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An efficient algorithm for solving nonlinear system of differential equations and applications

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Abstract: In this article, we apply Chebyshev collocation method to obtain the numerical solutions of nonlinear systems of differential equations. This method transforms the nonlinear systems of differential equation to nonlinear systems of algebraic equations. The convergence of the numerical method are given and their applicability is illustrated with some examples.

Keywords: Nonlinear differential system; collocation method; Chebsyhev polynomial; the logistic growth model; disease in a population.

1 Introduction

One of the fundamental classes of system equations is nonlinear differential equation systems (NDES). Such equations arise in many areas of natural science and play an important role in the modeling of real-life phenomena in other fields of science. Some of the most popular modelling are prey-predator model [1-6], epidemic model [7-10], kinetic model [11-12], ozone decomposition model [13-14], modelling of mosquito dispersal [15], modelling a thermal explosion [16], dynamical models of happiness [17]. Therefore, NDESs have recieved much attention in last years. Some of NDESs cannot be solved by known classical methods. Hence, it is desirable to present numerical methods to solve these equations numerically.

The last years, it was found that the spectral methods are a valid method to obtain approximations for some type equations such as differential equations,

Fredholm-Volterra integro differential equations(see [18-28]). In this paper, we present a method to approximate a nonlinear differential equations systems on interval by using shifted Chebyshev polynomials. Consider the following NDES

$$\sum_{j=0}^{m} P_{ij}(x, y_j, y_j^{(1)}, ..., y_j^{(k_i)}) = f_i(x) \quad x \in [-1,1] \tag{1}$$

for $i = 1,2,...,m$ and $x \in [a,b]$, we consider the supplementary conditions

$$\sum_{j=1}^{m}\sum_{k=1}^{n} (A_{jki} y^{(k_i-1)}(x_1) + B_{jki} y^{(k_j-1)}(x_2)) = \lambda_i, \;\; i = 1,2,...,m \tag{2}$$

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where $f_i(x)$ is analytic functions. The motivation of this paper is to illustrate the merits of the method in solving some systems of nonlinear ordinary differential equations. The collocation method is useful for obtaining exact and approximate solutions of linear differential equations. The availability of computer symbolic packages such as Maple give a mathematical tool to perform some complicated manipulations and to carry out some modifications on a method for a specific problem easily. In the present paper, we apply the collocation method for solving problem (1) and (2). Our method consists of reducing the solutions of Eq. (1) to a set of nonlinear algebraic equations by expanding as the truncated first kind shifted Chebyshev polynomials with unknown coefficients. The properties of the shifted Chebyshev polynomials are then utilized to evaluate the unknown coefficients. Note that we have computed the numerical results by Maple programming. The rest of the paper is as follows. First, in Section 2 we review some of the main properties of the shifted Chebyshev polynomials. In Section 3, we illustrate how the collocation method may be used to replace Eq.(1) by systems of nonlinear algebraic equations. In Section 4, we report our numerical results and demonstrate the efficiency and accuracy of the proposed numerical scheme by considering some numerical examples.

2 The Shifted Chebyshev polynomial properties

The shifted Chebshev polynomials which can be obtained with the aid of the following recurrence formula [29-30]:

$$T^*_{L,r+1}(x) = 2(\frac{2x}{L})T^*_{L,r}(x) - T^*_{L,r-1}(x), r = 1,2,...$$

where $T^*_{L,0} = 1$, $T^*_{L,1} = \frac{2x}{L} - 1$. The analytic form of the shifted Chebyshev polynomials $T^*_{L,r}(x)$ of degree r is given by

$$T^*_{L,r}(x) = r\sum_{p=0}^{r}(-1)^{r-p}\frac{(r+p-1)!2^{2p}}{(r-p)!(2p)!}x^p \tag{3}$$

where $T^*_{L,r}(0) = (-1)^r$, $T^*_{L,r}(L) = 1$. The orthogonality condition is

$$\int_0^L T^*_{L,j}(x)T^*_{L,i}(x)w_L(x)dx = h_k\delta_{ji}$$

where $w_L(x) = (Lx-x^2)^{-1/2}$ and $h_i = b_i\pi/2$, $b_0 = 2$, $b_i = 1$, $i \geq 1$. By Eq.(4), we have the $k-th$ derivatives of $T^*_{L,r}(x)$

$$(T^*_{L,r}(x))^{(k)} = T^{*,k}_{L,r}(x) = r\sum_{p=m}^{r}(-1)^{r-p}p(p-1)...(p-k+1)\frac{(r+p-1)!2^{2p}}{(r-p)!(2p)!}x^{p-k} \tag{4}$$

