

ANALYSIS
MODELING
APPLICATIONS
FOR A COMPLEX
WORLD

MODELS
IDEAS
A BRIGHTER
TOMORROW

ICAAMM 2026

13th International Conference on
Applied Analysis and Mathematical Modeling

$$\int_{\Omega} u(x) dx$$

$$\frac{\partial u}{\partial t} = \mathcal{L}(u)$$

$$\|u\|^2 \leq C$$

Abstract Book

PEOPLE
MATHEMATICS
A MORE
CONNECTED
TOMORROW



27–31 July 2026



Biruni University
Istanbul, Türkiye

DISCOVER
MODEL
SOLVE
SHAPE
A BRIGHTER
TOMORROW

*Where
Mathematics
Bridges Horizons*



İSTANBUL
TÜRKİYE
2026

ICAAMM 2026

13th International Conference on Applied Analysis and Mathematical Modeling

27–31 July 2026, Istanbul, Türkiye

About this book

This volume compiles the abstracts included in the ICAAMM 2026 conference materials. The abstracts are presented in a uniform conference-book format while preserving the scientific content, author information, affiliations, keywords, references, and figures supplied in the source document.

Conference scope

ICAAMM brings together researchers working in applied analysis, mathematical modeling, numerical methods, dynamical systems, optimization, statistics, artificial intelligence, engineering mathematics, computational science, and related interdisciplinary applications.

MESSAGE FROM THE CONFERENCE CHAIR

Dear Distinguished Colleagues, Researchers, and Participants,

It is my great pleasure and privilege to welcome you to the **13th International Conference on Applied Analysis and Mathematical Modeling (ICAAMM 2026)**, hosted by **Biruni University in Istanbul, Türkiye, from 27–31 July 2026**.

ICAAMM has developed into an international scientific platform that brings together researchers, academics, and professionals working across a broad spectrum of applied mathematics, mathematical modeling, nonlinear sciences, engineering, and interdisciplinary applications. Our primary objective is to provide an inspiring environment in which established scientists, young researchers, and graduate students can exchange new ideas, present recent results, initiate collaborations, and discuss emerging directions in contemporary mathematical sciences.

The scientific scope of ICAAMM 2026 reflects the increasing importance of mathematical analysis and modeling in understanding complex scientific and engineering problems. The conference program covers a wide range of topics, including applied analysis, applied mathematics and modeling, fractional calculus, nonlinear differential equations, nonlinear dynamical systems, numerical analysis, complex systems, mathematical methods in artificial intelligence, data-driven dynamical systems, mathematical and statistical methods, nonlinear optics, soliton and solitary waves, biological and biomedical complexity, synchronization and chaos control, and many other related areas.

One of the main strengths of ICAAMM is its interdisciplinary character. Modern scientific challenges increasingly require close interaction among mathematics, engineering, physics, computer science, biology, medicine, and data science. We therefore hope that ICAAMM 2026 will not only serve as a venue for presenting high-quality scientific work, but will also encourage new interdisciplinary collaborations and long-term academic partnerships.

We are especially honored to welcome distinguished plenary speakers and internationally recognized scientists whose contributions enrich the scientific quality of this conference. I would also like to express my sincere appreciation to the members of the International Program Committee, the Local Organizing Committee, our co-chairs, invited speakers, reviewers, authors, and all colleagues who have contributed their time and expertise to the organization of ICAAMM 2026.

I would like to extend my sincere gratitude to **Biruni University** for hosting this important scientific event in Istanbul. As one of the world's most historically and culturally significant cities, Istanbul provides a unique setting in which science, history, and culture meet. I sincerely hope that, alongside the scientific program, all participants will have the opportunity to experience the unique atmosphere and hospitality of this remarkable city.

This Abstract Book brings together the scientific contributions presented at ICAAMM 2026 and reflects the diversity, quality, and international character of the conference. I believe that these studies will stimulate further discussion, inspire new research questions, and contribute to future developments in applied analysis, mathematical modeling, and related interdisciplinary fields.

On behalf of the organizing committee, I warmly welcome all participants to ICAAMM 2026. I wish you a scientifically productive, intellectually stimulating, and memorable conference.

Prof. Dr. Mustafa Bayram
Conference Chair, ICAAMM 2026
Biruni University
Istanbul, Türkiye

CONFERENCE CHAIR

Honorary Chair

Adnan Yüksel

Rector of Biruni University, Türkiye

Conference Chair

Mustafa Bayram

Biruni University, Türkiye

Co-Chairs

Aydın Seğer — Türkiye

Amar Debbouche — Algeria

PLENARY SPEAKERS

Ali Akgül	Türkiye
Amar Debbouche	Algeria
Barbaros Çetin	Türkiye
Dumitru Baleanu	Lebanon
J. Alberto Conejero	Spain
Jamal-Odyseas Maaita	Greece
Juan R. Torregrosa	Spain
Luis Vazquez	Spain
Mohammad Hossein Heydari	Iran

INVITED SPEAKERS

Abdel-Salam Gomaa Abdel-Salam

Qatar

Adem Kilicman

Malaysia

Adil Jhangeer

Czech Republic

Amin Jajarmi

Iran

Babak Shiri

China

Celso Grebogi

UK

Irina Perfilieva

Czech Republic

Jan Awrejcewicz

Poland

Krishna Pada Das

India

Liliana Guran

Romania

Md. Nur Alam

Malaysia

Mikhail Smirnov

USA

Mir Sajjad Hashemi

Iran

Muhammad Bilal Riaz

Czech Republic

Muhammad Farman

Cyprus

Ömer Akin

TOBB University, Turkey

Praveen Agarwal

India

Raoul Nigmatullin

Russia

Sania Qureshi

Pakistan

Sunil Dutt Purohit

India

Sunil Kumar

India

Yeliz Karaca

USA

Zaid Odibat

Jordan

Zakia Hammouch

Morocco

COMMITTEES

International Program Committee

Committee Chairs

George Anastassiou — USA
Dumitru Baleanu — Romania
Ali Akgül — Türkiye

Committee Members

- Abdel-Salam Gomaa Abdel-Salam — Qatar University, Qatar
 Abdul Rouf Alghofari — University Brawijaya, Indonesia
 Adem Kilicman — Putra University, Malaysia
 Alexey Lukashov — Saratov State University, Russia
 Aly R. Seadawy — Taibah University, Saudi Arabia
 Barbaros Çetin — Bilkent University, Türkiye
 Burhaneddin İzgi — Istanbul Technical University, Türkiye
 Cemil Tunç — Van Yüüncü Yıl University, Türkiye
 Devendra Kumar — University of Rajasthan, India
 Elçin Yusufoglu — Uşak University, Türkiye
 Ercan Tunç — Tokat Gaziosmanpaşa University, Türkiye
 Faruk Ucar — Marmara University, Türkiye
 HongGuang Sun — Hohai University, China
 İdris Dağ — Eskişehir Osmangazi University, Türkiye
 Jamal-Odyseas Maaia — Greece
 Jose Francisco Gomez Aguilar — CONACyT-Tecnológico Nacional de México, México
 Juan J. Trujillo — University of La Laguna, Spain
 Kamyar Hosseini — Islamic Azad University, Iran
 Luis Vazquez — Universidad Complutense de Madrid, Spain
 Michal Fečkan — Comenius University, Slovakia
 Mir Sajjad Hashemi — Bonab University, Iran
 Muhammad Younis — University of Punjab, Pakistan
 Muslum Ozisik — Yıldız Technical University, Türkiye
 Mounir Zili — University of Monastir, Tunisia
 Naila Rozi — University Sindh Madrassatul Islam, Pakistan
 Necati Özdemir — Balıkesir University, Türkiye
 Norihan Binti Md Arifin — University Putra, Malaysia
 Praveen Agarwal — Anand International College of Engineering, India
 Ratno Bagus Edy Wibowo — University Brawijaya, Indonesia
 Reyad El-Khazali — Khalifa University of Science, UAE
 Selçuk Kutluay — İnönü University, Türkiye
 Shahram Rezapour — Azarbaijan Shahid Madani University, Iran
 Sunil Kumar — National Institute of Technology, India
 Tadeusz Kaczorek — Białystok University of Technology, Poland
 Tamer Ugur — Ataturk University, Türkiye
 Vishnu Narayan Mishra — Indira Gandhi National Tribal University, India
 Xiao-Jun Yang — China University of Mining and Technology, China
 Yong Zhou — Xiangtan University, China
 Zakia Hammouch — Moulay Ismail University, Morocco
 Abdon Atangana — University of the Free State, South Africa
 Abdurrahim Yılmaz — Middle East Technical University, Türkiye
 Agnieszka B. Malinowska — Białystok University of Technology, Poland
 Ali Akgül — Siirt University, Türkiye
 Andrei Shkalikov — Moscow State University, Russia
 Behzad Ghanbari — Kermanshah University, Iran
 Carlo Cattani — University of Tuscia, Italy
 Chaudry Masood Khaliq — North-West University, South Africa
 Dianchen Lu — Jiangsu University, China
 Ercan Çelik — Atatürk University, Türkiye
 Fairouz Tchier — King Saud University, Saudi Arabia
 Hari M. Srivastava — University of Victoria, Canada
 Hossein Jafari — University of South Africa, South Africa
 Jagdev Singh — JECRC University, India
 Jordan Hristov — University of Chemical Technology, Bulgaria
 Juan J. Nieto — University of Santiago de Compostela, Spain
 Jyoti Mishra — Gyan Ganga Institute of Technology and Sciences Jabalpur, India
 Kashif Ali Abro — Mehran University of Engineering and Technology, Pakistan
 Mahmoud Abdel-Aty — Sohag University, Egypt
 Michele Caputo — Accademia Nazionale Lincei University, Italy
 Muhammad Imran Asjad — University of Management and Technology, Pakistan
 Mustafa Bayram — Biruni University, Türkiye
 Mohammed Boualem — University of Béjaïa, Algeria
 M. Zaeem-ul-Haq Bhatti — Punjab University, Pakistan
 Nasser Sweilam — Cairo University, Egypt
 Nor Haniza Sarmin — University Technology, Malaysia
 Ozlem Deftlerli — Cankaya University, Türkiye
 Raoul R. Nigmatullin — Kazan State University, Russia
 Rachida Boudjerada — USTHB, Algeria
 Sachin Kumar — Thapar University, India
 Shaher Momani — Mutah University, Jordan
 Sunil Dutt Purohit — Rajasthan Technical University, India
 Suzeini Abdul Halim — University Malaya, Malaysia
 Tamaz Kaladze — Tbilisi University, Georgia
 Tasawar Hayat — King Abdulaziz University, Saudi Arabia
 Wen-Xiu Ma — University of South Florida, USA
 Yeliz Karaca — University of Massachusetts Chan Medical School, USA
 Yongguang Yu — Beijing Jiaotong University, China
 Zanariah Binti Abdul Majid — Putra University, Malaysia

Local Organizing Committee

Chairs

Huseyin Aydın — Biruni University, Türkiye
Yakup Yildirim — Biruni University, Türkiye

Committee Members

Burhan Tiryakioglu — Marmara University, Türkiye
Hakan Adiguzel — Sakarya University of Applied Science, Türkiye
Handenur Esen — Yildiz Technical University, Türkiye
Ismal Onder — Yildiz Technical University, Türkiye
Neslihan Ozdemir — Yildiz Technical University, Türkiye
Osman Ozcan — Firat University, Türkiye

Sebahat Ebru Das — Yildiz Technical University, Türkiye
Sevgi Esen Almali — Kırıkkale University, Türkiye
Tugcem Partal — Recep Tayyip Erdogan University, Türkiye

Gulsemay Yigit — Bahcesehir University, Türkiye
Handan Gursoy Demir — Iskenderun Technical University, Türkiye
Ibrahim Emiroglu — Yildiz Technical University, Türkiye
Melih Cinar — Yildiz Technical University, Türkiye
Nuran Guzel — Yildiz Technical University, Türkiye

Samet Kadioglu — Istanbul Technical University, Türkiye
Sertan Alkan — Iskenderun Technical University, Türkiye
Taylan Demir — Ankara University, Türkiye

CONTENTS

Modeling and Sustainability of Financial Risk Dynamics with Constant Proportional-Caputo (CPC) Fractional Operator	5
On the dynamics of a family of planar differential systems with cycles explicitly given	6
Several Performance Measures of Spherical Outlier Detection Test Statistics (ZA, Qk, Cn, En, Yn, and Xn): A Case Study on Ranau, Sabah Earthquake Neighbouring Data	7
Existence of Solutions for a Discrete Four-Point Φ -Laplacian Problem	8
Mathematical Modeling and Analysis of Surface Leakage Current in High-Voltage Insulation Systems	9
Development of an Artificial Intelligence-Based Diagnostic Method for Three-Phase Inverter Faults	10
Double k-Fractional Operators: Properties, Representations and Applications	11
Statistical evaluation of Sage manure (<i>Salvia officinalis</i> L.) as a biocontrol strategy against fungal diseases of greenhouse tomato	12
Multivariate analysis and diversity indices applied to ladybird beetle assemblages (Coleoptera: Coccinellidae) in different biotopes of northeastern Algeria	13
Boundary Element Method for Microfluidic Applications	15
Improvement of SUV vehicle ride comfort by controlling the MR dampers integrating the vehicle suspension	16
Adaptive fault tolerant control based disturbance observer for Quanser 2-DOF helicopter subject to model uncertainties and external disturbances	17
Comparative Analysis of the Randić and Estrada Indices for the Molecular Structures Based on Drugs Used in Cancer Treatment and Anti-Hypertension	18
Modeling and Control of an Artificial Pancreas	19
Nonlinear Mathematical Modelling of Channel Flow: Predicting Drag Coefficient by using Neural Networks	20
Dust-Acoustic Soliton Propagation in Maxwellian Polarized Dusty Plasmas	21
Influence of Dust Grain Size on Plasma Sheath Structure in the Presence of an Inclined Magnetic Field	22
Applications of Mathematics, Computing, and AI in Music and Art	23
On the Kernel of the Homomorphism of the Nonabelian Tensor Square in Six-Dimensional Bieberbach Group with Quaternion Point Symmetry	24
Biological and Chemical Complexity of Algerian Honeys: Integrated Melissopalynological, Physicochemical and Antioxidant Characterization	26
Hybrid Analytical-Numerical Modeling of Permanent Magnet Electrical Machines via Maxwell-Fourier and Finite Element Methods	27
The First and Second Zagreb Indices for Digraphs of Non-Normal Cyclic Subgroups of Dihedral Groups	28
Linear Canonical Boostlet Transform	30
Algebraic Modeling of CSS-Based Perfect Quantum Codes Derived from Eisenstein-Jacobi Integers	32
Deep Reinforcement Learning for QoS-Constrained Control of Unreliable Multi-Server Systems with Working Vacations and Impatient Customers	33
Modeling and optimization of airport runway capacity: A multi-server retrieval queueing approach enhanced by artificial intelligence	34
Dynamic bayesian game model for Ad hoc network security	35
On the equivalence to Smith form of multivariate polynomial matrices	36
Application of statistical models to identify road risk factors in Algeria	37
Intelligent RP Selection and Path Planning for Mobile Gateways in Energy-Constrained IoT	39
Analysis of deformation instability during severe shear of the PP during the ECAE process using DIC algorithms	40
A Primal Simplex Algorithm for Nonsmooth Support Vector Regression	41
Cubical Dimension of recursive Binary Trees into Hypercubes	42

Minimizing Empty Miles in Vehicle Routing: A Lookahead Heuristic Approach	43
Evaluation and modeling of hospital costs for lung cancer management in Algeria	44
A mathematical model for transport plan optimization and mileage reduction	46
Geometric Deformations of BTZ Black Hole Thermodynamics: Fractal Horizon Structure and Lorentz-Violating Riemannian Geometry in (2+1) Dimensions	47
ANFIS-Based Control of a Nonlinear Three-Tank System: Modeling and Simulation Study	49
Modeling and Control of Nonlinear Magnetic Levitation Systems Using Interval Type-2 Fuzzy Logic	50
Numerical Study of Mixed Convection Cooling of a Heat-Generating Element in an MHD Nanofluid-Filled Open Cavity	51
Behavior of A Binary Cement Based on Dam Sediments With a Low Kaolin Content	52
Inequalities for Rational Functions in the Complex plane	53
About Optimality in a Bilinear Programming Problem with Bounded Variables	55
Classification of minimal surfaces in Sol_3	56
Structural and Reactivity Study of Azo Naphthalenic Derivatives as Potential Antioxidant Agents: A DFT Investigation	58
Fractional Greeks in the Black-Scholes EDP	60
Analytical Investigation of Entropy Generation for Ternary Hybrid Nanofluid Flow in Concentric Cylinders with Particle Shape Variation	61
On a Compact 4th-Order Scheme for the General Acoustic Wave Equation	62
Instructor-Trained AI Tutor for Mathematics Education: Design and Evaluation of a Custom GPT	64
Permeability, Porosity and Mechanical Strength as Indicators of Concrete Durability	65
Predictive DFT Modeling of Structural Stability and Physical Properties of Double Half-Heusler $Hf_2InNiSb_2$ Alloy	66
Lie symmetry analysis of Merton equation	67
A Deniable Authentication Protocol Based on Multi Prime Power RSA	68
A High-Fidelity cDFIB ID FCC Framework for Moving-Boundary Simulations: Validation via an Oscillating Cylinder Case	69
Modelling Election-Induced Stock Market Volatility in Malaysia Using Merton's Jump-Diffusion Framework	71
Geometric properties of a free boundary problem	72
Based on modern comparative studies and surveys of deep learning architectures for HOI	73
Some Theorems on Starlike Functions of Complex Order with Respect to t-Symmetric Points	74
A Hybrid Richards–Temporal Fusion Transformer Framework for Modeling and Forecasting COVID-19 Dynamics in Türkiye	75
A new improvement on the class of constant coefficient sub-equation methods	76
Solar panel control system with bi-axial sun tracking and implementation model using Arduino	77
Integrating Convolutional Neural Networks with Ensemble Techniques for Superior Image Classification	78
A Simplified Real-Time Numerical Algorithm for Switching Angle Optimization Minimizing Total Harmonic Distortion in Cascade Multilevel Inverter	79
59 Years of Fractional Calculus with Caputo Operator	80
Bifurcation Analysis for a Stem-Cell Proliferation Model in a Hematological Disease under Continuous Small-Dose Treatment	81
A Data Mining Study on the Synthesis of Hydrogen from Hydrocarbons	84
Well-posedness and basic properties for an epidemic model with arbitrary order	85
Advanced integration of fractional calculus and AI for complex systems modeling	86
A Data-Driven Framework for Characterizing Nonlinear Dynamical Behaviors through Self-Organizing Maps and Cluster-Based Analysis	87
Reaction–Diffusion Dynamics in Heterogeneous Habitats with Time-Dependent Interfaces	88
Physics-Informed Neural Networks for Compact Star Modeling	89

First Type s -Orlicz Space : Completeness, Inclusion, and s -Hölder Inequality	90
On s -Orlicz–Morrey Spaces of Nakai’s Version and Their Inclusion Properties	92
Beyond Classical Fluid Modeling and Simulation	94
Nonlinear Dynamics of Anisotropic Matter in Curvature-Corrected Gravity	95
AI-Based Diagnosis of Lung Diseases from X-ray and CT Images	96
Statistical Characteristics of Collatz Sequences Generated from Different Probability Distributions	97
Analysis of the Conformity of Design Components to the Golden Ratio in Architectural Structures in the Eastern Black Sea Region	98
An Efficient mathematical Relation for Incident Radiation for solving radiation heat transfer equation	99
On the DGC Generalized k -Francois Sequence	100
Measuring Statistical Boundedness Degrees of Sequences	102
Optimal Constant Rebalanced Portfolios with Risk Constraints, Leverage and Multivariate Kelly Criterion	103
A different approach onto the generalized field theory	105
Soliton Solutions to the Conformable Nonlinear Soliton Neuron Equation via the New Kudryashov Scheme	106
Local and Global Well-Posedness Results for the Extended Davey-Stewartson System	108
A Data-Calibrated Fractional-Order One Health Framework for Modeling Nipah Virus Transmission with Memory Effects and Uncertainty Quantification	109
Generalized Piecewise Cardinal Functions for a Class of Third-Kind Singular FIDEs	111
Operational Matrix Method for Fractional Optimal Control Problems Subject to Third-Kind FIDEs	112
Monthly Sunspot Prediction Using an Oscillatory Second-Order Difference Equation and LSTM	113
Estuarine Dissolved Oxygen and Hypoxia Forecasting Using Neural ODEs	114
Neural Fractional Differential Equations for Rainfall–Runoff Modelling	115
Solution of Volterra Integro-Differential Equations via the Emad-Falih Transform	116
Thermal Radiation in optical Tomography	117
From the Lyapunov Characteristic Exponents to the Reversibility Error Method: The quest for identifying chaos	118
Fox H-Functions as Exact Solutions for Caputo Type Mass Spring Damper System Under Sumudu Transform	119
Model Based Study on the Dynamics of HIV Infection of CD4+ T-cells with Various Control Strategy	122
Generalized perturbed form of the nonlinear Schrödinger equation as the (2+1)-dimensional Biswas-Milovic model having the polynomial law and optical soliton solutions under the influence of white noise	123
On optical soliton solutions of the (2+1)-dimensional perturbed Biswas-Milovic equation under the parabolic law with nonlocal term	124
Optical Soliton Solutions of the Generalized Perturbed (2+1)-Dimensional Nonlinear Schrödinger Equation with Kerr Law	126
Predicting Glioblastoma Growth with Fractional-Order Modeling: A Hybrid Block-Product Integration Approach	127
On the Foundations of Generalized Non-Singular Fractional Calculus: Fundamental Theorems, Initial Value Problems, and Stability Analysis	128
Efficient Computation of Fractional Diffusion Equations with Hilfer Operators via Fuzzy Transform	129
Analytical study on the complex cubic–quintic Ginzburg–Landau equation with the anti-cubic law nonlinearity	130
Dark and Kink Solitons of the Perturbed Resonant (2+1)-Dimensional Maccari System via the Generalized Kudryashov Method	131
Analytical Bright and W-Shaped Soliton Solutions of the Sixth-Order Nonlinear Schrödinger Equation via the New Kudryashov Method	133
Flexible Memory-Kernel Methods for Fractional Dynamical Systems with Applications	135
A new method for characterizing nonlinear dynamical system behavior using self-organizing maps and clustering	137

Mathematical Neuron-Neural-Networks Coding Dynamics of Continuous- and Discrete-Time Models of Brain Complexity with Complex Feature Engineering in Biology, Nuclear Medicine and Artificial Intelligence	138
Optical Bullets and Domain Walls with Cross-Spatio Dispersion and Multiplicative White Noise	141
Fractional dynamics in anomalous diffusion processes: An artificial intelligence perspective	142

ABSTRACTS

ABSTRACT 1

Modeling and Sustainability of Financial Risk Dynamics with Constant Proportional-Caputo (CPC) Fractional Operator

Muhammad Farman¹, Evren Hincal¹, Zubaria Parvez², Ali Akgul³, and Dumitru Baleanu⁴

¹*Mathematics Research Center, Department of Mathematics,
Near East University, 99138 Nicosia, Turkey*

²*Department of Business Administration
Near East University, 99138 Nicosia, Turkey*

³*Art and Science Faculty, Department of Mathematics,
Siirt University, Siirt TR-56100, Turkey*

⁴*Department of Computer Science and Mathematics,
Lebanese American University, Beirut 1102 2801, Lebanon*

E-mail: farmanlink@gmail.com, evren.hincal@neu.edu.tr, zubariaparvez394@gmail.com,
aliakgul00727@gmail.com, dumitru.baleanu@gmail.com

Abstract: This study presents a fractional-order financial risk dynamical model utilizing the constant-proportional Caputo fractional derivative. We examine positivity, boundedness, existence, and uniqueness of solutions while evaluating a model's well-posedness. We use fixed-point concepts to establish the existence and uniqueness of the developed fractional model. The sensitivity indices for significant parameters are derived along with the economic threshold parameter. The equilibrium points' local stability is also investigated. Ulam-Hyers stability is used to validate its stability conditions, validating results, simulations, and chaos dynamics control of the system. Various fractional parameter values are investigated using an analytical-numerical technique that employs the Laplace transform for stability analysis. With its theoretical advancements in sustainable finance development, the paper examines the application of fractional calculus as a useful mathematical model for analyzing financial dynamics. In order to lessen financial risks, the research suggests that fractional models can provide a refined methodological approach to expand the understanding of the financial sector.

Keywords: Financial risk, Ulam-Hyers stability, Laplace transform, Fractional Operator.

ABSTRACT 2

**On the dynamics of a family of planar differential systems with cycles
explicitly given**

Rachid BOUKOUCCHA

Bejaia University, Dept. of Economic, Comercial and Management Sciences, Bejaia, Algeria
E-mail: rachid.boukoucha@univ-bejaia.dz

Abstract: In the study of planar differential system, it is not always possible to find explicit solutions for such systems, we resort to qualitative theory to seek information about solutions for non-linear systems to investigate their behavior. In the qualitative theory, a limit cycles, or isolated periodic solutions, were and still remain the most sought solutions when modeling physical systems in the plane. Most of the early examples in the theory of limit cycles in planar differential systems were commonly related to practical problems with mechanical and electronic systems, but periodic behavior appears in all branches of science, both the technological and natural sciences. Existence of limit cycles is one of the most difficult subjects in the qualitative theory of planar differential equations, even more difficult problem is to give an explicit expression of them. A large amount of references deals with the subject of limit cycle, for instance, the famous Hilbert's 16th problem motivated researchers to enter this domain of research. This was one of the main problems in the qualitative theory of planar differential equations in the 20th century.

In this work, we consider the family of the polynomial differential system. We prove that these systems are Liouville integrable. Moreover, we determine sufficient conditions for a polynomial differential system to possess an explicit algebraic or non-algebraic limit cycles. Concrete examples exhibiting the applicability of our result are introduced.

Keywords: Planar polynomial differential system, First integral, Limit cycle.

Mathematics Subject Classification: 34C05, 34C07, 37K10.

References

- [1] R. Boukoucha, On the dynamics of a family of planar differential systems with two cycles explicitly given, Rocky mountain journal of mathematics, 53 (2023), No. 6, 1709-1719.
- [2] R. Boukoucha, Explicit limit cycles of a family of polynomial differential systems, EJDE, Vol. 2017 (2017), No. 217, pp. 1-7.
- [3] R. Boukoucha, Explicit expression for a first integral for some classes of two-dimensional differential systems, SEMR, Vol 14 (2017), pp. 903-913.
- [4] D. Hilbert, Mathematische Probleme, Lecture, Second Internat. Congr. Math. (Paris, 1900), Nachr. Ges. Wiss. Göttingen Math. Phys. Kl. (1900) 253-297.

ABSTRACT 3

Several Performance Measures of Spherical Outlier Detection Test Statistics (ZA, Qk, Cn, En, Yn, and Xn): A Case Study on Ranau, Sabah Earthquake Neighbouring Data

Zulaikha Amran^{1,*}, Adzhar Rambli², Sayang Mohd Deni³

^{1,2,3}*Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, 40450 Shah Alam, Selangor, Malaysia*

E-mail: zulaikhaika915@gmail.com

Abstract: Outlier detection in spherical data is a fundamental challenge in directional statistics, particularly in geophysical applications where observations are constrained to the surface of a sphere. The research provides an evaluation of six spherical outlier detection test statistics out of ZA, Q^k , Cn, En, Yn, and Xn, with the help of control simulation and actual earthquake data obtained in Ranau, Sabah, Malaysia. Monte Carlo simulations are utilized with the von Mises–Fisher distribution to test performance for different sample sizes, concentration parameters, and levels of contamination. The effectiveness of each detection was assessed based on the previously mentioned performance statistics: P1 (power of detection for the presence of contamination), P3 (likelihood that the contaminant is extreme and classified as discordant), and P5 (likelihood of classifying a contaminant as discordant given is extreme). Critical values are obtained via quantile simulation at multiple significance levels. Simulation results indicate that ZA and Cn consistently achieve higher detection power in small samples and under low contamination, while En, Yn, Xn, and Q^k exhibit more moderate responses that stabilise as sample size and concentration increase. These findings provide both theoretical and empirical evidence on the relative strengths of classical and distance-based spherical discordancy tests. In particular, the ZA statistic show strong potential for practical deployment in geophysical and environmental studies involving directional data. The study contributes a unified performance assessment of multiple spherical outlier detectors and demonstrates their applicability to real seismic datasets, offering guidance for selecting appropriate methods under varying data conditions.

Keywords: Spherical data, Directional statistics; ZA statistic, Ranau earthquake, Von Mises–Fisher distribution, Geophysical data analysis.

References

- [1] Durdağ, U. M. (2024). Minimum-variance-based outlier detection in geodetic networks. *Geoscientific Model Development*. <https://doi.org/10.5194/gmd-17-2187-2024>.
- [2] Ley, C., & Verdebout, T. (2017). *Modern directional statistics*. Chapman and Hall/CRC.
- [3] Pewsey, A., & García-Portugués, E. (2021). Recent advances in directional statistics. *Test*, 30(1), 1-58.
- [4] Rambli A, Mohamed IB, Hussin AG (2022) A new outlier detection method for spherical data. *PLoS ONE* 17(8): e0273144. <https://doi.org/10.1371/journal.pone.0273144>.
- [5] Sau, M. F., & Rodriguez, D. (2018). Minimum distance method for directional data and outlier detection. *Advances in Data Analysis and Classification*, 12(3), 587-603.

ABSTRACT 4

Existence of Solutions for a Discrete Four-Point Φ -Laplacian Problem

Samira ZAHAR

Laboratory of applied mathematics LMA, Université de Bejaia, Algeria

E-mail: zahar-samira@yahoo.fr & samira.zahar@univ-bejaia.dz

Abstract: This work investigates a four-point boundary value problem for the discrete Φ -Laplacian eigenvalue problem, employing fixed-point theory for the sum of two operators. Under a general polynomial growth condition on the nonlinearity, we prove the existence of at least one nonnegative solution and provide numerical examples.

Keywords: Difference equation, Fixed point , Φ -Laplacian.

References

- [1] L. Bouchal and K. Mebarki. *New multiple fixed point theorems for sum of two operators and application to a singular generalized Sturm-Liouville multipoint BVP*, Matematički Vesnik, 76(3): 196–209, 2024.
- [2] A. Ghobadi and S. Heidarkhanii. *Critical point approaches for doubly eigenvalue discrete boundary value problems driven by ϕ_c -Laplacian operator*, Mathematical Communications, 30: 39-59, 2025.

ABSTRACT 5

Mathematical Modeling and Analysis of Surface Leakage Current in High-Voltage Insulation Systems**Hilal Rahali, Bilal djamal eddine Cherif, Samir Zeghlache, and Loutfi Benyettou**

Department of Electrical Engineering, Faculty of Technology, University of M'sila, Algeria Laboratoire de Génie Electrique M'sila, Algeria
E-mail: hilal.rahali@univ-msila.dz

Abstract: This research addresses discontinuous contamination layers developed on the surface of insulators. Two distinct zones are identified: a clean (dry) zone and a contaminated (wetted) zone. The length and position of the dry zone influence the leakage current. These were made on a plane experimental model under AC voltage. Three configurations were studied: pollution deposited near the HV electrode, pollution deposited near the ground electrode and pollution laid out with the center. This study was made using pollution layers with different conductivities. Based on the experimental results, a Mathematical model allowing us to calculate the gap impedance, the transferred (gap) voltage and the leakage current was established. This model enables us to predict the severity of pollution based on the sample's equivalent impedance.

References

- [1] D. Maadjoudj, O. Kherif, A. Mekhaldi and M. Tegar «Features Characterizing the Surface State of HV Insulator Glass Model under Desert Pollution», IEEE Transactions on Dielectrics and Electrical Insulation, Vol. 28 N°6, pp. 1964 - 1972, 2021.
- [2] Salem, A.A., Lau, K.Y., Rahiman, W. et al. Leakage current characteristics in estimating insulator reliability: experimental investigation and analysis. Sci Rep., vol(12), 14974,2022.
- [3] I. Ramirez, R. Hernandez and G. Montoya, "Measurement of leakage current for monitoring the performance of outdoor insulators in polluted environments," in IEEE Electrical Insulation Magazine, vol. 28, no. 4, pp. 29-34, 2012.
- [4] D. Maadjoudj, A. Mekhaldi and M. Tegar « Flashover Process and Leakage Current Characteristics of Insulator Model under Desert Pollution», IEEE Transactions on Dielectrics and Electrical Insulation, Vol. 25 N°6, pp. 2296 - 2304, 2018.
- [5] H. Zhang and R. Hacham, «Electrical surface resistance hydrophobicity and diffusion phenomena in PVC», IEEE, Trans. on elect. insul., Vol.6, N°1, pp. 73-83, 1999.
- [6] Z. Sahlia , A. Mekhaldi, R. Boudissa, S. Boudrahem « Prediction parameters of dimensioning of insulators under non-uniform contaminated conditions by multiple regression analysis» Electric Power Systems Research, 81, 821–829, 2011.

ABSTRACT 6

Development of an Artificial Intelligence-Based Diagnostic Method for Three-Phase Inverter Faults

Loutfi BENYETTOU, Samir ZEGHLACHE, Hilal RAHALI, Cherif Bilal Djamal Eddine

*Electrical Engineering Laboratory (LGE), Electrical Engineering Department, Faculty of Technology,
University of M'sila, 28000, M'sila, Algeria*
E-mail: loutfi.benyettou@univ-msila.dz

Abstract: The work presented here falls within the framework of detecting and diagnosing electronic open-circuit faults in a three-phase inverter associated with an induction machine. The adopted approach is based on the analysis of the statistical properties of phase currents. Two fault detection and diagnosis methods are evaluated. The first is based on analyzing the fault characteristics and their evolution in the time domain. Using this method, faults are detected with a detection probability greater than 80%. To improve performance, an artificial neural network algorithm is employed. This detection method proves to be highly effective, with a detection probability close to 100%.

Keywords: Diagnostic Method, Artificial Intelligence, Faults, Inverter.

Mathematics Subject Classification: 12X34, 56Y78.

References

- [1] I. Takahashi and T. Noguchi, «A New Quick Response and High Efficiency Control Strategy of Induction Motor», IEEE Trans. Ind. Electron, IE-22, Sept. /Oct. 1986.
- [2] I. Takahashi and. S. Asakawa, «Ultra-Wide Speed Control of Induction Motor Covered 10A6 range», IEEE Trans. Ind. Applicat., IA-25: 227-232, 1987.
- [3] T.G. Habetler and D.M. Divan, «Control Strategies for Direct Torque Control using Discrete Pulse Modulation», IEEE Trans. Ind. Applicat., IA-27(5): 893-901, 1991.
- [4] Farid Kadri, Said Drid, Fayçal Djeflal, « Méthode De Seuil Simple En Défaut Diagnostic Pour Onduleur Tension Source Dans Un Contrôle Direct De Couple Induction Motor Drive», IEEE Trans-Power Electr 2014 (EVER).

ABSTRACT 7

Double k -Fractional Operators: Properties, Representations and Applications

Meryem Meddahi*University of Hassiba Ben Bouali, Chlef*

E-mail: m.meddahi@univ-chlef.dz

Abstract: In this paper, we introduce Riemann–Liouville and Caputo k -fractional integrals and derivatives for bidimensional functions. We study the fundamental properties of the associated double fractional operators, with particular emphasis on the semigroup, inverse, and composition properties. The action of these operators on power functions is examined, and new formulas for partial k -fractional integrals and derivatives are derived using the double general transform.

Keywords: Double Riemann–Liouville k -fractional integrals, Double Riemann–Liouville k -fractional derivatives, Double Caputo k -fractional derivatives, Double general transform.

Mathematics Subject Classification: 26A33, 33E12, 44A10.

ABSTRACT 8

Statistical evaluation of Sage manure (*Salvia officinalis* L.) as a biocontrol strategy against fungal diseases of greenhouse tomato**Dalila Chelli-Tabti, Abdelmadjid Chelli and Naima Djouadi**

Research Laboratory in Ecology and Environment, Faculty of Natural and Life Sciences, University of Bejaia 06000 Bejaia, Algeria.

Laboratory of Applied Zoology, Faculty of Natural and Life Sciences, University of Bejaia 06000 Bejaia, Algeria.

Directorate of Agricultural Services, Bejaia, Algeria.

E-mail: dalila.tabti@univ-bejaia.dz, abdelmadjid.chelli@univ-bejaia.dz and dj29allhan@gmail.com

Abstract: This study highlights the importance of applied mathematics and statistical analysis in experimental agronomy, plant pathology, and sustainable crop protection. The excessive use of synthetic fungicides in vegetable production raises serious concerns regarding human health and environmental sustainability. In this context, plant-based biopesticides represent a promising alternative for the management of cryptogamic diseases. The present work aimed to statistically evaluate the effectiveness of sage manure (*Salvia officinalis* L.) against major fungal diseases affecting greenhouse tomato (*Solanum lycopersicum*), namely powdery mildew (*Leveillula taurica*) and late blight (*Phytophthora infestans*). The experiment was conducted in Boukhelifa (Algeria) from March 6 to May 28, 2025, using a completely randomized design with four concentrations of sage manure (T1: 10%, T2: 15%, T3: 20%, T4: 25%) and an untreated control. Weekly foliar applications were performed, and disease incidence and severity were recorded twice a week. Data were subjected to analysis of variance (ANOVA) to assess treatment effects. The results showed no statistically significant differences among treatments for either disease incidence ($p = 0.84$) or severity ($p = 0.92$). Although sage manure did not demonstrate significant antifungal efficacy under the tested conditions, these findings emphasize the necessity of rigorous statistical validation in biocontrol assessment studies. Further investigations involving different concentrations, application frequencies, and combined biocontrol strategies are recommended to enhance treatment efficiency.

Keywords: Statistical analysis, ANOVA, Biocontrol, Cryptogamic diseases, Greenhouse tomato, applied biostatistics.

References

- [1] Shankara, S., Rao, P., & Kumar, V. Morphological and reproductive characterization of *Solanum lycopersicum* L.: structure of bisexual flowers with pistil, androecium of 5–7 stamens, and ovary formed by two fused carpels. *Journal of Horticultural Botany*, 19(3), 201–209. 2005.
- [2] El Akel, M., Dupont, J., & Lefèvre, P. Étude de la pathologie de *Botrytis cinerea* sur la tomate en culture sous serre. Montpellier : Presses Agronomiques. 2001.
- [3] Fletcher, J. T., Gilligan, C. A., & Northover, J. First record of powdery mildew (*Oidium lycopersici*) on tomato in the United Kingdom. *Plant Pathology*, 37(3), 451–453. <https://doi.org/10.1111/j.1365-3059.1988.tb02096.x>. 1988
- [4] Mieslerová, B., & Lebeda, A. Taxonomic revision and naming of powdery mildew on tomato: *Oidium lycopersici*. *Mycotaxon*, 71, 23–32. 1999.

ABSTRACT 9

Multivariate analysis and diversity indices applied to ladybird beetle assemblages (Coleoptera: Coccinellidae) in different biotopes of northeastern Algeria**Abdelmadjid Chelli and Dalila Tabti-Chelli***University of Bejaia, Natural and Life Sciences Faculty 06000, Bejaia, Algeria*

E-mail: abdelmadjid.chelli@univ-bejaia.dz and Dalila.tabti@univ-bejaia.dz

Abstract: This study was conducted in the Kabylie region of northeastern Algeria between April 2022 and September 2023 to address the lack of detailed information on the local entomofauna, with a particular focus on ladybird beetles (Coleoptera: Coccinellidae). Ladybirds are widely recognized as key bioindicators of ecosystem health, and their global decline mirrors the broader decrease in insect biodiversity. The main objective of this research was to characterize the composition, diversity, and spatial-temporal distribution of ladybird assemblages across different biotopes. The results reveal an exceptionally high diversity of ladybird beetles in the Kabylie region. A total of 35 species belonging to 19 genera and 11 tribes were recorded, representing approximately 72% of the 49 species currently known from Algeria. This finding highlights Kabylie as a major reservoir of biodiversity for the family Coccinellidae at the national scale. The taxonomic structure of the assemblages is dominated by two tribes commonly reported in Algerian ecosystems: Scymnini, with 12 species, and Coccinellini, with 9 species. Together, these two tribes account for nearly 60% of the total species richness observed. In terms of abundance and distribution, the aphidophagous species *Hippodamia* (*Adonia*) *variegata* and *Coccinella* (*Coccinella*) *septempunctata* were the most dominant and widespread across all studied biotopes, reflecting their high ecological plasticity and adaptability to diverse environmental conditions. A noteworthy result of this study is the first record of the European species *Oenopia conglobata* (Linnaeus, 1758) in the Bejaia department. Previously known only from the Algiers region, its presence in Kabylie confirms the establishment of this species in Algeria and extends its known distribution eastwards. Statistical analyses demonstrated that total abundance, species richness, and diversity indices of ladybird beetles were significantly influenced by both habitat type and sampling month, as confirmed by Kruskal-Wallis tests and PERMANOVA ($p < 0.001$). Structurally complex habitats, particularly orchards and grasslands, supported the highest levels of species richness, Shannon diversity ($H' = 2.3\sim 2.8$), and Pielou evenness ($J' = 0.80\sim 0.90$). In contrast, simplified habitats such as cedar forests and wetlands exhibited lower diversity and abundance values. Beta diversity analyses using NMDS ordination and PERMANOVA further revealed that the composition of ladybird assemblages is significantly structured by habitat type ($R^2 = 0.228$) and sampling month ($R^2 = 0.158$). Moreover, the significant interaction between habitat and month indicates that temporal variations in community composition differ among biotopes, emphasizing the combined influence of spatial and seasonal factors on ladybird beetle assemblages.

