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On Weingarten tube surfaces with null curve in Minkowski 3-space

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Abstract: In this paper, we study a tube surfaces, consisted spline curve is null. Furthermore, we have investigated Weingarten conditions of this surface using the the mean curvatre H, the Gaussian curvature K and the second Gaussian curvature K_{II}, respectively.

Keywords: Tube surfaces, Weingarten conditions, null curve, curvatures.

1 Introduction

A surface S in either $\mathbb{R}^3$ or E_1^3 is called a Weingarten surface if there exists a non-trivial functional relation $\Omega(K,H)=0$ with respect to its Gaussian curvature K and its mean curvature H. The existence of a non-trivial functional relation $\Omega(K,H)=0$ on the surface S parametrized by $\Phi(u,v)$ is equivalent to the vanishing of the corresponding Jacobian determinant, namely

$$\frac{\partial(K,H)}{\partial(u,v)}=0$$

Several geometers have studied on Weingarten surfaces and obtained many interesting results. For study of these surfaces, in 1994 W. Kühnel and in 1999 G. Stamou, investigated ruled Weingarten surface in a Euclidean 3-space E^3. Also, in 1997 C. Baikoussis and Th. Koufogiorgos studied helicoidal (H,K_{II})-Weingarten surfaces. in 1999 F. Dillen and W. Kühnel, in 2005 F. Dillen and W.Sodsiri gave a classification of ruled Weingarten surface in a Minkowski 3-space E_1^3. In 2009 J. Suk Ro and D. Won Yoon studied tubular Weingarten and linear Weingaren surface in three dimension Euclidean space E^3.

In this paper we study a tube surface with null curve in a three dimension Minkowski space.

2 Preliminaries

Let $\mathbb{R}^3=\{(x,y,z):x,y,z\in\mathbb{R}\}$ be a 3-dimension space, and let $X=(x_1,x_2,x_3)$ and $Y=(y_1,y_2,y_3)$ be two vectors in $\mathbb{R}^3$. The Lorentz scalar product of X and Y is defined by

$$\langle X,Y\rangle=x_1y_1+x_2y_2-x_3y_3 \tag{2.1}$$

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$E_1^3 = (\mathbb{R}^3, \langle X,Y\rangle)$ is called Minkowski 3-space. Since the metric is indefinite, $x \in E_1^3$ has three causal characters: these are spacelike vector, null (lightlike) vector or timelike vector if $\langle x,x\rangle > 0$ or $x = 0, \langle x,x\rangle = 0$ and $x \neq 0$, $\langle x,x\rangle < 0$, respectively. For $x \in E_1^3$, the norm of the vector x is given by $\|x\| = \sqrt{|\langle x,x\rangle|}$. Therefore, x is a unit vector if $\langle x,x\rangle = \pm 1$. Similarly, an arbitrary curve $\alpha = \alpha(s) \subset E_1^3$ can locally be spacelike, timelike or null (lightlike) curve, if all of its velocity vectors $\alpha'(s)$ are respectively spacelike, timelike or null (lightlike) i.e., $\langle \alpha'(s), \alpha'(s)\rangle > 0$, $\langle \alpha'(s), \alpha'(s)\rangle < 0$, $\langle \alpha'(s), \alpha'(s)\rangle = 0$. So, $\alpha(s)$ is a unit speed curve if $\langle \alpha'(s), \alpha'(s)\rangle = \pm 1$, where s is the arc-length parameter of α. Any two vectors $X,Y \in E_1^3$ are called orthogonal if $\langle X,Y\rangle = 0$ (O'Neill, 1983).

The vector product of two vectors $X = (x_1,x_2,x_3)$, $Y = (y_1,y_2,y_3)$ belong to E_1^3, is defined as

$$X \wedge Y = (x_3y_2 - x_2y_3, x_1y_3 - x_3y_1, x_1y_2 - x_2y_1).$$

We also recall that the pseudosphere of radius 1 and center at the origin is the hyperquadric in E_1^3 defined by $S_1^2(1) = \{v \in E_1^3 : \langle v,v\rangle = 1\}$ (O'Neill, 1983).