3 Solution method

In order to solving Eq.(1) by using collocation method, first all, we approximate $y_j(x)$ as

$$y^j_N(x) = \sum_{r=0}^{N} a^j_r T^*_r(x) \tag{5}$$

where a_r^j, $r = 0,1,2,...,N$ are unknown shifted Chebyshev coefficients and N is chosen any positive integer. From (1) and (5), we have

$$\sum_{j=1}^{m} P_{ij}\left(x, \sum_{r=0}^{N} a_r^j T_r^*(x), \sum_{r=1}^{N} a_r^j T_r^{*,1}(x), ..., \sum_{r=k_i}^{N} a_r^j T_r^{*,k_i}(x)\right) = f_i(x) \tag{6}$$

Now, we collocate Eq.(6) at $(N-m+1)$ points x_p, $p = 0,1,2,...,N-m$ as:

$$\sum_{j=1}^{m} P_{ij}\left(x_p, \sum_{r=0}^{N} a_r^j T_r^*(x_p), \sum_{r=1}^{N} a_r^j T_r^{*,1}(x_p), ..., \sum_{r=k_i}^{N} a_r^j T_r^{*,k_i}(x_p)\right) = f_i(x_p) \tag{7}$$

Also, by substituting Eq.(5) in the conditions Eq.(2), we obtain k equations as follows:

$$\sum_{j=1}^{m}\sum_{k=1}^{n}\left(A_{jki}\sum_{r=k_i-1}^{N} a_r^j T_r^{*,k_i-1}(x_1) + B_{jki}\sum_{r=k_i-1}^{N} a_r^j T_r^{*,k_i-1}(x_2)\right) \tag{8}$$

Eq.(7) and Eq.(8) give a $(m\times(N+1))$-times non-linear algebraic equations. Solving this nonlinear algebraic system by aid of Maple 15, we obtain the unknown shifted Chebshev coefficients a_r^j, $r = 0,1,2,...,N$ Therefore, Using (5), obtain the approximate solutions for various N.

3.1 Error analysis

In this section, we present convergence analysis of the mention method. We assume that $y(x)$ is a sufficiently smooth function on $[0,1]$ and $I_N(x)$ is the interpolating polynomial to $y(x)$ at x_i where x_i, $i = 0,1,2,...,N$ are the Cbeyshev-Gauss grid points, then we have

$$y(x) - I_N(x) = \frac{y^{(N+1)}(\xi)}{(N+1)!}\prod_{i=0}^{N}(x-x_i), \xi \in [0,1]$$

Therefore, we have [29,32]

$$|y(x) - I_N(x)| \leq \frac{1}{2^{2N+1}}||y^{(N+1)}(x)||_\infty \tag{9}$$

Theorem 1. Suppose that the known functions in Eq.(1) are real $(N+1)$-times conti. differential functions on the $[0,1]$ and

$$y_N(x) = \sum_{r=0}^{N} a_r T_r^*(x)$$

are the shifted Chebyshev polynomials expansion of the exact solution. Let

$$\overline{y_N}(x) = \sum_{r=0}^{N} \overline{a_r} T_r^*(x)$$

be the approximate solution obtained by proposed method, then there exist real number α such that

$$||y(x) - y_N(x)|| \leq \alpha\frac{1}{2^{2N+1}}||y^{(N+1)}(x)||_\infty + \sqrt{\frac{3\pi}{8}}||A-\overline{A}||$$

where

$$A = \left[a_0\ a_1\ ...\ a_N\right]$$

and

$$\overline{A} = \left[\overline{a_0}\ \overline{a_1}\ \ldots\ \overline{a_N} \right].$$

Proof. Let $y_N(x)$ is real valued polynomials of degree$\leq N$ and $y_N(x)$ is the best approximation of $y(x)$. We can write

$$||y(x) - y_N(x)||_2 \leq ||y(x) - \overline{y_N}(x)||_2 + ||\overline{y_N}(x) - y_N(x)||_2$$