Keywords: Ladybird beetles, Spatial-temporal variation, Multivariate analysis, Algeria.

References

- [1] Hodek, I., Van Emden, H.F., Honek, A. (2012). Ecology and behaviour of the ladybird beetles (Coccinellidae). Ed. Wiley-Blackwell, Oxford UK., 532 p.
- [2] Iablokoff-Khnzorian, S.M. (1982). Les coccinelles, (Coléoptères- Coccinellidae), tribu Coccinellini des régions paléarctiques et orientales. Boubée, Paris. 568 p.

- [3] Nedvěd, O. (2015). Ladybird beetles (Coccinellidae) of Central Europe. Academia, Praha, 304 p.
- [4] Saharaoui, L. (1994). Inventaire et étude de quelques aspects bioécologiques des coccinelles entomophages (Coleoptera, Coccinellidae). *Journal of African Zoology* 108: 537-546.
- [5] Saharaoui, L., Hemptinne, J.L., Magro, A. (2014). Biogéographie des coccinelles (Coleoptera, Coccinellidae) d'Algérie. *Revue Faunistic Entomology*. 67: 147-164.

ABSTRACT 10

Boundary Element Method for Microfluidic Applications

Prof. Dr. Barbaros Çetin

Microfluidics & Lab-on-a-chip Research Group, Mech. Eng. Dept., İ.D. Bilkent University
E-mail: barbaros.cetin@bilkent.edu.tr

Abstract: Microfluidic systems require accurate prediction of flow and electric fields at small length scales, where strong confinement effects and particle–boundary interactions govern device performance. In many microfluidic applications, particularly those involving electrokinetic particle manipulation, the governing field equations reduce to Laplace or Poisson equations coupled with Stokes flow. Efficient and reliable numerical solutions to these equations are therefore a key requirement for the rational design of lab-on-a-chip platforms. This talk focuses on boundary element method (BEM) formulations developed for microfluidic applications, with particular emphasis on the solution of Laplace and Poisson equations arising in electrokinetic problems. Finite-sized particle models based on stress-tensor approaches are presented for electrophoretic and dielectrophoretic transport in confined microchannels, where particle–particle and particle–wall interactions play a dominant role. The boundary-only discretization and semi-analytical nature of BEM enable accurate evaluation of field gradients and forces without volumetric meshing, making it especially well-suited for microfluidic geometries with moving boundaries and strong near-field interactions. The presentation demonstrates how Laplace- and Poisson-type field solutions are integrated within a unified BEM framework to model electric potential distributions, induced forces, and resulting particle trajectories in microchannels. Selected examples from electrokinetic microfluidic systems are used to illustrate the numerical strategies, physical insights, and computational advantages of the method.

Keywords: Boundary element method, microfluidics.

Bio

Dr. Barbaros Çetin received his B.S. (2002) and M.S. (2005) in Mechanical Engineering at Middle East Technical University, Ankara, Turkey. He received his PhD (2009) in the Department of Mechanical Engineering at Vanderbilt University, where he focused on electrokinetic transport and particle manipulation in lab-on-a-chip devices for biomedical applications. Following his PhD, he became a faculty member at Middle East Technical University-Northern Cyprus Campus Mechanical Engineering Program. In 2011, he became a faculty member in the Mechanical Engineering Department at İ.D. Bilkent University, Ankara, Turkey. His current research interests include particle manipulation for microfluidic applications, modeling of particle motion using the boundary element method, and modeling, fabrication, and experimentation of flat-grooved heat pipes. Dr. Çetin is the recipient of the 2015 Bilkent University Distinguished Teacher Award, 2017 Outstanding Young Scientist Award of the Turkish Academy of Sciences (TÜBA-GEBİP), 2017 METU Prof. Dr. Mustafa N. Parlar Research Incentive Award and 2018 Science Academy Association Distinguished Young Scientist Award (BAGEP).

ABSTRACT 11

Improvement of SUV vehicle ride comfort by controlling the MR dampers integrating the vehicle suspension

Said Boukerroum¹, Nacer Hamzaoui²¹*USTHB/FGMGP, LMA, BP 32 El Allia 16111, Bab Ezzouar, Alger, Algérie*²*INSA de Lyon, LVA, Bt A de ST Exupéry, 25 bis avenue Jean Capelle 69621, France*

E-mail: Said.boukerroum@usthb.edu.dz, nacer.hamzaoui@insa-lyon.fr

Abstract: This study presents a comprehensive investigation and performance evaluation of magnetorheological (MR) fluid-based active isolators integrated into the suspension system of a sport utility vehicle (SUV), controlled using a comfort-oriented strategy. An eight-degree-of-freedom (8-DOF) multibody dynamic model of the vehicle was developed in MSC Adams and co-simulated with MATLAB/Simulink to implement a modified Skyhook control algorithm.

The effectiveness of the proposed semi-active control strategy was assessed through key ride comfort performance indices following parameter optimization. The methodology commenced with the development of a fully parametric virtual prototype in MSC Adams, incorporating all vehicle components modeled in SolidWorks to ensure accurate representation of the geometric and physical characteristics of the system. Real-time semi-active suspension control was achieved through control algorithms implemented in MATLAB/Simulink within a co-simulation framework. Simulation results demonstrate that the proposed control law significantly improves ride comfort performance, confirming the robustness and efficiency of the adopted approach.

Keywords: Semi-Active Suspension, Magnetorheological (MR) Damper, Virtual Model, Control law, Ride Comfort.

ABSTRACT 12

Adaptive fault tolerant control based disturbance observer for Quanser 2-DOF helicopter subject to model uncertainties and external disturbances

Samir Zeghlache¹, Hilal Rahali², Ali Djerioui², Loutfi Benyettou², Mohamed Fouad Benkhoris³

¹LASS, Laboratoire d'Analyse des Signaux et Systèmes, Department of Electrical Engineering, Faculty of Technology, University of M'sila University Pole, Road Bourdj Bou Arreiridj, M'sila 28000, Algeria

²LGE, Laboratoire de Génie Electrique, Department of Electrical Engineering, Faculty of Technology, University of M'sila University Pole, Road Bourdj Bou Arreiridj, M'sila 28000, Algeria

³Nantes Université, Institut de Recherche en Énergie Électrique de Nantes Atlantique, IREENA, UR 4642, Saint-Nazaire F-44600, France

E-mail: samir.zeghlache@univ-msila.dz, hilal.rahali@univ-msila.dz, ali.djerioui@univ-msila.dz, loutfi.benyettou@univ-msila.dz, mohamed-fouad.benkhoris@univ-nantes.fr

Abstract: The objective of this work is to offer a composite adaptive fault tolerant control technique for a quanser 2-degree of freedom (2-DOF) helicopter that can concurrently handle for actuator defects, model errors, and external disturbances. First, an integral sliding mode control (ISMC) is devised to provide the required system tracking performance while maintaining insensitivity to disturbances. This is accomplished by assuming that the constraints on external disturbances are known. Then, adaptive type 2 fuzzy logic control schemes are built and included into the ISMC in order to cope with actuator failures and model uncertainties of the quanser 2-DOF helicopter. Furthermore, in order to further strengthen the robustness of the suggested control strategy, a disturbance observer is constructed and synthesized with the control law. This is done in order to take into consideration for unknown external disturbances. Furthermore, type 1 fuzzy inference systems are used to mitigate the chattering effect resulting from the discontinuous control component of the ISMC. Lyapunov's approach is used to demonstrate that the tracking errors of the closed loop exhibit exponential stability when the stability analysis incorporates the type 1 fuzzy logic inference mechanism. Ultimately, a set of comparative simulation experiments are performed to verify the efficacy of the suggested control approach. These tests include subjecting quanser 2-DOF helicopter to fluctuations in inertial moment and varied levels of actuator failures. The simulation findings corroborate and validate the capabilities and benefits of the suggested control approach.

Keywords: Type 2 fuzzy system, Type 1 fuzzy system, Fault tolerant control, Integral sliding mode control, Adaptive control, disturbance observer, Quanser 2-DOF helicopter.

Mathematics Subject Classification: 12X34, 56Y78.

References

- [1] Boukadida W, Benamor A, Messaoud H, Siarry P, Multi-objective design of optimal higher order sliding mode control for robust tracking of 2-DoF helicopter system based on metaheuristics, *Aerospace Science and Technology*, 91(2019):442-455.
- [2] Lee H, Utkin V.I, Chattering suppression methods in sliding mode control systems, *Annu Rev Control*, 31 (2014): 179-188.
- [3] Veluvolu K.C, Soh Y.C, Cao W, Robust observer with sliding mode estimation for nonlinear uncertain systems, *IET Control Theory A*, 1,(2007) :1533-1540.

ABSTRACT 13

Comparative Analysis of the Randić and Estrada Indices for the Molecular Structures Based on Drugs Used in Cancer Treatment and Anti-Hypertension

Nur Idayu Alimon, Nor Haniza Sarmin

Faculty of Computer and Mathematical Sciences, Universiti Teknologi, MARA Johor Branch, Pasir Gudang Campus, 81750 Masai, Johor, Malaysia
Department of Mathematical Sciences, Faculty of Science
Universiti Teknologi Malaysia, 81310 Johor Bahru, Johor, Malaysia
E-mail: idayualimon@uitm.edu.my, nhs@utm.my

Abstract: Topological indices play an important role in mathematical chemistry by transforming molecular structures into numerical descriptors for quantitative structure–property relationship (QSPR) analysis. In this presentation, a comparative analysis between the Randić index and the Estrada index is presented with respect to selected physicochemical properties of drug molecules used in cancer treatment and antihypertensive. Three anticancer drugs and two antihypertensive drugs are modeled as simple connected graphs, where atoms and bonds are represented by vertices and edges, respectively. The Randić and Estrada indices are computed from the molecular graphs, and their correlations with physicochemical properties reported in the literature are statistically analyzed using SPSS. The findings aim to determine which invariant exhibits stronger association with the selected properties, thereby providing insight into the effectiveness of degree-based and spectral-based descriptors in modeling pharmaceutical compounds and supporting the application of graph-theoretical approaches in drug analysis and design.

Keywords: Randić index, Estrada index, molecular graph.

References

- [1] Randić, M. *Characterization of molecular branching*. *Journal of the American Chemical Society*. 97(23): 6609-6615, 1975.
- [2] Estrada, E. *Characterization of 3D molecular structure*. *Chemical Physics Letters*. 319: 713–718. 2000.
- [3] Havare, O.C. *Topological indices and QSPR modeling of some novel drugs used in the cancer treatment*. *International Journal of Quantum Chemistry*. 121(24): e26813, 2021.

ABSTRACT 14

Modeling and Control of an Artificial Pancreas

Fayssal Ouagueni¹, Kada Boureguig², Abdelghafour Herizi¹, Abderrzak Laib¹, Abdelhafid Benyounes¹

¹University of M'sila, M'sila, Algeria

²University Ibn Khaldoun, Tiaret, Algeria

E-mail: fayssal.ouagueni@univ-msila.dz, kada.boureguig@univ-tiaret.dz

Abstract: The artificial pancreas is a medical device for patients with type 1 diabetes. It replaces a failing pancreas by injecting insulin. It consists of a continuous blood glucose meter, an insulin pump and a control algorithm. The objective of this work is the study of the phenomena governing the dynamics of the artificial pancreas in order to develop a theoretical basis for the modeling one uses the approach of Bergman, the analysis and the design of the regulators to automatically adjust the blood glucose level, the two approaches used for the synthesis of the regulator are: The PID regulator and the adaptive fuzzy PID regulator.

Keywords: Type 1 diabetes, Artificial pancreas, Adaptive fuzzy PID controller.

References

- [1] Iqra Shafeeq Mughal, Luca Patanè, and Riccardo Caponetto. *A comprehensive review of models and nonlinear control strategies for blood glucose regulation in artificial pancreas*. Annual Reviews in Control, 57:100937, 2024.
- [2] Ronald Eric, Akif Rahmatillah, and Riries Rulaningtyas. *Control system modeling of artificial pancreas based on PID controller mechanism with Particle Swarm Optimization (PSO) tuning method*. The 11th International Conference on Theoretical and Applied Physics, AIP Conf. Proc. 2858, 030005, Surabaya, Indonesia, 2021.

ABSTRACT 15

Nonlinear Mathematical Modelling of Channel Flow: Predicting Drag Coefficient by using Neural Networks**Khalil Ur Rehman^{1,*}, and Wasfi Shatanawi¹**

¹*Department of Mathematics and Sciences, College of Sciences and Humanities, Prince Sultan University, Riyadh, 11586, Saudi Arabia*
E-mail: kurrehman@psu.edu.sa

Abstract: The fluid flow around obstacles subject to confined domains results in a non-linear differential system. Therefore, it is always difficult to get exact as well as appropriate solutions to narrate the flow field. Owing to such difficulties, the present work provides the solution strategy for fluid flow around a circular obstacle in a channel, indeed a well known configuration. In detail, Newtonian fluid flow is considered around an obstacle being installed in a channel, and the developed differential system is solved by using FEM. Two experiments are carried out to approximate the drag force faced by the obstacle. Neural networks model is constructed for predicting the drag coefficient subject to various values of Reynolds number. 70% data is slotted for training and 15% each for validation and testing. The Levenberg-Marquardt algorithm is used for training the neural networks. We have observed that the constructed ANN is best suited to predict the DC values, and one can adopt such a directory instead of wasting resources by performing experiments again and again.

Keywords: Non-linear PDEs, Channel, FEM, Drag Coefficient, Neural Networks.

References

- [1] Iftimie, Dragos, Milton C. Lopes Filho, and H. J. Nussenzweig Lopes. "Two-dimensional incompressible ideal flow around a small obstacle." (2003): 349-379.
- [2] Pichi, Federico, Francesco Ballarin, Gianluigi Rozza, and Jan S. Hesthaven. "An artificial neural network approach to bifurcating phenomena in computational fluid dynamics." *Computers & Fluids* 254 (2023): 105813.
- [3] Hajimirzaie, Seyed M. "Experimental observations on flow characteristics around a low-aspect-ratio wall-mounted circular and square cylinder." *Fluids* 8, no. 1 (2023): 32.

ABSTRACT 16

Dust-Acoustic Soliton Propagation in Maxwellian Polarized Dusty Plasmas

Mohamed Ouazene¹, Rabia Amour²¹*Faculty of Physics, University of Bab- Ezzouar, USTHB, B.P. 32, 16111 El Alia, Algiers, Algeria*²*Plasma Physics Group, Theoretical Physics Laboratory, Faculty of Physics, University of Bab-Ezzouar, USTHB, B.P. 32, 16111 El Alia, Algiers, Algeria*

E-mail: ouaz_moh@yahoo.fr, amouraz.80@gmail.com

Abstract: The introduction of charged dust grains significantly modifies the properties of a two-component plasma by introducing new physical mechanisms such as dust charge fluctuations, electron depletion, and unconventional energy dissipation processes. These effects enrich the collective dynamics of the plasma and lead to the emergence of new wave modes, particularly the dust-acoustic (DA) mode, which is a characteristic feature of dusty plasmas. In this work, we investigate the propagation of dust-acoustic waves in the presence of a Maxwellian polarization force self-generated by the complex plasma. This force, arising from electric field gradients and the collective response of the light plasma species, influences the dynamics of the charged dust grains and consequently modifies the propagation properties of the associated electrostatic perturbations. Special attention is devoted to the formation and evolution of large-amplitude dust-acoustic solitons. We analyze how the Maxwellian polarization force affects their fundamental characteristics, including their amplitude, width, and spatial structure. Our results show that the Maxwellian polarization force has a significant impact on the intrinsic properties of DA solitons, leading to notable modifications in their nonlinear structure. These findings provide further insight into nonlinear wave propagation in multispecies dusty plasmas.

Keywords: Dusty plasma, Dust acoustic Solitons, Maxwellian polarization force.

ABSTRACT 17

Influence of Dust Grain Size on Plasma Sheath Structure in the Presence of an Inclined Magnetic Field

Chekour Samir and Tahraoui Abdelatif*Quantum Electronic Laboratory, Faculty of Physics, USTHB University, BP 32 El-Alia, Bab Ezzouar
16111 Algiers*

E-mail: che-samir@hotmail.fr, schekour@usthb.dz

Abstract: This work presents a theoretical study of the influence of dust grain size on the structure of a plasma sheath in the presence of an inclined magnetic field. Based on a one-dimensional fluid model involving multiple species, we analyze how variations in dust grain radius affect the distributions of electrostatic potential, particle densities, and electric field. The fundamental equations include Poisson's equation and the continuity and momentum equations for the charged species. The magnetic field effect is modeled through its components parallel and perpendicular to the sheath plane. The study shows that grain size strongly influences the electrostatic sheath width, the potential profile, and the coupling between the magnetic field and dust charge.

Keywords: Plasma sheath, dust grains and magnetic field.

ABSTRACT 18

Applications of Mathematics, Computing, and AI in Music and Art

Dirk Schlingmann

University of South Carolina Upstate
E-mail: schlingm@uscupstate.edu

Abstract: In the presentation, I demonstrate how I apply mathematics, computing, and AI to analyze and synthesize sound and music and how I create algorithmic music and algorithmic music video art based on mathematical concepts. For example, I will show, how I use Fourier analysis to create mathematical models of digital sounds, and how I use general mathematics to intertwine pixel colors and sounds into algorithmic music video art where images become audible or sounds become visible. The work demonstrates how mathematics and algorithmic methods can extend artistic expression.

Advances in computational technology have made it possible to generate both sound and images entirely within the abstract domain of mathematics and code. These developments allow not only for the synthesis of new sounds but also for the integration of sound and images into unified multimedia experiences. Rather than performing directly on an instrument, the artist now designs algorithms that determine what the instrument will play and what pixel color will be used. Algorithmic art refers to the use of formalized procedures for the generation of artistic material. The central premise is that innovative music, images, and videos can be produced through mathematics, rules, equations, or computational processes rather than by traditional methods alone.

ABSTRACT 19

On the Kernel of the Homomorphism of the Nonabelian Tensor Square in Six-Dimensional Bieberbach Group with Quaternion Point Symmetry

Siti Afiqah Mohammad¹, Muhammad Hazwan A Rahman¹, Nor Haniza Sarmin²

¹*Faculty of Computer and Mathematical Sciences,
Universiti Teknologi MARA Johor Branch, Johor, Malaysia*

²*Department of Mathematical Sciences, Faculty of Science,
Universiti Teknologi Malaysia, Johor, Malaysia*

E-mail: sitiafiqah@uitm.edu.my, wanrahman01@gmail.com, nhs@utm.my

Abstract: Crystallography plays a central role in engineering and materials science, providing a mathematical foundation for understanding the symmetry and structure of crystalline solids. The classification of crystallographic groups, particularly Bieberbach groups, is essential for modeling periodic structures in higher dimensions and for applications from lattice design to computational materials analysis. The focus of this paper is to explicate and analyze the kernel of the homomorphism of the nonabelian tensor square for a specified group, which is a six-dimensional Bieberbach group with the nonabelian point group, specifically the quaternion point group of order six. The explicit results on the elements of the kernel of the homomorphism and its order are presented. These findings contribute to the algebraic characterization of crystallographic groups, offering insights into how nonabelian symmetries influence structural invariants. By bridging group-theoretic analysis with crystallographic applications, this study highlights the potential of algebraic methods to support engineering research in symmetry-based modeling and the design of advanced materials.

Keywords: Kernel of the homomorphism, Bieberbach group, nonabelian tensor square.

References

- [1] Afra J. Zomorodian. *Topology for Computing*. Cambridge University Press, United States of America, 2005.
- [2] Derek J. S. Robinson. *Graduate Texts in Mathematics: A Course in the Theory of Groups*. Springer New York, New York, 1982.
- [3] Michael R. Bacon, and Luise-Charlotte Kappe. *On Capable p-groups of Nilpotency Class Two*. Illinois Journal of Mathematics, 47:49-62, 2003.
- [4] Muhammad Hazwan A Rahman, Siti Afiqah Mohammad, and Muhanizah Abdul Hamid. *The Derived Subgroup of the Second Bieberbach Group of Dimension Six with the Quaternion Point Group of Order Eight*. AIP Conference Proceedings, 3184(1), 2024.
- [5] Muhammad Hazwan A Rahman, Siti Afiqah Mohammad, Nor Haniza Sarmin, Nurain Mieza Mahirah Mohd Sarip, Siti Nuraishah Mukhtar, and Ani Ayuni Zainal. *Transformation of Matrix Presentation for Bieberbach Groups into Polycyclic Presentations*. Malaysian Journal of Fundamental and Applied Sciences, 20(6):1375-1391, 2024.
- [6] Noraí Romeu Rocco. *On a Construction Related to the Non-abelian Tensor Square of a Group*. Boletim da Sociedade Brasileira de Matemática, 22(1):63-79, 1991.

- [7] Rohaidah Masri. *The Nonabelian Tensor Square of Certain Bieberbach Groups with Cyclic Point Group of Order Two*. Universiti Teknologi Malaysia, Malaysia, 2009.
- [8] Russell D. Blyth, and Robert Fitzgerald Morse. *Computing the Nonabelian Tensor Squares of Polycyclic Groups*. Journal of Algebra, 321:2139–2148, 2009.
- [9] Russell D. Blyth, Francesco Fumagalli, and Marta Morigi. *Some Structural Results on the Non-abelian Tensor Square of Groups*. Journal of Group Theory, 13:83-94, 2010.
- [10] Siti Afqah Mohammad. *The Homological Invariants of A Bieberbach Group of Dimension Six with Quaternion Point Group of Order Eight*. Universiti Teknologi Malaysia, Malaysia, 2018.
- [11] Siti Afqah Mohammad, Nor Haniza Sarmin, and Hazzirah Izzati Mat Hassim. *Polycyclic Presentations of the Torsion Free Space Group with Quaternion Point Group of Order Eight* Jurnal Teknologi (Sciences and Engineering), 77(33);151–156, 2015.

ABSTRACT 20

Biological and Chemical Complexity of Algerian Honeys: Integrated Melissopalynological, Physicochemical and Antioxidant Characterization

Mouhoubi-Tafnine, Zina, Djelili Farida, Soufi Ouahiba and Zenati Abel

Laboratoire de Biochimie Appliquée, Faculté des Sciences de la Nature et de la vie, Université de Bejaia, 06000, Algeria

E-mail: zina.mouhoubi@univ-bejaia.dz

Abstract: The objective of this work is to determine the botanical origin, some physicochemical criteria and the antioxidant activity of eleven honey harvested in the region of the wilaya of Bejaia. Determination of the botanical origin of the analyzed honeys showed the predominance of 3 monofloral species: Myrtaceae (M10), Ericaceae (M5), and Fabaceae (M4, M6, M7, M8). The rest of the samples are multifloral honeys. Most of the physicochemical parameters studied gave results meeting the standards proposed by the Codex Alimentarius Commission, thus testifying to the maturity and authenticity of the samples. The ash contents, the values of electrical conductivity, as well as the color confirm that these are indeed nectar honeys. Moreover, the studied honeys showed a considerable content of phenolic compounds and flavonoids, explaining their anti-radical power.

Keywords: Algerian honey, Melissopalynology, Biological complexity, Phenolic compounds, Antioxidant activity.

ABSTRACT 21

Hybrid Analytical-Numerical Modeling of Permanent Magnet Electrical Machines via Maxwell-Fourier and Finite Element Methods**Brahim LADGHEM-CHIKOUCHE***Department of Electrical Engineering, Faculty of Technology, University of M'sila, Road BBA, M'sila 28000, Algeria*

E-mail: brahim.lch@univ-msila.dz

Abstract: This work proposes an extended hybrid analytical method (HAM) based on a two-dimensional (2-D) coupling between the semi-analytical Maxwell-Fourier analysis and finite-element method (FEM) in polar coordinates. The proposed model is applied to any rotating electrical machines. The main objective of this paper is to establish the magnetic field solution in the whole machine by coupling an exact analytical model (AM), proposed for all regions having relative magnetic permeability equal to unity, with a FEM in ferromagnetic regions.

The AM and FEM are coupled in both directions in the edge separating teeth regions and all its adjacent regions by applied boundary conditions (BCs). The developed HAM gives accurate results on the magnetic flux density distribution and the cogging torque whatever the operating conditions, the magnetic or geometric parameters. All results obtained give very satisfactory agreement with 2-D finite-element analysis (FEA).

References

- [1] F. Dubas and C. Espanet, "Analytical solution of the magnetic field in permanent-magnet motors taking into account slotting effect: No-load vector potential and flux density calculation," IEEE Trans. Magn., vol. 45, no. 5, pp. 2097–2109, May 2009, doi:10.1109/TMAG.2009.2013245.
- [2] F. Dubas and K. Boughrara, "New scientific contribution on the 2-D subdomain technique in polar coordinates: Taking into account of iron parts," Math. Comput. Appl., vol. 22, no. 4, p. 42, Oct. 2017, doi:10.3390/mca22040042.
- [3] L. Roubache, K. Boughrara, F. Dubas, and R. Ibtouen, "New subdomain technique for electromagnetic performances calculation in radial-flux electrical machines considering finite soft-magnetic material permeability," IEEE Trans. Magn., vol. 54, no. 4, Apr. 2018, Art. no. 8103315, doi: 10.1109/TMAG.2017.2785254
- [4] B. Ladghem-Chikouche, K. Boughrara, F. Dubas, and R. Ibtouen, "2-D semi-analytical magnetic field calculation for flat permanentmagnet linear machines using exact subdomain technique," IEEE Trans. Magn., vol. 57, no. 6, Jun. 2021, Art. no. 8106211, doi:10.1109/TMAG.2021.3068326.
- [5] S. Ouagued, Y. Amara, and G. Barakat, "Comparison of hybrid analytical modelling and reluctance network modelling for pre-design purposes," Math. Comput. Simul., vol. 130, pp. 3–21, Dec. 2016, doi:10.1016/j.matcom.2016.05.001.

ABSTRACT 22

The First and Second Zagreb Indices for Digraphs of Non-Normal Cyclic Subgroups of Dihedral Groups

Nurin Alya Jhon Kenedi, Nor Haniza Sarmin, Nabilah Fikriah Rahin, Athirah Zulkarnain, Hazzirah Izzati Mat Hassim

*Department of Mathematical Sciences, Faculty of Science, 81310 UTM Johor Bahru, Johor, Malaysia
Centre for Defence Foundation Studies, UPNM, Kem Sg Besi, 57000 Kuala Lumpur, Malaysia
Department of Mathematics, Xiamen University Malaysia, Jalan Sunsuria, Bandar Sunsuria, 43900
Sepang, Selangor, Malaysia*

E-mail: nurinalya@graduate.utm.my, nhs@utm.my, nabilah@upnm.edu.my,
athirah.zulkarnain@xmu.edu.my, hazzirah@utm.my

Abstract: The Zagreb indices are fundamental graph invariants based on vertex degrees. For a graph, the first Zagreb index is defined as the sum of squares of vertex degrees (equivalently, the sum of the degree sums of adjacent vertices), while the second Zagreb index is defined as the sum of products of degrees of adjacent vertices. These indices are widely applied in mathematical chemistry. While they have been extensively studied for undirected graphs, their formulation and behavior for directed graphs (digraphs) remain underexplored. For a non-normal cyclic subgroup H of a group G , the digraph is defined as follows: the vertex set is G , and a directed edge from x to y exists if and only if $x \neq y$ and $xy \in H$. In this research, the general formulas for the first and second Zagreb indices are derived for the digraphs associated to the non-normal cyclic subgroups of dihedral groups of all orders. The approach is based on a detailed examination of the in-degree and out-degree distributions of all vertices within these digraphs. By applying the standard definitions of the Zagreb indices adapted for directed graphs, these topological descriptors are computed exactly. The results indicate that both the first and second Zagreb indices are strictly increasing functions of the group order, establishing a clear monotonic relationship between the group's algebraic structure and the graph-theoretic measures. This research contributes a significant advancement in the application of algebraic graph theory by establishing a direct link between group order and graph-based topological descriptors. These results lay the groundwork for using directed Zagreb indices as tools to analyze more complex algebraic structures and may inspire new computational methods for investigating nonlinear relationships in discrete mathematical models.

Keywords: Zagreb Indices, Non-Normal Cyclic Subgroups, Dihedral Groups.

Mathematics Subject Classification: 05C20, 05C25.

References

- [1] Gutman, I. and Trinajstić, N. Graph theory and molecular orbitals. Total π -electron energy of alternant hydrocarbons. *Chemical Physics Letters*, 1972. 17(4): 535–538.
- [2] Gutman, I., Trinajstić, N. and Wilcox, C. F. Graph theory and molecular orbitals. XII. Acyclic polyenes. *Journal of Chemical Physics*, 1975. 62(9): 3399–3405.
- [3] Monsalve, J. and Rada, J. Vertex-degree based topological indices of digraphs. *Discrete Applied Mathematics*, 2021. 295: 13–24.
- [4] Monsalve, J. and Rada, J. Sharp upper and lower bounds of VDB topological indices of digraphs. *Symmetry*, 2021. 13(10): 1903–1903.

- [5] Rahin, N. F., Sarmin, N. H. and Ilangovan, S. The non-normal subgroup graph of some dihedral groups. *ASM Science Journal*, 2020: 1–7.
- [6] Gao, W., Wang, W. and Farahani, M. R. Topological indices study of molecular structure in anticancer drugs. *Journal of Chemistry*, 2016: 1–8.
- [7] Kirmani, S. A. K., Ali, P. and Azam, F. Topological indices and QSPR/QSAR analysis of some antiviral drugs being investigated for the treatment of COVID-19 patients. *International Journal of Quantum Chemistry*, 2021. 121(9): e26594.

ABSTRACT 23

Linear Canonical Boostlet Transform

Owais Ahmad and Jasifa Fayaz

Department of Mathematics, National Institute of Technology, Hazratbal, Srinagar -190 006, Jammu and Kashmir, India.

E-mail: siawoahmad@gmail.com and jasifaitoo058@gmail.com

Abstract: This paper introduces the Linear Canonical Boostlet Transform, a novel integral transform that extends the classical boostlet framework by incorporating the parametric flexibility of linear canonical transforms. The key idea is to construct a new family of analyzing functions that are essentially ordinary boostlets mapped backward through a linear canonical transformation, allowing the transform to adapt to signals that have been warped, sheared, or chirped by physical systems such as misaligned optical lenses or moving radar platforms. The LCBT preserves the desirable properties of boostlets—parabolic scaling, directional sensitivity, and sparse representation of curved edges—while gaining the ability to operate directly in canonically distorted domains. The paper establishes the transform’s fundamental mathematical properties, including invertibility, energy preservation via a Plancherel-type formula, and an explicit reconstruction formula under an admissibility condition on the mother boostlet. A suite of uncertainty principles is extended to the LCBT setting, including Heisenberg, logarithmic, Pitt, and Nazarov inequalities, which quantify the fundamental trade-offs between localization in the transform domain and the original signal space. A concrete example using Gaussian functions on two-dimensional Minkowski space illustrates how the transform responds to changes in scale, Lorentz boost, and translation parameters. Numerical comparisons show that while the LCBT demands greater computational resources than the standard boostlet transform, it achieves high accuracy in demanding applications such as relativistic signal processing and quantum imaging.

Keywords: Boostlet transform, Linear Canonical transform, Minkowski space, Nazarov inequality, Pitt’s inequality, Logarithmic uncertainty principle.

Mathematics Subject Classification: 42C40, 42C15, 81R30, 42A38, 47G10.

References

- [1] O.Ahmad, J. Fayaz, Continuous Boostlet transform and associated uncertainty principles, *Computational and Applied Mathematics*, 45, 129 (2026).
- [2] O. Ahmad, A. A. Dar, Discrete Biquaternion Linear Canonical Transform, *Journal of Computational and Applied Mathematics*, 474, 117010 (2026)..
- [3] O.Ahmad, A. A. Dar, Wigner-Ville distribution associated with the biquaternion offset Linear canonical transform, *Computational and Applied Mathematics*, 45, 206 (2026).
- [4] O. Ahmad, A.A. Dar, Short-Time Biquaternion Quadratic Phase Fourier Transform. *Journal of Franklin Institute*, <https://doi.org/10.1016/j.jfranklin.2025.107709> (2025).
- [5] O. Ahmad, A.A. Dar, Biquaternion Linear Canonical Wavelet Transform: Theory and Applications, *Complex Analysis and Operator Theory*, 19, 192 (2025).
- [6] M. Bahri, R. Ashino, Some properties of windowed linear canonical transform and its logarithmic uncertainty principle, *International Journal of Wavelets, Multiresolution and Information Processing*, 14(3), 1640015 (2016).

- [7] W. Beckner, Pitt's inequality and the uncertainty principle, *Proceedings of the American Mathematical Society*, 123(6) 1897-1905 (1995).
- [8] B. Deng, R. Tao, Y. Wang, Convolution theorems for linear canonical transform and their applications, *Science China Information Sciences*, 49(5), 592-603 (2006).
- [9] W. G. Faris, Inequalities and uncertainty principles, *Journal of Mathematical Physics*, 19, 461-466 (1978).
- [10] Q. Feng, B. Z. Li, Convolution and correlation theorems for the two- dimensional linear canonical transform and its applications, *IET Signal Process*, 10 (2) (2016) 125-132.
- [11] G. B. Folland, A. Sitaram, The uncertainty principle: a mathematical survey, *The Journal of Fourier Analysis and Applications*, 3(3), 207-238 (1997).
- [12] V. Havin, B. Jorricke, The Uncertainty Principle in Harmonic Analysis, *Modern Surveys in Mathematics Springer-Verlag Berlin*, 28, (1994).
- [13] J. J. Healy, M. A. Kutay, H. M. Ozaktas, J. T. Sheridan, Linear canonical transforms: Theory and Applications, *Springer, New York*, 2016.
- [14] M. Moshinsky, C. Quense, Linear canonical transformations and their unitary representations, *Journal of Mathematical Physics*, 12(8), 1772-1783 (1971).
- [15] A. I. Zayed, Sampling of signals bandlimited to a Disc in the linear canonical transform, *IEEE Signal Process. Lett.*, 25 (12) (2018) 1765- 1769.
- [16] E. Zea and M. Laudato, On the representation of wave fronts localized in space-time and wavenumber-frequency domains, *Journal of the Acoustical Society of America Express Letters*, 1 (5), 054801 (2021). doi:10.1121/10.0004852.
- [17] E. Zea, M. Laudato and J. Andén, A continuous boostlet transform for acoustic waves in space-time, *arXiv preprint arXiv:2403.11362*, Available: <https://arxiv.org> (2024).
- [18] H. Zhao, Y. H Li, Y. Zhang, B. Z. Li, Theory and applications of the linear canonical transform in polar coordinates, *Digital Signal Processing*, (2026), doi: <https://doi.org/10.1016/j.dsp.2026.106070>
- [19] J. Zhao, R. Tao, Y. Wang, On signal moments and uncertainty relations associated with linear canonical transform, *Signal Processing*, 90(9), 2686-2689 (2010).

ABSTRACT 24

Algebraic Modeling of CSS-Based Perfect Quantum Codes Derived from Eisenstein-Jacobi Integers**Neriman Şolt, Murat Güzeltepe**

Department of Mathematics, Sakarya University, Sakarya, Türkiye
E-mail: neriman.solt2@ogr.sakarya.edu.tr, mguzeltepe@sakarya.edu.tr

Abstract: In quantum information theory, the construction of fault-tolerant communication and computation systems requires robust error-correcting mechanisms grounded in deep mathematical structures. This study presents a novel theoretical model for quantum error-correcting codes by integrating the algebraic properties of Eisenstein-Jacobi integers into applied coding theory. Specifically, a residue class ring generated by the product of prime Eisenstein-Jacobi integers is established as the foundational framework, upon which a new class of classical linear codes is constructed. By applying the Calderbank-Shor-Steane (CSS) construction, these classical algebraic structures are mapped to the quantum domain, resulting in the construction of perfect E -error-correcting quantum codes, which introduces a new definition to the existing literature. The presented algebraic methodology establishes a concrete bridge between abstract ring theory and fault-tolerant quantum coding practices, thereby expanding the potential of applied algebra in information theory.

Keywords: Eisenstein-Jacobi integers, CSS construction, Quantum error correction, Applied algebra, Residue class rings.

Acknowledgements: This work was supported by The Scientific and Technological Research Council of Türkiye (TÜBİTAK) with Project Number 125E211. The first author acknowledges the support of TÜBİTAK through the Directorate of Science Fellowships and Grant Program (BİDEB) under the 2211-A National PhD Fellowship Program.

ABSTRACT 25

Deep Reinforcement Learning for QoS-Constrained Control of Unreliable Multi-Server Systems with Working Vacations and Impatient Customers

Nassim Touche¹, Mohamed Boualem², Karima Bouibed¹, Aicha Anzi¹

¹ *University of Bejaia, Faculty of Exact Sciences, Research Unit LaMOS, 06000 Bejaia, Algeria*

² *University of Bejaia, Faculty of Technology, Research Unit LaMOS, 06000 Bejaia, Algeria*

E-mail: nassim.touche@univ-bejaia.dz , mohammed.boualem@univ-bejaia.dz,
karima.bouibed@univ-bejaia.dz, aicha.anzi@univ-bejaia.dz

Abstract: Ensuring quality of service (QoS) in real multi-server systems is challenging due to the simultaneous presence of failures, energy-saving policies, and impatient customer behavior. Analytical queueing models enable the evaluation of steady-state performance in such systems, but remain poorly suited for adaptive control when model complexity increases. In this work, we propose a dynamic control approach based on deep reinforcement learning for an unreliable multi-server system with working vacation policy and customer impatience. Building upon the analytical model of Bouchentouf et al. [11], we formulate the QoS control problem as a constrained Markov decision process. By adopting the Queue-Learning framework of Raeis et al. [9], the probabilistic constraint on end-to-end delay is integrated into the reward function via Lagrangian relaxation. A Deep Deterministic Policy Gradient (DDPG) agent is trained to dynamically adjust service rates. Simulation results show that the learned policy converges stably, strictly satisfies the QoS constraint, and achieves approximately 22% resource cost reduction compared to an analytically optimized static policy. These findings confirm the value of deep reinforcement learning for adaptive control of complex queueing systems under explicit probabilistic constraints.

Keywords: Multi-server queues; Quality of service; Deep reinforcement learning; Constrained Markov decision process; Queue-Learning; Working vacations.

Mathematics Subject Classification: 68T05, 60K25, 90B22, 90C40, 90C15.

References

- [1] M. Raeis, A. Tizghadam, and A. Leon-Garcia, "Queue-Learning: A Reinforcement Learning Approach for Providing Quality of Service," arXiv preprint arXiv:2101.04627, 2021.
- [2] A. A. Bouchentouf, M. Boualem, L. Yahiaoui and A. Hijaz , "A multi-station unreliable machine model with working vacation policy and customers' impatience," *Quality Technology & Quantitative Management*, vol. 19, no. 6, 2022, pp. 766-796.
- [3] M. Tang and V. W. S. Wong, "Deep reinforcement learning for task offloading in mobile edge computing systems," *IEEE Transactions on Mobile Computing*, vol. 21, no. 6, 2022, pp. 1985-1997.

ABSTRACT 26

Modeling and optimization of airport runway capacity: A multi-server retrial queueing approach enhanced by artificial intelligence**Louiza BERDJOUJ***University of A. Mira of Bejaia, Research Unit LaMOS, Bejaia, Algeria*

E-mail: louiza.berdjoudj@univ-bejaia.dz

Abstract: This paper addresses the challenge of optimal runway dimensioning in aerodromes using retrial queueing theory. Unlike classical models, the $M/G/s/K$ retrial system accurately captures the dynamics of holding patterns, where aircraft unable to land immediately join a virtual orbit to retry their approach after a random delay. We develop a mathematical framework based on Quasi-Birth-and-Death (QBD) processes to derive the system's stationary distribution. Furthermore, we explore how Artificial Intelligence (AI) integrates into this stochastic framework to transition from static modeling to dynamic, predictive management. An optimization analysis is conducted to determine the number of runways s that minimizes a global cost function, including infrastructure expenses, airline delay costs, and diversion penalties. Numerical results demonstrate that accounting for the retrial rate and service time variance is crucial to avoid underestimating airport congestion.

