>
<tr>
<td>CuO</td>
<td>6500</td>
<td>540</td>
<td>18</td>
<td>29</td>
<td>( 10^{-10} )</td>
</tr>
</tbody>
</table>
Table 2: The coefficient values of CuO – Water nanofluids.

| Coefficient values CuO – Water |  
|---------------------------------|---------|
| $a_1$                           | -26.593310846 |
| $a_2$                           | -0.403818333  |
| $a_3$                           | -33.3516805   |
| $a_4$                           | -1.915825591  |
| $a_5$                           | 6.42185846658E-02 |
| $a_6$                           | 48.40336955   |
| $a_7$                           | -9.787756683  |
| $a_8$                           | 190.245610009 |
| $a_9$                           | 10.9285386565 |
| $a_{10}$                        | -0.72009983664 |

Table 3: The values of shape factor of different shapes of nanoparticles.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical</td>
<td>3</td>
</tr>
<tr>
<td>Platelet</td>
<td>5.7</td>
</tr>
<tr>
<td>Cylinder</td>
<td>4.8</td>
</tr>
<tr>
<td>Brick</td>
<td>3.7</td>
</tr>
</tbody>
</table>

Table 4: Some of the basic operations of Differential transformation method.

<table>
<thead>
<tr>
<th>Original function</th>
<th>Transformed function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(\eta) = \alpha g(\eta) \pm \beta h(\eta)$</td>
<td>$F[k] = \alpha G[k] \pm \beta H[k]$</td>
</tr>
<tr>
<td>$f(\eta) = \frac{d^r g(\eta)}{d\eta^n}$</td>
<td>$F[k] = \frac{(k+n)!}{k!} G[k+n]$</td>
</tr>
<tr>
<td>$f(\eta) = g(\eta) h(\eta)$</td>
<td>$F[k] = \sum_{m=0}^{k} F[m] H[k-m]$</td>
</tr>
<tr>
<td>$f(\eta) = \sin(\sigma \eta + \alpha)$</td>
<td>$F[k] = \frac{\lambda k}{k!} \sin(\frac{\pi k}{2} + \alpha)$</td>
</tr>
<tr>
<td>$f(\eta) = \cos(\sigma \eta + \alpha)$</td>
<td>$F[k] = \frac{\lambda k}{k!} \cos(\frac{\pi k}{2} + \alpha)$</td>
</tr>
<tr>
<td>$f(\eta) = e^{\lambda \eta}$</td>
<td>$F[k] = \frac{\lambda^k}{k!}$</td>
</tr>
<tr>
<td>$F(\eta) = (1 + \eta)^m$</td>
<td>$F[k] = \frac{m(m-1)\ldots(m-k+1)}{k!}$</td>
</tr>
<tr>
<td>$f(\eta) = \eta^m$</td>
<td>$F[k] = \delta(k-m) = \begin{cases} 1, &amp; k = m \ 0, &amp; k \neq m \end{cases}$</td>
</tr>
</tbody>
</table>
Table 5: Nusselt number for various shape of the nanoparticles when $Ha = 1$, $Rd = 0.5$, $\lambda = 1$, $Ec = 0.5$, $\phi = 0.04$, $m = 5.7$, $Pr = 6.2$.

<table>
<thead>
<tr>
<th>R</th>
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<th>Platelet</th>
<th>Cylinder</th>
<th>Brick</th>
</tr>
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<tr>
<td>1</td>
<td>1.735301</td>
<td>1.815146</td>
<td>1.790366</td>
<td>1.757658</td>
</tr>
<tr>
<td>1.5</td>
<td>3.21957</td>
<td>3.259504</td>
<td>3.247116</td>
<td>3.230759</td>
</tr>
<tr>
<td>2</td>
<td>3.892636</td>
<td>3.91125</td>
<td>3.905465</td>
<td>3.897842</td>
</tr>
<tr>
<td>2.5</td>
<td>4.236621</td>
<td>4.241594</td>
<td>4.240018</td>
<td>4.237987</td>
</tr>
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