Using (9), we obtain

$$\begin{aligned} ||y(x) - y_N(x)||_2 &= \Big(\int_0^1 |y(x) - y_N(x)|^2 dx \Big)^{1/2} \leq \Big(\int_0^1 \frac{1}{2^{2N+1}(N+1)!} ||y^{(N+1)}(x)|| dx \Big)^{1/2} \\ &= \sqrt{L} \frac{1}{2^{2N+1}(N+1)!} ||y^{(N+1)}(x)||_\infty \end{aligned}$$

and we have

$$\begin{aligned} ||y(x) - \overline{y_N}(x)||_2 =& \Big(\int_0^1 \Big[\sum_{r=0}^{N} (a_r - \overline{a_r}) T_r^*(x) \Big]^2 dx \Big)^{1/2} \\ \leq& \Big(\int_0^1 \Big[\sum_{r=0}^{N} (a_r - \overline{a_r})^2 \Big] \Big[\sum_{r=0}^{N} |T_r^*(x)|^2 \Big] dx \Big)^{1/2} \\ =& \Big[\sum_{r=0}^{N} (a_r - \overline{a_r})^2 \Big]^{1/2} \Big(\sum_{r=0}^{N} \int_0^1 |T_r^*(x)|^2 dx \Big)^{1/2} \\ =& \sqrt{\frac{3\pi}{8}} ||A - \overline{A}||_2. \end{aligned}$$

Moreover, we can check the accuracy of the method. Since the truncated Chebyshev series (3) is an approximate solutions of Eq.(1), when the function $y_j^N(t)$, $j = 1, \ldots, m$ and its first derivatives are substituted in Eq.(1) the resulting equation must be satisfied approximately[32]; that is, for $t_i \in [0,1]$, $i = 0, 1, 2, , \ldots$

$$\Big| \sum_{j=1}^{m} P_{ij}(x_i, y_N^j, (y_N^j)', \ldots, (y_N^j)^{k_i}) - f_i(x_i) \Big| \cong 0,\ i = 1, 2, \ldots, m. \tag{10}$$

On the other hand, the error can be estimated by the function [32]

$$E_j^N(x) = \sum_{j=1}^{m} P_{ij}(x, y_N^j, (y_N^j)', \ldots, (y_N^j)^{k_i}) - f_i(x),\ i = 1, 2, \ldots, m. \tag{11}$$

4 Numerical results

In this section, we give some the numerical examples. The absolute errors in Tables are the values of $N_e^j = |y_j(x) - y_j^N(x)|$, those at selected points. In Tables $max|y_j(x) - y_j^N(x)|$ is maximum absolute errors. Moreover, we compare the absolute errors and L^2-norm is defined by

$$E_N^L = \Big(\int_0^1 (y(x) - y_N(x))^2 dx \Big)^{1/2}$$

where $y(x)$ and $y_N(x)$ denote the exact solution and the approximate solution obtained by the present method, respectively.

Example 1. Consider the nonlinear system of second-order boundary value problems:

$$y_1''(x) + xy_1'(x) + cos(\pi x)y_2'(x) = f_1(x)$$
$$y_2''(x) + xy_1'(x) + xy_1^2(x) = f_2(x)$$
$$y_1(0) = y_1(1) = 0,\ y_2(0) = y_2(1) = 0$$

where

$$f_1(x) = sin(x) + (x^2 - x + 2)cos(x) + (1 - 2x)cos(\pi x)$$
$$f_2(x) = -2 + xsin(x) + x(x-1)^2 sin^2(x) + (x^2 - x)cos(x).$$