Keywords: Retrial Queues, Runway Optimization, Stochastic Processes, Artificial Intelligence.

References

- [1] J.R. Artalejo. *Accessible bibliography on retrial queues: Progress in 2000–2010*. Mathematical and Computer Modelling, 51(9-10), 1071-1081, 2010.
- [2] C. Barnhart, and P. Belobaba. *Air Transport Optimization*. Handbook of Operations Research and Management Science, 2012.
- [3] A. Jacquillat, and A. R. Odoni. *An integrated scheduling and capacity management approach to airport congestion*. Transportation Science, 49(4), 816-834, 2015.
- [4] T. Phung-Duc. *Retrial queues with a finite number of sources and finite waiting seats*. Annals of Operations Research, 247(1), 3-34, 2017.

ABSTRACT 27

Dynamic bayesian game model for Ad hoc network security

Karima Bouibed, Aicha Anzi and Nouredine Khimoum

University of Bejaia, LaMOS Research Unit, Operational Research Department 06000 Bejaia, Algeria.

E-mail: karima.bouibed@univ-bejaia.dz

University of Bejaia, LaMOS Research Unit, Operational Research Department 06000 Bejaia, Algeria.

E-mail: aicha.anzi@univ-bejaia.dz

University of Bejaia, 06000 Bejaia, LaMOS Research Unit, Operational Research Department Algeria.

E-mail: noureddine.khimoum@univ-bejaia.dz

Abstract: Ad hoc networks present significant security challenges due to their decentralized architecture, dynamic topology, and absence of centralized control, which make them highly susceptible to a wide range of cyber threats. In this paper, we propose a novel security framework based on dynamic Bayesian games to model the strategic interactions between attackers and defenders in such uncertain environments. The proposed model incorporates incomplete information, allowing each player to maintain and update beliefs about the opponent's type based on the history of observed actions. By integrating sequential decision-making and belief updating, the framework enables both attackers and defenders to dynamically adapt their strategies and select optimal responses over time. Compared with traditional static game-theoretic approaches, the proposed model provides a more realistic and flexible representation of adversarial behavior in ad hoc networks. This work contributes to improved threat modeling and supports the development of more adaptive and intelligent defense mechanisms for securing decentralized wireless networks.

Keywords: Ad hoc Networks Security, Dynamic Bayesian Games, Intrusion Detection, Attacker-Defender Interaction.

Mathematics Subject Classification: 91Axx, 91A10, 91A06.

References

- [1] S. Zamir. *Bayesian Games: Games with Incomplete Information*. In: Sotomayor, M., Perez-Castrillo, D., Castiglione, F. (eds) *Complex Social and Behavioral Systems*. Encyclopedia of Complexity and Systems Science Series. Springer, New York, NY. <https://doi.org/10.1007/978-1-0716-0368-029>, 2020.
- [2] A. Meiwitz: *On the existence of equilibria to Bayesian games with non finit type and action space*, *Economics Letters*, 78(2): 213-218, 2003.

ABSTRACT 28

On the equivalence to Smith form of multivariate polynomial matrices

Mohamed S. Boudelloua

University of Constantine3, Dept. of Process Engineering, Constantine, Algeria

E-mail: mohamed.boudelloua@univ-constantine3.dz

Abstract: WIn this paper we give a new necessary and sufficient condition under which a class of multivariate polynomial matrices are equivalent to their Smith normal form. The class of polynomial matrices involved is one which corresponds to linear functional systems which can be reduced to a single equation in one unknown function.

Keywords: Polynomials and rational functions of several complex variables, Transformations.

Mathematics Subject Classification: 32A08 , 93B17.

References

- [1] M.S. Boudelloua. Further results on the equivalence to Smith form of multivariate polynomial matrices. *Control and Cybernetics*, 42(2):543–551, 2013.
- [2] M.S. Boudelloua and A. Quadrat. Serre’s reduction of linear functional systems. *Mathematics in Computer Science*, 4(2):289–312, 2010.
- [3] M. G. Frost and M. S. Boudelloua. Some further results concerning matrices with elements in a polynomial ring. *International Journal of Control*, 43(5):1534–1555, 1986.
- [4] M. G. Frost and C. Storey. Equivalence of a matrix over $R[s, z]$ with its Smith form. *International Journal of Control*, 28(5):665–671, 1978.
- [5] J. Guan, J. Liu, D. Li, and T. Wu. Further results on equivalence of multivariate polynomial matrices. *arXiv preprint*, June 2024.
- [6] E.B. Lee and S.H. Zak. Smith forms over $\mathbb{R}[z_1, z_2]$. *IEEE Trans. Autom. Control*, 28(1):115–118, 1983.
- [7] Z. Lin, M.S. Boudelloua, and L. Xu. On the equivalence and factorization of multivariate polynomial matrices. In *Proceedings of the 2006 international symposium of circuits and systems*, Island of Kos (Greece), 2006.
- [8] D. Lu, D. Wang, F. Xiao, and X. Zheng. On the equivalence problem of smith forms for multivariate polynomial matrices. *arXiv preprint*, July 2024.
- [9] A.C. Pugh, S.J. McInerney, M.S. Boudelloua, D.S. Johnson, and G.E. Hayton. A transformation for 2-D linear systems and a generalization of a theorem of Rosenbrock. *Int. J. Control*, 71(3):491–503, 1998.
- [10] A.C. Pugh, S.J. McInerney, and E.M.O. El-Nabrawy. Zero structures of n -D systems. *Int. J. Control*, 78(4):277–285, 2005.
- [11] A.C. Pugh, S.J. McInerney, M. Hou, and G.E. Hayton. A transformation for 2-D systems and its invariants. In *Proceedings of the 35th IEEE conference on decision and control*, pages 2157–2158, Kobe (Japan), 1996.
- [12] H. H. Rosenbrock. *State space and multivariable theory*. Nelson-Wiley, London, New York, 1970.

ABSTRACT 29

Application of statistical models to identify road risk factors in Algeria

Younes Hidra

*LIMED Laboratory, Faculty of Economics, Business, and Management Sciences, University of Bejaia,
06000 Bejaia, Algeria*

E-mail: younes.hidra@univ-bejaia.dz

Abstract: This study aims to analyze and quantify the influence of the main road risk factors on the number of accidents recorded annually in Algeria through a statistical modeling approach. First, a multivariate quantitative model is implemented to assess the impact of variables related to user behavior, vehicle characteristics, and the road environment, distinguishing between urban and rural contexts. The results show that the determinants of accidents differ according to the considered setting, except for certain common factors, particularly speeding violations and dangerous overtaking. Second, a binary (indicator) variable model is developed to analyze the effect of the implementation of legislative and regulatory measures on accident reduction. The results indicate a positive short-term impact, while highlighting a limited effectiveness in the medium term.

Furthermore, a field survey conducted among drivers in the County of Bejaia allows the inclusion of additional behavioral variables related to road users, traffic police, and stakeholders involved in driver training systems. The results confirm that the absence of prior traffic offenses is associated with a lower accident risk, while insufficient enforcement and certain interactions in mixed traffic (involving pedestrians, heavy goods vehicles, and motorcyclists) act as aggravating factors.

Finally, a logistic modeling approach shows that travel distance represents an exposure factor to accident risk, that recidivism related to negligent behavior significantly increases this risk, and that a high driving experience does not necessarily reduce the probability of accidents.

Keywords: Road accidents, modeling, road risk factors, user behavior, Algeria.

Mathematics Subject Classification: 62P20, 62M10.

References

- [1] Bencherif, H, Etude de l'insécurité routière en Algérie : de l'usage des modèles au développement des politiques publiques. Thèse de doctorat, Université de Batna, 2015.
- [2] Bougueroua, M., Carnis, L., Economic development, mobility and traffic accidents in Algeria, Accident Analysis & Prevention, vol.92, 2016.
- [3] CNPSR. Centre national de prévention et de sécurité routière 2004-2012. Etude statistique des accidents de circulation en Algérie, Edition des années 2003, 2004, 2005, 2006, 2007 2008, 2009, 2010, 2011, 2012, 2013, 2014.
- [4] DNSR. Délégation nationale de la sécurité routière 2021-2024. Données statistiques des accidents de circulation en Algérie, Edition des années 2021, 2022, 2023, 2024.
- [5] Gaudry M., Himouri, S., DRAG-ALZ-1, a first model of monthly total road demand, accident frequency, severity and victims, by category and of mean speed on highways, Algeria 1970-2007. Agora Jules Dupuit N° 140, Université de Montréal, p 17, Décembre 2012.

- [6] Gaudry, M., Fournier, F. & Simard R., DRAG-2, un modèle économétrique appliqué au kilométrage, aux accidents et à leur gravité au Québec. Partie 1 : Estimation et analyse du kilométrage et des victimes d'accidents de la route au Québec entre 1957 et 1989. Editions Société de l'Assurance Automobile du Québec, 153 pages, Octobre 1993.
- [7] Hidra. Y, Modélisation des indicateurs du risque routier en Algérie. Thèse de doctorat. Université de Bejaia, 2018.
- [8] Hidra, Y., & Kaid tlilane, N. Impact de la motorisation sur les accidents de la route en Algérie. Etude sur la période 2001-2022. Les cahiers du mecas, 19(1), 79-91. 2023.
- [9] Himouri, S. Modèle théorique de suivi de l'insécurité routière en Algérie 1970-2002. Thèse de Doctorat. Université des Sciences et Technologies Oran 2005.
- [10] Jaeger, L., L'évaluation du risque dans le système des transports routiers par le développement du modèle TAG. Thèse de Doctorat de Sciences Économiques, Faculté des Sciences Économiques et de Gestion, Université Louis Pasteur, Strasbourg. 1997.
- [11] OMS, Rapport sur la situation de la sécurité routière dans le monde 2015. Rapport 2015.

ABSTRACT 30

Intelligent RP Selection and Path Planning for Mobile Gateways in Energy-Constrained IoT

Louiza Belkhiri-Brahmi¹, Saloua Brahmi¹, Belkacem Brahmi¹

¹*Research Unit LaMOS (Modeling and Optimization of Systems), Faculty of Exact Sciences, Bejaia University, 06000, Bejaia, Algeria*
E-mail: louiza.belkhiri@univ-bejaia.dz

Abstract: The increasing proliferation of latency-sensitive Internet of Things (IoT) applications has established Fog Computing as a fundamental paradigm. This paradigm is used to bridge the gap between resource-constrained edge devices and remote Cloud infrastructures.

This study proposes an intelligent data collection strategy using a mobile gateway directed by a Q-Learning algorithm. The suggested method proactively optimizes rendezvous point selection and trajectory planning based on real-time network states, such as device energy levels and proximity, by modeling the gateway as an autonomous agent within the fog layer.

By reducing communication latency and energy consumption at the device level, the proposed approach aims to improve the Quality of Service (QoS) and extend the network lifetime. Simulation results demonstrate that the Q-learning-based mobile gateway approach outperforms comparative models and conventional static fog deployments in terms of Packet Delivery Ratio (PDR), end-to-end delay, and balanced energy consumption across the IoT network.

Keywords: Internet of Things (IoT), Mobile Gateway, Q-Learning, Energy Harvesting, QoS.

Mathematics Subject Classification: 68M1, 68M20.

References

- [1] Chen W, Qiu X, Cai T, Dai HN, Zheng Z and Zhang Y, "Deep reinforcement learning for Internet of Things: A comprehensive survey". In: IEEE Communications Surveys & Tutorials, vol. 23, no. 3, pp. 1659–1692, 2021.
- [2] Mehto A, Verma RK. and Jain S. , "Efficient trajectory planning and route adjustment strategy for mobile sink in WSN assisted IoT". In: 2022 IEEE Region 10 Symposium (TENSYP), Mumbai, India, 2022.
- [3] Belkhiri-Brahmi, L. and Yessad, S. and Semchedine, F. "Congestion control-based sink MOBility pattern for data gathering optimization in WSN". J Supercomput, 80, pp. 3441–3479, 2024.
- [4] Pandey OJ, Yuvaraj T, Paul JK, Nguyen HH, Gundepudi K and Shukla MK, "Improving energy efficiency and QoS of LPWANs for IoT using Q-learning based data routing". In: IEEE Transactions on Cognitive Communications and Networking, vol. 8, no. 1, pp. 365–379, 2021.
- [5] Bhimshetty S and Ikechukwu AV, "Energy-efficient deep Q-network: reinforcement learning for efficient routing protocol in wireless Internet of Things". In: Indonesian Journal of Electrical Engineering and Computer Science, vol. 33, no. 2, pp. 971–980, 2024.

ABSTRACT 31

Analysis of deformation instability during severe shear of the PP during the ECAE process using DIC algorithms

Boulahia Ramdane, Boukharouba Taoufik

USTHB, Algiers, Algeria

E-mail: ramdane.boulahia@usthb.edu.dz, taoufik.boukharouba@usthb.edu.dz

Abstract: Equal channel angular extrusion (ECAE) is one of the most efficient methods of severe plastic deformation of materials used to modify texture and microstructure without reducing the sample cross section. The application of a single ECAE pass to polypropylene (PP) was meticulously investigated at room temperature using a 90° die-angle tooling. Mechanical properties changes of the extruded samples due to backpressure and extrusion velocity effects were analyzed via uniaxial tensile tests. The tensile samples displayed multiple strain localizations in shear banded materials, whereas quite homogeneous deformation appeared for nonbanded ones. The digital image correlation technique suitable for large deformation was used to determine the full-field strain of the tensile samples in relation to the tensile strain and ECAE conditions. DIC measurements require an artificial random speckle pattern which was generated by green dots sprayed on the surface of each sample. The random speckle pattern was applied so that the speckles did not overlap. The studied zone was illuminated by a strong white-light beam. The illuminated random speckle pattern was captured during the deformation by a digital CCD camera placed in front of the sample and at a distance of 0.5 m from the sample. The images of the the sample surface were recorded at a frequency of 2 Hz. DIC method is based on comparing images of the sample surface in the undeformed (reference) and deformed states. In this way, a Lagrangian description is performed. A 10 image step was used for the image correlation. The zone of interest was divided into small square sub images of 64x64 pixels. The displacement vector was calculated using the corresponding sub-image pairs extracted from the reference and deformed states of the sample. By achieving the analysis on numerous subimages, the full-field contours of displacement were obtained. The analysis was performed with Davis software developed by Lavision.

Keywords: Equal channel angular extrusion, Large deformation, Digital image correlation.

References

- [1] Ramdane Boulahia, Fahmi Zaïri, Yuri Voznyak, and Jean-Michel Gloaguen. *Repeated equal-channel angular extrusion of polypropylene: Processing routes and back-pressure influence*. Mater. Today Commun, 26, 101754, (2021).
- [2] R. BOULAHIA, T. BOUKHAROUBA, J.M. GLOAGUEN. *Analyse au moyen de la corrélation d'images numériques de la déformation du Polypropylène extrudé dans une matrice ECAE à 135°*. 22ème Congrès Français de Mécanique, 24 au 28 Août 2015, Lyon-France.
- [3] R.BOULAHIA, T.BOUKHAROUBA, J-M GLAOGUEN, *The use of the DIC method to involve the strain instability occurred in an undergoing high shear during the ECAE process*. Design and Modeling of Mecahanical Sytems. LNME, Springer–Verlag Berlin Heidelberg, 567-574, (2013).
- [4] Qifeng Jiang, Ramdane Boulahia, Fahmi Zaïri, Iurii Vozniak, Zhengwei Qu, Jean-Michel Gloaguen and Xiaobing Liu. *Microstructure and Mechanical Properties of Severely Deformed Polypropylene in ECAE (Equal Channel Angular Extrusion) via Routes A and C*. Polymers, 14, 5287 (2022).

ABSTRACT 32

A Primal Simplex Algorithm for Nonsmooth Support Vector Regression

Belkacem BRAHMI

LaMOS Unit Research, Department of Operations Research, University of Bejaia, 06000, Bejaia, Algeria
 E-mail: belkacem.brahmi@univ-bejaia.dz

Abstract: Support Vector Regression (SVR) is a widely used and powerful method for nonlinear regression tasks. However, its application to large-scale problems remains challenging due to the high computational cost associated with its classical dual formulation, which requires $2n$ variables compared to the n variables used in standard SVM classification. In this paper, we propose PSA-SVR, a novel primal simplex algorithm that extends a recent primal approach for SVM classification to the regression setting.

PSA-SVR directly solves the nonsmooth dual SVR formulation that requires only n variable. It iteratively constructs a sequence of basic feasible solutions that converge to a global optimum while maintaining primal feasibility through efficient rank-one updates of the nonsmooth basis. A key advantage of our method is that, unlike classical SVR solvers that solve quadratic programming (QP) problems with $2n$ variables, PSA-SVR solves a reduced quadratic program with only n variables. This drastically reduces memory requirements and computational complexity, making it suitable for large-scale real-world regression tasks.

Extensive experiments on large-scale datasets demonstrate that PSA-SVR converges significantly faster than state-of-the-art solvers while delivering competitive or superior predictive accuracy, offering a scalable and practical alternative for nonlinear regression in data mining applications.

Keywords: Support vector regression, quadratic programming, nonsmooth basis, primal simplex method, basic feasible solution.

Mathematics Subject Classification: 90C20, 68T09.

References

- [1] N. Cristianini, J. Shawe-Taylor, "Introduction to Support Vector Machines and other kernel-based learning methods", Cambridge University Press, United Kingdom, 2000.
- [2] B. Brahmi, "An Efficient Primal Simplex Method for Solving Large-Scale Support Vector Machines", Neurocomputing, 599 (2024), Art. 128109.
- [3] S. Azib and B. Brahmi. "A working set algorithm for support vector regression problems", Journal of Numerical Analysis and Approximation Theory, Vol. 54 (2): 211-228, 2025.
- [4] D. R. Musicant and A. Feinberg, "Active Set Support Vector Regression", IEEE Transactions on Neural Networks, 15(2) (2004), pp. 268-275
- [5] B. Brahmi, M. O. Bibi, "Dual support method for solving convex quadratic programs", Optimization 59, pp. 851-872, 2010.
- [6] C. C. Chang and C. J. Lin, "LIBSVM: A Library for Support Vector Machines", ACM Transactions on Intelligent Systems and Technology, 2(3) (2011), pp. 1-27.

ABSTRACT 33

Cubical Dimension of recursive Binary Trees into Hypercubes

Kamal KABYL

University of Bejaia, Faculty of Technology, Research Unit LaMOS, 06000 Bejaia, Algeria
 E-mail: kamal.kabyl@univ-bejaia.dz

Abstract: The hypercube of dimension n , denoted Q_n is the graph whose 2^n vertices are the Boolean vectors of length n , such that two vertices are adjacent if and only if they differ in exactly one coordinate. An embedding of a graph G into a graph H is a mapping from $V(G)$ into $V(H)$ that preserves adjacency of vertices. Our particular interest concerns the case where H is a Hypercube and G is a recursive binary tree. The problem of embedding trees into the hypercube is much studied in the graph theory. In fact many efforts have been devoted to find sufficient conditions for which a tree T is a sub-graph of the hypercube Q_n . The problem consists of giving the smallest dimension of the hypercube in which a given tree T is embeddable. In this paper, we introduce new classes of recursive binary trees and we determine their cubical dimension.

Keywords: Binary Tree, Cubical dimension, Hypercube.

References

- [1] I. Havel and J. Mor'avek. *B-valuation of graphs*. Czech. Math. J, 22 (2), 338-351, 1972.
- [2] L. Nebesk'y. *On cubes and dichotomic trees*. Časopis Pěst. Mat, 99 (2), 164- 167, 1974.
- [3] I. Havel. *On hamiltonian circuits and spanning trees of hypercubes*. Časopis Pěst. Mat, 109 (2), 135-152, 1984.
- [4] S.A. Choudum and S. Lavanya. *Embedding a subclass of trees into hypercube*. Discrete Math. 311 (10-11), 866-871, 2011.
- [5] K. Kabyl, A. Berrachedi and É . Sopena. *A note on the cubical dimension of new classes of binary trees*. . Czech. Math. J. 65 (1), 151-160, 2015.

ABSTRACT 34

Minimizing Empty Miles in Vehicle Routing: A Lookahead Heuristic Approach**Noureddine KHIMOUM, Aicha ANZI and Karima BOUIBED**

University of Bejaia, Faculty of Exact Sciences, Research Unit LaMOS, Algeria,
E-mail: noureddine.khimoum@univ-bejaia.dz and aicha.anzi@univ-bejaia.dz and
karima.bouibed@univ-bejaia.dz

Abstract: This paper investigates a variant of the Vehicle Routing Problem (VRP) arising in real-world logistics, where the primary objective is to minimize empty travel distances between delivery operations while satisfying delivery demands between loading sites (clients) and unloading points (sales locations). The problem is modeled and defined on a bipartite structure linking loading sites (clients) and unloading points (sales locations), with an inherent alternation between loaded and empty trips. An exact formulation based on mixed-integer linear programming is first proposed to obtain optimal solutions for small instances. The MILP captures flow conservation, demand satisfaction, and route feasibility under distance constraints. To overcome scalability limitations, a constructive heuristic enhanced with a lookahead mechanism is developed. This approach anticipates future routing decisions to improve global solution quality.

Keywords: Integer Linear Programming, Lookahead Approach, Heuristic, Vehicle Routes.

Mathematics Subject Classification: 12X34, 56Y78.

References

- [1] Golden, Bruce L. and Raghavan, S. and Wasil, Edward A., "The Vehicle Routing Problem: Latest Advances and New Challenges", Springer, 2008.
- [2] Accorsi, L., Vigo, D., "A fast and scalable heuristic for the solution of large-scale capacitated vehicle routing problems", *Transportation Science*, 55, pp. 832-856, 2021.
- [3] Urquhart, N., "Heuristics for the Vehicle Routing Problem. In: *Nature Inspired Optimisation for Delivery Problems*", *Natural Computing Series*, Springer, 2022.
- [4] David Pisinger and Stefan Ropke, "A general heuristic for vehicle routing problems", *Computers and Operations Research*, 34(8), pp. 2403-2435, 2007.
- [5] Toth, Paolo and Vigo, Daniele, "The Vehicle Routing Problem", *Monographs on Discrete Mathematics and Applications*, SIAM, 2002.
- [6] Azi, N., Gendreau, M., Potvin, J.Y., "An exact algorithm for a vehicle routing problem with time windows and multiple use of vehicles", *European Journal of Operational Research* 202, pp. 756-763, 2010.

ABSTRACT 35

Evaluation and modeling of hospital costs for lung cancer management in Algeria

Moussa MESSAILI^{1,2}, Younes HIDRA^{1,2}, Smail CHALANE^{1,2}, Nabila KENDI^{1,2}, Houda BOUKHERIS³

¹*Medical Informatics and Dynamic and Intelligent Environments Laboratory (LIMED)*

²*Faculty of Economic Sciences, Commercial Sciences and Management Sciences, Abderrahmane Mira University, Bejaia, 06000, Algeria*

³*Faculty of Medicine, Abderrahmane Mira University, Bejaia, 06000, Algeria.*

E-mail: younes.hidra@univ-bejaia.dz, moussa.messaili@univ-bejaia.dz, chalane.smail@univ-bejaia.dz, nabila.kendi@uni-bejaia.dz

Abstract: Lung cancer is a major public health challenge, with a burden—both human and economic—that is constantly increasing worldwide. According to GLOBOCAN (2022) data, it is the most frequent cancer (2.48 million new cases) and the leading cause of cancer death (1.81 million deaths annually). In Algeria, the situation is equally alarming; according to the same source, the incidence of lung cancer is 11.4 new cases per 100,000 inhabitants, ranking as the 4th most frequent cancer in the country. Objective: The objective of this study is twofold: first, to conduct an evaluation of the direct costs of hospital management for lung cancer, and second, to model their determinants. Methodology: We conducted a retrospective study based on the exploitation of data from medical records of 100 and 56 lung cancer patients, respectively, at the Amizour Public Hospital and Bejaia University Hospital, followed between 2023 and 2024. For the statistical modeling of direct hospital cost determinants, we opted for a statistical analysis based on the Generalized Linear Model (GLM), primarily using Analysis of Variance (ANOVA) and Analysis of Covariance (ANCOVA). Results: Chemotherapy represented the main expenditure item, reaching up to 89% of the total cost for patients in advanced stages. Statistical modeling showed that advanced age, advanced disease stage, and expensive chemotherapy protocols were significantly associated with an increase in hospital spending ($p < 0.05$). Conclusion: This study highlights the significant economic burden of lung cancer on Algerian public hospitals and the need to adopt new, more cost-effective management methods. It also emphasizes the implementation of a prevention strategy based on early screening, not only to increase patient survival rates but also to alleviate the economic burden this disease places on the healthcare system.

Keywords: Lung cancer, Hospital costs, Algeria, Generalized Linear Model (GLM).

References

- [1] Boumehdi, B., & Boukhalfa, C. (2022). Le cancer du poumon attribuable au tabac : Analyse du coût. *International Journal of Accounting, Finance, Auditing, Management & Economics*, 3(3), 404–419.
- [2] Freddie Bray BSc, MSc, PhD1 and All Global cancer statistics: GLOBOCAN estimates of incidence and mortality worldwide for 36 cancers in 185 countries. 2022. DOI: 10.3322/caac.21834. 12 February 2024.
- [3] Kara L ; Meguenni.K ; Bouamra.A. Données du Réseau National des Registres du Cancer 2021-2022. 30/10/2024.

- [4] Mahmood Yousefi, Habib Jalilian, Somayeh Heydari, Farshad Seyednejad, Nazanin Mir, "Cost of Lung Cancer: A Systematic Review, Value in Health Regional Issues, Volume 33, 2023, Pages 17-26. <https://doi.org/10.1016/j.vhri.2022.07.007>.
- [5] Vergnenegre, A., et al. (2005). Les composantes du coût des stratégies de prise en charge du cancer du poumon en France. *Revue des Maladies Respiratoires*.

ABSTRACT 36

A mathematical model for transport plan optimization and mileage reduction**Anzi AICHA¹, Karima BOUIBED¹, Nouredine KHIMOUM¹**

¹*University A. Mira, Department of Operations Research, LaMOS Research Unit, Bejaia, Algeria*
E-mail: aicha.anzi@univ-bejaia.dz, karima.bouibed@univ-bejaia.dz, nouredine.khimoum@univ-bejaia.dz

Abstract: In the field of distribution logistics, optimally planning vehicle routes is a critical challenge for transport companies, particularly in their efforts to reduce empty trips, which lead to significant additional costs. This work addresses this issue by introducing an optimization model based on Vehicle Routing Problems and inspired by the real operational practices of an algerian logistic company having a single depot and multiple clients located across the national territory. Vehicle routes do not necessarily start or end at the depot. Vehicle capacity constraints are not applicable since orders are counted by vehicle rather than by load.

The aim is to develop a vehicle routing model adapted to the company's specific constraints, taking into account route flexibility (routes that do not necessarily start or end at the depot), driving time limitations, and, most importantly, the reduction of empty kilometers traveled. By incorporating these factors, the model seeks to enhance the overall efficiency of the transport plan while maximizing delivery profitability. To formulate the mathematical model, we need to introduce some notations. This study addresses the problem of reducing empty trips in road transport through route optimization. For this, a VRP-based mathematical program, including the empty trips constraints, is proposed. Computational experiments are conducted on real company data and the results are compared with the company real transportation plan.

Keywords: Transport Optimization, Vehicle Routing Problem, Empty Trips, Logistics Planning, Mathematical Modeling.

Mathematics Subject Classification: 90B06, 90C11.

References

- [1] Elatara, S., Abouelmehdib, K., Riffia, M.S. The vehicle routing problem in the last decade: variants, taxonomy and metaheuristics, *Procedia Computer Science*, Vol. 220, pp. 398 - 404, 2023.
- [2] Ghiani, G., Laporte, G., Musmanno, R. *Introduction to logistics systems management*, Wiley, Second Edition, 2013.
- [3] Kumar, S.N., Panneerselvam, R., A survey on the vehicle routing problem and its variants, *Intelligent Information Management*, Vol. 4, pp. 66 - 74, 2012.
- [4] Laporte, G. The vehicle routing problem: An overview of exact and approximate algorithms, *European Journal of Operational Research*, Vol. 59, pp. 345 - 358, 1992.

ABSTRACT 37

Geometric Deformations of BTZ Black Hole Thermodynamics: Fractal Horizon Structure and Lorentz-Violating Riemannian Geometry in (2+1) Dimensions

Prince A. Ganai¹, Khaja Zuber Amin¹

¹ *Department of Physics, National Institute of Technology, Srinagar, Kashmir-190006, India*
E-mail: princeganai@nitsri.ac.in, khajazuberamin@gmail.com

Abstract: We study the thermodynamic phase structure of the (2+1)-dimensional Bañados–Teitelboim–Zanelli (BTZ) black hole within a geometrically deformed gravitational framework that incorporates two independent modifications to the underlying Riemannian structure: (i) a Lorentz symmetry-breaking bumblebee vector field that alters the effective metric geometry and curvature coupling, and (ii) Barrow’s fractal entropy ansatz, which replaces the classical area law with a power-law dependence parametrised by a fractal deformation index $\Delta \in [0, 1]$, encoding quantum-gravitational corrections to the horizon geometry. Working in the York cavity formalism—a boundary-value framework in which the black hole is enclosed within a finite isothermal cavity of radius R —we derive the corrected thermodynamic potentials (entropy, Helmholtz and Gibbs free energies, internal energy, enthalpy, pressure, and heat capacity) as functions of the horizon radius, the Lorentz-violation parameter l_b , the fractal index Δ , and the cavity radius R . Our analysis reveals that l_b acts as a linear rescaling of the Riemannian metric structure, modifying thermodynamic quantities without altering their functional form, whereas Δ introduces genuinely non-linear, power-law deformations that reshape the phase structure, shift stability thresholds, and generate new critical behaviour absent in the classical geometry. The interplay between these two geometric parameters produces a rich two-parameter family of thermodynamic configurations with distinct stability domains and phase transitions regulated by the cavity boundary. These results demonstrate how geometric and analytical methods—Riemannian curvature, fractal geometry, and boundary-value techniques—can be combined to probe quantum-gravitational modifications of black hole thermodynamics in lower-dimensional spacetimes, with implications for holographic correspondences and mathematical models of gravitational phase transitions.

Keywords: BTZ black hole, Riemannian geometry, Barrow entropy, fractal horizon, bumblebee gravity, Lorentz symmetry breaking, York cavity formalism, black hole thermodynamics, phase transitions.

Mathematics Subject Classification: 83C57, 83C75, 53Z05, 83C80, 83D05.

References

- [1] S. W. Hawking, Particle Creation by Black Holes, *Comm. Math. Phys.* **43**, 199 (1975).
- [2] J. D. Bekenstein, Black Holes and Entropy, *Phys. Rev. D* **7**, 2333 (1973).
- [3] M. Bañados, C. Teitelboim, J. Zanelli, Black Hole in Three-Dimensional Spacetime, *Phys. Rev. Lett.* **69**, 1849 (1992).
- [4] M. Bañados, M. Henneaux, C. Teitelboim, J. Zanelli, Geometry of the 2+1 Black Hole, *Phys. Rev. D* **48**, 1506 (1993).
- [5] E. N. Saridakis, Barrow holographic dark energy, *Phys. Rev. D* **102**, 123525 (2020).

- [6] V. A. Kostelecký, S. Samuel, Spontaneous Breaking of Lorentz Symmetry in String Theory, *Phys. Rev. D* **39**, 683 (1989).
- [7] J. W. York, Black-hole thermodynamics and the Euclidean Einstein action, *Phys. Rev. D* **33**, 2092 (1986).
- [8] S. Carlip, The (2+1)-Dimensional Black Hole, *Class. Quantum Grav.* **12**, 2853 (1995).
- [9] R. M. Wald, The Thermodynamics of Black Holes, *Living Rev. Relativ.* **4**, 6 (2001).
- [10] J. D. Barrow, The Area of a Rough Black Hole, *Phys. Lett. B* **808**, 135643 (2020).
- [11] J. P. S. Lemos, Black holes and thermodynamics in $2 + 1$ dimensions, *Phys. Lett. B* **353**, 46 (1996).
- [12] M. H. Dehghani, S. H. Hendi, Thermodynamic schemes of charged BTZ-like black holes in modified gravity, *Results Phys.* **47**, 106047 (2023).
- [13] H. Quevedo, Geometric description of BTZ black hole thermodynamics, *Phys. Rev. D* **79**, 024012 (2009).
- [14] V. A. Kostelecký, R. Lehnert, Stability, causality, Lorentz and CPT violation, *Phys. Rev. D* **63**, 065008 (2001).
- [15] C. Teitelboim, Action and Entropy of extremal and nonextremal black holes, *Phys. Rev. D* **51**, 4315 (1995).

ABSTRACT 38

ANFIS-Based Control of a Nonlinear Three-Tank System: Modeling and Simulation Study

Abderrahmen Bouguerra¹, Keltoum Loukal¹

¹ *LGE Research Laboratory, Department of Electrical Engineering, University of M'sila, University Pole, Bordj Bou Arreridj, Road, M'sila 28000, Algeria*

E-mail: abderrahmen.bouguerra@univ-msila.dz, keltoum.loukal@univ-msila.dz

Abstract: This study investigates the application of an Adaptive Neuro-Fuzzy Inference System (ANFIS) for the modeling and control of a nonlinear coupled three-tank process. Initially, a mathematical model based on mass balance principles is developed to accurately describe the system's nonlinear dynamics and inter-tank interactions. Conventional control strategies often yield suboptimal performance due to the strong nonlinearities and coupling effects inherent in such systems. To address these challenges, an ANFIS-based controller is designed to achieve adaptive and reliable liquid level regulation. The proposed approach combines the linguistic reasoning of fuzzy logic with the learning capability of neural networks. Simulation results demonstrate that the ANFIS controller outperforms traditional methods in terms of tracking accuracy, overshoot reduction, and settling time. Moreover, the controller exhibits strong robustness against disturbances and parameter variations. These findings confirm the effectiveness of ANFIS in controlling complex nonlinear processes.

Keywords: ANFIS, Nonlinear Systems, Three-Tank Process, Mathematical Modeling, Intelligent Control.

Mathematics Subject Classification: 93-10, 60G65.

References

- [1] M. Ibrahim; M. Hussein; M. A. H. Abozied et al., Enhancing four-tank system through model predictive control-based adaptive neuro-fuzzy system, *International Journal of Fuzzy Systems*, (2025).
- [2] S. Kumar; M. Das; A. K. Karn, ANFIS robust control application and analysis for load frequency control with nonlinearity, *Journal of Electrical Systems and Information Technology*, (2024).
- [3] S. Ramadan et al., Solid oxide fuel cell control strategy enhancement by adaptive neuro-fuzzy inference system, *Scientific Reports*, (2024).
- [4] H. Espitia; I. Mach'on; H. L'opez, Proposal of a compact neuro-fuzzy adaptive controller for filling regulation of two coupled spherical tanks, *International Journal of Fuzzy Systems*, (2024).
- [5] C. Urrea; Y. Garcia-Garcia, Design and performance analysis of level control strategies in a nonlinear spherical tank, *Processes*, (2023).

ABSTRACT 39

Modeling and Control of Nonlinear Magnetic Levitation Systems Using Interval Type-2 Fuzzy Logic

Keltoum Loukal¹, Abderrahmen Bouguerra¹

¹*LGE Research Laboratory, Department of Electrical Engineering, University of M'sila, University Pole, Bordj Bou Arreridj, Road, M'sila 28000, Algeria*

E-mail: keltoum.loukal@univ-msila.dz, abderrahmen.bouguerra@univ-msila.dz

Abstract: Magnetic levitation (MagLev) systems require advanced control strategies due to their inherent instability and strong nonlinear behavior. This paper presents a comprehensive mathematical model describing the electromechanical dynamics of the MagLev system. System uncertainties and external disturbances are explicitly incorporated into the modeling framework. To address these challenges, a Type-2 Fuzzy Logic Controller (T2FLC) is designed to achieve robust stabilization. Unlike Type-1 fuzzy systems, the Type-2 approach effectively handles uncertainty in membership functions, leading to improved robustness. Simulation results demonstrate enhanced performance in terms of stability, disturbance rejection, and overall robustness. The proposed control strategy shows strong potential for real-time applications in nonlinear engineering systems.

Keywords: MagLev, Mathematical Modeling, Type-2 Fuzzy Logic Controller, Nonlinear Control Systems, Uncertainty Handling.

Mathematics Subject Classification: 93-10, 60G65.

References

- [1] S. I. Mohammad; M. A. Rahman; K. Ahmed, Robust speed and levitation control of high-speed trains using TSK type-2 fuzzy sliding mode strategy, *Scientific Reports*, 24846, (2025).
- [2] D. Sain, T. H. Lee, Novel mathematical modeling of IT2 Takagi–Sugeno fuzzy PID controllers with triangular FoUs and their application: A MagLev case study, *Applied Soft Computing*, 11110, (2026).
- [3] M. Abdollahzadeh, M. Pourgholi, Adaptive fuzzy sliding mode control of magnetic levitation system based on Interval Type-2 Fuzzy Neural Network Identification with an Extended Kalman–Bucy filter, *Engineering Applications of Artificial Intelligence*, 107645, (2024).
- [4] X. Zhou; L. Chen; Y. Li, A data-driven comprehensive evaluation method for electromagnetic suspension maglev control system, *Actuators*, 13(8), 314, (2024).
- [5] T. Zhang, S. Jiang, D. Shen, H. Xu , Adaptive finite-time fuzzy control for hybrid levitation system of maglev trains with active anti-lock constraints, *Journal of the Franklin Institute*, 360(4), 2635-2659, (2023).

ABSTRACT 40

Numerical Study of Mixed Convection Cooling of a Heat-Generating Element in an MHD Nanofluid-Filled Open Cavity

Messaoud HAMDI¹, Adel SAHI²

¹*Laboratoire de Physique Théorique, Faculté des Sciences Exactes, Université de Bejaia, 06000 Bejaia, Algérie*

²*Laboratoire de Mécanique Matériaux et Energétique, Faculté de Technologie, Université de Bejaia, 06000 Bejaia, Algérie*

E-mail: messaoud.hamdi@univ-bejaia.dz, adel.sahi@univ-bejaia.dz

Abstract: The efficiency of computer and electronic equipment is closely linked to the design of heat sinks. To remove heat from electronic equipment and reach an appropriate temperature, various techniques remain. In this context, the present study is devoted to the problem of cooling a heat source, by two-dimensional modeling of the dissipation performance of a solid element subjected to internal heat generation. The numerical simulation is based on the finite volume method of mixed convection inside an open cavity containing the heating element and filled with a nano-fluid of ferrous character, the whole is traversed by a vertical magnetic field. The dimensionless governing equations under Boussinesq's approximations are solved by the SIMPLE algorithm. The effects of the relevant parameters, Reynolds number and Hartmann number are studied by keeping the Rayleigh number constant ($Ra=106$), all this for a fixed size and position of the source. The numerical results are discussed in terms of current lines, isotherms, Nusselt number along the hot surface. The results highlight the performance status of the enclosure and revealed the optimal operating parameters.

Keywords: MHD, Mixed convection, Nanofluid, Heat-generating source, Open cavity.

References

- [1] Sameh E. Ahmed, M.A. Mansour, Ahmed Kadhim Hussein, S. Sivasankaran, Mixed convection from a discrete heat source in enclosures with two adjacent moving walls and filled with micropolar nanofluids, *Eng. Sci. Technol. Int. J.* 19 (2016) 364-376.
- [2] Arash Karimipour, Mohammad Hemmat Esfe, Mohammad Reza Safaei, Davood Toghraie Semiromic, S.N.Kazi, Mixed convection of copperwater nanofluid in a shallow inclined lid driven cavity using the lattice Boltzmann method, *Physica A* 402 (2014) 150-168.
- [3] Khalil M. Khanafer, Abdalla M. Al-Amiri, Ioan Pop, Numerical simulation of unsteady mixed convection in a driven cavity using an externally excited sliding lid, *European Journal of Mechanics B/Fluids* 26 (2007) 669-687.
- [4] M. Sheikholeslami, M. GorjiBandpy, Free convection of ferrofluid in a cavity heated from below in the presence of an external magnetic field, *Powder Technol.* 256(2014)490-498.
- [5] H. Aminfar, M. Mohammadpourfard, S.A.Zonouz, Numerical study of the ferrofluid flow and heat transfer through a rectangular duct in the presence of a non-uniform transverse magnetic field, *J.Magn.Magn.Mater.* 327(2013) 31-42.
- [6] S. V. Patankar, "Numerical heat transfer and fluid flow.," p. 214, 1980.