Let $\alpha = \alpha(u) : (a,b) \to E_1^3$ be a spacelike unit speed curve with a timelike binormal B, where u is the arc length parameter of α. Considering that $\|T\| = 1$, $B = \dfrac{T'}{\|T'\|}$ and $N = \dfrac{T \wedge T'}{\|T \wedge T'\|}$, we obtain the orthonormal frame field $\{T(u), N(u), B(u)\}$.

Consider M is a timelike tube surface parametrized by $\Psi : j \times \mathbb{R} \to E_1^3$. Then, the position vector of Ψ can be written in the following form

$$\Psi(u,v) = \alpha(u) + r(N(u)\cosh v + B(u)\sinh v) \tag{2.2}$$

where $\alpha, N, B : j \to E_1^3$ and $r : j \to \mathbb{R}$. We call α a base curve and a pair of two vectors N,B a director frame of the timelike tube surface Ψ. We must have

$$\|\Psi(u,v) - \alpha(u)\|^2 = r^2$$

The last equation expresses analytically the geometric fact that $\Psi(u,v)$ lies on a Lorentzian sphere $S_1^2(u)$ of radius r centered at $\alpha(u)$ (Abdel-Aziz and Saad, 2011).

Theorem 2.1. Let Ψ be a timelike tube surface in Minkowski 3-space E_1^3 defined in (2.2) satisfying the Jacobi condition

$$\Omega(K,H) = 0 \tag{2.3}$$

for the Gaussian curvature K and the mean curvature H of Ψ. Then, Ψ is a Weingarten surface (Abdel-Aziz and Saad, 2011).

Theorem 2.2. Let Ψ be a timelike tube surface with non-degenerate second fundamental form in Minkowski 3-space E_1^3 satisfying the Jacobi equation

$$\Omega(K_{II},K) = 0 \tag{2.4}$$

for the second Gaussian curvature K_{II} and the Gaussian curvature K. Then, the surface Ψ is a Weingarten surface (Abdel-Aziz and Saad, 2011).

3 Tubular weingarten surface with a null curve

A parametric equation of a Ψ tube surface with $C(t):(a,b)\to E_1^3$ null curve is given by

$$\Psi(t,\theta)=C(t)+\theta N(t)+\theta^2 B(t) \tag{3.1}$$

where T, N, B vectors are null, spacelike and null vectors respectively. The orthonormal frame is $\{T(t),N(t),B(t)\}$ and we have the following conditions

$$\begin{aligned}\langle T,T\rangle &= \langle B,B\rangle &= 0\\ \langle N,N\rangle &= \langle T,B\rangle &= 1\\ \langle N,B\rangle &= \langle N,T\rangle &= 0\end{aligned}$$

$\kappa(t)$ and $\tau(t)$ are curvature and torsion respectively and we can give differential formulas for this system as below

$$\begin{aligned}&T'(t)=N\\ &N'(t)=\tau T-B\\ &B'(t)=-\tau N\end{aligned}$$

therefore, differentiation of the tube surface Ψ can be calculate as follows,

$$\begin{aligned}&\Psi_t=(1+\tau\theta)\,T(t)+\left(-\tau\theta^2\right)N(t)+(-\theta)\,B(t)\\ &\Psi_\theta=N(t)+2\theta B(t)\\ &\Psi_t\times\Psi_\theta=\left(-2\tau\theta^3+\theta\right)T(t)+\left(2\theta+2\tau\theta^2\right)N(t)+(-1-\tau\theta)\,B(t)\\ &\|\Psi_t\wedge\Psi_\theta\|=\sqrt{8\tau^2\theta^4+12\tau\theta^3-2\tau\theta^2+4\theta^2-2\theta}\end{aligned}$$

we find the the unit normal vector field of the tube surface Ψ express by

$$\xi(u)=\frac{\Psi_t\wedge\Psi_\theta}{\|\Psi_t\wedge\Psi_\theta\|}=\frac{1}{\lambda}\left(-2\tau\theta^3+\theta\right)T(t)+\left(2\theta+2\tau\theta^2\right)N(t)+(-1-\tau\theta)\,B(t)$$

where $\lambda=\sqrt{8\tau^2\theta^4+12\tau\theta^3-2\tau\theta^2+4\theta^2-2\theta}$. furthermore components of the first fundamental form of Ψ are

$$\begin{aligned}&E=\langle\Psi_t,\Psi_t\rangle=-2\theta\,(1+\tau\theta)+\tau^2\theta^4\\ &F=\langle\Psi_t,\Psi_\theta\rangle=\tau\theta^2+2\theta\\ &G=\langle\Psi_\theta,\Psi_\theta\rangle=1\end{aligned}$$