The exact solutions are $y_1(x) = x - x^2$ and $y_2(x) = (x-1)sin(x)$. From numerical algorithm, we have, for $N = 6$

$$\sum_{r=2}^{6} a_r^1 T_r^{*,2}(x_q) + x_q \sum_{r=1}^{6} a_r^1 T_r^{*,1}(x_q) + cos(\pi x_q) \sum_{r=1}^{6} a_r^2 T_r^{*,1}(x_q) = f_1(x_q) \tag{12}$$

$$\sum_{r=2}^{6} a_r^2 T_r^{*,2}(x_q) + x_q \sum_{r=1}^{6} a_r^1 T_r^{*,1}(x_q) + x_q \Big(\sum_{r=0}^{6} a_r^1 T_r^{*}(x_q) \Big)^2 = f_2(x_q) \tag{13}$$

with $q = 0, 1, 2, 3, 4$, where x_q are roots of the shifted Chebyshev polynomials T_5^* and conditions

$$y_1^6(0) = a_0^1 - a_1^1 + a_2^1 - a_3^1 + a_4^1 - a_5^1 + a_6^1 = 0 \tag{14}$$
$$y_1^6(1) = a_0^1 + a_1^1 + a_2^1 + a_3^1 + a_4^1 + a_5^1 + a_6^1 = 0 \tag{15}$$
$$y_2^6(0) = a_0^2 - a_1^2 + a_2^2 - a_3^2 + a_4^2 - a_5^2 + a_6^2 = 0 \tag{16}$$
$$y_2^6(1) = a_0^2 + a_1^2 + a_2^2 + a_3^2 + a_4^2 + a_5^2 + a_6^2 = 0 \tag{17}$$

Thus, we obtain 14 nonlinear algebraic equations with the 14 unknown by Eqs.(9)-(13). Now solving equations, we have unknown coefficients whose are substituting into Eq.(5), we get the approximate solution for $N = 6$

$$y_6^1(x) = x - x^2 \tag{18}$$
$$y_6^2(x) = -0.998619x + x^2 + 0.166038x^3 - 0.166375x^4 - 0.008922x^5 + 0.007313x^6 \tag{19}$$

Results for Example 1 is reported in Table 1. Table 1 are compared with the results obtained by the Variational iteration method (VIM) and present method between exact solution $y_2(x)$ and computational solution $y_N^2(x)$ for $N = 6, 8$. The behavior of the absolute errors obtained by present method are shown in Fig.1. In Fig.2, we plotted error estimation functions for $N = 8$.

Table 1. Numerical results of Example 1.

		Present Method					
x	Exact Solution	$N=6$	$N_e=6$	$N=8$	$N_e=8$	VIM	Error of VIM
0.1	-0.089850	-0.089712	0.138E-3	-0.089841	0.441E-5	-0.27844	0.0003
0.2	-0.158935	-0.158659	0.275E-3	-0.158927	0.846E-5	-0.47274	0.025
0.3	-0.206864	-0.206451	0.412E-3	-0.206853	0.110E-4	-0.57415	0.0078
0.4	-0.233651	-0.233105	0.545E-3	-0.233642	0.830E-5	-0.58722	0.0166
0.5	-0.239712	-0.239047	0.665E-3	-0.239720	0.813E-5	-0.52768	0.0277
0.6	-0.225856	-0.225099	0.757E-3	-0.225907	0.509E-4	-0.41910	0.0387
0.7	-0.193265	-0.192474	0.791E-3	-0.193394	0.129E-3	-0.28860	0.0459
0.8	-0.143471	-0.142748	0.488E-3	-0.143697	0.226E-3	-0.16242	0.0449
0.9	-0.078332	-0.077844	0.488E-3	-0.078591	0.258E-3	-0.06184	0.0309
1.0	0.000000	-0.57E-10	0.57E-10	0.58E-11	0.58E-11	0.0	0.0

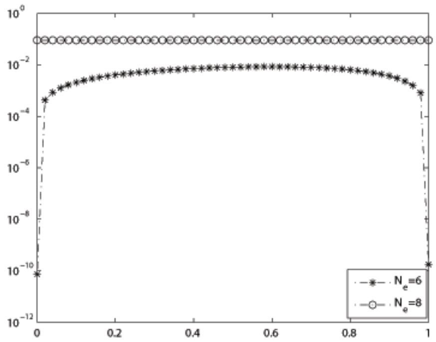

Fig. 1: Comparison of absolute errors.

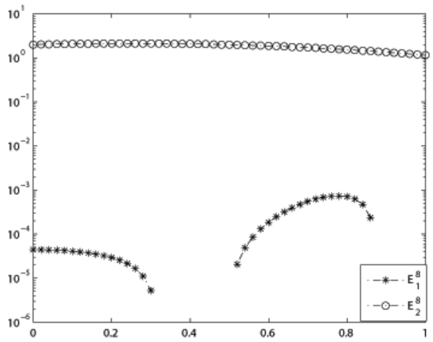

Fig. 2: Error function for $N = 8$.