ABSTRACT 41

Behavior of A Binary Cement Based on Dam Sediments With a Low Kaolin Content

Mekki BIBI¹, Mohamed Aziz CHIKOUCHE², Elhem GHORBEL³

^{1,2}*Laboratory of Materials and Mechanics of structures, Civil engineering Department, Technology Faculty, University of M'sila, University Pole, Road Bordj Bou Arreridj (Algeria)*

³*L2MGC, University of Cergy Pontoise, Cergy Pontoise Cedex, France*

E-mail: mekki.bibi@univ-msila.dz

Abstract: In Algeria, semi-arid regions are characterised by steep slopes and soft rocks. This lithology is periodically subjected to water and wind erosion, thus promoting sediment deposition in dams. Simultaneously, cement plants consume hectares of clay and limestone daily, leading to ecosystem destruction, landscape degradation, and other negative impacts. To address these problems and reduce environmental impact, it is proposed to incorporate dam sediments as a partial substitute for clinker in the production of a binary cement. Heat treatments at 600 and 800 °C disrupted the structure of the phyllosilicates present in the sediments. The different cements exhibited a range of (0.26–0.29) to reach normal consistency, and the setting time of the cements increased by 50 minutes. The compressive strengths of the modified cements met the standards but were lower than the strength of the CEMI. The transformation of kaolin into metakaolin enabled the development of a pozzolan from sediment that had become cumbersome due to repeated storage in special dikes. Improving the fineness of the material or using a chemical activator should be recommended to enhance the study.

Keywords: Eco-cement, Dredged sediment, Heat treatment, Pozzolana.

Mathematics Subject Classification: 12X34, 56Y78.

References

- [1] Khoudour D, Bensefia S, & Bidi Z. (2025). "Soil erosion risk assessment in arid Mediterranean regions using GIS and the MEDALUS Model: The Soubella sub-Catchment, Algeria". *Geodynamics*, 2, 16-28.
- [2] Chikouche M A, Ghorbel E, & Bibi M. (2016). "The possibility of using dredging sludge in manufacturing cements: Optimization of heat treatment cycle and ratio replacement". *Construction and Building Materials*, 106, 330-341.

ABSTRACT 42

Inequalities for Rational Functions in the Complex plane

Idrees Qasim

Department of Mathematics, National Institute of Technology, Srinagar, India, 190006

E-mail: idreesf3@nitsri.ac.in

Abstract: This paper investigates inequalities for rational functions with prescribed poles and zeros located inside or outside a disk of radius k . We extend classical polynomial inequalities due to Govil (1973, 1980) to the setting of rational functions. In particular, we establish a relationship between the maximum modulus of a rational function on the circle of radius k (where $k \geq 1$) and on the unit circle. Our findings refine inequalities for rational functions previously obtained by Li, Mohapatra, and Rodriguez (1993), providing sharper bounds under certain conditions.

Keywords: Bernstein inequality, Rational functions, Poles, Zeros.

Mathematics Subject Classification: 30A10, 30C15.

References

- [1] I. Ahmad, Ab Liman, W.M. Shah, A Note on Rational Functions with Prescribed Poles and Restricted Zeros, Vietnam J. Math. 43 (2015) 159–162.
- [2] A. Aziz, Inequalities for the derivative of a polynomial, Proc. Amer. Math. Soc. 89(2) (1983) 259-266.
- [3] A. Aziz and N.A. Rather, Growth of maximum modulus of rational functions with prescribed poles, Math. Inequal. Appl. 2 (1999) 165-173.
- [4] A. Aziz and W. M. Shah, Some refinements of Bernstein-Type inequalities for rational functions, Glas. Mat. 32(52) (1997) 29-37.
- [5] A. Aziz and B. A. Zarger, Some properties of rational functions with prescribed poles, Canad. Math. Bull. 44 (1999) 417–426.
- [6] S. N. Bernstein, Sur l'ordre de la meilleure approximation des fonctions continues par des polynômes de degré donné, Mem. Acad. R. Belg. 4 (1912) 1–103.
- [7] R.B. Gardner, N.K. Govil, G.V. Milovanović, Extremal Problems and Inequalities of Markov-Bernstein Type for Algebraic Polynomials, Mathematical Analysis and Its Applications. Elsevier/Academic Press, London, 2022.
- [8] A. Giroux, Q.I. Rahman and G. Schmeisser, On Bernstein's Inequality, Can. J. Math. 31(2)(1979) 347-353.
- [9] N.K. Govil, On a theorem of S. Bernstein, J. Math. Phys. Sci. 14 (1980) 183-187.
- [10] N.K. Govil, On the derivative of a polynomial, Proc. Amer. Math. Soc. 41 (1973) 543-546.
- [11] P. D. Lax, Proof of a conjecture of P. Erdős on the derivative of a polynomial, Bull. Amer. Math. Soc. (N.S) 50 (1944) 509-513.
- [12] Xin Li, R. N. Mohapatra and R. S. Rodriguez, Bernstein inequalities for rational functions with prescribed poles, J. London Math. Soc. 51 (1995) 523-531.

- [13] M.A. Malik, On the derivative of a polynomial, J. London Math. Soc. 1 (1969) 57-60.
- [14] A. Mir and S. Hans, Inequalities concerning rational functions in the complex domain, Sib. Math. J. 63 (5) (2022) 1012–1022.
- [15] I. Qasim, A. Liman, Bernstein type Inequalities for Rational Functions, Indian J. Pure Appl. Math. 46(3) (2015) 337-348.
- [16] W.M. Shah, A Generalization of a Theorem of Paul Turán, Journal of Ramanujan Society 1 (1996) 29-35.
- [17] P. Turán, Über die ableitung von polynomen, Compos. Math. 7 (1939) 89-95.

ABSTRACT 43

About Optimality in a Bilinear Programming Problem with Bounded Variables

Kheddouci Ouarda, Khimoum Nouredine, Bibi Mohand Ouamer*LaMOS Research Unit, University of Bejaia, Bejaia, Algeria*E-mail: ouarda.kheddouci@univ-bejaia.dz, nouredine.khimoum@univ-bejaia.dz,
mdouamer.bibi@univ-bejaia.dz

Abstract: The main objective of this work is to develop a constructive method for solving bilinear programming problems with bounded variables by exploiting the particular structure of their feasible region. Bilinear programming problems are known to be non-convex and generally difficult to solve due to the presence of bilinear terms. To address this challenge, we build upon theoretical results, establishing an equivalence between the original bilinear programming problem and an associated concave minimization programming formulation. Using this equivalence, we propose an approach that transforms the initial problem into a more tractable form, allowing for a systematic exploration of the feasible region. The proposed method takes advantage of the geometric and structural properties of the problem to efficiently identify the optimal candidate solutions. This framework provides new insights into the resolution of bounded bilinear programming problems and contributes to the development of more effective solution techniques in non-convex optimization.

Keywords: Bilinear programming, Optimality equivalence, Quadratic programming.

References

- [1] Andjouh, A. and Bibi, M. O.: Adaptive Global Algorithm for Solving Box-Constrained Non-convex Quadratic Minimization Problems. *Journal of Optimization Theory and Applications*, **192**, 360-378 (2022).
- [2] Konno, H.: A Cutting Plane Algorithm for Solving Bilinear Programs. *Mathematical Programming*, **11**(1), 14-27 (1976) .

ABSTRACT 44

Classification of minimal surfaces in Sol_3

Mehraj Ahmad Lone

Department of Mathematics, National Institute of Technology, Hazratbal, Srinagar -190 006, Jammu and Kashmir, India

E-mail: mehrajlone@nitsri.ac.in

Abstract: A significant discovery made in the field of differential geometry is that of a minimal surface. A surface M in E^3 is called minimal if its mean curvature (H) is zero. In other words, a minimal surface is a local area minimizer. . In this paper, we will discuss the classification of surfaces based on the curvature in particular zero mean curvature surfaces (ZMC) using Gauss Map.

Keywords: Affine translation surfaces, Gauss map, Sol_3 .

Mathematics Subject Classification: 53A10, 53A35.

References

- [1] M. Bekkar, Exemples de surfaces minimales dans l'espace de Heisenberg, Rend. Sem. Fac. Sci. Univ. Cagliari, 61(2), 1991, 123–130.
- [2] M. Bekkar and T. Sari, Surfaces minimales réglées dans l'espace de Heisenberg H_3 , Rend. Sem. Mat. Univ. Pol. Torino, 50, 1992, 243–254.
- [3] A. Borisenko and E. Petrov, Surfaces in the three-dimensional Heisenberg group on which the Gauss map has bounded Jacobian, Mathematical Notes, 89(5-6), 2011, 746–748.
- [4] C. Figueroa, On the Gauss map of a minimal surface in the Heisenberg group, Mat. Contemp., 33(139-156), 2007, 10-21711.
- [5] C. Figueroa, The Gauss map of Minimal graphs in the Heisenberg group, Journal of Geometry and Symmetry in Physics, 25, 2012, 1-21.
- [6] C. Figueroa, Translation surfaces in the Heisenberg Group, arXiv preprint arXiv:2510.23916, 2025.
- [7] D. Gans, A new model of the hyperbolic plane, The American Mathematical Monthly, 73(3), 1966, 291–295.
- [8] J. Inoguchi, Flat translation invariant surfaces in the 3-dimensional Heisenberg group, Journal of Geometry, 82(1-2), 2005, 83–90.
- [9] J. I. Inoguchi, R. López and M.I. Munteanu, Minimal translation surfaces in the Heisenberg group Nil_3 , Geometriae Dedicata, 161(1), 2012, 221-31.
- [10] H. Liu and Y. Yanhua, Affine translation surfaces in Euclidean 3-space, Proc. Japan Acad. Ser., 89, 2013, 111-113.
- [11] S. D. Pauls, Minimal surfaces in the Heisenberg group, Geometriae Dedicata, 104(1), 2004, 201-31.
- [12] A. Sanini, Gauss Map of a Surface of Heisenberg Group, Bolletino U.M.I, 11B, 1997, 79–73.

- [13] R. Xue, Minimal Affine Translation Surfaces in Nil_3 Space, Journal of Northeastern University (Natural Science), 45(10), 2024, 1513.
- [14] D. W. Yoon, C. W. Lee and M. K. Karacan, Some translation surfaces in the 3-dimensional Heisenberg group, Bulletin of the Korean Mathematical Society, 50(4), 2013, 1329-1343.

ABSTRACT 45

Structural and Reactivity Study of Azo Naphthalenic Derivatives as Potential Antioxidant Agents: A DFT Investigation**BENABILA Nabila¹, CHETIOUI Souheyla^{2,3}**¹*Laboratory of Therapeutic Organic Substances and Sustainable Processes (LTOCSP) Mohamed Boudiaf University of M'Sila, 28000 M'sila, Algeria*²*VEHDD Laboratory, University of M'sila, 28000 M'sila, Algeria*³*Research Unit for Chemistry of the Environment and Molecular Structural, University of Constantine 1, Constantine 25000, Algeria*

E-mail: nabila.bennabila@univ-msila.dz

Abstract: Phenolic compounds are among the most important natural antioxidants, acting by transferring the phenolic hydrogen atom to peroxy radicals and thereby inhibiting oxidative chain propagation. In this work, we report a DFT study of two azo naphthalenic derivatives, p MeOH and p ClOH, which show promising antioxidant activity in experimental assays. Both compounds contain a phenolic OH group in ortho position relative to the azo/hydrazone bridge, and differ only by the para substituent (methyl or chloro), allowing a direct structure–reactivity comparison.

Density functional theory calculations were performed using the hybrid functionals M05 2X and LC ω PBE with the 6-31+G(d,p) basis set. The geometries of the neutral molecules, their corresponding radicals, cationic radicals, and anions were optimized and confirmed as true minima by vibrational frequency analysis. Unrestricted calculations were used for open shell species, and spin contamination was monitored by $\langle S^2 \rangle$ values. The IEFPCM solvation model was applied to evaluate the effect of polar (water, DMSO, methanol) and non polar (benzene) solvents on structural and electronic properties.

Structural analysis revealed that the electronic distribution around the azo/hydrazone bridge and the phenolic ring is sensitive to the substituent, explaining the differences in reactivity between p MeOH and p ClOH. MEP maps and HOMO LUMO analysis indicate that p MeOH exhibits higher electron donating ability, whereas p ClOH shows a more selective reactivity pattern. Global reactivity descriptors and spin density distributions for the radicals further support the experimentally observed antioxidant behavior.

These results suggest that azo naphthalenic phenolic derivatives represent promising scaffolds for the rational design of new antioxidant agents.

Keywords: Azo naphthalenic derivatives, DFT, antioxidant activity, M05 2X, LC ω PBE.

Mathematics Subject Classification: 12X34, 56Y78.

References

- [1] Halliwell, B., & Gutteridge, J. M. C. (2015). *Free Radicals in Biology and Medicine* (5th ed.). Oxford University Press.
- [2] Bakhouch, I., Aliat, T., Boubellouta, T., Gali, L., Şen, A., & Bellik, Y. (2021). Phenolic contents and in vitro antioxidant, anti-tyrosinase, and anti-inflammatory effects of leaves and roots extracts of the halophyte *Limonium delicatulum*. *South African Journal of Botany*, 139, 42-49.
- [3] Walker, R. (1970). The metabolism of azo compounds: a review of the literature. *Food and Cosmetics Toxicology*, 8(6), 659-676.

- [4] Gičević, A., Hindija, L., & Karačić, A. (2019, May). Toxicity of azo dyes in pharmaceutical industry. In International Conference on Medical and Biological Engineering (pp. 581-587). Cham: Springer International Publishing.
- [5] Concilio, S., Sessa, L., Petrone, A. M., Porta, A., Diana, R., Iannelli, P., & Piotto, S. (2017). Structure modification of an active azo-compound as a route to new antimicrobial compounds. *Molecules*, 22(6), 875.
- [6] Piotto, S., Concilio, S., Sessa, L., Diana, R., Torrens, G., Juan, C., ... & Iannelli, P. (2017). Synthesis and antimicrobial studies of new antibacterial azo-compounds active against staphylococcus aureus and listeria monocytogenes. *Molecules*, 22(8), 1372.
- [7] Rezaei-Seresht, E., Mireskandari, E., Kheirabadi, M., Cheshomi, H., Rezaei-Seresht, H., & Aldaghi, L. S. (2017). Synthesis and anticancer activity of new azo compounds containing extended Π -conjugated systems. *Chemical Papers*, 71(8), 1463-1469.

ABSTRACT 46

Fractional Greeks in the Black-Scholes EDP

Baya Takhedmit¹, Leila Abbaci¹, Bahia Barache², Siham Kebaili²¹Research Unit LaMOS, University of Bejaia, Algeria²Laboratory of LMA, University of Bejaia, Algeria

E-mail: baya.takhedmit@univ-bejaia.dz, leila.abbaci@univ-bejaia.dz, bahia.barache@univ-bejaia.dz, siham.kebaili@univ-bejaia.dz

Abstract: The classical Black-Scholes PDE, based on integer-order partial derivatives, assumes Markovian dynamics incompatible with the non-local behaviors observed in real markets (volatility clustering, persistent correlations). Our work is focus to extend this PDE using fractional partial derivatives to define non-local sensitivity indices (fractional $\theta^\alpha, \Delta^\alpha, \gamma^\beta$) adapted to memory dynamics. The time-fractional PDE employs the Caputo operator of order $\alpha \in (0, 1)$, while spatial operators of order $\beta \in (1, 2)$ generalize directional and convexity sensitivities. These fractional Greeks converge to their classical counterparts while preserving stability.

Analytical solutions to the fractional Black-Scholes equation, along with those incorporating fractional Greeks, were graphically illustrated. The impact of fractional derivative orders α and β on hedging performance, particularly for European options, was thoroughly examined.

Keywords: Fractional PDEs, non-local Greeks, Black-Scholes model.

References

- [1] Shokrollahi, C., et al. Fractional delta hedging strategy for pricing currency options with transaction costs. *arXiv:1702.00037*, 2017.
- [2] Cartea, Á. and del Castillo-Negrete, D. Fractional calculus in financial modeling. *Risk Management and Asset Pricing*, 2011.
- [3] Mrad, M., et al. Analyzing the Black-Scholes equation with fractional derivatives. *International Journal of Optimization and Control: Theories & Applications*, 2022.
- [4] Leonenko, N., et al. Time-fractional diffusion equation for Black-Scholes formalism. *Fractional Dynamics and Control*, 2013.

ABSTRACT 47

Analytical Investigation of Entropy Generation for Ternary Hybrid Nanofluid Flow in Concentric Cylinders with Particle Shape Variation**Abdelhakim DJALAB¹, Mohamed Kezzar², Mohamed Rafik SARI³**

¹*Mechanical Engineering Department, Mohamed Boudiaf University of M'sila, Po Box 166 Ichebilia, 28000 M'sila, Algeria*

²*Mechanical Engineering Department, Faculty of Technology, University of Skikda, El Hadaiek Road, B. O. 26, 21000 Skikda, Algeria*

³*Laboratory of Industrial Mechanics, Mechanical Engineering Department, Badji Mokhtar University of Annaba, B. O. 12, 23000 Sidi Amar Annaba, Algeria*
E-mail: abdelhakim.djalab@univ-msila.dz

Abstract: Hybrid nanofluids with three different materials are becoming very important for engineering applications such as compact heat exchangers, chemical reactors, and cooling systems for electronics. This study investigates the flow of a ternary hybrid nanofluid (MoS_2 , GO, and Cu dispersed in acetic acid–water) between two concentric cylinders that experience particle shape fracture (spherical, cylindrical, and platelets). A mathematical model is developed that includes chemical reaction, thermal radiation, and the effect of particle shape on heat transfer and fluid behavior. The governing nonlinear equations are reduced to ordinary differential equations and solved using the Adomain Decomposition Method with results refined via the Homotopy Analysis Method. The model shows good agreement with available benchmarks. Results indicate that increasing the Grashof number enhances fluid velocity, while the radiation parameter can reduce fluid temperature. Plate-shaped nanoparticles generate higher entropy than spherical ones, highlighting the strong influence of buoyancy, Brownian motion, and thermal effects on flow and heat transfer.

Keywords: Nanofluid, Chemical reaction, Brownian motion, Heat transfer, Mathematical modeling, Adomain, Thermal radiation.

References

- [1] Abduljeel MA, Yasin NJ, Ghadhban SA, Soomro SA, Performance improving for the flat plate solar collectors by using nanofluids: review study. *J Tech* 6(1):52–68, 2024.
- [2] Elnaqeeb T, Animasaun IL, Shah NA, Ternary-hybrid nanofluids: significance of suction and dual-stretching on three-dimensional flow of water conveying nanoparticles with various shapes and densities. *Zeitschrift Für Naturforschung A* 76(3):231–243, 2021.
- [3] Younes H, Mao M, Murshed SS, Lou D, Hong H, Peterson GP, Nanofluids: key parameters to enhance thermal conductivity and its applications. *Appl Therm Eng* 207:118202, 2022.

ABSTRACT 48

On a Compact 4th-Order Scheme for the General Acoustic Wave Equation

Alexander Zlotnik

Higher School of Economics University, Dept. of Mathematics, 109028, Moscow, Russia

E-mail: azlotnik@hse.ru

Abstract: We deal with the general multidimensional acoustic wave equation, where both the speed of sound and density of a medium are variable, that is most physically relevant [6]. For the first time, we develop a new compact scheme for this equation which is three-level in time, three-point in each spatial direction and has the fourth truncation order $\mathcal{O}(|h|^4 + h_t^4)$. In addition, the scheme is semi-explicit in time and easily implementable. It non-trivially generalizes a semi-explicit vector compact scheme constructed and studied recently in the much simpler case of the constant density [2, 3]. Although other high-order numerical methods for the general acoustic wave equation were known, the construction of a compact high-order scheme remained an unsolved problem for a long time, and only the particular case of the constant density has previously been covered, in particular, see [4]–[8].

Results of computations confirm the high precision of the scheme and its fourth error order not only in the mesh C norm but in the mesh C^1 norm as well that is of value in applications. The details of this research can be found in [8].

The study was implemented in the framework of the Basic Research Program at HSE University (HSE-BR-2025-002).

Keywords: Acoustic wave equation, Variable speed of sound and density, Semi-explicit three-level scheme, Compact fourth-order scheme, Practical error analysis.

Mathematics Subject Classification: 65M06, 74S20, 76M12.

References

- [1] L. M. Brekhovskikh and O. A. Godin, “Acoustics of layered media I. Plane and quasi-plane waves”, Springer, 2012.
- [2] Y. Jiang, Y. Ge, “An explicit high-order compact difference scheme for the three-dimensional acoustic wave equation with variable speed of sound”, *Int. J. Comput. Math.*, 100(2), pp. 321–341, 2023.
- [3] A. Zlotnik, T. Lomonosov, “On stability and error bounds of an explicit in time higher-order vector compact scheme for the multidimensional wave and acoustic wave equations”, *Appl. Numer. Math.*, 195, 54–74, 2024.
- [4] F. Smith, S. Tsynkov, E. Turkel, “Compact high order accurate schemes for the three dimensional wave equation”, *J. Sci. Comput.*, 81(3), 1181–1209, 2019.
- [5] K. Li, W. Liao, Y. Lin, “A compact high order alternating direction implicit method for three dimensional acoustic wave equation with variable coefficient”, *J. Comput. Appl. Math.*, 361(1), pp. 113–129, 2019.
- [6] A. Zlotnik, R. Čiegis, “On higher-order compact ADI schemes for the variable coefficient wave equation”, *Appl. Math. Comput.*, 412, article 126565, 2022.
- [7] A. Zlotnik, R. Čiegis, “On construction and properties of compact 4th order finite-difference schemes for the variable coefficient wave equation”, *J. Sci. Comput.*, 95(1), article 3, 2023.

- [8] A. Zlotnik, T. Lomonosov, A compact 4th-order scheme for the general acoustic wave equation, Lobachevskii J. Math., 47(5), 2026 (to appear).

ABSTRACT 49

Instructor-Trained AI Tutor for Mathematics Education: Design and Evaluation of a Custom GPT**Diana M. Audi***American University of Sharjah (AUS), Department of Mathematics, Sharjah, UAE*
E-mail: daudi@aus.edu

Abstract: Recent advances in generative artificial intelligence have sparked growing interest in AI-assisted learning; however, the use of generic AI tools in mathematics education remains problematic due to inconsistent reasoning, limited pedagogical alignment, and frequent mathematical inaccuracies. This paper presents the design and evaluation of a custom instructor-trained AI tutor (Math-GPT) developed specifically to support university-level mathematics learning. Unlike general-purpose AI systems, the proposed Math-GPT is trained exclusively on instructor-curated materials, including lecture notes, handwritten derivations, worked examples, and structured solution templates, ensuring alignment with established mathematics pedagogy. The study adopts a mixed-methods research design, using a Calculus II course as a pilot context to investigate the impact of the Math-GPT on student learning outcomes, engagement, and confidence. Quantitative data include pre- and post-conceptual assessments, quiz and exam performance, and usage analytics, while qualitative data are gathered through student surveys, reflections, and focus groups. Statistical hypothesis testing is employed to evaluate learning gains and identify relationships between AI usage patterns and performance outcomes. Preliminary findings indicate that students using the Math-GPT demonstrate improved conceptual understanding, higher problem-solving accuracy, and increased self-efficacy compared to non-users. Students also report greater clarity in understanding multi-step procedures and reduced frustration when studying independently. The paper further discusses design principles for instructor-aligned AI tutors and outlines a scalable framework for integrating custom AI support into mathematics curricula. These results suggest that discipline-specific, pedagogy-aware AI tutors can play a meaningful role in enhancing mathematics learning when thoughtfully designed and rigorously evaluated.

Keywords: Artificial Intelligence in Education, Mathematics Education, Generative AI, Intelligent Tutoring Systems, Instructor-Trained AI, Calculus Education, AI-Supported Learning, Pedagogy-Aware AI.

Mathematics Subject Classification: 97U70, 97C70, 68T07.

References

- [1] Gaeta, A., Orciuoli, F., Pascuzzo, A., & Peduto, A. (2025). Enhancing traditional ITS architectures with large language models for generating motivational feedback. *Computers and Education: Artificial Intelligence*, 9, 100433. <https://doi.org/10.1016/j.caeai.2025.100433>
- [2] Kuklick, L., Greiff, S., & Lindner, M. A. (2023). Computer-based performance feedback: Effects of error message complexity on cognitive, metacognitive, and motivational outcomes. *Computers & Education*, 200, 104785. <https://doi.org/10.1016/j.compedu.2023.104785>
- [3] Meyer, J., Jansen, T., Schiller, R., Liebenow, L. W., Steinbach, M., Horbach, A., & Fleckenstein, J. (2024). Using LLMs to bring evidence-based feedback into the classroom: AI-generated feedback increases secondary students' text revision, motivation, and positive emotions. *Computers & Education: Artificial Intelligence*, 6, 100199. <https://doi.org/10.1016/j.caeai.2023.100199>

ABSTRACT 50

Permeability, Porosity and Mechanical Strength as Indicators of Concrete Durability**Laid Baali¹, Mohmaed Bencheikh², Mohamed Aziz Chikouche¹**¹*Laboratory of Materials and Mechanics of Structures, Department of civil Engineering, University Med Boudiaf of M'sila, Algeria*²*Department of Civil Engineering & Faculty of Technology & University Med Boudiaf of M'sila. Algeria*
E-mail:

Abstract: In general, the ability of concrete to resist the penetration of harmful substances or any other liquid can be a good indication of its durability, although this is linked to several parameters, among these are the porosity and air, water and chemical permeability, which are probably the most important parameters that can influence the durability of concrete. This article describes a method based on an experimental procedure for evaluating the water permeability of concrete made from local materials (M'sila region). An experimental device is suitable for this method. Tests are carried out under air pressure of 5 bars through an existing opening inside the test piece to exit by external surface. Darcy's law is applied to determine its permeability. In addition, the porosity of the concrete is evaluated according to the ASTM standard designation C642 (standard test method for specific gravity, absorption, and voids in hardened concrete). The main objective of this work is to succeed in developing the existing correlation between the different parameters studied; namely: permeability, porosity and mechanical resistance of hardened concrete as a function: of the age of the concrete matrix and of the aggressive "NaCl" "Gypsum" medium. It is therefore a question of determining the criteria to be imposed on these parameters in order to achieve the production of a concrete based on local materials, the least permeable possible, the least porous and therefore characterized by an acceptable durability.

Keywords: Permeability, Porosity, Mechanical Strength, Concrete.**Mathematics Subject Classification:** 12X34, 56Y78.**References**

- [1] M.Buil, J.P.Ollivier « Conception des bétons: La structure poreuse »- La durabilité des bétons, Presses de l'école nationale des ponts et chaussées, ISBN 2-85978-184-6, Paris1992, pp: 57 - 97.
- [2] D.Whiting « Permeability of sélecte concrete » -Permeability of concrete- David whiting, Arthur walitt editors, ACI Publication, SP 108-11, Editorial production: Patricia J.Kost, N: 88-71000, USA 1988, pp: 195 - 222.
- [3] N.Bakir « Influence du climat chaud sur les propriétés du béton à l'état durci »-Thèse d'ingénieur, M'sila 1990, page 71.
- [4] Recueil de normes françaises- AFNOR, Béton et constituants du béton, Paris 1991.
- [5] R.Dupain, R.Lanchon et J.C.Saint-Arroman « Granulats, Sols, Ciments et Béton », Casteilla, 1995, ISBN2-7135-1471-1, pp : 3-33
- [6] G.Dreux et J.Festa « Nouveaux guide de béton »,7ème Edition 1995, Eyrolles, Paris, ISBN : 2-212-10230-5, pp : 156-203
- [7] B.R.Sullivan « Permeability testing system for Grout, Concrete, and Rock » -Permeability of concrete- David whiting, Arthur walitt editors, ACI Publication, SP 108-9, Editorial production: Patricia J.Kost, N: 88-71000, USA 1988, pp: 159 - 173.

ABSTRACT 51

Predictive DFT Modeling of Structural Stability and Physical Properties of Double Half-Heusler Hf₂InNiSb₂ Alloy**Khadidja Berarma¹, Amina Raghdi²**

¹ *University of M'sila, Department of Chemistry, Laboratory of Inorganic Materials, , M'sila 28000, Algeria*

² *University of M'sila, Department of Chemistry, Laboratory for organo-therapeutic substances and sustainable processes,, M'sila 28000, Algeria*
E-mail: khadidja.berama@univ-msila.dz

Abstract: Full Potential Linear Augmented Plane Wave (FP-LAPW), based on density functional theory(DFT) [1], is applied to calculate the structural stability, electrical, optical, and thermal behavior of double half-Heusler alloys (Hf₂InNiSb₂) using the WIEN2k code [2]. Double half-Heusler alloys, like Heusler alloys, are highly promising materials that have attracted the attention of scientists as they offer significant potential for applications in thermoelectricity [3], UV devices, and, more widely, in optoelectronics for the development of sensors and components operating in extended energy ranges [4]. The compound investigated has a direct energy bandgap, as well as a significant absorption coefficient $160 \times 10^4 \text{ cm}^{-1}$ in the UV region, rendering it particularly suitable for optoelectronic applications in the UV range. In addition, Debye's quasi-harmonic model is applied to estimate the dependence of pressure and temperature on thermodynamic properties, including volume, compressibility moduli, and heat capacity, over a pressure range of 0 to 10 GPa and a temperature range of 0 to 1000 K.

Keywords: DFT, Electronic structure ,FP-LAPW, Absorption Coefficient.

Mathematics Subject Classification: 81V70, 82D25, 78A40.

References

- [1] Hohenberg P and Kohn W , "Density functional theory (DFT) " Phys. Rev. 136 B864, 1964..
- [2] Blaha P et al "WIEN2k: an APW+ lo program for calculating the properties of solids" J. Chem. Phys. 152 074101, 2020.
- [3] Shutoh N and Sakurada S "Thermoelectric properties of the $\text{Ti}_x(\text{Zr}_{0.5}\text{Hf}_{0.5})_{1-x}\text{NiSn}$ half-Heusler compounds " J. Alloys Compd. 389 204–8, 2005
- [4] Žutić I, Fabian J and Sarma S D "Spintronics: fundamentals and applications" Rev. Mod. Phys. 76 323, 2004.

ABSTRACT 52

Lie symmetry analysis of Merton equation

C. Sophocleous

Department of Mathematics and Statistics, University of Cyprus, Nicosia CY 1678, Cyprus

E-mail: sophocleous.christodoulos@ucy.ac.cy

Abstract: The pioneering works of [1, 2] and [3] ushered in a new era in the mathematical modeling of financial markets and optimal investment problems. In the present study, we investigate the original financial model introduced by Merton in 1971, which describes the optimal allocation of wealth between a risky asset and a risk-free asset in order to maximize an investor's lifetime utility. The model is governed by the nonlinear partial differential equation

$$J_t - \frac{(\alpha - r)^2}{\alpha\sigma^2} \frac{J_w^2}{J_{ww}} + rwJ_w + \frac{(1 - \gamma)\eta}{\beta} J_w + \frac{(1 - \gamma)^2 e^{-\rho t}}{\gamma} \left[\frac{e^{\rho t} J_w}{\beta} \right]^{\frac{\gamma}{\gamma-1}} = 0,$$

where the parameters characterize the financial market and the investor's preferences. We analyze this equation within the framework of Lie symmetry theory. First, the equivalence transformations admitted by the model are determined and subsequently employed to simplify the governing equation. In particular, using a special case of the equivalence transformations, Merton's equation can be mapped onto the simplified equation

$$u_t - \frac{\kappa u_x^2}{u_{xx}} + xu_x + u_x^n = 0.$$

We then perform a complete classification of its Lie point symmetries and utilize the resulting symmetry generators to obtain invariant solutions. More importantly, we employ Lie symmetries to solve the nonlinear model subject to prescribed terminal conditions. In addition, we derive the contact symmetries and contact transformations admitted by Merton's equation and demonstrate how these transformations can be used to linearize the nonlinear model. In particular, the Legendre transformation is shown to provide a natural mapping to a linear representation. Moreover, we establish a connection between Merton's equation and the classical linear diffusion equation, thereby revealing a deeper structure underlying the model. Finally, we discuss several related nonlinear financial equations that have appeared in the recent literature and indicate how the methodology developed in the present work can be extended to analyze these models in a systematic manner.

Keywords: Merton financial model, Equivalence transformations, Lie symmetries, Contact transformations, Contact symmetries, Linearization, Exact solutions.

Mathematics Subject Classification: 35A30, 35K55, 58J70.

References

- [1] R. C. Merton, "Optimum consumption and portfolio rules in a continuous time model", *J. Econ. Theory*, **3**, 373–413 (1971).
- [2] R. C. Merton, "Theory of rational option pricing", *Bell Journal of Economic and Management Sciences*, **4**, 141–183 (1973).
- [3] F. Black and M. Scholes, "The pricing of options and corporate liabilities", *Journal of Political Economy*, **81**, 637–654 (1973).

ABSTRACT 53

A Deniable Authentication Protocol Based on Multi Prime Power RSA

Normahirah Nek Abd Rahman¹, Eddie Shahril Ismail²¹*Pusat PERMATA@Pintar Negara, Universiti Kebangsaan Malaysia, Selangor, Malaysia*²*Department of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, Selangor, Malaysia*

E-mail: normahirah@ukm.edu.my, esbi@ukm.edu.my

Abstract: A deniable authentication protocol allows the receiver to verify the source of a message while preventing them from being able to prove the sender's identity to any third party. This essential feature is highly valuable for secure negotiations and other privacy-sensitive applications. Due to the reduced communication overhead, non-interactive deniable authentication protocol have become a central focus in recent cryptographic research using diverse cryptographic primitives. As a result, the development of an non-interactive deniable authentication protocol that effectively balances both security and efficiency remains a substantial challenge [1]. To address this issue, this study introduces a new non-interactive deniable authentication protocol based on multi-prime power RSA. Multi-Prime Power RSA is a cryptographic variant of RSA that uses a modulus of the form $(N = p^r q^s)$, where $(r \geq s \geq 2)$ [2]. This structure introduces efficiency improvements in decryption and modular exponentiation while maintaining security comparable to traditional RSA under certain conditions and can be used as a stepping stone to construct an efficient non-interactive deniable authentication protocol. Overall, the proposed protocol is completed and maintained the deniability property. Besides that, the protocol's robustness has been formally proven against several attack vectors, including forgery, impersonation, known message attack and man-in-the-middle attacks.

Keywords: Deniable authentication protocol, multi-prime power RSA, non-interactive protocol.

References

- [1] Zhu, H. *A simple and secure non-interactive deniable authentication scheme with privacy protection using quantum bits*. Journal of Computers, 29(3), 83-93. 2018.
- [2] Nitaj, A., Susilo, W., and Tonien, J. *A generalized attack on the multi-prime power RSA*. [I In International Conference on Cryptology in Africa]. Cham: Springer Nature Switzerland, pp. 537-549, July 2022.

ABSTRACT 54

A High-Fidelity cDFIB IDFCC Framework for Moving-Boundary Simulations: Validation via an Oscillating Cylinder Case

Yan-Ting Lin^{1,3}, Pao-Hsiung Chiu², Huei Chu Weng³

¹*National Atomic Research Institute, Taoyuan, Taiwan*

²*Institute of High Performance Computing, A*STAR, Singapore*

³*Department of Mechanical Engineering, Chung Yuan Christian University, Taiwan*

E-mail: yangtinglin@nari.org.tw, chiuph@a-star.edu.sg and hcweng@cycu.edu.tw

Abstract: This study evaluates the accuracy and robustness of an integrated numerical framework combining a convolutional direct forcing immersed boundary method (cDFIB) [1], a differential interpolation (DI) scheme, and an enhanced divergence-free-condition compensated coupled (IDFCC) approach for solving incompressible flows with moving boundaries. The classic benchmark case of a circular cylinder undergoing large-amplitude harmonic oscillations in a fluid at rest ($Re = 100$, $KC = 5$) is investigated for rigorous quantitative and qualitative validation. Quantitatively, the normalized horizontal ($u/U_{\max}$) and vertical ($v/U_{\max}$) velocity profiles are extracted across multiple cross-sections at critical phase angles ($\phi = 180^\circ$, 210° , and 330°). The numerical results exhibit exceptional agreement with the benchmark experimental data of Dütsch et al. [2] and prior reference numerical solutions of Nicolaou et al. [3], accurately capturing the sharp velocity gradients within the dense boundary layers.

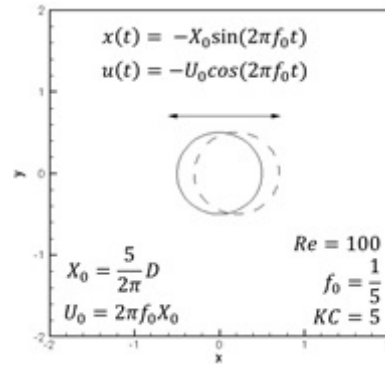


Figure 1: Definition of the benchmark validation case of a circular cylinder oscillating harmonically in a fluid at rest.

References

- [1] Pao-Hsiung Chiu, "cDFIB: A convolutional direct forcing immersed boundary method for solving incompressible flows with time-varying geometries", *Journal of Computational Physics*, 487, 112178, 2023.
- [2] H. Dütsch, F. Durst, S. Becker, H. Lienhart, "Low-Reynolds-number flow around an oscillating circular cylinder at low Keulegan-Carpenter numbers", *Journal of Fluid Mechanics*, 360, pp. 249-271, 1998.
- [3] L. Nicolaou, S.Y. Jung, T.A. Zaki, "A robust direct-forcing immersed boundary method with enhanced stability for moving body problems in curvilinear coordinates", *Computers & Fluids*, 119, pp. 101-114, 2015.

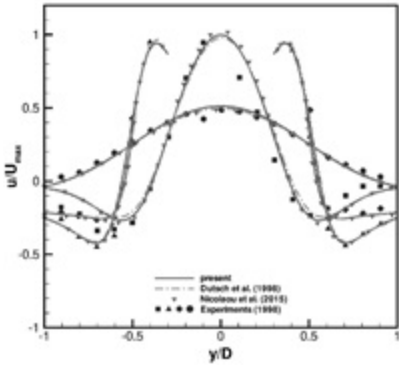


Figure 2: Comparison of the normalized horizontal velocity profile for the oscillating cylinder benchmark.

ABSTRACT 55

Modelling Election-Induced Stock Market Volatility in Malaysia Using Merton's Jump-Diffusion Framework

Mohamed Faris Laham¹, Hafizah Bahaludin², Siti Nur Iqmal Ibrahim³

¹*Institute for Mathematical Research, Universiti Putra Malaysia,
UPM Serdang, Malaysia*

²*International Islamic University Malaysia, Kuantan, Malaysia*

³*Institute for Mathematical Research, Universiti Putra Malaysia,
UPM Serdang, Malaysia*

E-mail: mohdfaris@upm.edu.my, hafizahbahaludin@iiu.edu.my, iqmal@upm.edu.my

Abstract: Political uncertainty arising from general elections often generates significant fluctuations in financial markets, particularly in emerging economies where investor sentiment is highly sensitive to political developments [1]. Traditional financial models such as the Geometric Brownian Motion (GBM) model assume continuous stock price movements and normally distributed returns which may be insufficient in capturing abrupt market shocks and discontinuous price behaviour during election periods. This study examines the impact of election-related jump events on stock prices and volatility in Malaysian stock market through the application of Merton's Jump-Diffusion model (MJD). The study focuses on the Malaysian stock market during the 12th, 13th, 14th and 15th General Elections. Daily stock price data and market index data from the FTSE Bursa Malaysia Kuala Lumpur Composite Index (FBMVKLCI) are used to examine stock return dynamics before, during and after the election period. The findings demonstrate that election periods are associated with increased market volatility and significant jump occurrences in stock returns and reveal a positive relationship between jump intensity and market uncertainty during elections. Overall, this study improves the understanding of stochastic stock price dynamics in emerging markets and highlights the importance of incorporating jump components into financial modeling frameworks during periods of increased uncertainty.

Keywords: Merton jump-diffusion model, Bursa Malaysia, Stochastic modeling.

References

- [1] Pástor, L'uboř and Veronesi, Pietro *Political uncertainty and risk premia.* " *Journal of financial Economics* 110, no. 3 (2013): 520-545.
- [2] Ibrahim, Siti Nur Iqmal, Masnita Misiran, and Mohamed Faris Laham. *Geometric fractional Brownian motion model for commodity market simulation.* *Alexandria Engineering Journal* 60.1 (2021): 955-962.