Components of the second fundamental form of Ψ are

$$\begin{aligned}P=\langle\Psi_{tt},\sigma\rangle\ &=\frac{1}{\lambda}[\left(1+2\tau\theta-\tau'\theta^2\right)\left(2\theta+2\tau\theta^2\right)\\ &+\left(1+\tau'\theta-\tau^2\theta^2\right)(-1-\tau\theta)+\left(\tau\theta^2\right)\left(-2\tau\theta^3+\theta\right)]\end{aligned}$$

$$Q=\langle\Psi_{t\theta},\sigma\rangle\ =\frac{1}{\lambda}\left[\tau\,(-1-\tau\theta)+(-2\tau\theta)\left(2\theta+2\tau\theta^2\right)+\left(2\tau\theta^3-\theta\right)\right]$$

$$W=\langle\Psi_{\theta\theta},\sigma\rangle=\frac{1}{\lambda}\left[-4\tau\theta^3+2\theta\right]$$

The Gaussian curvature K, second Gaussian curvature K_{II} and the mean curvature H are given by

$$
\begin{aligned}
K=\frac{PW-Q^2}{EG-F^2} &= \tfrac{1}{2}[-8\tau^3\theta^8+8\tau^2\tau'\theta^7+\left(12\tau^2+8\tau\tau'+20\tau^4\right)\theta^6 \\
&\quad+\left(-4\tau^2\tau'+36\tau^3-8\tau^2-4\tau\tau'\right)\theta^5 \\
&\quad+\left(-4\tau'-4\tau\tau'+16\tau^2-14\tau-6\tau^3-8\tau^4\right)\theta^4 \\
&\quad+\left(-6\tau^2+2\tau\tau'-16\tau^3+4\tau\right)\theta^3+\left(-6\tau^2+2\tau'+\tau^4+5\right)\theta^2 \\
&\quad+\left(2\tau^3+2\tau\right)\theta+\tau^2]/\left(\theta\left(1+2\tau\theta^2+(\tau+2)\theta\right)\right)
\end{aligned}
$$

and

$$
\begin{aligned}
H=\frac{EW-2FQ+GP}{2(EG-F^2)} &= -\frac{1}{4}[4\tau^3\theta^6+(-4\tau^2+8\tau^3)\theta^4+(24\tau^2-2\tau\tau')\theta^3 \\
&\quad+(-2\tau'+17\tau+4\tau^2-3\tau^3)\theta^2+(-7\tau^2+6\tau+\tau\tau')\theta \\
&\quad-4\tau+2+\tau']/(1+2\tau\theta^2+(\tau+2)\theta).
\end{aligned}
$$