Example 2. Consider the nonlinear system of first-order boundary value problems:

$$y_1'(x) - y_2'(x) + y_1(x)y_2(x) = f_1(x)$$
$$y_2'(x) + y_1'(x) - y_1(x)y_2(x) = f_2(x)$$
$$y_1(0) = y_2(0) = 0$$

where

$$f_1(x) = e^{-x}(1-x) + e^x(x+1) - x^2$$
$$f_2(x) = e^{-x}(x-1) + e^x(x+1) + x^2$$

and exact solutions of this problem $y_1(x) = xe^x$ and $y_2(x) = xe^{-x}$. We compare our computational solutions obtained by present method with the exact solutions in Tables 2 and 3 for $y_N^1(x)$ and $y_N^2(x)$ respectively. It can be seen from the Tables and Figures that, our approximate solutions are in very good agreement with the exact solutions. Moreover, the results of the corresponding absolute errors are presented in the same tables, using only the five, seven and nine order terms of the approximate solutions. We find that the absolute errors are very small, which reveals that the speed of convergence of present method is very fast, and the overall errors can be made very small by computing more terms in approximations.

Table 2. Numerical results of Example 2.

		Present Method					
x	Exact Solution	$N=5$	$N_e=5$	$N=7$	$N_e=7$	$N_e=9$	$N_e=9$
0.1	0.110517	0.110514	0.230E-5	0.110517	0.224E-8	0.110517	0.477E1-2
0.2	0.244280	0.244278	0.205E-5	0.244280	0.493E-8	0.244280	0.311E-12
0.3	0.404957	0.404960	0.275E-5	0.404957	0.287E-9	0.404957	0.464E-11
0.4	0.596729	0.596734	0.413E-5	0.596729	0.785E-8	0.596729	0.623E-11
0.5	0.824360	0.824358	0.173E-5	0.824360	0.155E-8	0.824360	0.133E-11
0.6	1.093271	1.093262	0.887E-5	1.093271	0.126E-7	1.093271	0.962E-11
0.7	1.409626	1.409621	0.547E-5	1.409626	0.417E-8	1.409626	0.116E-10
0.8	1.780432	1.780444	0.114E-4	1.780432	0.169E-7	1.780432	0.546E-11
0.9	2.213642	2.213651	0.907E-5	2.213642	0.327E-7	2.213642	0.145E-12
1.0	2.718281	2.718160	0.121E-3	2.718281	0.233E-6	2.718281	0.251E-9

Table 3. Numerical results of Example 2.

		Present Method					
x	Exact Solution	$N=5$	$N_e=5$	$N=7$	$N_e=7$	$N_e=9$	$N_e=9$
0.1	0.090483	0.090484	0.101E-5	0.090483	0.134E-8	0.090483	0.187E-12
0.2	0.163746	0.163747	0.905E-6	0.163746	0.161E-8	0.163746	0.128E-12
0.3	0.222245	0.222244	0.110E-5	0.222245	0.239E-9	0.222245	0.182E-11
0.4	0.268128	0.268126	0.165E-5	0.268128	0.340E-8	0.268128	0.240E-11
0.5	0.303265	0.303265	0.653E-6	0.303265	0.210E-8	0.303265	0.482E-11
0.6	0.329286	0.329290	0.340E-5	0.329286	0.435E-8	0.329286	0.364E-11
0.7	0.347609	0.347612	0.231E-5	0.347609	0.179E-8	0.347609	0.415E-10
0.8	0.359463	0.359459	0.350E-4	0.359463	0.677E-8	0.359463	0.168E-11
0.9	0.365912	0.365909	0.329E-5	0.365912	0.989E-8	0.365912	0.469E-12
1.0	0.367879	0.367915	0.362E-4	0.367879	0.739E-7	0.367879	0.810E-10

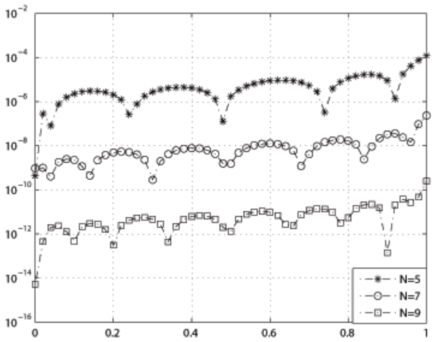

Fig. 3: Comparison of absolute errors.