ABSTRACT 56

Geometric properties of a free boundary problem

Martin Lukarevski*Goce Delcev University, Department of Mathematics and Statistics, Stip, North Macedonia*

E-mail: martin.lukarevski@ugd.edu.mk

Abstract: We study the geometric properties of a particular type of a free boundary problem. The introduction of a relaxation function allows for a priori estimates for the surface area. The evolution of the free boundary in time is not arbitrary and we show that the model is volume-preserving and area-shrinking.

Keywords: Free boundary problem, Geometric properties, Evolution equations.

Mathematics Subject Classification: 35XX.

ABSTRACT 57

Based on modern comparative studies and surveys of deep learning architectures for HOI

Lahouaoui Lalaoui¹, Ahd Lakhnach¹, Abd elhak Djalab¹

¹*Mohamed Boudiaf University, Deprt of Electronic Laboratory of Electrical Engineering, Faculty of Technology, , PO Box 166 Ichebilia, 28000 M'sila, Algeria*

E-mail: lahouaoui.lalaoui@univ-msila.dz, ahd.lakhnach@univ-msila.dz, abdelhak.djalab@univ-msila.dz

Abstract: Human–Object Interaction (HOI) detection serves as a cornerstone for advanced visual scene understanding, necessitating the concurrent localization of individuals, target objects, and the semantic classification of their relationships. Although deep learning paradigms have substantially advanced baseline detection metrics, sustaining high fidelity in unstructured, adversarial environments remains a critical bottleneck. This paper delivers a rigorous comparative evaluation of prominent deep learning architectures—encompassing YOLOv5, Faster R-CNN, Single Shot MultiBox Detector (SSD), and UNet++—tailored for HOI-centric tasks under degraded imaging conditions. We systematically quantify architectural resilience against the confounding effects of spatial occlusion, extreme illumination volatility, and stochastic image noise. Our empirical findings map the Pareto-optimal frontiers governing localization accuracy, structural robustness, and computational throughput (inference latency). Consequently, this work establishes a definitive benchmark and taxonomy, providing critical design guidelines for deploying dependable HOI models in real-world, resource-constrained environments.

References

- [1] Z. Cao, G. Hidalgo, T. Simon, S. E. Wei, and Y. Sheikh, "OpenPose: Realtime multi-person 2D pose estimation using Part Affinity Fields," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 43, no. 1, pp. 172–186, Jan. 2021.
- [2] G. Gkioxari, R. Girshick, P. Dollár, and K. He, "Detecting and recognizing human-object interactions," in *Proc. IEEE Conf. Comput. Vis. Pattern Recognit. (CVPR)*, pp. 8359–8367, 2018.
- [3] P. Zhou, B. Li, Q. Hou, and M. M. Cheng, "Label decoupling framework for human-object interaction detection," *IEEE Trans. Image Process.*, vol. 31, pp. 5389–5402, 2022.
- [4] Y. Li, S. Zhou, Z. Huang, and Y. Zhang, "HOI Transformer: Human-object interaction detection with transformers," *arXiv preprint arXiv:2104.13682*, 2021.
- [5] H. Xu, Y. Zhu, C. B. Choy, and L. Fei-Fei, "Learning to detect and classify human-object interactions," *Int. J. Comput. Vis.*, vol. 127, no. 3, pp. 258–275, Mar. 2019.
- [6] Hou, Q., Zhou, P., & Cheng, M. M.. HOI Analysis: Understanding human-object interaction by modeling object semantics. In *Proceedings of the IEEE/CVF International Conference on Computer Vision (ICCV)*, pp. 3027–3036, (2021).
- [7] R. Jain, H. Gandhi, and P. Bhattacharya, "Lightweight human object interaction detection for edge computing," *IEEE Access*, vol. 10, pp. 92846–92859, 2022.

ABSTRACT 58

Some Theorems on Starlike Functions of Complex Order with Respect to t -Symmetric Points

Öznur Özkan Kılıç

Başkent University, Department of Technology and Knowledge Management, 06790, Ankara, Turkey
 E-mail: oznur@baskent.edu.tr

Abstract: In this work, we introduce a new subclass $t - SSP$ of starlike functions defined with respect to t -symmetric points. Based on this construction, we further define two subclasses, $t - SSP(\gamma, \beta)$ and $t - SSP(A, B)$, which extend the classical theory of starlike functions in the unit disk. For functions belonging to these classes, we establish several results concerning majorization. As consequences of our main results, a number of corollaries are derived, which include and generalize several earlier known results in the literature.

Keywords: Starlike function, t -Symmetric points, Majorization.

Mathematics Subject Classification: Primary 30C45.

References

- [1] Altıntaş, O.; Owa, S. Majorization and quasi-subordination for certain analytic functions. *Proc. Japan Acad. Ser. A Math. Sci.*, **1992**, 68(7), 181-185.
- [2] Altıntaş, O.; Özkan, Ö.; Srivastava, H.M. Majorization by starlike functions of complex order. *Complex Variables*, **2011**, Vol 46, 207-218.
- [3] Altıntaş, O. Majorization for certain analytic functions. *European J. of Pure and App. Math.*, **2012**, 5(1), 16-24.
- [4] Dziok, J.; Murugusundaramoorthy, G.; Janani, T. Majorization problems for classes of analytic functions. *Journal of Mathematics and Applications*, **2015**, 38, pp 49-57.
- [5] Goel, R.M.; Mehrotra, B.C. A subclass of starlike functions with respect to symmetric points. *Tamkang J. Math.*, **1982**, 13, 11-24.
- [6] Özkan Kılıç, Ö.; Altıntaş, O. Majorization by starlike functions. *J. Nonlinear Sci. Appl.*, **2017**, 10, 1215-1220.
- [7] Selvaraj, C.; Vasanthi, N. Subclasses of analytic functions with respect to symmetric and conjugate points. *Tamkang Journal of Mathematics*, **2011**, 42(1), , 87-94.

ABSTRACT 59

A Hybrid Richards–Temporal Fusion Transformer Framework for Modeling and Forecasting COVID-19 Dynamics in Türkiye

Begüm ÇIĞŞAR

*Adana Alparslan Türkeş Science and Technology University, Dept. of Data Science and Analytics,
Adana, Türkiye*

E-mail: bcigsar@atu.edu.tr

Abstract: Forecasting epidemics presents significant challenges due to the nonlinear and highly fluctuating nature of pandemic dynamics. Although growth models such as the Richards model provide a mathematically interpretable and representative framework, they often struggle to adapt to functional breakpoints and irregular fluctuations. On the other hand, purely data-driven machine learning approaches, while highly flexible, generally lack the epidemiological foundation and interpretability required to estimate disease-related parameters beyond the observed data.

To address these complementary limitations, this study proposes a novel two-stage hybrid framework. In the first stage, the Richards growth model is fitted to the cumulative COVID-19 case data of Türkiye. After extracting the deterministic epidemic trend captured by the model, the resulting residual series is passed to the second stage. In this stage, a deep learning architecture based on the Temporal Fusion Transformer (TFT) is employed to model the residual dynamics. The performance of the proposed hybrid model is evaluated and compared with the Richards model, Prophet, and TFT approaches using MAPE, RMSE, adjusted R^2 , and MAE as performance metrics.

Keywords: Richards Growth Model, Temporal Fusion Transformer, Machine Learning, COVID-19.

Mathematics Subject Classification: 00A71, 62P12.

References

- [1] F. J. Richards, “A Flexible Growth Function for Empirical Use”, *Journal of Experimental Botany*, 10(2), pp. 290-301, 1959.
- [2] Rafat Zreiq, Souad Kamel, Sahbi Boubaker, Asma A Al-Shammari, Fahad D Algahtani, Fares Alshammari, “Generalized Richards model for predicting COVID-19 dynamics in Saudi Arabia based on particle swarm optimization Algorithm”, *AIMS Public Health*, 7(4), pp. 828-843, 2020.
- [3] Takeshi Miyama, Sung-mok Jung, Katsuma Hayashi, Asami Anzai, Ryo Kinoshita, Tetsuro Kobayashi, Natalie M. Linton, Ayako Suzuki, Yichi Yang, Baoyin Yuan, Taishi Kayano, Andrei R. Akhmetzhanov, Hiroshi Nishiura, “Phenomenological and mechanistic models for predicting early transmission data of COVID-19”, *Mathematical Biosciences and Engineering*, 19(2), pp. 2043-2055, 2022.
- [4] B. Lim, S.Ö. Arık, N. Loeff, T. Pfister, “Temporal fusion transformers for interpretable multi-horizon time series forecasting”, *International journal of forecasting*, 37(4), pp. 1748-1764, 2021.
- [5] S. Jiao, Y. Wang, X. Ye, L. Nagahara, T. Sakurai, “Spatio-temporal epidemic forecasting using mobility data with LSTM networks and attention mechanism”, *Sci Rep*, 15(9603), 2025.

ABSTRACT 60

A new improvement on the class of constant coefficient sub-equation methods

Mir Sajjad Hashemi*Department of Mathematics, University of Bonab, Bonab, Iran*

E-mail: hashemi_math396@yahoo.com

Abstract: his work introduces a significant improvement to the class of constant coefficient sub-equation methods by proposing a new analytical technique termed the "Variable Coefficient Generalized Abel Equation Method." Unlike existing sub-equation approaches that rely on constant coefficient ordinary differential equations (ODEs) or auxiliary ODEs, the proposed method employs variable coefficient ODEs, thereby offering greater flexibility and power in solving nonlinear equations. The effectiveness of this new method is demonstrated by applying it to various equations. This advancement extends the applicability of variable coefficient sub-equation techniques to a wide range of models in mathematical physics.

Keywords: Generalized Abel equation method, Exact solutions, Differential equations.

Mathematics Subject Classification: 83C15, 35C08.

References

- [1] M.S. Hashemi, "A variable coefficient third degree generalized Abel equation method for solving stochastic Schrödinger–Hirota model", *Chaos, Solitons & Fractals*, 180, 114606, (2024).
- [2] M.S. Hashemi, M. Bayram, M.B. Riaz, D. Baleanu, "Bifurcation analysis, and exact solutions of the two-mode Cahn–Allen equation by a novel variable coefficient auxiliary equation method", *Results in Physics* 64, 107882, (2024).
- [3] E.M.E. Zayed, B.M.M. Saad, M.S. Hashemi, M.A.S. Murad, A.H. Arnous, M. Bayram, "Exact solutions of stochastic generalized long–short wave resonance equations with cubic–quintic nonlinearity via the new sub-ODE method", *Nonlinear Analysis: Modelling and Control*, 31, 1-22, (2026).
- [4] H. Biçer, "A variable coefficient second-degree Abel equation method for the complex Ginzburg–Landau equation with anti-cubic nonlinearity", *Communications in Theoretical Physics*, 78(6), 065004, (2026).
- [5] N Günhan Ay, E Yasar, "Analytical Study of Perturbed Nonlinear Schrödinger Equations: Optical Soliton Dynamics Via Generalized Abel Equation and Collective Variables Approaches", *Mathematical Methods in the Applied Sciences*, 49(3), 1998–2006, (2026).

ABSTRACT 61

Solar panel control system with bi-axial sun tracking and implementation model using Arduino**Hadda Tiouiri, Leila Bechane, Hakim Baaziz***Laboratory of physics and chemistry of materials, University of M'sila, Algeria*

E-mail: hadda.tiouiri@univ-msila.dz leila.bechane@univ-msila.dz, hakim.baaziz@univ-msila.dz

Abstract: This study focuses on the design and implementation of an automatic dual-axis solar tracking system controlled by an Arduino board in order to improve the efficiency of photovoltaic panels. The developed system automatically tracks the sun's position along two rotational axes, maximizing solar radiation capture throughout the day. The main objective is to develop a precise, cost-effective, and accessible solution capable of increasing energy production by 25% to 35% compared to traditional fixed systems. The device combines an astronomical algorithm to calculate the theoretical position of the sun with photoelectric sensors that provide real-time adjustment. The system is controlled by an Arduino Mega 2560 microcontroller driving stepper motors to ensure accurate positioning of the photovoltaic panels. Simulation results show an average improvement in energy efficiency of about 30% over a year, with gains reaching up to 40% during intermediate seasons. The estimated cost of the experimental prototype is approximately €250, with a return on investment estimated between 3 and 4 years depending on installation and operating conditions.

References

- [1] H. Kalpen, W. Honle, M. Somer, U. Schwarz, K. Peters and H. Georg
- [2] von Schnering, Z. anorg. allg. Chem. 1998, 624.
- [3] A. D. Becke and E. R. Johson, J. chem. Phys. 124, 2006, 221101.
- [4] J. Chao et al., Mater. Res. Bull. 98, 2018, 194–199.
- [5] A. Sarkar et al., J. Colloid Interface Sci. 483, 2016, 49–59.
- [6] J. Ni et al., Adv. Energy Mater. 4, 2014, 1400798.
- [7] Y. Cheng et al., Angew. Chem., Int. Ed. 57, 2018, 246–251.

ABSTRACT 62

Integrating Convolutional Neural Networks with Ensemble Techniques for Superior Image Classification

Abd elhak Djalab, Lahouaoui Lalaoui, Abdelghani Lakhdari

Mohamed Boudiaf University, Dept of Electronic Laboratory of Electrical Engineering, Faculty of Technology, , PO Box 166 Ichebilia, 28000 M'sila, Algeria

E-mail: abdelhak.djalab@univ-msila.dz, lahouaoui.lalaoui@univ msila.dz,
abdelghani.lakhdari@univ-msila.dz

Abstract: This study introduces a novel hybrid architecture that synergizes Convolutional Neural Networks (CNNs) with advanced ensemble techniques to significantly elevate the performance of image classification tasks. The proposed framework employs a multi-stage pipeline, integrating feature extraction from state-of-the-art pre-trained CNNs with the predictive strengths of diverse machine learning classifiers. Through systematic experimentation on benchmark image datasets, we demonstrate that the hybrid model achieves state-of-the-art accuracy while maintaining robustness against over fitting. Additionally, we explore the impacts of data augmentation and hyper parameter optimization on model performance. Our findings highlight not only the improvements in classification accuracy but also the potential for enhanced interpretability through ensemble model contributions. This work paves the way for further research into hybrid methodologies, offering a promising avenue for future advancements in computer vision applications.

References

- [1] G. Nuneti, R. P. Ch, R. Kumar K, and K. Swamy E, "AI-Assisted Breast Cancer Prediction, Classification, and Future Directions: A Narrative Review Involving Histopathological Image Datasets," *The Open Public Health Journal*, vol. 18, pp. 1–15, Jan. 2025.
- [2] A. Manzoor, G. Fargetta, A. Ortis, and S. Battiato, "ShieldNet: A Novel Adversarially Resilient Convolutional Neural Network for Robust Image Classification," *Applied Sciences*, vol. 16, no. 3, pp. 1254–1267, Feb. 2026.
- [3] A. B. Abdusalomov, M. Mukhiddinov, and T. K. Whangbo, "Brain Tumor Detection Based on Deep Learning Approaches and Magnetic Resonance Imaging," *Cancers*, vol. 15, no. 16, pp. 4172–4191, Aug. 2023.
- [4] S. Sharma and K. Singh, "Hybrid Deep Learning Architectures for Image Classification: A Survey of Feature Extraction and Ensemble Methods," *IEEE Access*, vol. 12, pp. 45120–45135, Mar. 2024.
- [5] M. R. Ibrahim, J. A. Lane, and T. O. Santos, "Hyperparameter Optimization and Data Augmentation Strategies for Robust Convolutional Neural Networks," *Int. J. Comput. Vis.*, vol. 132, no. 4, pp. 512–529, Apr. 2024.
- [6] Y. Zhang, L. Wang, and X. Dynamic, "Ensemble Convolutional Neural Networks for Benchmark Image Datasets: Overfitting Mitigation and Interpretability Analysis," in *Proc. IEEE/CVF Conf. Comput. Vis. Pattern Recognit. (CVPR)*, pp. 11425–11434, 2025..

ABSTRACT 63

A Simplified Real-Time Numerical Algorithm for Switching Angle Optimization Minimizing Total Harmonic Distortion in Cascade Multilevel Inverter

Yu-Chi Wu, Chia-Huang Li, Tsung-Wei Tsai, Shih-Hai Chien

National United University, Dept. of Electrical Engineering, 360, Miao-Li, Taiwan

E-mail: ycwu@nuu.edu.tw

Abstract: This paper presents a simplified real-time numerical algorithm for solving the nonlinear switching angle optimization problem in cascade multilevel inverters, with the objective of minimizing Total Harmonic Distortion (THD) in the output voltage waveform. The optimal switching angles are determined by a system of nonlinear transcendental equations derived from the Optimized Harmonic Stepped-Waveform (OHSW) modulation strategy. Existing real-time solutions to this nonlinear system require square-root operations at each iteration, imposing a computational bottleneck that limits the modulation update rate in digital controllers. The proposed algorithm reformulates the iterative numerical procedure through algebraic substitution, eliminating the square-root operation and reducing per-iteration computation time while preserving the THD minimization property. The algorithm is applicable to both equal and unequal DC voltage source configurations. Numerical verification is conducted on an 11-level cascade multilevel inverter, and Hardware-in-the-Loop (HIL) validation is performed using a TMS320F28335 DSP and a Xilinx Virtex-6 FPGA integrated with the RT-LAB real-time digital simulator. Experimental results demonstrate that the proposed algorithm achieves a 4.3–4.5% reduction in average CPU processing time compared to existing methods, while maintaining equivalent THD minimization performance. The FPGA implementation further achieves a per-iteration computation latency of 0.6 μ s, confirming the feasibility of the proposed numerical approach for high-speed real-time power electronics control applications.

Keywords: Nonlinear Equations, Numerical Application, Applied Mathematics.

Mathematics Subject Classification: 65H10, 65K05, 94C05.

References

- [1] Y. Liu, H. Hong, and A. Q. Huang, "Real-time calculation of switching angles minimizing THD for multilevel inverters with step modulation," *IEEE Trans. Ind. Electron.*, 56(2), pp. 285–293, 2009.
- [2] Y. Liu, H. Hong, and A. Q. Huang, "Real-time algorithm for minimizing THD in multilevel inverters with unequal or varying voltage steps under staircase modulation," **IEEE Trans. Ind. Electron.**, vol. 56, no. 2, pp. 294–303, 2009.
- [3] S. Urgan, H. Yigit, and S. Mirjalili, "Investigation of Recent Metaheuristics Based Selective Harmonic Elimination in Multilevel Inverters," *Electronics*, vol. 12, no. 4, p. 1058, 2023.
- [4] P. Kala, A. Sharma, V. Jatuly, J. Joshi, and Y. Yang, "An Initial Solution Based Selective Harmonic Elimination Method for Multilevel Inverter," *CPSS Trans. Power Electron.*, 9 (1), pp. 63-78, 2024.
- [5] Y. Bektas, "Real-time control of Selective Harmonic Elimination in a multilevel inverter for renewable energy systems," *Alexandria Eng. J.*, 5(6), 2024.
- [6] K. Rangasamy and Rani Thottungal, "FPGA Based Hardware Implementation of WTHD Minimisation in Asymmetric Cascaded Multilevel Inverter," **Istanbul Univ. J. Electr. Electron. Eng.*, 14(2), 2025.

ABSTRACT 64

59 Years of Fractional Calculus with Caputo Operator

Dumitru Baleanu

*Department of Computer Science and Mathematics, Lebanese American University,
Beirut, Lebanon*

E-mail: dumitru.baleanu@gmail.com

Abstract: Fractional calculus represents an extension of meaning and it has a long history in both science and engineering. In this talk I will present some important applications of fractional calculus in the Caputo sense in Physics, Engineering and Biology.

Keywords: Fractional calculus, Caputo operator, MABC operator.

References

- [1] Dumitru Baleanu, Kai Diethelm, Enrico Scalas and Juan J. Trujillo, *Fractional calculus: models and numerical methods*. World Scientific, New York, Second edition, 2016.
- [2] Michele Caputo. *Linear Models of Dissipation whose Q is almost Frequency Independent-II* Geophys. J. R. Astr. Soc. 529-539, 1967.

ABSTRACT 65

Bifurcation Analysis for a Stem-Cell Proliferation Model in a Hematological Disease under Continuous Small-Dose Treatment

Olfat Al Lakkies, A. Halanay

*Department of Mathematics and Informatics, National University for Science and Technology
Politehnica Bucharest, 060042, Bucharest, Romania.*

E-mail: olfat.al@stud.fsa.upb.ro and andrei.halanay@upb.ro

Abstract: This work investigates a delayed mathematical model for the proliferation of affected pluripotent stem cells in a hematological disease under a continuous small-dose treatment scheme. The treatment is introduced through pharmacokinetic variables representing the plasmatic and bone-marrow concentrations of the active substance, while the proliferative phase of hematopoietic stem cells is described by a distributed delay. We analyze the existence and local stability of the positive equilibrium and identify conditions under which the delay destabilizes the equilibrium and produces a Hopf bifurcation. Using center manifold theory and normal-form calculations for retarded functional differential equations, the first Lyapunov coefficient is obtained in order to determine the direction and stability of the bifurcating periodic solutions. Numerical verification shows that increasing the treatment intensity reduces the equilibrium level of affected stem cells, but also changes the critical delay and the stability margin of the delayed feedback system. The results emphasize that treatment design should take into account both dose intensity and the biological cell-cycle delay.

Keywords: Stem-cell proliferation, hematological disease, distributed delay, continuous treatment, Hopf bifurcation, stability analysis, Lyapunov coefficient, mathematical biology.

Mathematics Subject Classification: 34K18, 34K20, 37G15, 92C37.

References

- [1] M. Adimy, Integrated semigroups and delay differential equations, *J. Math. Anal. Appl.* 177(1), 125–124, 1993.
- [2] M. Adimy, F. Crauste and S. Ruan, Stability and Hopf bifurcation in a mathematical model of pluripotent stem cell dynamics, *Nonlinear Analysis: Real World Applications*, 6(4) (2005), 651–670.
- [3] M. Adimy, F. Crauste and S. Ruan, A mathematical study of the hematopoiesis process with applications to chronic myelogenous leukemia, *SIAM Journal on Applied Mathematics*, 65(4) (2005), 1328–1352.
- [4] M. Adimy, F. Crauste, A. Halanay, M. Neamțu and D. Opriș, Stability of limit cycles in a pluripotent stem cell dynamics model, *Chaos, Solitons & Fractals*, 27(4) (2006), 1091–1107.
- [5] S. Bernard, J. Belair and M. C. Mackey, Sufficient conditions for stability of linear differential equations with distributed delay, *Disc. Cont. Dyn. Sys. Ser. B*, 1, 233–256, 2001.
- [6] G. Bradford, B. Williams, R. Rossi and I. Bertinello, Quiescence, cycling, and turnover in the primitive haematopoietic stem cell compartment, *Exper. Hematol.* 25, 445–453, 1997.
- [7] F. J. Burns and I. F. Tannock, On the existence of a G_0 phase in the cell cycle, *Cell. Tissue Kinet.* 19, 321–334, 1970.

- [8] J. Carr, Applications of Center Manifold Theory, Springer, Berlin, 1981.
- [9] N. Chafee, A bifurcation problem for a functional differential equation of finitely retarded type, J. Math. Anal. and Appl. 35, 312–348, 1991.
- [10] J. Clairambault, D. Claude, E. Filipski, T. Granda and F. Levi, Toxicité et efficacité antitumorale de l'oxaliplatine sur l'osteosarcome de Glasgow induit chez la souris: un modele mathematique, Patologie Biologie, 51 (2003), 212–215.
- [11] C. Colijn and M. C. Mackey, Bifurcation and bistability in a model of hematopoietic regulation, SIAM Journal on Applied Dynamical Systems, 6(2) (2007), 378–394.
- [12] T. Faria and L. Magalhaes, Normal Forms for Retarded Functional Differential Equations and Applications to Bogdanov-Takens singularity, J. Diff. Eq. 122(2), 201–224, 1995.
- [13] J. Hale and S. M. Verduyn Lunel, Introduction to Functional Differential Equations, Applied Mathematical Sciences, Vol. 99, Springer, New York, 1993.
- [14] A. Halanay, Differential Equations: Stability, Oscillations, Time Lags, Academic Press, New York, 1966.
- [15] C. Haurie, D. C. Dale and M. C. Mackey, Cyclical neutropenia and other periodic hematological diseases: A review of mechanisms and mathematical models, Blood 92, 2629–2640, 1998.
- [16] B. D. Hassard, N. D. Kazarinoff and Y.-H. Wan, Theory and Applications of Hopf Bifurcation, London Mathematical Society Lecture Note Series 41, Cambridge University Press, 1981.
- [17] H. Kellermann and M. Hieber, Integrated Semigroup, J. Fund. Anal. 84, 160–180, 1989.
- [18] M. C. Mackey, A unified hypothesis on the origin of aplastic anaemia and periodic hematopoiesis, Blood 51, 946–956, 1978.
- [19] M. C. Mackey, Dynamic hematological disorders of stem cell origin, in “Biophysical and Biochemical Information Transfer in Recognition”, eds. by J. G. Vassileva-Popova and E. V. Jensen, Plenum Press, New York, pp. 373–409, 1979.
- [20] M. C. Mackey, C. Haurie and J. Belair, Cell replication and control, in “Nonlinear Dynamics in Physiology and Medicine”, eds. by A. Beuter, L. Glass, M. C. Mackey and M. S. Titcombe, Springer, New York, pp. 233–269, 2003.
- [21] L. Pujo-Menjouet, S. Bernard and M. C. Mackey, Long period oscillations in a G_0 model of hematopoietic stem cells, SIAM Journal on Applied Dynamical Systems, 4(2) (2005), 312–332.
- [22] L. Pujo-Menjouet and M. C. Mackey, Contribution to the study of periodic chronic myelogenous leukemia, C. R. Biologies 327, 235–244, 2004.
- [23] L. Sachs, The molecular control of hematopoiesis and leukemia, C. R. Acad. Sci. Paris 316, 882–891, 1993.
- [24] G. Stepan, Retarded dynamical systems: stability and characteristic functions, Pitman Research Notes in Math. Ser., Longman, Harlow, 1989.
- [25] D. C. De Souza and A. R. Humphries, Dynamics of a mathematical hematopoietic stem-cell population model, SIAM Journal on Applied Dynamical Systems, 18(2) (2019), 808–852.
- [26] R. K. Pedersen, M. Andersen, T. Stiehl and J. T. Ottesen, Understanding hematopoietic stem cell dynamics—insights from mathematical modelling, Current Stem Cell Reports, 9 (2023), 9–16.

- [27] I. Abdulrashid and X. Han, A mathematical model of chemotherapy with variable infusion, *Communications on Pure and Applied Analysis*, 19(7) (2020), 3539–3558.
- [28] J. Hesse, N. Nelson and A. Relógio, Shaping the future of precision oncology: integrating circadian medicine and mathematical models for personalized cancer treatment, *Current Opinion in Systems Biology*, 37 (2024), 100506.
- [29] C. Zhu and T. Stiehl, Modelling post-chemotherapy stem cell dynamics in the bone marrow niche of AML patients, *Scientific Reports*, 14 (2024), 25060.
- [30] L. G. Parajdi, X. Bai, D. Kegyes and C. Tomuleasa, A mathematical model of clonal hematopoiesis explaining phase transitions in myeloid leukemia, *Mathematical Medicine and Biology: A Journal of the IMA*, 42(3) (2025), 253–288.
- [31] M. Brunetti, I. A. Iasenza, A. L. Jenner, N. J.-M. Raynal, K. Eppert and M. Craig, Mathematical modelling of clonal reduction therapeutic strategies in acute myeloid leukemia, *Leukemia Research*, 140 (2024), 107485.
- [32] S. Chulián, A. Martínez-Rubio, M. Rosa and V. M. Pérez-García, Mathematical models of leukaemia and its treatment: a review, *SeMA Journal*, 79 (2022), 441–486.
- [33] C. Colijn and M. C. Mackey, A mathematical model of hematopoiesis–I. Periodic chronic myelogenous leukemia, *Journal of Theoretical Biology*, 237(2) (2005), 117–132.
- [34] C. Colijn and M. C. Mackey, A mathematical model of hematopoiesis–II. Cyclical neutropenia, *Journal of Theoretical Biology*, 237(2) (2005), 133–146.
- [35] A. Halanay, Stability analysis for a mathematical model of chemotherapy action in hematological diseases, *Bulletin of the Mathematical Society of Sciences Mathematiques de Roumanie*, 53(101), no. 1 (2010), 3–10.
- [36] A. Halanay, Treatment induced periodic solutions in some mathematical models of tumoral cell dynamics, *Mathematical Reports*, 12(62), no. 4 (2010), 329–339.
- [37] I. Badralexi, A.-M. Bordei, A. Halanay and I. R. Rădulescu, Dynamics of Chronic Myeloid Leukemia under Imatinib Treatment: A Study of Resistance Development, *Mathematics*, 12 (2024), 3937. <https://doi.org/10.3390/math12243937>.

ABSTRACT 66

A Data Mining Study on the Synthesis of Hydrogen from Hydrocarbons

Nadera Settouti, Bachir Liani

Laboratory of Theoretical Physics, university of Tlemcen, Algeria

E-mail: nd.settouti@gmail.com, bliani@yahoo.fr

Abstract: The synthesis of hydrogen from hydrocarbons represents a critical technology for sustainable energy systems, with steam methane reforming (SMR) currently producing 95% of hydrogen in the United States. This data mining study analyzes machine learning applications across hydrocarbon-to-hydrogen conversion processes, including SMR, partial oxidation, and CO₂ hydrogenation via Fischer–Tropsch synthesis. We compiled and analyzed literature data from over 100 peer-reviewed publications, revealing that artificial neural networks (ANNs) achieve R^2 values of 0.9987 for SMR process optimization, while Random Forest models predict Fischer–Tropsch catalyst performance with mean average errors of 3.7–10.9%. Key findings demonstrate that temperature, pressure, BET surface area, and catalyst composition are the primary descriptors governing hydrogen production efficiency. This study establishes data mining as a transformative approach for optimizing hydrocarbon-to-hydrogen conversion, reducing experimental costs, and accelerating the development of low-carbon hydrogen production technologies.

References

- [1] Y. Wang, “Machine Learning-Based Predictive Control of an Electrically-Heated Steam Methane Reforming,” PhD thesis, University of California, Los Angeles, 2024.
- [2] Y. Wang *et al.*, “Machine Learning-based Energy Optimization for On-site SMR Hydrogen Production,” *Journal of Hydrogen Energy*, 2021.
- [3] J. Hu *et al.*, “Parametric analysis of CO₂ hydrogenation via Fischer–Tropsch synthesis: A review based on machine learning for quantitative assessment,” *International Journal of Hydrogen Energy*, vol. 59, pp. 1023–1041, 2024. doi: 10.1016/j.ijhydene.2024.02.055.
- [4] S. G. Mallat, “A theory for multiresolution signal decomposition: The wavelet representation,” *Transactions of the American Mathematical Society*, vol. 315, no. 1, pp. 69–88, 1989.
- [5] A. Fedorov and D. Linke, “Witchcraft with small data: AI models for Fischer–Tropsch process based on CO₂,” Leibniz-Institut für Katalyse, 2025.

ABSTRACT 67

Well-posedness and basic properties for an epidemic model with arbitrary order

Amar Debbouche*Department of Mathematics, Guelma University, Guelma, 24000 Algeria*

E-mail: amar_debbouche@yahoo.fr

Abstract: We investigate a specific SIR model with quarantine and vaccination compartments. We start to establish the well-posedness and boundedness results of the considered system. For the main results, a generation matrix and equilibrium points of the model are obtained. In order to manage the transmission of infection within the outlined model, we employ the strategy of optimal control. This approach involves implementing control measures and interventions guided by mathematical optimization techniques to minimize the spread of disease. These control strategies may encompass vaccination campaigns, quarantine protocols, social distancing measures and other preventive actions. Further, to evaluate the effectiveness of proposed model and the applied optimal control strategy, we conduct a series of numerical simulations. Computational results involve running the model under different scenarios, considering a range of parameters and meticulously analyzing the resulting outcomes.

Keywords: Epidemic model, well-posedness, sensitivity analysis, optimal control, numerical simulation.

References

- [1] S. Hariharan, L. Shangerganesh, A. Debbouche, V. Antonov, Dynamic behaviors for fractional epidemiological model featuring vaccination and quarantine compartments, *Journal of Applied Mathematics and Computing*, **71**(1) (2025) 489–509.
- [2] H. Ismail, S. Hariharan, L. Shangerganesh, A. Debbouche, S.V. Kashtanova, Dynamic properties and computational studies for a nonlinear multi-delay epidemic model, *Computational Mathematics and Modeling*, (2025) 1–30.

ABSTRACT 68

Advanced integration of fractional calculus and AI for complex systems modeling

Zakia Hammouch^{1,2,3}

¹*Department of Sciences, ENS Moulay Ismail University, Morocco*

²*Department of Electrical and Electronics Engineering*

³*The Faculty of Engineering and Natural Sciences and Natural Sciences Fenerbahçe University, Türkiye
Department of Medical Research, China Medical University Taichung Taiwan*

E-mail:

Abstract: This presentation explores the integration of fractional calculus and artificial intelligence in the mathematical modeling of complex dynamical systems, with applications in electrical and epidemiological contexts. We develop novel fractional differential models to describe population interactions and the spread of infectious diseases, as well as behavioral dynamics such as drug addiction, capturing long-term memory and hereditary properties through advanced fractional operators. Additionally, hybrid AI frameworks combining neural networks, genetic algorithms, and optimal control techniques are employed to enhance the predictive accuracy and scalability of these models. Numerical simulations and sensitivity analyses demonstrate the effectiveness of the proposed approaches in capturing system behaviors, stability, and bifurcations, providing valuable insights for system optimization and public health strategies. This interdisciplinary work highlights the potential of combining fractional calculus and artificial intelligence to address complex real-world challenges and improve decision-making processes.

References

- [1] Z. Hammouch, M. Ouazzani Jamil, C. Unlu *et al.*, “Dynamics investigation and numerical simulation of fractional-order predator-prey model with Holling type II functional response,” *Discrete and Continuous Dynamical Systems-S*, vol. 18, no. 5, pp. 1230–1266, 2025.
- [2] N. Madani and Z. Hammouch, “A new approach combining tangent hyperbolic and logistic sigmoid neural networks to model and solve tumor growth dynamics,” *Knowledge-Based Systems*, p. 114679, 2025.
- [3] N. Madani and Z. Hammouch, “A hybrid approach using logistic sigmoid, radial basis, and hyperbolic tangent neural networks to model M-pox disease dynamics,” *Journal of Computational and Applied Mathematics*, p. 117146, 2025.
- [4] N. Madani, Z. Hammouch, and N. Ozdemir, “Hybrid GA-IPA and neural network-based solution for fractional-order nonlinear predator-prey dynamical systems,” *Nonlinear Dynamics*, vol. 113, no. 23, pp. 32979–33004, 2025.

ABSTRACT 69

A Data-Driven Framework for Characterizing Nonlinear Dynamical Behaviors through Self-Organizing Maps and Cluster-Based Analysis**Syeda Sarwat Kazmi^{1,2}, Muhammad Bilal Riaz¹**¹*IT4Innovations, VSB – Technical University of Ostrava, Ostrava, Czech Republic*²*Faculty of Electrical Engineering and Computer Science, VSB - Technical University of Ostrava, Ostrava, Czech Republic*

E-mail: syeda.sarwat.kazmi@vsb.cz, muhammad.bilal.riaz@vsb.cz

Abstract: Understanding complex nonlinear dynamical systems remains challenging due to the intricate dependence of system behavior on parameter variations. This work presents a data-driven framework for identifying and characterizing distinct dynamical regimes through the integration of self-organizing maps (SOMs) and hierarchical clustering techniques. A large parameter space is systematically explored by generating trajectory data from a nonlinear dynamical system. The resulting high-dimensional dynamical responses are projected onto a topology-preserving SOM, allowing the extraction of latent behavioral structures and the classification of similar trajectories into coherent groups. To investigate the dynamical characteristics of the identified clusters, phase portraits, bifurcation structures, Poincaré maps, and Lyapunov exponent analyses are employed. The results reveal multiple behavioral patterns ranging from regular oscillatory dynamics to chaotic responses, together with transition regions separating distinct dynamical states. The proposed framework provides an efficient approach for exploring parameter-dependent behaviors, uncovering hidden structures in complex systems, and supporting the analysis of nonlinear phenomena arising in applied mathematics, physics, and engineering.

Keywords: Nonlinear dynamical systems, Self-organizing maps, Clustering, Bifurcation analysis, Chaos.

Mathematics Subject Classification: 37M10, 37N99, 68T09.

References

- [1] T. Kohonen, *Self-Organizing Maps*, vol. 30, Springer Science & Business Media, 2012.
- [2] L. Kaufman and P. J. Rousseeuw, *Finding Groups in Data: An Introduction to Cluster Analysis*, John Wiley & Sons, 2009.
- [3] F. Nielsen and F. Nielsen, “Hierarchical clustering,” in *Introduction to HPC with MPI for Data Science*, pp. 195–211, 2016.
- [4] S. H. Strogatz, *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering*, Chapman and Hall/CRC, 2024.
- [5] G. C. Layek, *An Introduction to Dynamical Systems and Chaos*, vol. 1, Springer India, West Bengal, India, 2015.

ABSTRACT 70

Reaction–Diffusion Dynamics in Heterogeneous Habitats with Time-Dependent Interfaces

N. DIB-BAGHDADLI

Department of Mathematics, Laboratory of Nonlinear Analysis and Applied Mathematics, Abou Bekr Belkaid University, 13000 Tlemcen, Algeria
E-mail: nabahatadiba.dib@univ-tlemcen.dz

Abstract: Spatial heterogeneity and time-dependent interfaces play a critical role in the persistence and distribution of populations in ecological and epidemiological systems. In this work, we extend classical reaction–diffusion models to heterogeneous domains separated by moving interfaces with dynamic transmission conditions. We establish the well-posedness of the resulting system using maximal L^p -regularity theory and illustrate the influence of spatial heterogeneity, interface evolution, and asymmetric fluxes through numerical simulations. The proposed framework is applicable to arbitrary spatial dimensions and provides insights into population persistence, spatial spread, and stability thresholds in complex environments.

Keywords: Reaction–diffusion, Heterogeneous habitats, Moving interfaces, Maximal L^p -regularity, Population persistence, Numerical simulations.

Mathematics Subject Classification: 35K57, 35R37, 92D25, 92C60.

References

- [1] J. Prüss and G. Simonett, *Moving Interfaces and Quasilinear Parabolic Evolution Equations*, Birkhäuser, 2016.
- [2] R. S. Cantrell and C. Cosner, “On the effects of spatial heterogeneity on the persistence of populations,” *Journal of Mathematical Biology*, vol. 48, no. 6, pp. 585–614, 2004.
- [3] H. Amann, *Linear and Quasilinear Parabolic Problems. Volume I: Abstract Linear Theory*, Birkhäuser, Basel, Switzerland, 1995.
- [4] N. Dib-Baghdadli, R. Labbas, T. Mahdjoub, and A. Medeghri, “On some reaction-diffusion equations generated by non-domiciliated triatominae, vectors of Chagas disease,” *Discrete & Continuous Dynamical Systems – Series B*, vol. 26, no. 12, pp. 6091–6115, 2021.

ABSTRACT 71

Physics-Informed Neural Networks for Compact Star Modeling

Ayesha Khalid, Muhammad Bilal Riaz

IT4Innovations, VSB – Technical University of Ostrava, Ostrava, Czech Republic

E-mail: kha0057@vsb.cz, muhammad.bilal.riaz@vsb.cz

Abstract: In this work, we study anisotropic compact stars using a physics-informed neural network (PINN) framework in a static, spherically symmetric spacetime governed by the Einstein field equations. The model is trained on a known analytical solution to examine how accurately PINNs can learn anisotropic stellar configurations. A modified Van der Waals equation of state and neural corrections to the Tolman VII metric potentials are used to improve flexibility while maintaining physical consistency. The obtained density, radial pressure, tangential pressure, and anisotropy profiles remain smooth, stable, and physically well behaved throughout the stellar interior. The close agreement with the analytical solution confirms the reliability and accuracy of the supervised PINN approach. This study shows that PINNs can learn physically consistent compact star solutions and may be extended to more complex cases where analytical solutions are unavailable.

Keywords: Physics-Informed Neural Networks for Anisotropic Compact Star Modeling.

Mathematics Subject Classification: 12X34, 56Y78.

References

- [1] M. Raissi, P. Perdikaris, and G. E. Karniadakis, “Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations,” *Journal of Computational Physics*, vol. 378, pp. 686–707, 2019.
- [2] G. E. Karniadakis, I. G. Kevrekidis, L. Lu, P. Perdikaris, S. Wang, and L. Yang, “Physics-informed machine learning,” *Nature Reviews Physics*, vol. 3, pp. 422–440, 2021.
- [3] Z.-H. Li, C.-Q. Li, and L.-G. Pang, “Solving Einstein equations using deep learning,” arXiv:2309.07397 [gr-qc], 2023.