Also, we get

$$
\begin{aligned}
K_{II} = &-\frac{1}{4}[(64\tau^3(\tau')^2+64\tau^4\tau'')\theta^{13}+(-32\tau^4\tau'''-64(\tau')^3\tau^2-160\tau^3\tau'\tau'')\theta^{12} \\
&+(-280\tau^5-64\tau'\tau^2\tau''+\tau'''\tau^3\tau'-32\tau^3\tau'''-208\tau^5\tau''+240\tau^6+96(\tau')^2\tau''\tau^2 \\
&+128\tau^3(\tau')^2-256\tau^4(\tau')^2-48\tau^3\tau''-64(\tau')^3\tau+128\tau^4\tau''-192(\tau')^2\tau^2)\theta^{11} \\
&+(96(\tau')^3\tau-320\tau^4\tau''+48\tau^3\tau'''-96(\tau')^3\tau^2+16\tau^4\tau'''-240\tau^5\tau'+80\tau^5\tau''' \\
&+288\tau^4\tau'\tau''+160\tau'\tau^2\tau''+96(\tau')^2\tau\tau''+64\tau'\tau'''\tau^2+64(\tau')^2\tau^2-80\tau^3\tau' \\
&+96(\tau')^3\tau^3+160\tau^3\tau''+160\tau^5+744\tau^4\tau'-320\tau^3(\tau')^2-48\tau^3\tau'\tau'')\theta^{10} \\
&+(564\tau^4+192\tau^5+128\tau^3\tau'+208\tau(\tau')^2-376(\tau')^2\tau^2-544\tau^4\tau'+1852\tau^6 \\
&+64(\tau')^3-192\tau^7+240\tau^5(\tau')^2-352\tau^4(\tau')^2-288\tau^3(\tau')^2+128(\tau')^3\tau^2 \\
&-96(\tau')^3\tau+512\tau^3\tau''-288\tau'\tau^2\tau''-32\tau'''\tau^3\tau'-96(\tau')^2\tau''\tau^2-48(\tau')^2\tau\tau'' \\
&-16\tau'\tau'''\tau^2+16\tau'\tau\tau''+32\tau'\tau\tau'''+288\tau^4\tau''+224\tau^4\tau'''-16\tau^{3\tau'''}-160\tau^5\tau'' \\
&-200\tau^3\tau''+40\tau^2\tau''+120\tau^6\tau''+48\tau^2\tau''')\theta^9+(216\tau^4+60\tau^2\tau'+3196\tau^5 \\
&-1432\tau^3\tau'-288\tau(\tau')^2-600(\tau')^2\tau^2+488\tau^4\tau'-296\tau^6-16(\tau')^3+120\tau^6\tau' \\
&-1072\tau^5\tau'+512\tau^4(\tau')^2-752\tau^3(\tau')^2-48(\tau')^3\tau^3+16(\tau')^3\tau^2+48(\tau')^3\tau^3 \\
&+16(\tau')^3\tau^2+48\tau^3\tau-32\tau^3\tau'\tau''+208\tau'\tau^2\tau''-240\tau^4\tau'\tau''-96(\tau')^2\tau\tau'' \\
&-64\tau'\tau'''\tau^2-256\tau'\tau\tau''-32\tau'\tau\tau'''-480\tau^4\tau''-24\tau^4\tau'''+184\tau^3\tau'''+296\tau^5\tau'' \\
&+504\tau^3\tau''-72\tau^5\tau'''-8\tau^2\tau''-88\tau^2\tau'''-48(\tau')^2\tau'')\theta^8+(-370\tau^3+1876\tau^4 \\
&-184\tau^2\tau'-180\tau^5+1360\tau^3\tau'-96(\tau')^2-48\tau(\tau')^2-180(\tau')^2\tau^2-2808\tau^4\tau' \\
&+1272\tau^6+32(\tau')^3-578\tau^7-96\tau^5(\tau')^2+320\tau^5\tau'+128\tau^4(\tau')^2+768\tau^3(\tau')^2 \\
&-64(\tau')^3\tau^2+36\tau^8-488\tau^3\tau'\tau''+48\tau'\tau^2\tau''+8\tau'''\tau^3\tau'+24(\tau')^2\tau''\tau^2 \\
&+48(\tau')^2\tau\tau''+16\tau'\tau'''\tau^2+152\tau'\tau\tau''-32\tau'\tau\tau'''+208\tau^4\tau''-208\tau^4\tau''' \\
&-32\tau^3\tau'''+4\tau^5\tau''-684\tau^3\tau''+352\tau^2\tau''-48\tau^6\tau''+56\tau^2\tau'''+44\tau\tau'' \\
&+8\tau'\tau''-56\tau\tau'''-16\tau'\tau''')\theta^7+(236\tau\tau'+528\tau^3+396\tau^4+1970\tau^2\tau' \\
&+4188\tau^5-2376\tau^3\tau'+96\tau^2+532\tau(\tau')^2+716(\tau')^2\tau^2+472\tau^4\tau'-1468\tau^6 \\
&-8(\tau')^3+84\tau^7-58\tau^6\tau'+624\tau^5\tau'-224\tau^4(\tau')^2+324\tau^3(\tau')^2+24(\tau')^3\tau^2 \\
&+258\tau^6+8(\tau')^3-5\tau^7+4\tau^6\tau'-92\tau^5\tau'+28\tau^4\tau'-74\tau^3(\tau')^2-50\tau''+48\tau'\tau^2\tau'' \\
&-6\tau^4\tau'\tau''+8\tau'\tau\tau''-8\tau'\tau\tau'''+56\tau^3\tau'''+16\tau^5\tau''-2\tau^3\tau''-2\tau^5\tau'''+114\tau^2\tau'' \\
&+12\tau^2\tau'''-12(\tau')^2+48\tau\tau''-54\tau'\tau''-10\tau\tau''')\theta^4+(-31\tau\tau'-46\tau+416\tau^2 \\
&-226\tau^3-234\tau'-858\tau^4+212\tau^2\tau'+158\tau^5-80\tau^3\tau'-97(\tau')^2+10\tau(\tau')^2 \\
&-28(\tau')^2\tau^2-226\tau^4\tau'+22\tau^6-6\tau^7+7\tau^5\tau'+\tau^4(\tau')^2+20\tau^3(\tau')^2-10\tau'''
\end{aligned}
$$