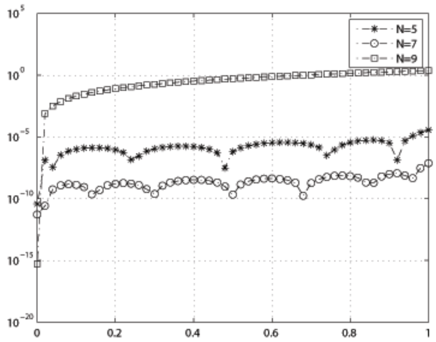

Fig. 4: Comparison of absolute errors.

Example 3. Let us consider the nonlinear stiff problem [33-34]

$$y_1'(x) = -1002 y_1(x)$$
$$y_2'(x) = y_1(x) - y_2(x) - (y_2(x))^2$$
$$y_1(0) = y_2(0) = 1$$

The exact solution of Example 3 is

$$y_1(x) = e^{-2x},\ y_2(x) = e^{-x}$$

The obtained results are summarized in Tables 4 and 5. The Variational iteration method (VIM) is better than Adomian decomposition method (ADM). The our approximate solutions and VIM are in very good agreement.

Table 4. Numerical comparison of Example 5.3 for $y_1(x)$.

x	VIM errors [33]	ADM errors [34]	PM errors
1.0	0.1785E-16	0.2556E-4	0.0975E-16
1.5	0.1972E-16	0.3487E-5	0.1045E-16
2.0	0.2003E-16	0.1638E-3	0.1256E-16
2.5	0.2416E-16	0.2417E-3	0.1689E-16
3.0	0.2942E-16	0.3764E-2	0.2563E-16

Table 5. Numerical comparison of Example 3 for $y_2(x)$.

x	VIM errors [33]	ADM errors [34]	PM errors
1.0	0.1785E-15	0.2895E-4	0.1001E-16
1.5	0.1942E-15	0.4135E-4	0.1112E-16
2.0	0.2105E-15	0.1638E-3	0.1561E-16
2.5	0.2743E-15	0.2417E-3	0.1674E-16
3.0	0.3013E-15	0.1735E-2	0.2256E-16

Example 4. Let us consider the following nonlinear system of Lane-Emden equation:

$$y_1''(x)+\frac{2}{x}y_1'(x)-y_2(x)+e^{y_2(x)}=f_1(x)$$
$$y_2''(x)+\frac{2}{x}y_2'(x)+y_1(x)+e^{y_1(x)}=f_2(x)$$
$$y_1(0)=y_2(0)=1$$

where

$$f_1(x)=1+x^2+(4x^2-6)e^{-x^2}-ln(1+x^2)$$
$$f_2(x)=ln(1+x^2)+exp(e^{-x^2})+\frac{6}{1+x^2}-\frac{4x^2}{(1+x^2)^2}$$

subject to initial conditions

$$y_1(0)=1,\ y_1'(0)=y_2(0)=y_2'(0)=0$$

which has the following analytic solution:

$$y_1(x)=e^{-x^2},\ y_2(x)=ln(1+x^2)$$

Numerical results are given in Tables 6 and 7.

Table 6. Numerical results of Example 4.

		Present Method					
x	Exact Solution	$N=5$	$N_e=5$	$N=8$	$N_e=8$	$N_e=9$	$N_e=9$
0.2	0.960789	0.960780	0.928E-5	0.960789	0.389E-7	0.960789	0.389E-9
0.4	0.852143	0.852179	0.352E-4	0.852143	0.535E-7	0.852143	0.535E-9
0.6	0.697676	0.697651	0.249E-4	0.697674	0.143E-5	0.697674	0.143E-7
0.8	0.527292	0.527317	0.252E-4	0.527287	0.501E-5	0.527287	0.501E-7
1.0	0.367879	0.369850	0.197E-2	0.367912	0.332E-4	0.367912	0.332E-6