ABSTRACT 72

First Type s -Orlicz Space : Completeness, Inclusion, and s -Hölder Inequality

Sofihara Al Hazmy

*Faculty of Mathematics and Natural Science Education, Universitas Pendidikan Indonesia, 40154
Bandung, Indonesia*

E-mail: sofiharaalhazmy@upi.edu

Abstract: By replacing the convexity of Young's function with first type s -convexity, we construct a novel function space that generalize the classical Orlicz space, which we called as s -Orlicz space. Several essential properties such as completeness, inclusion, and Hölder's type inequality have been established. This novel concept unlock a new branch in function space arising in harmonic analysis study field.

Keywords: First Type s -Orlicz Space, Completeness, Inclusion, s -Hölder Inequality.

Mathematics Subject Classification: 42B35.

References

- [1] W. Orlicz, *Linear Functional Analysis*, Series in Real Analysis, World Scientific, 1992.
- [2] H. Hudzik and L. Maligranda, "Some remarks on s -convex functions," *Aequationes Mathematicae*, vol. 48, pp. 100–111, 1994.
- [3] I. M. R. Pinheiro, "Second note on the definition of s -convexity," *Advances in Pure Mathematics*, vol. 05, pp. 127–130, 2015.
- [4] I. M. R. Pinheiro, "Exploring the concept of s -convexity," *Aequationes Mathematicae*, vol. 74, pp. 201–209, 12 2007.
- [5] I. M. R. Pinheiro, "Jensen's inequality in detail and s -convex functions," *International Journal of Mathematical Analysis*, vol. 3, pp. 95–98, 2009.
- [6] S. A. Hazmy, S. Fātimah, and A. A. Masta, "Some notes about boundedness of fractional integral on orlicz spaces," in *AIP Conference Proceedings*, vol. 2734, p. 080002, AIP Publishing, 2023.
- [7] S. Fātimah, S. A. Hazmy, and A. A. Masta, "Necessary condition for boundedness of stein-Weiss operator on orlicz spaces," *Journal of Engineering Science and Technology*, vol. 16, pp. 3792–3800, 2021.
- [8] S. Fātimah, A. A. Masta, S. A. Hazmy, C. Kustiwan, and I. Rukmana, "Discrete orlicz-morrey spaces and their inclusion properties," *Journal of Engineering Science and Technology*, vol. 16, pp. 2018–2027, 2021.
- [9] A. A. Masta, H. Gunawan, and W. Setya-Budhi, "On inclusion properties of two versions of orlicz-morrey spaces," *Mediterranean Journal of Mathematics*, vol. 14, p. 228, 12 2017.
- [10] E. Nakai, "Orlicz-morrey spaces and the hardy-littlewood maximal function," *Studia Mathematica*, vol. 188, no. 3, pp. 193–221, 2008.
- [11] M. T. Al Azhary Masta, Siti Fātimah, "Third version of weak orlicz-morrey spaces and its inclusion properties," *Indonesian Journal of Science and Technology*, vol. 4, no. 2, pp. 257–262, 2019.

- [12] J. Havrda and F. Charvát, “Restriction of risk space and its dense space,” *Asian-European Journal of Mathematics*, vol. 14, p. 2150114, 7 2021.
- [13] S. Gala, Y. Sawano, and H. Tanaka, “A remark on two generalized orlicz-morrey spaces,” *Journal of Approximation Theory*, vol. 198, pp. 1–9, 10 2015.
- [14] Y. Sawano, S. Sugano, and H. Tanaka, “Orlicz-morrey spaces and fractional operators,” *Potential Analysis*, vol. 36, pp. 517–556, 5 2012.
- [15] T. Yajing, B. Harzika, S. A. Mohiuddin, et al., “Norm inequalities involving upper bounds for certain matrix operators on orlicz ellipses and orlicz-taylor double sequence spaces,” *Journal of Analysis*, vol. 31, pp. 2817–2834, 2023.
- [16] K. Raj, M. Devi, and M. Mursaleen, “New product type operators acting between weighted bergman-orlicz spaces and some weighted-type spaces,” *Journal of Analysis*, vol. 32, pp. 2299–2327, 2024.
- [17] A. Manna, “Norm inequalities involving upper bounds for certain matrix operators in orlicz-type sequence spaces,” *Journal of Analysis*, vol. 27, pp. 761–779, 2019.
- [18] W. A. J. Luxemburg, *Banach Function Spaces*, Thesis, Technische Hogeschool te Delft, 1955.
- [19] A. A. Masta, H. Gunawan, and W. Setya-Budhi, “On inclusion properties of two versions of orlicz-morrey spaces,” *Mediterranean Journal of Mathematics*, vol. 14, Dec. 2017.
- [20] A. A. Masta, Ifronika, and S. Fātimah, “Sufficient and necessary conditions for holders inequality in weighted orlicz spaces,” *Int. J. Appl. Phys. Math.*, vol. 10, no. 1, pp. 25–32, 2020.
- [21] H. G. Al Azhary Masta and W. S. Budhi, “Inclusion properties of orlicz and weak orlicz spaces,” *J. Math. Fundam. Sci.*, vol. 48, pp. 193–203, Dec. 2016.
- [22] E. W. Kreyszig and H. L. Heyden, *Real Analysis*, Upper Saddle River, NJ: Pearson, 4th ed., Jan. 2010.

ABSTRACT 73

On s-Orlicz–Morrey Spaces of Nakai’s Version and Their Inclusion Properties

Al Azhary Masta¹, Rifkhi Akbar Putra Rustandi¹, Javid Ali², Sofihara Al Hazmy¹

¹Kanal-F Group, Mathematics Study Program, Indonesia University of Education, Jl. Dr. Setiabudi, Bandung, 40154, Indonesia

²Department of Mathematics, Aligarh Muslim University, Aligarh, India

E-mail: alazhari.masta@upi.edu, rakbarputrar@gmail.com, javid.mm@amu.ac.in, sofiharaalhazmy@upi.edu

Abstract: Lebesgue spaces are one of the fundamental topics in analysis that have been widely studied and developed. Among their generalizations are Orlicz spaces, introduced by Birnbaum and Orlicz (1931), and Morrey spaces, introduced by Morrey (1938). Nakai (2006) further developed a version of Morrey spaces, which led to various extensions and combinations with other function spaces, known as Orlicz–Morrey spaces. In this paper, we study s-Orlicz–Morrey spaces in the sense of Nakai’s version by incorporating an extended Young function. We investigate the structural properties of these spaces and establish their inclusion relations. In particular, we obtain several necessary and sufficient conditions that guarantee inclusion between different s-Orlicz–Morrey spaces. The results presented in this paper contribute to the development of more general function spaces and provide a framework for further analysis in this area.

Keywords: Generalized Orlicz–Morrey space, Orlicz–Morrey space, Young function, Generalized Young function, Inclusion property.

Mathematics Subject Classification: 26D15; 46B25, 46E30.

References

- [1] A. A. Bakery and A. R. A. Elmatty, “Some properties of pre-quasi norm on Orlicz sequence space,” *Journal of Inequalities and Applications*, vol. 2020, no. 1, pp. 1–13, 2020.
- [2] R. Dermawan, S. Fatimah, S. A. Hazmy, A. A. Masta, and C. Kustiawan, “Generalization of Young’s function,” *AIP Conference Proceedings*, vol. 2734, no. 1, 2023, <https://doi.org/10.1063/5.0156150>.
- [3] S. Fatimah, A. A. Masta, S. Al Hazmy, C. Kustiawan, and I. Rukmana, “Discrete Orlicz–Morrey spaces and their inclusion properties,” *Journal of Engineering Science and Technology*, vol. 16, no. 3, pp. 2018–2027, 2021.
- [4] C. Kustiawan, D. Dasep, E. Sumiaty, S. Fatimah, and S. Al Hazmy, “Generalized Orlicz Sequence Spaces,” *BAREKENG: Jurnal Ilmu Matematika dan Terapan*, vol. 17, no. 1, pp. 427–438, 2023.
- [5] C. Léonard, *Orlicz Spaces*, 2007.
- [6] W. A. Luxemburg, *Banach Function Spaces*, PhD Thesis, Technische Hogeschool te Delft, 1955.
- [7] A. A. Masta, S. Fatimah, and M. Taqiyuddin, “Third version of Weak Orlicz–Morrey spaces and its inclusion properties,” *Indonesian Journal of Science and Technology*, vol. 4, no. 2, pp. 257–262, 2019.
- [8] A. A. Masta, E. Sumiaty, M. Taqiyuddin, and I. Pradipta, “The sufficient condition for inclusion properties of discrete weighted Lebesgue spaces,” *Journal of Physics: Conference Series*, vol. 1013, no. 1, article 012152, 2018.

- [9] A. A. Masta, T. Ifronika, M. Taqiyuddin, and R. Rosjanuardi, "Inclusion properties of weighted weak Orlicz spaces," *Journal of Engineering Science and Technology*, vol. 16, no. 5, pp. 3974–3986, 2021.
- [10] E. Nakai, "Orlicz–Morrey spaces and some integral operators," Research Report, pp. 144–156, 2004.
- [11] E. Nakai, "On Orlicz–Morrey spaces (The structure of Banach spaces and Function spaces)," *RIMS Kokyûroku*, vol. 1520, pp. 78–88, 2006.
- [12] M. Rao and Z. Ren, *Theory of Orlicz Spaces*, New York: Marcel Dekker, Inc., 1991.
- [13] A. W. Roberts and D. E. Varberg, *Convex Functions*, New York and London: Academic Press, Inc., 1973.
- [14] Y. Sawano, S. Sugano, and H. Tanaka, "Orlicz–Morrey spaces and fractional operators," *Potential Analysis*, vol. 36, no. 4, pp. 517–556, 2012.
- [15] K. L. Tseng, S. R. Hwang, and S. S. Dragomir, "On some new inequalities of Hermite–Hadamard–Fejér type involving convex functions," *Demonstratio Mathematica*, vol. 40, no. 1, pp. 51–64, 2007.

ABSTRACT 74

Beyond Classical Fluid Modeling and Simulation

Muhammad Bilal Riaz

IT4Innovations, VSB-Technical University of Ostrava, Ostrava, Czech Republic
E-mail: muhammad.bilal.riaz@vsb.cz

Abstract: In the framework of fractional calculus, fluid models can be developed mainly through three approaches. The first approach is the direct replacement of classical integer-order derivatives by fractional derivatives in the governing equations. This method introduces memory effects through time-fractional operators and nonlocal transport through space-fractional operators. The second approach is based on the fractional tensor formulation, where classical tensorial quantities such as the velocity gradient, strain-rate tensor, deformation tensor, and stress tensor are reformulated using fractional gradient operators. This provides a more systematic fractional continuum-mechanics framework. The third approach is the fractional constitutive-law formulation, in which the conservation laws are retained in their classical form, but the stress-strain or flux-gradient relations are generalized using fractional derivatives. This method is particularly useful for modeling viscoelastic fluids, hereditary effects, relaxation phenomena, and complex non-Newtonian behavior.

Keywords: Fractional Fluid Modeling, Memory-Driven Flows; Nonlocal Transport, Fractional Constitutive Laws, Complex Fluid Simulation.

Mathematics Subject Classification: 76A05, 76A10, 76Mxx, 26A33, 35R11, 35Q35.

References

- [1] S. Momani and Z. Odibat, “Analytical solution of a time-fractional Navier–Stokes equation by Adomian decomposition method,” *Applied Mathematics and Computation*, vol. 177, no. 2, pp. 488–494, 2006.
- [2] R. Stern, R. Effenberger, H. Fichtner *et al.*, “The space-fractional diffusion-advection equation: Analytical solutions and critical assessment of numerical solutions,” *Fractional Calculus and Applied Analysis*, vol. 17, pp. 171–190, 2014.
- [3] C. S. Drapaca and S. Sivaloganathan, “A fractional model of continuum mechanics,” *Journal of Elasticity*, vol. 107, pp. 105–123.
- [4] K. A. Lazopoulos and A. K. Lazopoulos, “Fractional vector calculus and fractional continuum mechanics,” *Progress in Fractional Differentiation and Applications*, vol. 2, no. 2, pp. 85–104, 2016.
- [5] R. L. Bagley and P. J. Torvik, “Fractional calculus—a different approach to the analysis of viscoelastically damped structures,” *AIAA Journal*, vol. 21, pp. 741–748, 1983.
- [6] H. Qi and M. Xu, “Stokes’ first problem for a viscoelastic fluid with the generalized Oldroyd-B model,” *Acta Mechanica Sinica*, vol. 23, pp. 463–469, 2007.

ABSTRACT 75

Nonlinear Dynamics of Anisotropic Matter in Curvature-Corrected Gravity

Anam Zahra and Muhammad Bilal Riaz

IT4Innovations, VSB-Technical University of Ostrava, Ostrava, Czech Republic
 E-mail: anam.zahra@vsb.cz; muhammad.bilal.riaz@vsb.cz

Abstract: We have investigated the nonlinear evolution of anisotropic self-gravitating matter configurations in the framework of $f(R)$ gravity, adopting the Starobinsky model $f(R) = R + \alpha R^2$ to incorporate higher-order curvature corrections. The analysis focuses on the compact object Her X-1 described through the generalized Tolman–Kuchowicz metric in the presence of dark matter. The results show that dark matter significantly modifies the radial and tangential pressure profiles, thereby influencing the internal structure and dynamical behavior of the system. The generalized metric parameter n is found to play a crucial role in controlling the physical characteristics of the compact configuration. The model exhibits singularity-free behavior and satisfies key physical viability requirements, including energy conditions, hydrostatic equilibrium, causality, adiabatic stability, Herrera’s cracking criterion, and the mass–radius constraint.

Keywords: $f(R)$ Gravity; Anisotropic Matter; Dark Matter Effects; Tolman–Kuchowicz Metric; Compact Stars.

Mathematics Subject Classification: 12X34, 56Y78.

References

- [1] Kausar, H. R., and Noureen, I. (2014). Dissipative spherical collapse of charged anisotropic fluid in $f(R)$ gravity. *The European Physical Journal C*, 74, 1-8.
- [2] Rizwana Kausar, H., Noureen, I., and Shahzad, M. U. (2015). Dynamical analysis of charged anisotropic spherical star in $f(R)$ gravity. *The European Physical Journal Plus*, 130(10), 204.
- [3] Naz, T., Usman, A., and Shamir, M. F. (2021). Embedded class-I solution of compact stars in $f(R)$ gravity with Karmarkar condition. *Annals of Physics*, 429, 168491.
- [4] Nashed, G. G. L., and Capozziello, S. (2021). Anisotropic compact stars in $f(R)$ gravity. *The European Physical Journal C*, 81(5), 1-20.
- [5] Nojiri, S. I., and Odintsov, S. D. (2007). Introduction to modified gravity and gravitational alternative for dark energy. *International Journal of Geometric Methods in Modern Physics*, 4(01), 115-145.

ABSTRACT 76

AI-Based Diagnosis of Lung Diseases from X-ray and CT Images

Irem Nur Kivircik, Tuğcem Partal

Recep Tayyip Erdoğan University, Dept. of Computer Engineering, 53020, Rize, Turkey
 E-mail: iremnur_kivircik20@erdogan.edu.tr and tugcem.partal@erdogan.edu.tr

Abstract: In this study, we propose a hybrid artificial intelligence model that integrates Convolutional Neural Networks (CNN) and Support Vector Machines (SVM) for the automatic detection of lung diseases from chest radiography and computed tomography (CT) images. Deep features obtained through transfer-learning-based CNN architectures (VGG-16, ResNet-50) are combined using an ensemble approach and classified with an SVM, thereby improving the model's accuracy and generalization performance. Model performance is evaluated using metrics including accuracy, F1-score, Cohen's Kappa, Matthews Correlation Coefficient (MCC), and ROC analysis. The anticipated results indicate that the proposed system can serve as a clinical decision-support tool that accelerates the diagnostic process, reduces the misdiagnosis rate, and contributes to early detection.

Keywords: Convolutional Neural Networks, Support Vector Machines, Lung Diseases, Machine Learning, Medical Imaging.

Mathematics Subject Classification: 68T07, 92C55.

References

- [1] S. M. Fati, E. M. Senan, and N. ElHakim, "Deep and hybrid learning technique for early detection of tuberculosis based on X-ray images using feature fusion," *Applied Sciences*, vol. 12, no. 14, p. 7092, 2022.
- [2] D. Sharifrazi et al., "Fusion of convolution neural network, support vector machine and Sobel filter for accurate detection of COVID-19 patients using X-ray images," *Biomedical Signal Processing and Control*, vol. 68, p. 102622, 2021.
- [3] D. C. R. Novitasari et al., "Detection of COVID-19 chest X-ray using support vector machine and convolutional neural network," *Communications in Mathematical Biology and Neuroscience*, vol. 2020, 2020.
- [4] A. Bhandary et al., "Deep-learning framework to detect lung abnormality—A study with chest X-ray and lung CT scan images," *Pattern Recognition Letters*, vol. 129, pp. 271–278, 2020.
- [5] D. Rismawati, S. Surono, and A. Thobirin, "Hybrid feature fusion from multiple CNN models with Bayesian-optimized machine learning classifiers," *Computer Science and Information Technologies*, vol. 6, no. 3, pp. 315–325, 2025.
- [6] D. Varshni, K. Thakral, L. Agarwal, R. Nijhawan, and A. Mittal, "Pneumonia detection using CNN based feature extraction," in *Proc. 2019 IEEE Int. Conf. Electrical, Computer and Communication Technologies (ICECCT)*, 2019, pp. 1–7.

ABSTRACT 77

Statistical Characteristics of Collatz Sequences Generated from Different Probability Distributions

Melikenur YILDIRIM, Zafer BEKIRYAZICI

Department of Mathematics, Recep Tayyip Erdogan University, Rize, Türkiye
E-mail: melikenuryildirim13@gmail.com, zafer.bekiryazici@erdogan.edu.tr

Abstract: The Collatz conjecture, introduced by Lothar Collatz, states that every natural number greater than 1 will eventually reach 1 through the repeated application of a prescribed iterative procedure. This study focuses on the statistical properties of the sequences when the initial number generating the Collatz sequence is randomly selected from certain probability distributions. Expected sequence lengths and the standard deviation of the number of elements in the sequences are investigated in relation to the type of discretization used to obtain the initial element which generates the sequence. The results are used to comment on how the changes in probability densities affect the statistical characteristics of the generated sequences.

Keywords: Collatz conjecture, Simulation, Uniform distribution, Triangular distribution, Beta distribution.

References

- [1] David Applegate and Jeffrey C. Lagarias. *Lower Bounds for the Total Stopping Time of $3x + 1$ Iterates*. Mathematics of Computation, 72(242):1035–1049, 2003.
- [2] David Barina. *Improved Verification Limit for the Convergence of the Collatz Conjecture*. The Journal of Supercomputing, 81(7):810, 2025.
- [3] Shalom Eliahou. *The $3x + 1$ Problem: New Lower Bounds on Nontrivial Cycle Lengths*. Discrete Mathematics, 118(1–3):45–56, 1993.
- [4] Catherine Forbes, Merran Evans, Nicholas Hastings, and Brian Peacock. *Statistical Distributions*. John Wiley & Sons, Hoboken, New Jersey, 2011.
- [5] Jeffrey C. Lagarias. *The $3x + 1$ Problem and Its Generalizations*. The American Mathematical Monthly, 92(1):3–23, 1985.
- [6] Keith R. Matthews. *Generalized $3x + 1$ Mappings: Markov Chains and Ergodic Theory*. In Jeffrey C. Lagarias (Ed.), *The Ultimate Challenge: The $3x + 1$ Problem*, pp. 79–103. American Mathematical Society, Providence, Rhode Island, 2010.
- [7] John Simons and Benne de Weger. *Theoretical and Computational Bounds for m -Cycles of the $3n + 1$ Problem*. Acta Arithmetica, 117(1):51–70, 2005.

ABSTRACT 78

Analysis of the Conformity of Design Components to the Golden Ratio in Architectural Structures in the Eastern Black Sea Region

Pelin UZAN, Zafer BEKIRYAZICI

Department of Mathematics, Recep Tayyip Erdogan University, Rize, Türkiye
E-mail: pelinuzannn@gmail.com, zafer.bekiryazici@erdogan.edu.tr

Abstract: The Golden Ratio has frequently been associated with aesthetics in nature and art. This study investigates the correspondence between the Golden Ratio and various selected architectural structures in the Eastern Black Sea region of Türkiye. Architectural components such as towers, gates, façades, and decorative elements in such structures are examined to investigate whether they conform to the Golden Ratio. Statistical analyses are employed to quantify deviations from the Golden Ratio and to explore the distributional characteristics of these deviations. Prominent structures, including Zilkale and Sümela Monastery, along with selected regional examples from Samsun to Batumi are used to identify common aesthetic and geometric features. The findings aim to contribute to the mathematical analysis of cultural heritage and architectural aesthetics while providing a quantitative perspective on regional architectural design. Numerical characteristics such as rates of adherence to the Golden Ratio and deviation from the ratio are used to interpret the aesthetic feature the studied architectural structures.

Keywords: Golden ratio, Architecture, Deviation from the Golden Ratio.

References

- [1] Md. Akhtaruzzaman and A. A. Shafie. *Geometrical Substantiation of Phi, the Golden Ratio and the Baroque of Nature, Architecture, Design and Engineering*. International Journal of Arts, 1(1):1–22, 2011.
- [2] Hassan Ghorbani. *Golden Ratio: The Mathematics of Beauty*. Mathematics Interdisciplinary Research, 6(2):159–170, 2021.
- [3] G. B. Thapa and R. Thapa. *The Relation of Golden Ratio, Mathematics and Aesthetics*. Journal of the Institute of Engineering, 14(1):188–199, 2018.
- [4] A. Zeina. *The Golden Ratio and Its Impact on Architectural Design*. International Design Journal, 12(2):77–90, 2022.
- [5] T. O. Omotehinwa and S. O. Ramon. *Fibonacci Numbers and Golden Ratio in Mathematics and Science*. International Journal of Computer and Information Technology, 2(4):630–638, 2013.

ABSTRACT 79

An Efficient mathematical Relation for Incident Radiation for solving radiation heat transfer equation**Amir Dehghanian**

¹*Shiraz University of Technology, Department of Mechanical Engineering, Shiraz, Iran*
E-mail: dehghanian@sutech.ac.ir

Abstract: In this research, radiative heat transfer in participating media is evaluated using discrete transfer methods. A relatively simple and efficient relation is employed to calculate incident radiation values, enabling rapid and accurate analysis of key parameters. The results obtained from this method are compared with those from the conventional discrete transfer method, and significant agreement is observed between them, demonstrating the reliability and accuracy of the proposed approach. The effect of albedo coefficient on radiation transfer is investigated, and its role in determining the absorption, reflection, and transmission of radiative energy is analyzed. Reflectance and transmittance quantities are also calculated and evaluated as important indicators in this study. These quantities contain valuable signals that can be utilized in inverse heat transfer methods, particularly in retrieving radiative properties in thermal environments. The findings of this research indicate that the proposed approach can serve as an effective tool in modeling and optimizing thermal-radiative systems, paving the way for the development of more accurate methods in the analysis of transient radiative heat transfer.

Keywords: Radiative heat transfer, Discrete transfer method, Participating media, Transient radiation.

Mathematics Subject Classification: 12X34, 56Y78.

References

- [1] S. C. Mishra, P. Chugh, P. Kumar, and K. Mitra, "Development and comparison of the DTM, the DOM and the FVM formulations for the short-pulse laser transport through a participating medium," *International Journal of Heat and Mass Transfer*, vol. 49, pp. 1820–1832, 2006.
- [2] S. M. Hosseini Sarvari, "Solution of multi-dimensional radiative heat transfer in graded index media using the discrete transfer method," *International Journal of Heat and Mass Transfer*, vol. 112, pp. 1098–1112, 2017.
- [3] J. M. Wang and C. Y. Wu, "Transient radiative transfer in a scattering slab with variable refractive index and diffuse substrate," *International Journal of Heat and Mass Transfer*, vol. 53, pp. 3799–3806, 2010.
- [4] Z. Wang, Q. Cheng, and H. Zhou, "The DRESOR method for transient radiation transfer in 1-D graded index medium with pulse irradiation," *International Journal of Thermal Sciences*, vol. 68, pp. 127–135, 2013.
- [5] L. H. Liu and P. F. Hsu, "Analysis of transient radiative transfer in semitransparent graded index medium," *Journal of Quantitative Spectroscopy and Radiative Transfer*, vol. 105, pp. 357–376, 2007.

ABSTRACT 80

On the $\mathcal{DGC}$ Generalized k -Francois SequenceMehmet Arslan¹, Gülsüm Yeliz Saçlı²¹*Istanbul Gelisim University, Faculty of Engineering and Architecture
Department of Computer Engineering, 34310, Istanbul, Türkiye*²*Yildiz Technical University, Faculty of Arts and Sciences
Department of Mathematics, 34220, Istanbul, Türkiye
E-mail: meharslan@gelisim.edu.tr, yeliz.sacli@yildiz.edu.tr*

Abstract: In this study, we introduce the dual-generalized complex ($\mathcal{DGC}$) numbers with coefficients from the generalized k -Francois sequence for $p, k \in \mathbb{R}$. Depending on the value of p , the $\mathcal{DGC}$ generalized k -Francois sequence reduces to several special cases. It reduces to the dual-complex generalized k -Francois sequence for $p = -1$, hyper-dual generalized k -Francois sequence for $p = 0$, and dual-hyperbolic generalized k -Francois sequence for $p = 1$. Furthermore, different subclasses are obtained by specifying the parameter k . For example, it reduces to the $\mathcal{DGC}$ Francois sequence for $k = 1$. After introducing the n th $\mathcal{DGC}$ generalized k -Francois number, we present its recurrence relation and investigate the fundamental relationships between the $\mathcal{DGC}$ generalized k -Francois, $\mathcal{DGC}$ Fibonacci, and $\mathcal{DGC}$ Lucas numbers. Then, we give the Binet formula, the generating function, the exponential generating function, and various summation formulas for this special generalized sequence.

Keywords: Francois sequence, recurrence relation, Binet formula, generating function.

Mathematics Subject Classification: 11B37, 11B39, 11B83.

References

- [1] Thomas Koshy. *Fibonacci and Lucas Numbers with Applications*. John Wiley & Sons, New York, 2001.
- [2] Steven Vajda. *Fibonacci and Lucas Numbers and the Golden Section: Theory and Applications*. Ellis Horwood Limited, Chichester, 1989.
- [3] Verner E. Hoggatt Jr. *Fibonacci and Lucas Numbers*. The Fibonacci Association, Santa Clara, CA, 1969.
- [4] Richard A. Dunlap. *The Golden Ratio and Fibonacci Numbers*. World Scientific Publishing, Singapore, 1997.
- [5] Paula Catarino, and Anabela Borges. *On Leonardo Numbers*. Acta Math. Univ. Comenian., 89(1):75–86, 2020.
- [6] Orhan Dışkaya, Hamza Menken, and Paula Catarino. *On the Hyperbolic Leonardo and Hyperbolic Francois Quaternions*. Journal of New Theory, 42:74–85, 2023.
- [7] Yasemin Alp. *On the Generalized Francois Numbers*. Turkish Journal of Mathematics and Computer Science, 16(2):346–353, 2024.
- [8] Eduard Study. *Geometrie der Dynamen*. B. G. Teubner, Leipzig, 1903.
- [9] Isaac Moiseevich Yaglom. *Complex Numbers in Geometry*. Academic Press, New York and London, 1968.

- [10] Ettore Pennestri and Roberto Stefanelli. *Linear Algebra and Numerical Algorithms Using Dual Numbers*. Multibody System Dynamics, 18(3):323–344, 2007.
- [11] Isaiah L. Kantor, and Alexander S. Solodovnikov. *Hypercomplex Numbers*. Springer-Verlag, New York, 1989.
- [12] Andrew A. Harkin, and Joseph B. Harkin. *Geometry of Generalized Complex Numbers*. Mathematics Magazine, 77(2):118–129, 2004.
- [13] Mutlu Akar, Salim Yüce, and Serdal Şahin. *On the Dual Hyperbolic Numbers and the Complex Hyperbolic Numbers*. Journal of Computer Science and Computational Mathematics, 8(1):1–6, 2018.
- [14] Vladimir Majerník. *Multicomponent Number Systems*. Acta Physica Polonica A, 90(3):491–498, 1996.
- [15] Farid Messelmi. *Dual-Complex Numbers and Their Holomorphic Functions*. HAL Preprint, hal-01114178, 2015.
- [16] Jeffrey A. Fike and Juan J. Alonso. *Automatic Differentiation through the Use of Hyper-Dual Numbers for Second Derivatives*. Lecture Notes in Computational Science and Engineering, 87:163–173, 2012.
- [17] Nurten Gürses, Gülsüm Yeliz Şentürk, and Salim Yüce. *A Study on Dual-Generalized Complex and Hyperbolic-Generalized Complex Numbers*. Gazi University Journal of Science, 34(1):180–194, 2021.
- [18] Nurten Gürses, Gülsüm Yeliz Şentürk, and Salim Yüce. *A Comprehensive Survey of Dual-Generalized Complex Fibonacci and Lucas Numbers*. Sigma Journal of Engineering and Natural Sciences, 40(1):179–187, 2022.
- [19] Gülsüm Yeliz Şentürk, Nurten Gürses, and Salim Yüce. *Fundamental properties of extended Horadam numbers*. Notes Number Theory Discret. Math, 27:219–235, 2021.

ABSTRACT 81

Measuring Statistical Boundedness Degrees of Sequences

Fatih Temizsu

Sivas University of Science and Technology, Department of Engineering Fundamental Sciences, 58000,
Sivas, Turkey

E-mail: ftemizsu@sivas.edu.tr

Abstract: : In this study, we introduce and investigate a parameterized family of sequence spaces defined by a structural measurement that assigns a degree of statistical boundedness within the interval $[0,1]$. This family consists of all sequences whose degree does not exceed a fixed threshold parameter, providing a continuous interpolation between the spaces of bounded and statistically bounded sequences. We explore the fundamental sequence space structure and algebraic properties of these spaces. Finally, we establish a fine-grained classification framework for the asymptotic behavior of sequences, contributing a more refined structure to the theory of sequence spaces via statistical boundedness.

Keywords: Statistical convergence, Statistical boundedness.

Mathematics Subject Classification: 40A35, 40G15.

References

- [1] J. A. Fridy and C. Orhan, “Statistical limit superior and limit inferior,” *Proceedings of the American Mathematical Society*, vol. 125, no. 12, pp. 3625–3631, 1997.
- [2] V. K. Bhardwaj and S. Gupta, “On some generalizations of statistical boundedness,” *Journal of Inequalities and Applications*, vol. 2014, article 12, 2014.
- [3] R. Çolak, “Statistical convergence of order α ,” in *Modern Methods in Analysis and Its Applications*, New Delhi, India: Anamaya Pub., pp. 121–129, 2010.
- [4] J. Boos, *Classical and Modern Methods in Summability*, Oxford: Oxford University Press, 2000.
- [5] A. D. Gadjiev and C. Orhan, “Some approximation theorems via statistical convergence,” *Rocky Mountain Journal of Mathematics*, vol. 32, no. 1, pp. 129–138, 2002.

ABSTRACT 82

Optimal Constant Rebalanced Portfolios with Risk Constraints, Leverage and Multivariate Kelly Criterion

Mikhail Smirnov¹, Antonio Dapporto²

¹Columbia University, 2990 Broadway, New York, NY, 10027, USA

²Columbia University and Morgan Stanley, 1585 Broadway, New York, NY, 10036, USA

E-mail: msmirnov@columbia.edu, ad4243@columbia.edu

Abstract: The Kelly criterion gives an optimal fixed-fraction betting strategy for favorable games. It was successfully applied by Edward Thorp to a variety of gambling and investment situations. Treating the US stock market as a favorable game, E. Thorp and L. Rotando in 1992 examined the total returns of the S&P 500 for 1926–1984 and determined the optimal Kelly fraction to be 1.17, indicating that a slightly leveraged portfolio with 117% allocation to the S&P 500 index and annual rebalancing will give the highest compounded annual return. That portfolio performed in-sample from 1926 to 1984 with about 9% annual compounded return, and out-of-sample from 1985 to 2025 with a little over 12% annual compounded return. We propose a direct numerical optimization approach to Kelly leverage calculation as opposed to the formula-based approach. This numerical approach can be used in both single- and multi-asset cases. It can also incorporate additional constraints on each asset and total portfolio leverage, drawdowns, and other risk measures, extending the work of Chekhlov–Uryasev–Zabaranin. We present empirical log-return curves for different fractions f and different rebalancing frequencies. These curves are useful for fractional Kelly investors who consider various rebalancing frequencies. Direct numerical optimization typically provides lower and more realistic Kelly fractions compared to continuous-time model formulas. It accounts for discrete rebalancing intervals and increases sensitivity to tail negative returns. Using this approach, we then compute the optimal Kelly leverage for different sub-periods from 1926 to 2025 and for different indices, including the S&P 500, Nasdaq 100, and multi-asset portfolios containing the S&P 500, 10-year U.S. Treasury Bonds, and Baa-rated corporate bonds.

Keywords: Games, Stochastic decision problems, Stochastic models: economic and financial models, including capital growth and investment sizing, Kelly Criterion, Growth-optimal portfolios, Optimal Leverage, Logarithmic Utility.

Mathematics Subject Classification: 91A60, 91B70, 90C30, 60G40.

References

- [1] A. Chekhlov, S. Uryasev, and M. Zabaranin, “Drawdown measure in portfolio optimization,” *International Journal of Theoretical and Applied Finance*, vol. 8, no. 1, pp. 13–58, 2005.
- [2] A. Garivaltis, “Game-theoretic optimal portfolios for jump diffusions,” *Games*, vol. 10, no. 8, pp. 1–9, 2019.
- [3] S. Grossman and Z. Zhou, “Optimal investment strategies for controlling drawdowns,” *Mathematical Finance*, vol. 3, pp. 241–276, 1993.
- [4] J. L. Kelly, “A new interpretation of information rate,” *Bell System Technical Journal*, vol. 35, pp. 917–926, 1956.
- [5] L. M. Rotando and E. O. Thorp, “The Kelly criterion and the stock market,” *The American Mathematical Monthly*, pp. 922–931, 1992.

- [6] L. C. MacLean, E. O. Thorp, and W. T. Ziemba, Eds., *The Kelly Capital Growth Investment Criterion: Theory and Practice*, World Scientific Handbook in Financial Economics, Vol. 3, World Scientific Publishing Co., 2011.

ABSTRACT 83

A different approach onto the generalized field theory

Ayşe Dilara Şensoy¹, Mustafa Emre Kansu²¹Kütahya Dumlupınar University, Graduate Schools of Sciences, Dept. of Physics, 43100, Kütahya, Türkiye²Kütahya Dumlupınar University, Faculty of Art and Sciences, Dept. of Physics, 43100, Kütahya, Türkiye

E-mail: adilaratetik@gmail.com and memre.kansu@dpu.edu.tr

Abstract: In this study, a concise representation of the Bopp-Podolsky-Maxwell-Proca equation set, which is a form of generalized electromagnetic field equations, is constructed using quaternion algebra. Firstly, a quaternion differential operator is defined for the equation set, and a new electromagnetic field term is proposed for the simplified generation of the source equation. Furthermore, a general quaternion potential term similar to the source equation is presented, and the accuracy of the previously proposed quaternion field equation is verified by applying the operator to this potential. The relationship between quaternion potential and source expressions is easily expressed using quaternion algebra. In conclusion, it is understood that the Bopp-Podolsky-Maxwell-Proca equation can be written in a short, easy and elegant quaternion form, an analogy is established between equation set, and the results are consistent with both quaternion and classical forms.

Keywords: Quaternion algebra, Electromagnetic theory, Field theory.

Mathematics Subject Classification: : 11R52, 78A25.

References

- [1] D. J. Griffiths, *Introduction to Electrodynamics*, Prentice Hall Inc., 1999.
- [2] W. R. Hamilton, *Elements of Quaternions*, Chelsea Publishing, 1969.
- [3] A. Proca, “[Article title not available],” *Comptes Rendus*, vol. 190, p. 1377, 1930.
- [4] G. A. Proca, *Oeuvre Scientifique Publiée*, S. I. A. G., Rome, 1988.
- [5] F. Bopp, “[Article title not available],” *Annalen der Physik*, vol. 430, p. 345, 1940.
- [6] B. Podolsky, “[Article title not available],” *Physical Review*, vol. 62, p. 68, 1942.
- [7] E. Hebey and P. D. Thizy, “[Article title not available],” *Journal of Partial Differential Equations*, vol. 31, p. 119, 2018.

ABSTRACT 84

Soliton Solutions to the Conformable Nonlinear Soliton Neuron Equation via the New Kudryashov Scheme

Neslihan Ozdemir¹, Muslum Ozisik^{2,3}, Aydin Secer⁴, Mustafa Bayram⁴

¹Department of Mathematics, Yildiz Technical University, Istanbul, Turkey

²Department of Mathematical Engineering, Yildiz Technical University, Istanbul, Turkey

³Department of Mathematics, Faculty of Engineering and Natural Sciences, Istinye University, Istanbul, Turkey

⁴Department of Computer Engineering, Biruni University, Istanbul, Turkey

E-mail: ozdemirn@yildiz.edu.tr, ozisik@yildiz.edu.tr, asecer@biruni.edu.tr, mustafabayram@biruni.edu.tr

Abstract: This study is about the nonlinear soliton neuron equation with the conformable derivative. The primary motivation of this study is to examine the optical soliton solutions of the model using the new Kudryashov approach. The second motivation is to investigate in detail the effects of the model parameters on the dynamics of the obtained soliton solutions. Bright and dark soliton structures are derived, and their characteristics are illustrated through 2D and 3D surface plots. In addition, the conformable derivative framework enables the model to capture fractional-order effects while preserving the simplicity of classical differential operators, making it an effective tool for analyzing nonlinear wave propagation. These findings provide a deeper understanding of the physical behavior of the nonlinear soliton neuron equation and demonstrate the sensitivity of wave dynamics to fractional-order effects. The graphical representations facilitate the visualization of the behavior of the obtained bright and dark soliton solutions under the influence of different model parameters. The consistency of the analytical solutions with the governing equation confirms the validity of the proposed methodology and the accuracy of the obtained results. Therefore, the outcomes of this study may serve as a valuable reference for future research on conformable fractional differential equations, mathematical physics, and related engineering applications.

Keywords: Soliton pulse, Bright soliton, Dark soliton, Parameter effects.

Mathematics Subject Classification: 35C08, 35E05.

References

- [1] Muslum Ozisik, Aydin Seer, Mustafa Bayram, and Huseyin Aydın. *An encyclopedia of Kudryashov's integrability approaches applicable to optoelectronic devices*, Optik (ISSN: 0030-4026) 265 169499, 2022.
- [2] Drazin, Philip G., and Robin Stanley Johnson. *Solitons: an introduction*, Vol. 2. Cambridge university press, 1989.
- [3] Hasegawa, Akira. *Optical solitons in fibers*, Springer, Berlin, Heidelberg, 1989. 1-74.
- [4] Kodama, Yuji, and Akira Hasegawa. *Nonlinear pulse propagation in a monomode dielectric guide*, IEEE Journal of Quantum Electronics 23.5, 510-524, 1987.
- [5] Kudryashov, Nikolay A. *One method for finding exact solutions of nonlinear differential equations*, Communications in Nonlinear Science and Numerical Simulation 17.6, 2248-2253, 2012.

- [6] Younas, Usman, and Jan Muhammad. *Nonlinear wave dynamics of soliton neuron model: a high-fidelity computational investigation*, Modeling Earth Systems and Environment 12.3 164, 2026.
- [7] Seadawy, Aly R., Asghar Ali, and Ahmet Bekir. *Solitary wave solutions of the nonlinear fractional soliton neuron model via application of five mathematical methods*, Modern Physics Letters B 39.23 2550098, 2025.
- [8] Abdeljawad, Thabet. *On conformable fractional calculus*, Journal of Computational and Applied Mathematics 279, 57-66, 2015.

ABSTRACT 85

Local and Global Well-Posedness Results for the Extended Davey-Stewartson System

Yasin Hasanoglu, Cihangir Özemir*Dept. of Mathematics, Istanbul Technical University, Istanbul, Türkiye*

E-mail: hasanoglu18@itu.edu.tr, ozemir@itu.edu.tr

Abstract: Evolution of internal waves in a three-dimensional two-layered fluid with distinct velocity potentials and densities in each layer is governed by the $(2+1)$ -dimensional Extended Davey–Stewartson (EDS) system [1]. We present local and global well-posedness results for the EDS system by reducing it to the Almost Cubic Nonlinear Schrödinger (ACNLS) framework developed by Eden and Kuz [2].