$$-12\tau^3\tau'\tau''-12\tau'\tau\tau''+14\tau^4\tau''-6\tau^4\tau'''-\tau^5\tau''-12\tau^3\tau''+8\tau^2\tau''+12\tau^2\tau'''$$
$$+17\tau\tau''-4\tau\tau''')\theta^3+(18\tau(\tau')^2-4\tau\tau'''-2\tau^4\tau''-6\tau^3\tau'''+4(\tau')^2\tau^2-10\tau^6$$
$$-44\tau^2-5\tau''-6\tau^3\tau'''+4(\tau')^2\tau^2-10\tau^6-44\tau^2-5\tau''-252\tau^3+3\tau^4\tau'+4\tau^3\tau''$$

$$-48(\tau')^3\tau+48\tau^3\tau'\tau''-216\tau'\tau^2\tau''+60\tau^4\tau'\tau''+24\tau^2\tau\tau''+16\tau'\tau'''\tau^2$$
$$+112\tau'\tau\tau''+32\tau'\tau\tau'''+68\tau^4\tau''+12\tau^4\tau'''-192\tau^3\tau'''-128\tau^5\tau''+60\tau^3\tau''$$
$$+20\tau^5\tau'''-504\tau^2\tau''+20\tau^2\tau'''+48(\tau')^2\tau''+64\tau\tau''+108\tau'\tau''+36\tau\tau''')\theta^6$$
$$+(640\tau\tau'+187\tau^2+264\tau^3+4851\tau^4-1150\tau^2\tau'-1396\tau^5+384\tau^3\tau'$$
$$+134(\tau')^2+216\tau(\tau')^2-28(\tau')^2\tau^2+1590\tau^4\tau'-291\tau^6-32(\tau')^3+120\tau^7$$
$$+12\tau^5(\tau')^2-136\tau^5\tau'-34\tau^4(\tau')^2-160\tau^3(\tau')^2+32(\tau')^3\tau-3\tau^8-30\tau''$$
$$20\tau'''+120\tau^3\tau'\tau''+56\tau'\tau^2\tau''-12(\tau')^2\tau\tau''-4\tau'\tau'''\tau^2-56\tau'\tau\tau''+8\tau'\tau\tau'''$$
$$-90\tau^4\tau''+60\tau^4\tau'''+24\tau^3\tau'''+2\tau^5\tau''+156\tau^3\tau''+78\tau^2\tau''+6\tau^6\tau''-56\tau^2\tau'''$$
$$-206\tau\tau''-4\tau'\tau''+28\tau\tau'''+16\tau'\tau''')\theta^5+(-768\tau\tau'-114\tau^2+2239\tau^3-107\tau'$$
$$-672\tau^4+16\tau^2\tau'-962\tau^5+1154\tau^3\tau'-20(\tau')^2-338\tau(\tau')^2-28(\tau')^2-137\tau^4\tau'$$