Table 7. Numerical results of Example 4.

x	Exact Solution	Present Method					
		$N=5$	$N_e=5$	$N=8$	$N_e=8$	$N_e=9$	$N_e=9$
0.2	0.039280	0.039280	0.683E-4	0.039220	0.623E-5	0.039220	0.623E-6
0.4	0.148420	0.148628	0.208E-4	0.148420	0.408E-5	0.148420	0.408E-6
0.6	0.307484	0.307790	0.305E-4	0.307484	0.362E-5	0.307484	0.362E-6
0.8	0.494696	0.495315	0.619E-3	0.494696	0.475E-4	0.494696	0.475E-5
1.0	0.693147	0.699363	0.621E-2	0.693147	0.982E-3	0.693147	0.982E-5

5 Application of method

Modelling of ecological systems has received a great deal of attention from theoretical ecologists in the last few decades. Much focus has been on mathematical models of these systems, since they have substantially contributed to the understanding of the dynamics of systems by forging strong links between models and available data. We first consider the logistic growth in a population as a single species model to be governed by [35-39]

$$\frac{dy}{dx} = ry(1-y/k),\ y(0)=\alpha,\ r>0,\ k>0 \tag{20}$$

where $y=y(x)$ represents the population of the species at time x, $ry(1-y/k)$ is the per capita growth rate and k is the carrying capacity of the environment. For numerical purpose we take $r=k=1$ and $\alpha=2$ in $[36-37]$,then exact solution is

$$y(x)=\frac{2}{2-e^{-x}}.$$

Comparison of various numerical methods are given in Table 8. For various, absolute errors of approximate solutions are plotted in Fig..

Table 8. Comparison of numerical methods.

x	Present method	Homotopy per. method[39]	Adomian decomp. method[37]	Bessel col. method[36]
0.0	2.0000000000	2.0000000000	2.0000000000	2.0000000000
0.2	1.6930940588	1.6932805333	1.6924480000	1.6929496880
0.4	1.5041219473	1.5236181333	1.4703360000	1.5040518406
0.6	1.3781801288	1.6597808000	1.0528640000	1.3781410954
0.8	1.2897638171	3.1168085333	-0.293248000	1.2896532734
1.0	1.2253849971	8.9083333333	-4.100000000	1.2276056502

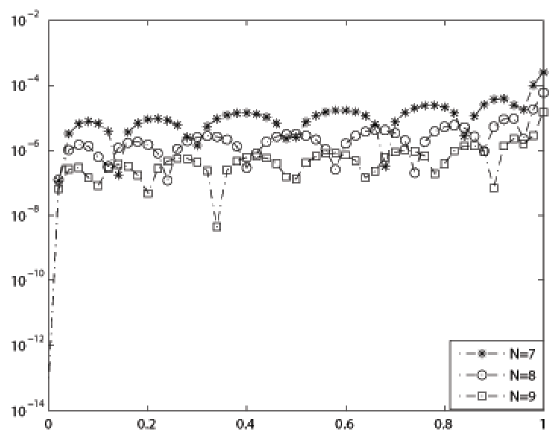

Fig. 5: Comparison of absolute errors for the logistic model.

Secondly, we consider the modelling of spreading of a non-fatal disease in a population which is assumed to have constant size over the period of the epidemic is considered in [40-41]. It was created a model in which they considered a fixed population with only three compartments: susceptible $y_1(x)$, infected $y_2(x)$ and recovered $y_3(x)$. The compartments used for this model consist of three classes:

1. $y_1(x)$: susceptible population, those so far uninfected and therefore liable to infection;

2. $y_2(x)$: infective population, those who have the disease and are still at large;

3. $y_3(x)$: isolated population, or who have recovered and are therefore immune.