Keywords: Well-posedness, Davey–Stewartson, almost cubic nonlinear Schrödinger.

References

- [1] J. Sun, X. Tang, Y. Chen, *Exploring two-dimensional internal waves: A new three-coupled Davey–Stewartson system and physics-informed neural networks with weight assignment methods*, Physica D **459** (2024), 134021.
- [2] A. Eden, E. Kuz, *Almost cubic nonlinear Schrödinger equation: existence, uniqueness and scattering*, Commun. Pure Appl. Anal. **8** (2009), 1803–1823.

ABSTRACT 86

A Data-Calibrated Fractional-Order One Health Framework for Modeling Nipah Virus Transmission with Memory Effects and Uncertainty Quantification

Taylan Demir¹, Zainab Abdullah Waheed², Afiaa Raheem Khudhair³

¹Ankara University, Dept. of Mathematics, 06100, Ankara, Turkey

²Middle Technical University, Institute of Medical Technology-AI Mansour, 10013, Baghdad, Iraq

³Thi-qar University, Dept. of Economic, 64001, Thi-qar, Iraq

E-mail: taylandemir@ankara.edu.tr, zainab_waheed@mtu.edu.iq, afya-rahim@utq.edu.iq

Abstract: The resurgence of Nipah virus (NiV) represents a consistent public health threat due to its high fatality rate, repeated zoonotic transmission and increasing incidences of human-to-human and hospital-acquired infections. Taking into account the recent outbreaks of Nipah virus, classical epidemic models seem insufficient to describe time delays in surveillance, heterogeneous transmission routes and long memory properties exhibited by NiV. Therefore, this paper suggests a new fractional-order model of the epidemic which combines biological knowledge, data and uncertainty quantification with the One Health approach. The suggested model covers zoonotic infection, community transmission, hospitalization, isolation, recovery and death in the structure of compartments with the use of the Caputo derivative as a model of memory in the epidemic transmission process. The parameters of the model are estimated based on the epidemiological data on number of outbreaks, hospitalizations, incubation period, mortality and contact tracing data. Also, the data-driven estimation of parameters and uncertainty propagation through the model help in the evaluation of the predictive accuracy of the model in the situation of the incomplete and possibly erroneous outbreak data. The qualitative analysis provides the essential features of the considered fractional system, i.e., positivity, boundedness, existence and uniqueness of solutions, and viability of the epidemic. Numerical experiments demonstrate the effect of the fractional order on the system dynamics, hospitalization rates, and mortality, and consider the effect of uncertainty in the prediction of epidemic dynamics with the help of confidence intervals.

Keywords: Nipah virus, Fractional-order epidemic model, Caputo derivative, Zoonotic spillover, Healthcare-associated transmission, Memory effects, Parameter estimation, Uncertainty quantification, Mathematical epidemiology.

Mathematics Subject Classification: 26A33, 34A08, 92D30, 92C60, 37N25.

References

- [1] P. Dutta, G. Samanta, and J. J. Nieto, “Nipah virus transmission dynamics: equilibrium states, sensitivity and uncertainty analysis,” *Nonlinear Dynamics*, vol. 113, no. 9, pp. 10617–10657, 2025.
- [2] K. Shah, A. Khan, B. Abdalla, T. Abdeljawad, and K. A. Khan, “A mathematical model for Nipah virus disease by using piecewise fractional order Caputo derivative,” *Fractals*, vol. 32, no. 2, article 2440013, 2024.
- [3] P. Venkatesan, “Nipah virus outbreaks in South Asia,” *The Lancet Microbe*, vol. 7, no. 6, 2026.
- [4] N. Gupta, E. Gkrania-Klotsas, J. F. Drexler, S. Banerjee, S. Tsiodras, Y. Ray, *et al.*, “Nipah virus in South Asia: from emergence to enduring preparedness challenges,” *Clinical Microbiology and Infection*, 2026.

- [5] R. Madhukalya, U. Yadav, H. A. Parray, N. Raj, S. S. Lupitha, V. Kumar, *et al.*, “Nipah virus: pathogenesis, genome, diagnosis, and treatment,” *Applied Microbiology and Biotechnology*, vol. 109, no. 1, article 158, 2025.

ABSTRACT 87

Generalized Piecewise Cardinal Functions for a Class of Third-Kind Singular FIDEs

Tooba Baghban¹, Mohammad Hossein Heydari¹, Dumitru Baleanu²¹*Shiraz University of Technology, Dept. of Mathematical, Shiraz, Iran*²*Lebanese American University, Dept. of Computer Science and Mathematics, Beirut, Lebanon*

E-mail: t.baghban@sutech.ac.ir, heydari@sutech.ac.ir, dimitru.baleanu@lau.edu.lb

Abstract: A spectral-collocation method using piecewise φ -Chebyshev cardinal functions is presented for third-kind weakly singular FIDEs with φ -tempered fractional derivatives. The Kronecker delta property avoids numerical integration for coefficients, and an explicit operational matrix for the φ -tempered fractional integral operator, derived via confluent hypergeometric functions, reduces the problem to a nonlinear algebraic system. Existence, uniqueness, and exponential convergence are rigorously established. The method unifies several tempered fractional operators and demonstrates robust performance across various $\varphi(t)$, α , and λ settings, with stability unaffected by the tempering parameter. Setting $\lambda = 0$ recovers known non-tempered formulations.

Keywords: φ -tempered fractional derivative, Weakly singular kernel, Spectral-collocation method, Chebyshev cardinal functions, Fractional integro-differential equations.

Mathematics Subject Classification: 65M70, 26A33.

References

- [1] A. D. Mali, K. D. Kuche, A. Fernandez, and H. M. Fahad, "On tempered fractional calculus with respect to functions and the associated fractional differential equations," *Mathematical Methods in the Applied Sciences*, vol. 45, no. 17, pp. 11134–11157, 2022.
- [2] M. H. Heydari, T. Baghban, M. Bayram, and M. A. Zaky, "A piecewise cardinal method for a class of third-kind fractional quadratic integro-differential equations involving Caputo tempered derivative," *Chaos, Solitons & Fractals*, vol. 202, article 117466, 2026.
- [3] T. Baghban, M. H. Heydari, M. Bayram, and M. A. Zaky, "A piecewise logarithmic Jacobi cardinal scheme for nonlinear third-kind fractional integro-differential equations," *Applied Numerical Mathematics*, vol. 220, pp. 167–188, 2026.
- [4] P. Rahimkhani, "A numerical method for ψ -fractional integro-differential equations by Bell polynomials," *Applied Numerical Mathematics*, vol. 207, pp. 244–253, 2025.

ABSTRACT 88

**Operational Matrix Method for Fractional Optimal Control Problems
Subject to Third-Kind FIDEs****Tooba Baghban¹, Mohammad Hossein Heydari¹, Mustafa Bayram²**¹*Shiraz University of Technology, Dept. of Mathematical, Shiraz, Iran*²*Biruni University, Dept. of Computer Engineering, Istanbul, Turkey*

E-mail: t.baghban@sutech.ac.ir, heydari@sutech.ac.ir, mustafabayram@biruni.edu.tr

Abstract: A numerical scheme using generalized hat functions is introduced for fractional optimal control problems governed by a third-kind Volterra-Fredholm fractional integro-differential equation with a singular coefficient vanishing at the initial time. The method employs an explicit operational matrix for the Hadamard fractional integral operator, expressed via confluent hypergeometric functions, reducing the infinite-dimensional problem to a finite-dimensional constrained optimization problem. The fractional dynamics are enforced via collocation, and Lagrange multipliers incorporate the constraints, yielding a coupled algebraic system. Numerical experiments confirm satisfactory convergence and robust performance across different fractional orders, effectively handling singular coefficients and hereditary integrals.

Keywords: Fractional optimal control, Third-kind FIDE, Generalized hat functions, Operational matrix, Lagrange multipliers.

Mathematics Subject Classification: 26A33, 49M25.

References

- [1] O. P. Agrawal, "A general formulation and solution scheme for fractional optimal control problems," *Nonlinear Dynamics*, vol. 38, no. 1, pp. 323–337, 2004.
- [2] T. Baghban, M. H. Heydari, M. Bayram, and M. A. Zaky, "A reliable strategy for a category of third-kind nonlinear fractional integro-differential equations," *Chaos, Solitons & Fractals*, vol. 200, article 117022, 2025.
- [3] M. H. Heydari, T. Baghban, M. Bayram, and M. A. Zaky, "A piecewise cardinal method for a class of third-kind fractional quadratic integro-differential equations involving Caputo tempered derivative," *Chaos, Solitons & Fractals*, vol. 202, article 117466, 2026.
- [4] Sh. Karami, M. H. Heydari, D. Baleanu, and M. Bayram, "Modified hat functions for constrained fractional optimal control problems with ψ -Caputo derivative," *Communications in Nonlinear Science and Numerical Simulation*, vol. 143, article 108657, 2025.

ABSTRACT 89

Monthly Sunspot Prediction Using an Oscillatory Second-Order Difference Equation and LSTM

Aydin Secer¹, Hakan Adiguzel² and Mustafa Bayram¹¹ *Biruni University, Department of Software Development, 34015, Istanbul, Türkiye*² *Bursa Technical University, Dept. of Data Science and Analytics, 16310, Bursa, Türkiye*

E-mail: asecer@biruni.edu.tr, hakan.adiguzel@btu.edu.tr and mustafabayram@biruni.edu.tr

Abstract: This study uses publicly available monthly mean sunspot numbers and transforms the series into a sign-changing sequence for oscillation analysis. An LSTM model is trained for one-step-ahead prediction, and the residual of an oscillatory second-order difference equation is included as a penalty term in the loss function. The model is compared with a standard LSTM trained without the equation-based penalty. The study examines whether this penalty improves prediction while maintaining consistency with the difference equation.

Keywords: Sunspot prediction, oscillatory difference equations, LSTM, time-series forecasting

Mathematics Subject Classification: 39A21, 68T07.

ABSTRACT 90

Estuarine Dissolved Oxygen and Hypoxia Forecasting Using Neural ODEs

Zehra Yılmaz¹, Mustafa Enes KAYHAN¹, Hakan Adiguzel¹ and Aydin Secer²

¹Bursa Technical University, Dept. of Data Science and Analytics, 16310, Bursa, Türkiye

²Biruni University, Department of Software Development, 34015, Istanbul, Türkiye

E-mail: 25458668006@ogr.btu.edu.tr, 25458668013@ogr.btu.edu.tr, hakan.adiguzel@btu.edu.tr and asecer@biruni.edu.tr

Abstract: Dissolved oxygen in estuarine waters varies with temperature, salinity, depth, mixing and biological processes. This study uses quality-controlled, 10-minute water-column observations from NOAA hypoxia buoys in Chesapeake Bay. The observations are aggregated to hourly resolution and modelled using a controlled Neural ODE. The model forecasts dissolved oxygen and the probability of hypoxia at 6, 12 and 24 h horizons. Hypoxia is defined as $DO < 2 \text{ mg L}^{-1}$. The study examines whether continuous-time modelling improves short-term forecasting when sensor observations contain gaps.

Keywords: neural ordinary differential equations, dissolved oxygen, hypoxia forecasting, estuarine water quality

Mathematics Subject Classification: 68T07, 34A34, 86A05.

ABSTRACT 91

Neural Fractional Differential Equations for Rainfall–Runoff Modelling

Mustafa Enes KAYHAN¹, Zehra Yılmaz¹, Hakan Adiguzel¹ and Aydin Secer²

¹*Bursa Technical University, Dept. of Data Science and Analytics, 16310, Bursa, Türkiye*

²*Biruni University, Department of Software Development, 34015, Istanbul, Türkiye*

E-mail: 25458668013@ogr.btu.edu.tr, 25458668006@ogr.btu.edu.tr, hakan.adiguzel@btu.edu.tr and asecer@biruni.edu.tr

Abstract: Rainfall–runoff dynamics depend on current meteorological conditions and antecedent catchment storage. This study applies a Neural Fractional Differential Equation model to daily hydro-meteorological records from CAMELS-DE. The data are divided chronologically into training, validation and test periods. The study examines whether fractional memory improves rainfall–runoff modelling, particularly during recession and low-flow periods, without reducing overall prediction accuracy.

Keywords: Neural fractional differential equations, rainfall–runoff modelling, hydrological forecasting.

Mathematics Subject Classification: 34A08, 68T07, 86A05.

ABSTRACT 92

Solution of Volterra Integro-Differential Equations via the Emad-Falih Transform

Serpil Salinan

Pamukkale University, Dept. of Mathematics, Faculty of Science, 20160, Denizli, Turkey
E-mail: ssalinan@pau.edu.tr

Abstract: In this study, solutions of linear Volterra integro-differential equations are obtained using the Emad-Falih transform. Several examples are presented to demonstrate the simplicity and effectiveness of the method.

Keywords: Emad-Falih Transform, Volterra integro-differential equations, Integral transforms.

Mathematics Subject Classification: 45J05, 65R10.

References

- [1] E. Kuffi and S. F. Maktoof, “Emad-Falih Transform: A New Integral Transform,” *Journal of Interdisciplinary Mathematics*, vol. 24, no. 8, pp. 2381–2390, 2021.
- [2] A. Mısır and F. B. Aktaş, “Volterra İntegro-Diferensiyel Denklemlerin Kharrat-Toma Dönüşümü ile Çözümü,” *GÜ Fen Bilimleri Dergisi*, vol. 6, no. 2, pp. 234–243, 2025.
- [3] H. Bozburun and H. A. Peker, “Solution of Linear Volterra Integro-Differential Equations of Second Kind Using Shehu Transform,” *Tbilisi Mathematical Journal*, vol. 8, Special Issue, pp. 177–186, 2021.
- [4] A. M. Wadi and N. A. Sulaiman, “Applying Pourreza Transform to Solve Volterra Integro-Differential Equations of the Second Kind,” *Eurasian Journal of Science and Engineering*, vol. 11, no. 2, pp. 272–277, 2025.

ABSTRACT 93

Thermal Radiation in optical Tomography

Amir Dehghanian, Omid Bavi

Department of Mechanical Engineering, Shiraz University of Technology, Shiraz, Iran
E-mail: dehghanian@sutech.ac.ir, o.bavi@sutech.ac.ir

Abstract: This study investigates thermal radiation tomography for estimating the radiative properties of participating media. The transient radiation transfer equation should be solved, while a suitable method is applied to the inverse analysis. Numerical results demonstrate that the proposed approach can accurately estimate absorption and scattering properties, even in the presence of measurement errors. However, the nonlinear nature of the problem increases the computational cost.

References

- [1] H. K. Kim and A. Charette, "Frequency domain optical tomography using a conjugate gradient method without line search," *Journal of Quantitative Spectroscopy and Radiative Transfer*, vol. 104, no. 2, pp. 248–256, 2007.
- [2] W. An, L. M. Ruan, and H. Qi, "Inverse radiation problem in one-dimensional slab by time-resolved reflected and transmitted signals," *Journal of Quantitative Spectroscopy and Radiative Transfer*, vol. 107, no. 1, pp. 47–60, 2007.
- [3] L. Y. Wei, H. Qi, X. L. Zhang, S. Wen, M. A. Islam, and L. M. Ruan, "Reconstruction of radiative properties fields in participating media by using the sequential quadratic programming combined with regularization technique," *Journal of Heat Transfer*, vol. 141, no. 2, 2019.
- [4] A. Dehghanian and S. M. Hosseini Sarvari, "Transient radiative transfer in variable index media using the discrete transfer method," *Journal of Quantitative Spectroscopy and Radiative Transfer*, vol. 255, article 107259, 2020.

ABSTRACT 94

From the Lyapunov Characteristic Exponents to the Reversibility Error Method: The quest for identifying chaos

Jamal-Odyseas Maaita*Democritus University of Thrace, Kavala, Greece*

E-mail: tmaaita@physics.duth.gr

Abstract: Distinguishing between regular and chaotic behavior remains challenging. Although Lyapunov Characteristic Exponents dominate the literature, new indices and methods for identifying the nature of dynamical systems continue to be introduced. In this talk, we discuss chaos and several chaos indicators, such as the maximal Lyapunov Exponent, the Smaller Alignment Index (SALI), and the Reversibility Error Method (REM). We present a conjecture for classifying dynamical behavior when the maximal Lyapunov Characteristic Exponent is close to zero. Furthermore, we apply REM, using an appropriate numerical solver, to non-Hamiltonian, non-symplectic conservative systems, extending the method to a broader class of conservative systems.

Keywords: Chaos index, Maximal Lyapunov Characteristic Exponent, Smaller Alignment Index, Reversibility Error.

References

- [1] J.-O. Maaita and D. Prousalis, “A comparison between four chaotic indicators in systems with hidden attractors,” *Journal of Computational and Nonlinear Dynamics*, vol. 20, no. 1, article 011008, 2025.
- [2] F. Panichi, L. Ciotti, and G. Turchetti, “Fidelity and reversibility in the restricted three body problem,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 35, pp. 53–68, 2016.
- [3] F. Panichi, K. Goździewski, and G. Turchetti, “The reversibility error method (REM): a new, dynamical fast indicator for planetary dynamics,” *Monthly Notices of the Royal Astronomical Society*, vol. 468, no. 1, pp. 469–491, 2017.

ABSTRACT 95

Fox H-Functions as Exact Solutions for Caputo Type Mass Spring Damper System Under Sumudu Transform

Sania Qureshi^{1,2}

¹*Department of Basic Sciences and Related Studies, Mehran University of Engineering & Technology, Jamshoro – 76062, Pakistan*

²*Research Center of Applied Mathematics, Khazar University, Baku 1009, Azerbaijan*
E-mail: sania.qureshi@faculty.muet.edu.pk

Abstract: Closed form solutions for mathematical systems are not easy to find in many cases. In particular, linear systems such as the population growth/decay model, RLC circuit, mixing problems in chemistry, first-order kinetic reactions, and mass-spring-damper system in mechanical and mechatronic engineering can be handled with tools available in the theoretical study of linear systems. One such linear system has been investigated in the present research study.

The second-order linear ordinary differential equation called the mass-spring-damper system is explored under the Caputo-type differential operator while using the Sumudu integral transform. The closed-form solution has been found in terms of the Fox H -function, wherein different aspects of the solution can be obtained with variation in $\alpha \in (1, 2]$ and $\beta \in (0, 1]$. The classical mass-spring-damper model is retrieved for $\alpha = \beta = 1$.

References

- [1] A. Akgül, “A novel method for a fractional derivative with non-local and non-singular kernel,” *Chaos, Solitons & Fractals*, vol. 114, pp. 478–482, 2018.
- [2] A. Akgül and E. Bonyah, “Reproducing kernel Hilbert space method for the solutions of generalized Kuramoto–Sivashinsky equation,” *Journal of Taibah University for Science*, vol. 13, no. 1, pp. 661–669, 2019.
- [3] A. Akgül, A. Cordero, and J. R. Torregrosa, “Solutions of fractional gas dynamics equation by a new technique,” *Mathematical Methods in the Applied Sciences*, vol. 43, no. 3, pp. 1349–1358, 2020.
- [4] A. Akgül and E. Karatas Akgül, “A novel method for solutions of fourth-order fractional boundary value problems,” *Fractal and Fractional*, vol. 3, no. 2, article 33, 2019.
- [5] E. K. Akgül, “Solutions of the linear and nonlinear differential equations within the generalized fractional derivatives,” *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 29, no. 2, article 023108, 2019.
- [6] K. K. Ali, C. Cattani, J. F. Gómez-Aguilar, D. Baleanu, and M. S. Osman, “Analytical and numerical study of the DNA dynamics arising in oscillator-chain of Peyrard–Bishop model,” *Chaos, Solitons & Fractals*, vol. 139, article 110089, 2020.
- [7] K. K. Ali, A. M. Wazwaz, M. S. Mehanna, and M. S. Osman, “On short-range pulse propagation described by $(2+1)$ -dimensional Schrödinger’s hyperbolic equation in nonlinear optical fibers,” *Physica Scripta*, vol. 95, no. 7, article 075203, 2020.

- [8] R. Almeida and S. Qureshi, “A fractional measles model having monotonic real statistical data for constant transmission rate of the disease,” *Fractal and Fractional*, vol. 3, no. 4, article 53, 2019.
- [9] O. A. Arqub, M. S. Osman, A. H. Abdel-Aty, A. B. A. Mohamed, and S. Momani, “A numerical algorithm for the solutions of ABC singular Lane–Emden type models arising in astrophysics using reproducing kernel discretization method,” *Mathematics*, vol. 8, no. 6, article 923, 2020.
- [10] A. Atangana and A. Akgül, “Can transfer function and Bode diagram be obtained from Sumudu transform,” *Alexandria Engineering Journal*, 2020.
- [11] D. Baleanu, A. Fernandez, and A. Akgül, “On a fractional operator combining proportional and classical differintegrals,” *Mathematics*, vol. 8, no. 3, article 360, 2020.
- [12] E. Bas, B. Acay, and R. Ozarslan, “Fractional systems with singular and non-singular kernels for energy efficient buildings,” *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 29, no. 2, article 023110, 2019.
- [13] V. Gill, K. Modi, and Y. Singh, “Analytic solutions of fractional differential equation associated with RLC electrical circuit,” *Journal of Statistics and Management Systems*, vol. 21, no. 4, pp. 575–582, 2018.
- [14] J. Huang, S. Ruan, X. Wu, and X. Zhou, “Seasonal transmission dynamics of measles in China,” *Theory in Biosciences*, vol. 137, no. 2, pp. 185–195, 2018.
- [15] D. Kumar, M. Kaplan, M. Haque, M. S. Osman, and D. Baleanu, “A variety of novel exact solutions for different models with conformable derivative in shallow water,” *Frontiers in Physics*, vol. 8, article 177, 2020.
- [16] P. Kumar and S. Qureshi, “Using Shehu integral transform to solve fractional order Caputo type initial value problems,” *Journal of Applied Mathematics and Computational Mechanics*, vol. 18, no. 2, pp. 75–83, 2019.
- [17] P. Kumar and S. Qureshi, “Laplace–Carson integral transform for exact solutions of non-integer order initial value problems with Caputo operator,” *Journal of Applied Mathematics and Computational Mechanics*, vol. 19, no. 1, pp. 57–66, 2020.
- [18] M. S. Osman and K. K. Ali, “Optical soliton solutions of perturbing time-fractional nonlinear Schrödinger equations,” *Optik*, article 164589, 2020.
- [19] M. S. Osman, A. Korkmaz, H. Rezazadeh, M. Mirzazadeh, M. Eslami, and Q. Zhou, “The unified method for conformable time fractional Schrodinger equation with perturbation terms,” *Chinese Journal of Physics*, vol. 56, no. 5, pp. 2500–2506, 2018.
- [20] M. S. Osman, A. Zafar, K. K. Ali, and W. Razzaq, “Novel optical solitons to the perturbed Gerdjikov–Ivanov equation with truncated M-fractional conformable derivative,” *Optik*, article 165418, 2020.
- [21] I. Podlubny, *Fractional Differential Equations: An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of Their Solution and Some of Their Applications*, Elsevier, 1998.
- [22] S. Qureshi, “Periodic dynamics of rubella epidemic under standard and fractional Caputo operator with real data from Pakistan,” *Mathematics and Computers in Simulation*, vol. 178, pp. 151–165, 2020.
- [23] S. Qureshi and P. Kumar, “Using Shehu integral transform to solve fractional order Caputo type initial value problems,” *Journal of Applied Mathematics and Computational Mechanics*, vol. 18, no. 2, pp. 75–83, 2019.

- [24] S. Qureshi and A. Yusuf, “Modeling chickenpox disease with fractional derivatives: From Caputo to Atangana–Baleanu,” *Chaos, Solitons & Fractals*, vol. 122, pp. 111–118, 2019.
- [25] S. Qureshi, A. Yusuf, A. A. Shaikh, M. Inc, and D. Baleanu, “Fractional modeling of blood ethanol concentration system with real data application,” *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 29, no. 1, article 013143, 2019.
- [26] D. G. Zill, *Differential Equations with Boundary-Value Problems*, Nelson Education, 2016.

ABSTRACT 96

Model Based Study on the Dynamics of HIV Infection of CD4⁺ T-cells with Various Control Strategy

Krishna Pada Das

Department of Mathematics Mahadevananda Mahavidyalaya Barrackpore Kol-120

E-mail: krishnaisi@yahoo.co.in

Abstract: Human Immunodeficiency Virus (HIV) is a retrovirus that attacks the human immune system, particularly targeting CD4⁺ T-cells, which play a crucial role in maintaining the body's ability to fight infections. HIV enters CD4⁺ T-cells by binding to their surface receptors and subsequently injecting its viral RNA into the host cell. Inside the cell, the reverse transcriptase (RT) enzyme converts HIV RNA into viral DNA. This viral DNA integrates with the host cell's DNA, transforming the infected CD4⁺ T-cell into a virus-producing factory. The infected cells become activated and undergo division, while newly formed mature HIV particles are released from the host cell and infect additional CD4⁺ T-cells. Over time, this continuous replication process spreads the infection and progressively weakens the immune system. The progression of HIV infection is generally categorized into three main stages.

Stage 1 (Acute HIV Infection): This stage occurs approximately 2–4 weeks after exposure, during which the virus rapidly multiplies and flu-like symptoms may appear. The CD4⁺ T-cell count remains within the normal range, typically between 500 and 1600 cells/mm³.

Stage 2 (Chronic HIV Infection: Asymptomatic or Latent Stage): During this stage, HIV continues to reproduce at relatively low levels. Individuals may not exhibit noticeable symptoms; however, they can still transmit the virus. The CD4⁺ T-cell count gradually decreases and generally ranges between 200 and 500 cells/mm³.

Stage 3 (AIDS: Acquired Immunodeficiency Syndrome): In the final stage of HIV infection, the CD4⁺ T-cell count falls below 200 cells/mm³, resulting in severe immune system impairment. Consequently, individuals become vulnerable to opportunistic infections and other life-threatening complications. Without effective treatment, this stage may lead to severe health consequences. Considering a high external viral load from macrophages, represented by $g = 1.26$, and using the parameter values given in Table 1.1, the corresponding numerical simulations are obtained. The results presented in the following figures demonstrate that a high external HIV viral load can destabilize the system, leading to limit-cycle oscillations. The increase in external viral pressure drives the immune system toward an unstable state. However, excessive increases in external viral load may also cause the system to return to a disease-free equilibrium through a backward bifurcation (BP). This indicates that the immune system may regulate the external viral pressure and restore stability under certain conditions.

ABSTRACT 97

Generalized perturbed form of the nonlinear Schrödinger equation as the $(2 + 1)$ -dimensional Biswas-Milovic model having the polynomial law and optical soliton solutions under the influence of white noise

Elif Tufan Gov, Muslum Ozisik

Yildiz Technical University, Graduate School of Science and Engineering, Dept. of Mathematics, 34220, Istanbul, Turkey

Yildiz Technical University, Department of Mathematical Engineering, 34220, Istanbul, Turkey
E-mail: elif.tufan@std.yildiz.edu.tr and ozisik@yildiz.edu.tr

Abstract: This paper investigates the optical soliton dynamics of the generalized perturbed form of the $(2+1)$ -dimensional Biswas-Milovic model governed by the nonlinear Schrödinger equation with the polynomial law of nonlinearity in the presence of white noise. The considered model incorporates higher-order perturbation effects together with stochastic fluctuations, providing a more realistic framework for describing pulse propagation in nonlinear optical media subjected to random environmental disturbances. By employing an appropriate traveling-wave transformation and an efficient integration scheme, bright, dark, and singular, stochastic optical soliton solutions are derived. The influence of white noise on the soliton characteristics is examined through stochastic simulations. The results reveal that low-intensity noise preserves the principal soliton characteristics, whereas increasing noise intensity induces amplitude fluctuations, waveform distortions, and phase perturbations. These observations demonstrate the robustness of the derived solutions within a certain stochastic regime while highlighting the significant role of random perturbations in nonlinear wave propagation. The analytical findings, supported by illustrative numerical simulations and graphical visualizations, provide new insights into stochastic optical soliton dynamics in generalized perturbed $(2 + 1)$ -dimensional Biswas-Milovic model and may contribute to the design and optimization of nonlinear optical communication and photonic devices operating under noisy environments.

Keywords: Optical fiber, Biswas-Milovic equation, Perturbation, Stochastic, Noise effect.

Mathematics Subject Classification: 35C08, 35E05, 35R60.

References

- [1] Anjan Biswas, Daniela Milovic, "Bright and dark solitons of the generalized nonlinear Schrödinger's equation," *Communications in Nonlinear Science and Numerical Simulation*, vol. 15, no. 6, pp. 1473-1484, 2010. <https://doi.org/10.1016/j.cnsns.2009.06.017>
- [2] Muslum Ozisik, "Novel $(2 + 1)$ and $(3 + 1)$ forms of the Biswas-Milovic equation and optical soliton solutions via two efficient techniques," *Optik*, vol. 269, 169798, 2022. <https://doi.org/10.1016/j.ijleo.2022.169798>
- [3] Al-sawalha M. M., Noor, S., Shah, R., Yasmin H., "Chaotic analysis, Hopf bifurcation and collision of optical periodic solitons in $(2+1)$ -dimensional degenerated Biswas-Milovic equation with Kerr law of nonlinearity," *Scientific Reports*, vol. 15, 44322, 2025. <https://doi.org/10.1038/s41598-025-30009-1>
- [4] Muslum Ozisik, Aydin Secer, Mustafa Bayram, Huseyin Aydin, "An encyclopedia of Kudryashov's integrability approaches applicable to optoelectronic devices," *Optik*, vol. 265, 169499, 2022. <https://doi.org/10.1016/j.ijleo.2022.169499>

ABSTRACT 98

On optical soliton solutions of the (2+1)-dimensional perturbed Biswas-Milovic equation under the parabolic law with nonlocal term

Hasan Cakicioglu¹, Muslum Ozisik², and Aydin Secer³

¹Graduate School of Science and Engineering, Department of Mathematics, Yildiz Technical University, Istanbul, Turkey

²Department of Mathematical Engineering, Yildiz Technical University, 34220 Istanbul, Turkey

³Computer Engineering, Biruni University, Istanbul, Turkey.

E-mail: hasan.cakicioglu@std.yildiz.edu.tr

Abstract: The generalized nonlinear Schrödinger equation plays a fundamental role in describing the propagation of nonlinear optical pulses in dispersive media. As one of its important extensions, the Biswas-Milovic equation provides an effective mathematical framework for modeling nonlinear wave dynamics in optical fiber communication systems. In this study, the (2+1)-dimensional perturbed Biswas-Milovic equation with a nonlocal term under the parabolic law of self-phase modulation is investigated. By employing an appropriate traveling-wave transformation, the governing model is reduced to a nonlinear ordinary differential equation. Subsequently, the reduced equation is analyzed by means of the new Kudryashov method and the unified Riccati equation expansion method to derive analytical solutions. A variety of optical soliton structures, including bright, dark, singular, are successfully derived under suitable parametric values. The existence conditions of these solutions are explicitly determined, and the influence of the model parameters on the wave profiles is discussed. To further illustrate the physical characteristics of the obtained solutions, representative 3D and 2D graphical simulations are presented. The results demonstrate that reported solutions enrich the existing solution spectrum of the perturbed Biswas-Milovic equation and provide additional insight into nonlinear wave propagation in optical media governed by parabolic-law nonlinearity and nonlocal effects.

Keywords: Generalization of the Schrödinger equation, Optic solitons, Perturbation, Nonlocal law, Self-steepening.

Mathematics Subject Classification: 35C08, 35E05, 35R60.

References

- [1] A. Biswas and D. Milovic, "Bright and dark solitons of the generalized nonlinear Schrödinger's equation," *Communications in Nonlinear Science and Numerical Simulation*, vol. 15, no. 6, pp. 1473–1484, 2010.
- [2] Q. Zhou, M. Ekici, A. Sonmezoglu, et al., "Optical solitons with Biswas–Milovic equation by extended trial equation method," *Nonlinear Dynamics*, vol. 84, pp. 1883–1900, 2016.
- [3] L. Kaur and A. M. Wazwaz, "Optical soliton solutions of variable coefficient Biswas–Milovic (BM) model comprising Kerr law and damping effect," *Optik*, vol. 266, article 169617, 2022.
- [4] M. Ozisik, "Novel (2 + 1) and (3 + 1) forms of the Biswas–Milovic equation and optical soliton solutions via two efficient techniques," *Optik*, vol. 269, article 169798, 2022.
- [5] Sirendaoreji, "Unified Riccati equation expansion method and its application to two new classes of Benjamin–Bona–Mahony equations," *Nonlinear Dynamics*, vol. 89, no. 1, pp. 333–344, 2017.

- [6] M. Ozisik, A. Secer, M. Bayram, and H. Aydin, “An encyclopedia of Kudryashov’s integrability approaches applicable to optoelectronic devices,” *Optik*, vol. 265, article 169499, 2022.
- [7] S. Li and X. Du, “Study on dynamic features and traveling wave solutions of perturbed nonlinear Biswas–Milovic equation combining Kudryashov’s law of refractive index,” *Mathematical Methods in the Applied Sciences*, vol. 47, pp. 4367–4380, 2024.

ABSTRACT 99

Optical Soliton Solutions of the Generalized Perturbed (2+1)-Dimensional Nonlinear Schrödinger Equation with Kerr Law

Hasan Cakicioglu¹, Muslum Ozisik²

¹Graduate School of Science and Engineering, Department of Mathematics, Yildiz Technical University, Istanbul, Turkey

²Department of Mathematical Engineering, Yildiz Technical University, 34220 Istanbul, Turkey
E-mail: hasan.cakicioglu@std.yildiz.edu.tr

Abstract: In this study, we analyzed (2+1)-dimensional perturbed Biswas-Milovic equation with Kerr law. We derived optical soliton solutions via a sub version of the new extended auxiliary equation and the unified Riccati equation expansion methods. The main steps of the proposed methodology include the formulation of the mathematical model, the introduction and application of an appropriate wave transformation to derive the reduced nonlinear ordinary differential equation, and the implementation of the analytical method by constructing its series solution and auxiliary differential equation, through which the resulting algebraic system is obtained and the unknown parameters of the problem are determined. We constructed bright, dark, and kink soliton for suitable parameter values. We examined the influence of the governing parameters on the soliton profile of the resulting wave structures and illustrate their 2D and 3D visualizations. The findings confirm that the perturbed (2+1)-dimensional Biswas-Milovic equation provides an effective generalized Schrödinger framework for generating Kerr law optical solitons and modeling nonlinear wave dynamics.

Keywords: (2+1)-Biswas-Milovic equation, Soliton solution, Unified Riccati equation expansion scheme, Perturbation, Complex wave transformation.

Mathematics Subject Classification: 35C08, 35E05, 35R60.

References

- [1] A. Biswas and D. Milovic, "Bright and dark solitons of the generalized nonlinear Schrödinger's equation," *Communications in Nonlinear Science and Numerical Simulation*, vol. 15, no. 6, pp. 1473–1484, 2010.
- [2] N. A. Kudryashov, "First integrals and general solutions of the Biswas-Milovic equation," *Optik*, vol. 210, Art. 164490, 2020.
- [3] A. Hasegawa, "Optical soliton: Review of its discovery and applications in ultra-high-speed communications," *Frontiers in Physics*, vol. 10, 2022.
- [4] M. M. Alsawalha, S. Noor, R. Shah, *et al.*, "Chaotic analysis, Hopf bifurcation and collision of optical periodic solitons in (2+1)-dimensional degenerated Biswas-Milovic equation with Kerr law of nonlinearity," *Scientific Reports*, vol. 15, Art. 44322, 2025.
- [5] M. Ozisik, "Novel (2+1) and (3+1) forms of the Biswas-Milovic equation and optical soliton solutions via two efficient techniques," *Optik*, vol. 269, Art. 169798, 2022.
- [6] M. Ozisik, A. Secer, M. Bayram, and H. Aydin, "An encyclopedia of Kudryashov's integrability approaches applicable to optoelectronic devices," *Optik*, vol. 265, Art. 169499, 2022.

ABSTRACT 100

Predicting Glioblastoma Growth with Fractional-Order Modeling: A Hybrid Block-Product Integration Approach

F. Afiatdoust¹, M. H. Heydari¹, O. Bavi²

¹Department of Mathematics, Shiraz University of Technology, Shiraz, Iran

²Department of Mechanical Engineering, Shiraz University of Technology, Shiraz, Iran

E-mail: f.afiadoust@sutech.ac.ir, heydari@sutech.ac.ir, o.bavi@sutech.ac.ir

Abstract: Glioblastoma multiforme (GBM) is one of the most aggressive and invasive forms of brain cancer, and predicting its progression remains a major challenge due to its complex infiltration patterns and limited therapeutic response. To achieve more reliable growth prediction, this study integrates early-stage MRI data into a fractional-order reaction-diffusion Proliferation-Invasion (PI) model. In order to reflect the heterogeneous composition of brain tissue, the model assigns distinct diffusion coefficients to gray matter, white matter, and cerebrospinal fluid. The nonlinear time-fractional governing equation is reformulated into an equivalent Volterra integro-differential form and then solved numerically through a hybrid temporal technique that combines a six-step block algorithm with a product-integration scheme. For the spatial domain, the finitedifference method is employed. Unlike conventional time-marching methods, which are inherently sequential, the block-based framework reduces this dependence and enables the simultaneous computation of multiple unknown solution values at different grid points. To the best of our knowledge, this work is the first to couple a block algorithm with a product-integration scheme for the numerical solution of a fractional PI model in the context of GBM growth prediction. The effectiveness of the proposed approach is verified through time-step independence analysis, demonstrating stable and convergent behavior with relatively low computational cost. These properties make the method attractive for both large-scale simulations and patient-specific modeling. In addition, comparison with experimental murine MRI data shows close agreement between the numerical predictions and the observed tumor evolution. Overall, the findings indicate that fractional dynamics can provide a more realistic description of GBM progression. The proposed framework enables accurate estimation of tumor growth at arbitrary time points without requiring repeated imaging, which can reduce both computational expense and clinical cost while offering a promising basis for personalized prognosis and treatment planning.

References

- [1] M. Ghahramani and O. Bavi, "Biomechanical modeling of glioblastoma progression: a comprehensive review from classic mathematical frameworks to data-driven strategies," *Biomechanics and Modeling in Mechanobiology*, vol. 25, no. 1, pp. 1, 2026.
- [2] A. M. Maleki, M. Hosseininia, M. R. Ghahremani, O. Bavi, and M. H. Heydari, "Personalized Prognosis of Glioblastoma via Fractional-Order/Biomechanical Modeling: Bridging Anomalous Diffusion, Continuum Mechanics, and MRI Data," *Results in Engineering*, Art. 109059, 2026.
- [3] O. Bavi, M. Hosseininia, M. Hajishamsaei, and M. H. Heydari, "Glioblastoma multiforme growth prediction using a Proliferation-Invasion model based on nonlinear time-fractional 2D diffusion equation," *Chaos, Solitons & Fractals*, vol. 170, Art. 113393, 2023.

ABSTRACT 101

**On the Foundations of Generalized Non-Singular Fractional Calculus:
Fundamental Theorems, Initial Value Problems, and Stability Analysis****Ali Akgül**

Siirt University, Art and Science Faculty, Department of Mathematics, 56100 Siirt, Turkey
E-mail: aliakgul@siirt.edu.tr

Abstract: The authoritative mathematical foundation for fractional calculus based on generalized non-singular kernels is provided in this article, "Charting the realm of smooth memory." The foundation of this calculus is established by extending the Caputo-Fabrizio and Atangana-Baleanu operators to arbitrary order $\alpha \in (n - 1, n)$: the fundamental theorem that guarantees their associated fractional integrals act as exact left inverses, thereby ensuring consistent initial conditions for differential problems. Additionally, we demonstrate that these generalized operators are not straightforward realizations of either the standard Caputo fractional derivative or integer-order derivatives; instead, their Laplace transform reveals a convolution structure that distinguishes them as different families of fractional differential operators while being fully compatible with the calculus fundamental theorem. Thus, by combining definition, theorem, and application into a cohesive whole and accurately charting the stable regions and intrinsic boundaries of non-singular fractional calculus, this study settles fundamental disputes.

References

- [1] T. H. Gronwall, "Note on the derivatives with respect to a parameter of the solutions of a system of differential equations," *Annals of Mathematics*, vol. 20, no. 2, pp. 292–296, 1919.
- [2] G. H. Hardy, J. E. Littlewood, and G. Pólya, *Inequalities*, Cambridge University Press, Cambridge, 1934.
- [3] M. I. Voitsekhovskii, "Minkowski inequality," in *Encyclopedia of Mathematics*, EMS Press, 2001.
- [4] A. Atangana, "The Atangana Integral: Resolving Decades of Confusion in Fractional Calculus and Bridging the Riemann–Liouville–Caputo Divide," 2026.