$$+18\tau^4+2\tau^3(\tau')^2-142\tau^3\tau'+4\tau^5\tau'+54\tau\tau'-11\tau^2\tau''+68\tau+48\tau^5-17\tau^2\tau'$$
$$-6\tau'\tau^2\tau'')\theta^2+(-\tau^3\tau''+(\tau')^2-8\tau^2\tau'+10\tau^4\tau'-4\tau^5+8+6\tau'-4\tau^3+28\tau^4$$
$$-2\tau^2\tau'''-\tau\tau''+3(\tau')^2-4\tau^2+28\tau^4-2\tau^2\tau'''-\tau\tau''+3(\tau')^2-4\tau^2)\theta+6\tau^3\tau'$$
$$+4\tau^3-2\tau\tau'+4\tau)]\ /[\ (-8\tau^3)\theta^8+(9\tau^2\tau')\theta^7+(20\tau^4+8\tau\tau'+12\tau^2)\theta^6+(-4\tau^2\tau'$$
$$+36\tau^3-8\tau^2-4\tau\tau')\theta^5(-4\tau\tau'+16\tau^2-8\tau^4-14\tau-4\tau'-6\tau^3)\theta^4+(4\tau-6\tau^2$$
$$-16\tau^3+2\tau\tau')\theta^3+(5-6\tau^2+2\tau'+\tau^4)\theta^2+(2\tau^3+2\tau)\theta+\tau^2)^2].$$

We have investigated the Weingarten condition for the Gaussian curvature, Mean curvature and the second Gaussian curvature by taking $\tau=1$, under this condition we can give the following theorems.

Theorem 3.1. Let Ψ be a tube surface in Minkowski 3-space E_1^3 defined in (3.1) satisfying the Jacobi condition

$$\Omega(K,H)=0$$

for the Gaussian curvature K and the mean curvature H of Ψ. Then, Ψ is a Weingarten surface.

Proof. Let Ψ be a tube surface in E_1^3 parametrized by (3.1) and satisfying Jacobi condition. Then, we have,

$$\frac{\partial K}{\partial u}\frac{\partial H}{\partial v}-\frac{\partial K}{\partial v}\frac{\partial H}{\partial u}=0 \tag{3.2}$$

differentiating K and H with respect to t,

$$\begin{aligned}\frac{\partial K}{\partial t}=\ &\tfrac{1}{2}[-16\tau^3\tau'\theta^9+(8\tau^3\tau''+8\tau^2(\tau')^2-24\tau^2\tau')\theta^8\\
&+(16\tau^2\tau''+60\tau^4\tau'+16\tau(\tau')^2+12\tau^2\tau')\theta^7\\
&+(152\tau^3\tau'-8\tau^2\tau'-4\tau^2(\tau')^2+8(\tau')^2-4\tau^3\tau''-4\tau^2\tau''+8\tau\tau''+24\tau\tau')\theta^6\\
&+(124\tau^2\tau'-16\tau\tau'-8\tau^2\tau''-8\tau\tau''-24\tau^4\tau'-12\tau^3\tau'-8\tau(\tau')^2)\theta^5\\
&+(32\tau\tau'-4\tau\tau''-14\tau'-4\tau''-64\tau^3\tau'-24\tau^2\tau'-4(\tau')^2+2\tau^2\tau'')\theta^4\\
&+(-12\tau\tau'+3\tau^4\tau'+4\tau\tau''-\tau'-54\tau^2\tau')\theta^3+(8\tau^3\tau'-12\tau\tau'+2\tau'')\theta^2\\
&(2\tau'+7\tau^2\tau')\theta+2\tau\tau']/((\tau\theta+1)(1+2\tau\theta^2+(\tau+2)\theta)\theta),\end{aligned}$$