Assume that there is a steady constant rate between susceptible population and infective population and that a constant proportion of these constant results in transmission. Following system determines the progress of the disease

$$\frac{dy_1}{dx} = -\beta y_1(x)y_2(x)$$
$$\frac{dy_2}{dx} = \beta y_1(x)y_2(x) - \gamma y_2(x)$$
$$\frac{dy_2}{dx} = \gamma y_2(x)$$

with initial conditions

$$y_1(0) = \alpha_1,\ y_2(0) = \alpha_2,\ y_3(0) = \alpha_3.$$

where population is a fixed that is $R = y_1(x)+y_2(x)+y_3(x)$. For more details and numerical treatments can see in Ref.[40-46]. For numerical results the following values, for parameters are considered[42]: $\alpha_1 = 20$ Initial population of $y_1(x)$, who are susceptible. $\alpha_2 = 15$ Initial population of $y_1(x)$, who are infective. $\alpha_3 = 10$ Initial population of $y_1(x)$, who are immune. $\beta = 0.01$ Rate of change of susceptible to infective population. $\gamma = 0.02$ Rate of change of infective to immune population.

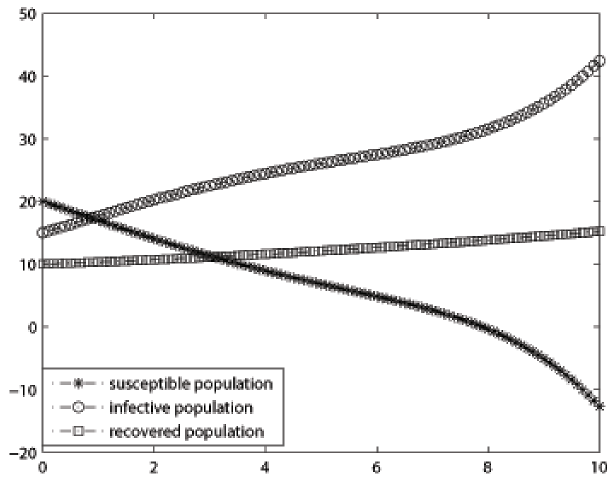

Fig. 6: Plot of susceptible, infective and recovered population with seven terms.

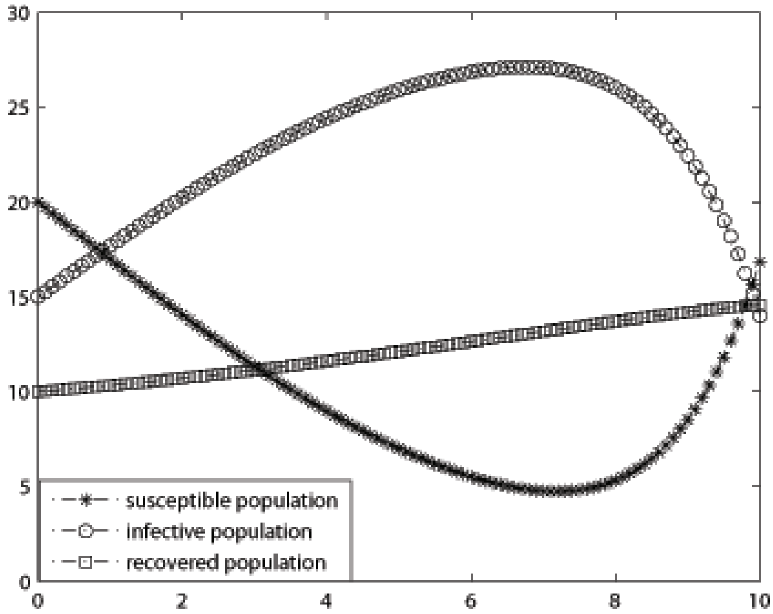

Fig. 7: Plot of susceptible, infective and recoveredpopulation with nine terms.

The given Figs. 6 and 7 show the relations between the population number of susceptible, infective and recovered versus time. Numerical results are the same line in Ref.[7].As the plots show while the number of susceptibles increases the population of who are infective decreases in the period of the epidemic, meanwhile the number of immune population increases [7].

6 Conclusion

In this paper, we have proposed a collocation method based on the shifted Chebyshev polynomials to numerically solve nonlinear differential equations. This method uses the shifted Chebyshev-Gauss nodes to reduce the considered nonlinear differential equations to the solution of a nonlinear algebraic equation with unknown the shifted Chebyshev polynomials. It is seen from the numerical experiments that the convergence rate of the numerical solutions are increas, when terms of polynomial increases. Moreover, we give two numerical application of proposed method as consider the logistic growth in a population as a single species and spreading of a non-fatal disease in a population.

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