and

$$\begin{aligned}\frac{\partial H}{\partial t} = & -\frac{1}{4}[8\tau^3\tau'\theta^7 + 12\tau^2\tau'\theta^6 + (-4\tau^2\tau' + 16\tau^3\tau')\theta^5 + (-8\tau\tau' + 48\tau^2\tau' - 2\tau^2\tau'')\theta^4 \\ & (48\tau\tau' - 6\tau^3\tau' - 4\tau\tau'' + 4\tau^2\tau')\theta^3 + (\tau^2\tau'' - 16\tau^2\tau' + 17\tau' + 8\tau\tau' - 2\tau'')\theta^2 \\ & (2\tau\tau'' - 14\tau\tau' + 4\tau')\theta + \tau'' - 4\tau']/[(\tau\theta+1)(1+2\tau\theta^2+(\tau+2)\theta],\end{aligned}$$

differentiating K and H with respect to θ,

$$\begin{aligned}\frac{\partial K}{\partial \theta} = & \tfrac{1}{2}[-80\tau^4\theta^{10} + (-96\tau^3 - 48\tau^4 + 64\tau^3\tau')\theta^9 + (16\tau^3 + 120\tau^5 + 128\tau^2\tau' + 40\tau^3\tau')\theta^8 \\ & +(96\tau^2 - 16\tau^3\tau' + 64\tau^2\tau' + 16\tau^3 + 80\tau^5 + 304\tau^4 + 64\tau\tau')\theta^7 \\ & +(224\tau^3 + 8\tau\tau' + 196\tau^4 - 16\tau^2 - 44\tau^2\tau' - 16\tau^5 - 12\tau^3\tau')\theta^6 \\ & +(4\tau^2 - 24\tau^2\tau' - 44\tau^4 - 56\tau + 152\tau^3 - 16\tau^5 - 40\tau\tau' - 16\tau')\theta^5 \\ & +(-40\tau^4 - 44\tau - 12\tau' + 2\tau^2\tau' - 44\tau^3 + 40\tau^2 - 2\tau^5 - 12\tau\tau')\theta^4 \\ & +(-20\tau^2 + 8\tau + 4\tau\tau' - 32\tau^3 - 8\tau^4)\theta^3 + (-10\tau^3 - \tau^4 - 4\tau + 2\tau' + 5 - 8\tau^2)\theta^2 \\ & (-2\tau^3 - 4\tau^2)\theta - \tau^2]/(\theta^2(1+2\tau\theta^2+(\tau+2)\theta)^2),\end{aligned}$$

and

$$\begin{aligned}\frac{\partial H}{\partial \theta} = & -\frac{1}{4}[32\tau^4\theta^7 + (40\tau^3 + 20\tau^4)\theta^6 + (32\tau^4 + 8\tau^3)\theta^5(-4\tau^2\tau' + 24\tau^4 + 84\tau^3 - 24\tau^2)\theta^4 \\ & (-4\tau^2\tau' + 80\tau^2 + 80\tau^3 - 8\tau\tau')\theta^3 + (34\tau - 8\tau\tau' - 2\tau - 4\tau' - 6\tau^3 - 4\tau\tau')\theta \\ & -4 + 12\tau - 2\tau' - 3\tau^2]/[1+2\tau\theta^2+(\tau+2)\theta]^2.\end{aligned}$$

From the above equations (3.2) is satisfied, and then, the surface Ψ is a Weingarten surface.

Theorem 3.2. Let Ψ be a tube surface with non-degenerate second fundamental form in Minkowski 3-space E_1^3 satisfying the Jacobi condition

$$\Omega(K_{II}, K) = 0 \tag{3.3}$$

for the second Gaussian curvature K_{II} and the Gaussian curvature K. Then, the surface Ψ is a Weingarten surface.

Proof. For the proof, we can use the similar way with the theorem 3.1. When we take the partial derivative of the Gaussian curvature K and the second Gaussian curvature K_{II}, we can find the following equation

$$\frac{\partial K}{\partial t}\frac{\partial K_{II}}{\partial \theta} - \frac{\partial K}{\partial \theta}\frac{\partial K_{II}}{\partial t} = 0. \tag{3.4}$$

Then, Ψ is a Weingarten tube surface.

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