ABSTRACT 102

Efficient Computation of Fractional Diffusion Equations with Hilfer Operators via Fuzzy Transform**Irina Perfilieva***University of Ostrava, 30 dubna, 22, Ostrava, Czech Republic*

E-mail: irina.Perfilieva@osu.cz

Abstract: The Hilfer fractional derivative is a two-parameter operator that interpolates between the Riemann–Liouville and Caputo derivatives using a configurable type parameter. This operator is fundamental in the analysis of fractional differential equations in mathematical physics, engineering, economics, and control systems. We consider a fractional diffusion equation in which the partial derivative with respect to time is the Hilfer fractional derivative. Since the solutions are singular as $t \rightarrow +0$, numerical approximations are obtained from a sequence of convergent solutions in a weighted continuous function space. We propose an alternative method based on operational matrices. Our approach considers the case where the equation is subject to Dirichlet boundary conditions. In this case, we first transform the equation into a form involving only the Volterra and Riemann–Liouville integral operators. This form is obtained by applying a pair of F -transforms, each defined by the corresponding integral operator and parameterized by the corresponding fuzzy partition of the domain. The main result is a matrix equation involving two operational matrices corresponding to the two integral operators. This equation and its solution correspond to the given fractional differential equation and its approximate solution, respectively.

References

- [1] V. Novak, I. Perfilieva, and A. Dvorak, *Insight into Fuzzy Modeling*, Wiley, Hoboken, NJ, 2016.
- [2] I. Perfilieva, S. Ziari, R. Nuraci, and T. M. T. Pham, “F-transform utility in the operational-matrix approach to the Volterra integral equation,” *Fuzzy Sets and Systems*, vol. 475, Art. 108764, 2024.
- [3] I. Perfilieva and T. M. T. Pham, “On weakly singular Volterra equations in the direct sum of Hilbert spaces: Operational-matrix approach,” submitted.
- [4] Y. Zhang, Z. Sun, and H. Liao, “Finite difference methods for the time fractional diffusion equation on non-uniform meshes,” *Journal of Computational Physics*, pp. 195–210, 2014.

ABSTRACT 103

Analytical study on the complex cubic–quintic Ginzburg–Landau equation with the anti-cubic law nonlinearity

Neslihan Ozdemir

*Yildiz Technical University, Faculty of Arts and Sciences
Department of Mathematics, 34220, Istanbul, Türkiye
E-mail: ozdemirn@yildiz.edu.tr*

Abstract: An analysis of the complex cubic–quintic Ginzburg–Landau equation with the anti-cubic law nonlinearity is presented, highlighting its application to wave propagation in optical fibers. Through the application of the subversion of the new extended auxiliary equation method (SAEM26), diverse analytical wave structures are successfully uncovered. To characterize the wave dynamics, the evolution of these solitons is mapped via 2D, 3D, and contour plots, explicitly demonstrating how physical parameters influence their behavior. These results expand the current understanding of nonlinear wave theory and establish a foundational framework for subsequent investigations in nonlinear optics.

Keywords: Soliton behavior, Anti-cubic law nonlinearity, Parameter effects.

Mathematics Subject Classification: 35C08, 35E05, 35Q56.

References

- [1] Drazin, Philip G., and Robin Stanley Johnson. *Solitons: an introduction*, Vol. 2. Cambridge university press, 1989.
- [2] Muslum Ozisik, Aydin Seçer, Mustafa Bayram, and Huseyin Aydın. *An encyclopedia of Kudryashov's integrability approaches applicable to optoelectronic devices*, Optik (ISSN: 0030-4026) 265 169499, 2022.
- [3] Kudryashov, Nikolay A. *One method for finding exact solutions of nonlinear differential equations*, Communications in Nonlinear Science and Numerical Simulation 17.6, 2248-2253, 2012.
- [4] Cinar, Melih, et al. "Retrieval of optical solitons: Complex cubic–quintic Ginzburg–Landau equation augmented with the anti-cubic law." Optik 289 (2023): 171232.

ABSTRACT 104

Dark and Kink Solitons of the Perturbed Resonant (2+1)-Dimensional Maccari System via the Generalized Kudryashov Method

Selvi Altun Durmus¹, Aydin Secer², Muslum Ozisik^{3,4}, Mustafa Bayram²

¹*Mudanya University, Common Courses Coordination Unit, Bursa-Turkey,*

²*Biruni University, Computer Engineering Department, Istanbul-Turkey,*

³*Yildiz Technical University, Mathematical Engineering Department, Istanbul-Turkey*

⁴*Istinye University, Department of Mathematics, Faculty of Engineering and Natural Sciences, Istanbul-Turkey*

E-mail: selvi.durmus@mudanya.edu.tr

Abstract: This study investigates the resonant (2+1)-dimensional Maccari system incorporating perturbation effects arising from self-steepening, nonlinear dispersion, and a full-nonlinear perturbation term. By employing an appropriate traveling wave transformation, the governing nonlinear partial differential equations are reduced to a system of nonlinear ordinary differential equations. The resulting equations are analytically solved through the generalized Kudryashov method (gKM), which provides an efficient framework for constructing exact closed-form solutions of nonlinear evolution equations. As a consequence of the proposed methodology, novel dark and kink soliton solutions are successfully derived under suitable parametric constraints. The existence conditions of these soliton structures are discussed in terms of the physical parameters governing the model. To reveal the dynamical characteristics of the obtained solutions, comprehensive three-dimensional (3D), two-dimensional (2D), and contour plots are presented. These graphical simulations demonstrate the localization, stability, amplitude variation, and propagation behavior of the wave profiles in the resonant Maccari medium. Furthermore, the influence of the perturbation parameters on the morphology of the soliton structures is analyzed, illustrating their significant role in controlling the shape and propagation characteristics of nonlinear waves.

The obtained exact solutions enrich the family of analytical solutions available for perturbed resonant Maccari systems and demonstrate the effectiveness of the gKM in solving higher-dimensional nonlinear evolution equations encountered in nonlinear optics, plasma physics, and fluid dynamics. The analytical findings together with the graphical interpretations provide further insight into the propagation mechanisms of nonlinear localized waves in resonant media.

Keywords: Resonant Maccari system, Generalized Kudryashov method, Dark soliton, Kink soliton, Nonlinear optics.

Mathematics Subject Classification: 35C08, 35E05, 35Q56.

References

- [1] A. Maccari ; The Kadomtsev–Petviashvili equation as a source of integrable model equations, *Journal of Mathematical Physics*, (2026), 37(12), 6207–6222.
- [2] N.A. Kudryashov; One method for finding exact solutions of nonlinear differential equations, *Communications in Nonlinear Science and Numerical Simulation*, (2012), 17, 2248–2253.
- [3] Y.L. Ma, A.M. Wazwaz, B.Q. Li; Soliton resonances, soliton molecules, soliton oscillations and heterotypic solitons for the nonlinear Maccari system, (2023) *Nonlinear Dynamics*, 111, 18331–1834.
- [4] F Mahmud, M Samsuzzoha, MA Akbar ; The generalized Kudryashov method to obtain exact traveling wave solutions of the PHI-four equation and the Fisher equation, *Results in Physics*, 7, (201), 4296–4302 (2020).

- [5] A. Maccari ; The Maccari system as model system for rogue waves, Physics Letters A, 384, 28, (2020), 126740.
- [6] M. Ozisik, A. Secer, M. Bayram, H. Aydin; An encyclopedia of Kudryashov's integrability approaches applicable to optoelectronic devices, Optik, 265 (2022) 169499.

ABSTRACT 105

Analytical Bright and W-Shaped Soliton Solutions of the Sixth-Order Nonlinear Schrödinger Equation via the New Kudryashov Method

Selvi Altun Durmus

Mudanya University, Common Courses Coordination Unit, Bursa-Turkey

E-mail: Corresponding author:selvi.durmus@mudanya.edu.tr

Abstract: The sixth-order nonlinear Schrödinger equation is an important mathematical model for describing the propagation of ultra-short optical pulses in nonlinear dispersive media, where higher-order dispersion and nonlinear effects cannot be neglected. Constructing exact analytical solutions of this equation is essential for understanding the dynamics of nonlinear wave propagation. In this study, the new Kudryashov Method (nKM) is employed to obtain exact solutions of the sixth-order nonlinear Schrödinger equation. By applying a traveling wave transformation, the governing equation is reduced to an ordinary differential equation, and analytical solutions are derived systematically. As a result, two different classes of localized wave solutions, namely bright and W-shaped solitons, are obtained. To illustrate the propagation characteristics of the obtained solutions, three-dimensional (3D), two-dimensional (2D), and contour plots are presented. These graphical representations clearly demonstrate the localization and propagation behavior of the soliton profiles. In addition, the effects of the higher-order coefficients on the wave structures are investigated through a parametric analysis. The results show that variations in these coefficients significantly influence the amplitude, width, and shape of the solitons. The higher-order terms are therefore shown to play an important role in controlling the balance between dispersion and nonlinearity, leading to different propagation characteristics. The obtained results indicate that the nKM is an efficient and reliable approach for constructing exact solutions of higher-order nonlinear evolution equations. The analytical solutions and their physical interpretations may contribute to future studies in nonlinear optics and the propagation of ultra-short optical pulses.

Keywords: Sixth-order nonlinear Schrödinger equation, New Kudryashov Method, Exact solutions, Bright soliton, W-shaped soliton, Nonlinear optics.

Mathematics Subject Classification: 35C08, 35E05, 35Q56.

References

- [1] M Arshad, AR Seadawy, D Lu ; Bright–dark solitary wave solutions of generalized higher-order nonlinear Schrödinger equation and its applications in optics, *Journal of Electromagnetic Waves and Applications* (2017), 31, 16.
- [2] N.A. Kudryashov; One method for finding exact solutions of nonlinear differential equations, *Communications in Nonlinear Science and Numerical Simulation*, (2012), 17, 2248–2253.
- [3] M Wang, X Li, J Zhang ; Sub-ODE method and solitary wave solutions for higher order nonlinear Schrödinger equation, *Physics Letters A* (2007), 363, 1–2, 96-101.
- [4] M Arshad, AR Seadawy, D Lu; Elliptic function and solitary wave solutions of the higher-order nonlinear Schrödinger dynamical equation with fourth-order dispersion and cubic-quintic nonlinearity and its stability, *The European Physical Journal Plus* (2017), 132, 371.

- [5] D.S. Mou, C.Q. Dai; Nondegenerate solitons and collision dynamics of the variable-coefficient coupled higher-order nonlinear Schrödinger model via the Hirota method, *Applied Mathematics Letters* (2022), 133, 108230.
- [6] M. Ozisik, A. Secer, M. Bayram, H. Aydin; An encyclopedia of Kudryashov's integrability approaches applicable to optoelectronic devices, *Optik*, 265 (2022) 169499.

ABSTRACT 106

Flexible Memory-Kernel Methods for Fractional Dynamical Systems with Applications

Amin Jajarmi

Department of Electrical Engineering, University of Bojnord, Bojnord, Iran

E-mail: a.jajarmi@ub.ac.ir, a.jajarmi@gmail.com

Abstract: Memory, hereditary behavior, anomalous evolution, and nonlocal interactions are often observed in many natural, biological, physical, and engineering systems, and their description by conventional integer-order differential equations is inadequate. Fractional calculus is a powerful mathematical tool for describing such phenomena by generalizing the operations of differentiation and integration to non-integer orders [1]. Through this extension, fractional models can account for the influence of past states on present dynamics and can therefore provide a more realistic description of systems with complex temporal behavior. However, the effectiveness of a fractional model is determined not only by the fractional order but also by the form of the memory kernel governing the distribution of historical information. This presentation explores flexible memory-kernel methods for modeling and analyzing fractional dynamical systems. Particular emphasis is placed on generalized fractional operators defined with respect to an auxiliary increasing function. This function introduces an additional degree of freedom into the model. It allows the effective time scale, memory intensity, and temporal distribution of past influences to be adjusted within a unified mathematical formulation [2-4]. Thus, the generalized fractional operators are more flexible in comparison to the classical fractional derivatives while keeping the main variables, structure, and physical meaning of the original dynamical system. In the presentation, we will cover the main analytical and numerical issues in this modeling framework. It includes the mathematical background to guarantee the well-posedness of the obtained fractional models and the meaningfulness of their solutions. Numerical schemes for approximating such solutions in a form suitable for practical implementation are also discussed. The computational approach, together with the theoretical analysis, provides a direct link between generalized fractional operators and the simulation of complex dynamical behavior. Representative applications to observational data illustrate the approach's practical relevance. These examples show the influence of the choice of fractional order and the kernel-generating function on the model's performance. They also show that jointly selecting these quantities can improve agreement between mathematical predictions and observed system behavior. This flexibility makes the proposed methodology suitable for a broad range of applications in which memory and nonlocal effects play an important role. In general, the presentation shows that choosing a memory kernel is an important part of developing and identifying fractional models. Kernel-based methods are a general framework that combines mathematical theory, numerical approximation, and data validation. Such a perspective can enable the creation of more accurate, flexible, and interpretable models of complex dynamical systems in many fields of science and engineering.

References

- [1] S. G. Samko, A. A. Kilbas, and O. I. Marichev, *Fractional Integrals and Derivatives: Theory and Applications*, CRC Press, 1993.
- [2] D. Baleanu, P. Shekari, L. Torkzadeh, H. Ranjbar, A. Jajarmi, and K. Nouri, "Stability analysis and system properties of Nipah virus transmission: A fractional calculus case study," *Chaos, Solitons & Fractals*, vol. 166, Art. 112990, 2023.

- [3] A. Jajarmi, D. Baleanu, S. S. Sajjadi, and J. J. Nieto, “Analysis and some applications of a regularized Ψ -Hilfer fractional derivative,” *Journal of Computational and Applied Mathematics*, vol. 415, Art. 114476, 2022.
- [4] R. Almeida, “A Caputo fractional derivative of a function with respect to another function,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 44, pp. 460–481, 2017.

ABSTRACT 107

A new method for characterizing nonlinear dynamical system behavior using self-organizing maps and clustering

Adil Jhangeer*IT4Innovations, VSB – Technical University of Ostrava, Ostrava-Poruba, Czech Republic*

E-mail: adil.jhangeer@vsb.cz

Abstract: Complex nonlinear dynamical systems are difficult to analyze due to the interactions among multiple parameters. In this work, we apply Self-Organizing Maps (SOMs) to cluster dynamical behaviors based on parameter variations and perform a dynamical analysis of the resulting clusters. Using the FitzHugh–Nagumo model as a case study, we identify latent behavioral patterns and group parameter sets that produce similar dynamics. The SOM demonstrates strong topology preservation and clustering quality, confirming its effectiveness as a reliable tool for exploring parameter-dependent behaviors in complex dynamical systems.

Keywords: Parameter distribution map, Cluster distance analysis, Dynamical analysis.

Mathematics Subject Classification: 37M10, 68T10, 68T05, 37N30.

ABSTRACT 108

Mathematical Neuron-Neural-Networks Coding Dynamics of Continuous- and Discrete-Time Models of Brain Complexity with Complex Feature Engineering in Biology, Nuclear Medicine and Artificial Intelligence

Yeliz Karaca

*Department of Mathematics and Department of Neurology, University of Massachusetts (UMass),
Worcester, MA, USA*

E-mail: yeliz.karaca@ieee.org

Abstract: The complexity of the brain does not lie merely in the quantity of its components, rather in the highly nonlinear ways they are involved in interactions. Thus, the prevalence of complex entities concerning the brain requires various levels of observation in medical data analysis and coding in the domains of neuroscience, neuroimaging, biology and mathematical neuroscience, to name some. Since it may not always be possible to capture the subtle details in various diseases proving to be life-threatening, mathematical models come to the foreground owing to providing abstractions by reducing the problem to its fundamental characteristics while addressing the flow of interacting elements involved. To these ends, mathematical models generally operate across three scales, which are single neuron level referred to as microscopic, neural populations, as the mesoscopic level and lastly brain networks, known as the macroscopic level. Advanced mathematical means attempt to analyze the dynamic equilibrium at which the brain operates by modeling the interactions between excitatory and inhibitory neurons. Furthermore, complex systems also include multiple interacting variables and emerging behaviors which are not possible to be predicted beforehand or comprehended merely from their related properties, which requires the extracting of the association between the components of complex systems. Considering the emergent conditions, high levels of ambiguity, nonlinearity, unstable and uncertainty in complex systems, certain engineering processes are entailed as a reality. At this stage, feature engineering and selection provide tools for the adjusting, reworking and re-representation of predictors so that the tools in the context can be placed within an accurate robust predictive modeling, which enables the uncovering of predictor-response relationships. Mathematical dynamical ensemble modeling of neuron and neural network dynamics entails the precise capturing of the functions and complexities in multi-criteria decision-making processes while being capable of inferring and reconstructing the dynamics and structure of complex entities through serving the observability and evaluation emerging amid different flows of information. Mathematical modeling with continuous-time and discrete-time neural network aspects embody distinct approaches to dynamics with continuous time models employed for capturing fine-grained temporal dynamics and handling irregularly sampled data, while discrete time models being more aligned for regularly sampled or sequential step-based tasks. Conducive to the process of formulating an abstract model in terms of mathematical language to describe the complex behavior of a real system, modeling neuron and neural network dynamics in conjunction with artificial intelligence (AI) methods can be utilized for capturing the neural networks' functioning processes along with their subtle features in complex systems that are often maintained by flows of information. Facilitation of multifaceted decision-making, optimized outcome prediction and addressing the neurological diseases' complexity in the case of stroke is the aim of another study which proposes mathematical model adopted the application of multifractal regularization method, 10 different algorithms composed of deep learning, ensemble methods (i.e. bagging, boosting and stacking), fundamental and clustering methods as well as comparison of predictive accuracy rates. The established mathematical model proposed has been projected to be beneficial and advantageous for the accurate classification, evaluation, multicriteria decision-making and prediction to address uncertainty, unpredictability as well as variability [1]. High degrees of complexity in various areas, including debilitating and life-threatening medical concerns and conditions in particular, have obligated the emergence of applications to be more intelligent for catering

the related critical decision-making processes. A primary objective of imaging including neuroimaging is to understand the neurobiology underlying cognition and other features. Machine learning models trained on imaging data can be utilized as means of predicting various related phenotypes, which could enhance the capabilities of precision medicine, improving generalizability compared to conventional Magnetic Resonance Imaging (MRI) processes [1]. Given the complex structure of the human brain with a myriad diversity of nonlinear interactions between different units of its system displaying a unique nature/exemplum, brain function, characterized by connectomes between neurons and different brain regions, it is critical to generate multi-step and multi-layer network mathematical models at the pedestal of complexity and theoretical tenets for being able to model complex systems. One study focuses on computational analyses relying on advanced mathematical models employing multifaceted methods for the extraction of information from the Diffusion Magnetic Resonance Imaging (dMRI), allowing the reconstruction of connectivity network structures, with SpinDoctor into neuronal dynamics, as proposed, which can provide a reliable pathway to generate accurate quantitative parametric instruments to reflect the brain tissues' properties focusing on the precise connection between the tissue microstructure and signals [2]. Coalesced with accurate neuron geometry models and numerical DMRI simulations, a novel extended and multifaceted predictive mathematical model based on SpinDoctor and Bloch–Torrey partial differential equation (BTPDE) with the Caputo fractional-order derivative (FOD) with three-parameter (α, β, δ) Mittag-Leffler function (MLF) is proposed and developed accordingly by extending for the application on Brain Neuron Spin Unit dataset with the relevant multi-stage application-related steps. The feedforward neural networks (FFNNs) with BFGS Quasi-Newton equation, as one of the artificial neural network (ANN) algorithms, are applied on BTPDE with Caputo fractional-order derivative for the neurons and their algorithmic complexity is computed [2]. Hidden Markov Model (HMM), as a stochastic process by which implicit processes can be inferred indirectly through a sequence of observed states, is an applicable mathematical model with regard to uncertain phenomena in describing and computing complex dynamical behaviors. HMM and multifractal methods within the framework of predictive quantization complexity models are addressed in the subsequent work in relation to the differential prognosis and differentiation of the subgroups with respect to multiple sclerosis (MS), an autoimmune degenerative disease with time and space related dissemination, coupled with some subtle features that could remain unnoticed in clinical or medical practices [3]. The study attains accurate prediction, diagnosis and prognosis of MS subgroups by HMM with Viterbi algorithm and Forward–Backward algorithm as the dynamic and efficient products of knowledge-based and AI-based systems in precision medicine. Moreover, Viterbi algorithm's recursion enables maximization, uncovering the most probable hidden state sequence. Computational complexity has been computed for both the original and generated datasets [3]. Given these aspects, it can be noted that complex systems research and AI display a close interconnection, which can provide a common denominator for a comprehensive range of interdisciplinary applications and interactions. Integrated multistage mathematical models reinforced with imaging aim to ensure the intersection of deep learning, machine learning and ensemble methods in AI [1]. The other study [4], concerned with cloud computing, proposes a model based on the Android based mobile phones for stroke patients with different stroke subtypes. The system set up has two basic application elements, which are mobile and server application. While Artificial Neural Network (ANN) module is beneficial for classifying stroke subtypes, server application is for saving the data from the patients. In view of these considerations, the lecture aims to facilitate multifaceted decision-making processes and optimized outcome prediction by addressing the complexity of diseases and accurate identification of the pre-determined features by proper selection and extraction of variables vis-à-vis the nature of the related discipline based on pertinent criteria so that the required mathematical model can be constructed appropriately [4]. These subtleties emphasize the integration of interdisciplinary regimes including the medical, clinical and computer-based aspects based on mathematical modeling taking into account their distinctive contributions that need to be carried out appropriately in a well-poised balance and knowledge-based scheme. Hence, the identification of the external factors triggering the diseases along with their past history can be realized, which could bring about accurate and timely prediction, diagnostics, disease trajectory, progression modeling, treatment, follow-up, efficient management procedures and processes beside maintaining the patients' life quality with optimal outcomes including

(self-)control and survival, as health, ultimately, is at stake.

Keywords: Mathematical neuroscience and biology, Nuclear medicine, Neuro imaging, Fractional calculus, Coding dynamics; Complex feature engineering, Bloch–Torrey partial differential equation, Fractal intelligence systems, Computational complexity, Predictive bias, Precision medicine, Neuro-computing, and deep learning, neural networks, Spatiotemporal brain neural dynamics and stochastic networks’ complexity.

References

- [1] Yeliz Karaca and Majaz Moonis. *Mathematical Neuron-Neural-Networks Dynamics Modeling and Brain Complexity of Multicriteria Fractal-Based Ensemble Predictive Algorithmic Systems*. Journal of Computational and Applied Mathematics, Article 117319, 2026.
- [2] Yeliz Karaca. *Fractional Calculus Operators–Bloch–Torrey Partial Differential Equation–Artificial Neural Networks–Computational Complexity Modeling of the Micro–Macrostructural Brain Tissues with Diffusion MRI Signal Processing and Neuronal Multi-Components*. Fractals, 31(10):2340204, 2023.
- [3] Yeliz Karaca, Dumitru Baleanu, and Rana Karabudak. *Hidden Markov Model and Multifractal Method-Based Predictive Quantization Complexity Models vis-à-vis the Differential Prognosis and Differentiation of Multiple Sclerosis’ Subgroups*. Knowledge-Based Systems, 246:108694, 2022.
- [4] Yeliz Karaca, Majaz Moonis, Yu-Dong Zhang, and Cem Gezgez. *Mobile Cloud Computing Based Stroke Healthcare System*. International Journal of Information Management, 45:250–261, 2019.

ABSTRACT 109

Optical Bullets and Domain Walls with Cross-Spatio Dispersion and Multiplicative White Noise**Yakup Yıldırım^{1,2}**¹*Department of Computer Engineering, Biruni University, Istanbul-34010, Turkey*²*Mathematics Research Center, Near East University, 99138 Nicosia, Cyprus*

E-mail: yyildirim@biruni.edu.tr

Abstract: This study presents a comprehensive investigation into the formation and behavior of optical bullets and domain walls in nonlinear optical media, taking into account the influences of spatio-dispersive effects and chromatic dispersion along all three orthogonal spatial directions. These effects are further examined under the influence of multiplicative white noise, which introduces a stochastic component to the model, simulating random fluctuations often present in real-world optical systems. To derive exact analytical solutions, the well-established F-expansion method is employed. Through this approach, a complete spectrum of optical soliton solutions is successfully constructed, encompassing bright, dark, singular, and other nonlinear wave structures. These results contribute to the theoretical understanding of complex optical phenomena and offer potential applications in high-capacity optical communication systems and photonic technologies.

Keywords: Bullets, Domain walls, White noise.

References

- [1] Wazwaz, A. M. (2025). Bright and Dark Optical Soliton Solutions for a Nonlinear Schrödinger Equation With Kerr Law Nonlinearity and Weak Nonlocality. *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields*, 38(3), e70063.
- [2] Ali, K. K., Wazwaz, A. M., Mehanna, M. S., & Osman, M. S. (2020). On short-range pulse propagation described by $(2+1)$ -dimensional Schrödinger's hyperbolic equation in nonlinear optical fibers. *Physica Scripta*, 95(7), 075203.

ABSTRACT 110

Fractional dynamics in anomalous diffusion processes: An artificial intelligence perspective

J. Alberto Conejero

IUMPA, Universitat Politècnica de Valencia, Spain

E-mail: alberto.conejero@upv.es

Abstract: Anomalous diffusion, in which the mean squared displacement grows as a power law in time, $MSD \propto t^\alpha$, is ubiquitous across physical and biological systems, from cellular transport to porous media. Its origin often lies in memory and non-locality, the same features captured mathematically by fractional calculus, linking the physical phenomenon of anomalous diffusion to fractional dynamical systems as its natural mathematical generalization.

This talk traces a research program applying machine learning to both sides of that connection. It begins with the AnDi Challenge [1], a community benchmark that established, over a shared dataset of diverse diffusion models and realistic noise conditions, that ML methods outperform classical statistical estimators at inferring the diffusion exponent and classifying the underlying process from short, noisy trajectories - a benchmark later extended [2] to detect motion changes within single trajectories, closer to real single-molecule experiments. We then present two competing architectures developed for this problem: a convolutional-bidirectional LSTM model operating directly on trajectory sequences [3], and a Gramian Angular Field encoding that reformulates trajectories as images for pretrained computer-vision models [4], each achieving state-of-the-art or near state-of-the-art accuracy through different design philosophies.

Finally, we show these same architectures transfer to the discrete fractional logistic (Wu-Baleanu) map, recovering its fractional-order and chaos parameters from single trajectories, including under noise and missing data [5,6], demonstrating that AI-based parameter inference generalizes across both anomalous diffusion and fractional dynamical systems.

Keywords: Anomalous diffusion, fractional calculus, fractional dynamical systems, machine learning, deep learning, single-particle tracking, Wu-Baleanu map.

References

- [1] Munoz-Gil, G., Volpe, G., Garcia-March, M. A., Aghion, E., Argun, A., Hong, C. B., et al. (2021). Objective comparison of methods to decode anomalous diffusion. *Nature Communications*, 12, 6253. <https://doi.org/10.1038/s41467-021-26320-w>
- [2] Munoz-Gil, G., Bachimanchi, H., Pineda, J., Midtvedt, B., Fernandez-Fernandez, G., Requena, B., et al. (2025). Quantitative evaluation of methods to analyze motion changes in single-particle experiments. *Nature Communications*, 16, 6749. <https://doi.org/10.1038/s41467-025-61949-x>
- [3] Garibo-i-Orts, O., Baeza-Bosca, A., Garcia-March, M. A., & Conejero, J. A. (2021). Efficient recurrent neural network methods for anomalously diffusing single particle short and noisy trajectories. *Journal of Physics A: Mathematical and Theoretical*, 54, 504002. <https://doi.org/10.1088/1751-8121/ac3707>
- [4] Garibo-i-Orts, O., Firbas, N., Sebastia, L., & Conejero, J. A. (2023). Gramian angular fields for leveraging pretrained computer vision models with anomalous diffusion trajectories. *Physical Review E*, 107, 034138. <https://doi.org/10.1103/PhysRevE.107.034138>

- [5] Conejero, J. A., Garibo-i-Orts, O., & Lizama, C. (2023). Inferring the fractional nature of Wu Baleanu trajectories. *Nonlinear Dynamics*, 111, 12421-12431. <https://doi.org/10.1007/s11071-023-08463-1>
- [6] Garibo-i-Orts, O., Lizama, C., Akgul, A., & Conejero, J. A. (2024). Predictive deep learning models for analyzing discrete fractional dynamics from noisy and incomplete data. *Chinese Journal of Physics*, 89, 1276-1285. <https://doi.org/10.1016/j.cjph.2024.04.010>

INDEX

Page numbers refer to the corresponding abstracts in this book.

Author Index

Leila Abbaci	60	Hasan Cakicioglu	124, 126
Tahraoui Abdelatif	22	Prof. Dr. Barbaros Çetin	15
Zenati Abel	26	Smail CHALANE	44
Hakan Adiguzel	113, 114, 115	Abdelmadjid Chelli	12, 13
F. Afiatdoust	127	Dalila Chelli-Tabti	12
Owais Ahmad	30	Bilal djamal eddine Cherif	9
Anzi AICHA	46	CHETIOUI Souheyla	58
Ali Akgul	5, 128	Shih-Hai Chien	79
Javid Ali	92	Mohamed Aziz CHIKOUCHE	52, 65
Nur Idayu Alimon	18	Pao-Hsiung Chiu	69
Khaja Zuber Amin	47	Begüm ÇIĞŞAR	75
Rabia Amour	21	J. Alberto Conejero	142
Zulaikha Amran	7	Antonio Dapporto	103
Aicha Anzi	33, 35, 43	Krishna Pada Das	122
Mehmet Arslan	100	Amar Debbouche	85
Diana M. Audi	64	Amir Dehghanian	99, 117
Laid Baali	65	Taylan Demir	109
Hakim Baaziz	77	Sayang Mohd Deni	7
Tooba Baghban	111, 112	N. DIB-BAGHDADLI	88
Hafizah Bahaludin	71	Abd elhak Djalab	73, 78
Dumitru Baleanu	5, 80, 111	Abdelhakim DJALAB	61
Bahia Barache	60	Ali Djerioui	17
O. Bavi	127	Naima Djouadi	12
Omid Bavi	117	Selvi Altun Durmus	131, 133
Mustafa Bayram	106, 112, 113, 131	Cherif Bilal Djamal Eddine	10
Leila Bechane	77	Djelili Farida	26
Zafer BEKIRYAZICI	97, 98	Muhammad Farman	5
Louiza Belkhiri-Brahmi	39	Jasifa Fayaz	30
BENABILA Nabila	58	Prince A. Ganai	47
Mohmaed Bencheikh	65	Elhem GHORBEL	52
Mohamed Fouad Benkhoris	17	Elif Tufan Gov	123
Loutfi Benyettou	9, 10, 17	Murat Güzeltepe	32
Abdelhafid Benyounes	19	A. Halanay	81
Khadidja Berarma	66	Messaoud HAMDI	51
Louiza BERDJOU DJ	34	Zakia Hammouch	86
Mekki BIBI	52	Nacer Hamzaoui	16
Mohamed Boualem	33	Yasin Hasanoglu	108
Mohamed S. Boudelloua	36	Mir Sajjad Hashemi	76
Abderrahman Bouguerra	49, 50	Hazzirah Izzati Mat Hassim	28
Karima Bouibed	33, 35, 43, 46	Sofihara Al Hazmy	90, 92
Said Boukerroum	16	Abdelghafour Herizi	19
Houda BOUKHERIS	44	M. H. Heydari	127
Rachid BOUKOUCHA	6	Mohammad Hossein Heydari	111, 112
Kada Boureguig	19	Younes Hidra	37, 44
Belkacem Brahmi	39, 41	Evren Hincal	5
Saloua Brahmi	39	Siti Nur Iqmal Ibrahim	71

Eddie Shahril Ismail	68	Zubaria Parvez	5
Amin Jajarmi	135	Irina Perfilieva	129
Adil Jhangeer	137	Idrees Qasim	53
Kamal KABYL	42	Sania Qureshi	119
Mustafa Emre Kansu	105	Amina Raghdhi	66
Yeliz Karaca	138	Hilal Rahali	9, 10, 17
Mustafa Enes KAYHAN	114, 115	Nabilah Fikriah Rahin	28
Syeda Sarwat Kazmi	87	Muhammad Hazwan A Rahman	24
Siham Kebaili	60	Normahirah Nek Abd Rahman	68
Nabila KENDI	44	Adzhar Rambli	7
Nurin Alya Jhon Kenedi	28	Boulahia Ramdane	40
Mohamed Kezzar	61	Khalil Ur Rehman	20
Ayesha Khalid	89	Muhammad Bilal Riaz	87, 89, 94, 95
Noureddine Khimoum	35, 43	Rifkhi Akbar Putra Rustandi	92
Nouredine KHIMOU	46	Gülsüm Yeliz Sağl	100
Afiaa Raheem Khudhair	109	Adel SAHI	51
Öznur Özkan Kılıç	74	Serpil Salinan	116
Irem Nur Kivircik	96	Chekour Samir	22
Brahim LADGHEM-CHIKOUCHE	27	Mohamed Rafik SARI	61
Mohamed Faris Laham	71	Nor Haniza Sarmin	18, 24, 28
Abderrzak Laib	19	Dirk Schlingmann	23
Abdelghani Lakhdari	78	Aydin Secer	106, 113, 114, 115, 124, 131
Ahd Lakhnach	73	Ayşe Dilara Şensoy	105
Olfat Al Lakkies	81	Nadera Settouti	84
Lahouaoui Lalaoui	73, 78	Wasfi Shatanawi	20
Chia-Huang Li	79	Mikhail Smirnov	103
Bachir Liani	84	Neriman Şolt	32
Yan-Ting Lin	69	C. Sophocleous	67
Mehraj Ahmad Lone	56	Dalila Tabti-Chelli	13
Keltoum Loukal	49, 50	Baya Takhedmit	60
Martin Lukarevski	72	Boukharouba Taoufik	40
Yakup Yi ldi ri m	141	Fatih Temizsu	102
Jamal-Odyseas Maaita	118	Hadda Tiouiri	77
Al Azhary Masta	92	Nassim Touche	33
Meryem Meddahi	11	Tsung-Wei Tsai	79
Moussa MESSAILI	44	Pelin UZAN	98
Siti Afiqah Mohammad	24	Zainab Abdullah Waheed	109
Mouhoubi-Tafnine	26	Huei Chu Weng	69
Khimoum Noureddine	55	Yu-Chi Wu	79
Fayssal Ouagueni	19	Melikenur YILDIRIM	97
Soufi Ouahiba	26	Zehra Yilmaz	114, 115
Bibi Mohand Ouamer	55	Samira ZAHAR	8
Kheddouci Ouarda	55	Anam Zahra	95
Mohamed Ouazene	21	Samir Zeghlache	9, 10, 17
Neslihan Ozdemir	106, 130	Zina	26
Cihangir Özemer	108	Alexander Zlotnik	62
Muslum Ozisik	106, 123, 124, 126, 131	Athirah Zulkarnain	28
Tuğçem Partal	96		

Keyword Index

(2+1)-Biswas-Milovic equation	126	Caputo derivative	109
-Laplacian	8	Caputo operator	80
-tempered fractional derivative	111	Channel	20
Absorption Coefficient	66	Chaos	87
Acoustic wave equation	62	Chaos index	118
Ad hoc Networks Security	35	Chebyshev cardinal functions	111
Adaptive control	17	Chemical reaction	61
Adaptive fuzzy PID controller	19	Cluster distance analysis	137
Adomain	61	Clustering	87
Affine translation surfaces	56	Coding dynamics	138
AI-Supported Learning	64	Collatz conjecture	97
Algeria	13, 37, 44	Compact fourth-order scheme	62
Algerian honey	26	Compact Stars	95
almost cubic nonlinear Schrodinger	108	Completeness	90
and deep learning	138	Complex feature engineering	138
ANFIS	49	Complex Fluid Simulation	94
Anisotropic Matter	95	Complex wave transformation	126
Anomalous diffusion	142	Computational complexity	138
ANOVA	12	Concrete	65
Anti-cubic law nonlinearity	130	Constrained Markov decision process	33
Antioxidant activity	26, 58	Contact symmetries	67
Applied algebra	32	Contact transformations	67
applied biostatistics	12	continuous treatment	81
Applied Mathematics	79	Control law	16
Architecture	98	Convolutional Neural Networks	96
Artificial Intelligence	10, 34	COVID-19	75
Artificial Intelligence in Education	64	Cryptogamic diseases	12
Artificial pancreas	19	CSS construction	32
Attacker-Defender Interaction	35	Cubical dimension	42
Azo naphthalenic derivatives	58	Dark Matter Effects	95
Barrow entropy	47	Dark soliton	106, 131
basic feasible solution	41	Davey–Stewartson	108
Bernstein inequality	53	deep learning	142
Beta distribution	97	Deep reinforcement learning	33
Bieberbach group	24	Deniable authentication protocol	68
Bifurcation analysis	87	Deviation from the Golden Ratio	98
Bilinear programming	55	DFT	58, 66
Binary Tree	42	Diagnostic Method	10
Binet formula	100	Difference equation	8
Biocontrol	12	Differential equations	76
Biological complexity	26	Digital image correlation	40
Biswas-Milovic equation	123	Dihedral Groups	28
black hole thermodynamics	47	Directional statistics	7
Black-Scholes model	60	Discrete transfer method	99
Bloch–Torrey partial differential equation	138	dissolved oxygen	114
Boostlet transform	30	distributed delay	81
Boundary element method	15	disturbance observer	17
Bright soliton	106, 133	Domain walls	141
Brownian motion	61	Double Caputo k -fractional derivatives	11
BTZ black hole	47	Double general transform	11
Bullets	141	Double Riemann–Liouville k -fractional derivatives	11
bumblebee gravity	47	Double Riemann–Liouville k -fractional integrals	11
Bursa Malaysia	71	Drag Coefficient	20
Calculus Education	64	Dredged sediment	52

Dust acoustic Solitons	21	Healthcare-associated transmission	109
dust grains and magnetic field	22	Heat transfer	61
Dusty plasma	21	Heat treatment	52
Dynamic Bayesian Games	35	Heat-generating source	51
Dynamical analysis	137	hematological disease	81
Eco-cement	52	Heterogeneous habitats	88
Eisenstein-Jacobi integers	32	Heuristic	43
Electromagnetic theory	105	Hopf bifurcation	81
Electronic structure	66	Hospital costs	44
Emad-Falih Transform	116	hydrological forecasting	115
Empty Trips	46	Hypercube	42
Energy Harvesting	39	hypoxia forecasting	114
Epidemic model	85	including capital growth and investment sizing	103
Equal channel angular extrusion	40	Inclusion	90
Equivalence transformations	67	Inclusion property	92
Estrada index	18	Instructor-Trained AI	64
estuarine water quality	114	Integer Linear Programming	43
Evolution equations	72	Integral sliding mode control	17
Exact solutions	67, 76, 133	Integral transforms	116
f(R) Gravity	95	Intelligent Control	49
Fault tolerant control	17	Intelligent Tutoring Systems	64
Faults	10	Internet of Things (IoT)	39
FEM	20	Intrusion Detection	35
Field theory	105	Inverter	10
Financial risk	5	Kelly Criterion	103
First integral	6	Kernel of the homomorphism	24
First Type s-Orlicz Space	90	Kink soliton	131
Fixed point	8	Ladybird beetles	13
FP-LAPW	66	Lagrange multipliers	112
fractal horizon	47	Laplace transform	5
Fractal intelligence systems	138	Large deformation	40
Fractional calculus	80, 138, 142	LC	58
Fractional Constitutive Laws	94	Lie symmetries	67
fractional dynamical systems	142	Limit cycle	6
Fractional Fluid Modeling	94	Linear Canonical transform	30
Fractional integro-differential equations	111	Linearization	67
Fractional Operator	5	Logarithmic uncertainty principle	30
Fractional optimal control	112	Logarithmic Utility	103
Fractional PDEs	60	Logistics Planning	46
Fractional-order epidemic model	109	Lookahead Approach	43
Francois sequence	100	Lorentz symmetry breaking	47
Free boundary problem	72	LSTM	113
Games	103	Lung cancer	44
Gauss map	56	Lung Diseases	96
Generalization of the Schrödinger equation	124	Lyapunov coefficient	81
Generalized Abel equation method	76	M05 2X	58
Generalized hat functions	112	MABC operator	80
Generalized Kudryashov method	131	Machine Learning	75, 96, 142
Generalized Linear Model (GLM)	44	MagLev	50
Generalized Orlicz-Morrey space	92	Magnetorheological (MR) Damper	16
Generalized Young function	92	Majorization	74
generating function	100	mathematical biology	81
Generative AI	64	Mathematical epidemiology	109
Geometric properties	72	Mathematical Modeling	46, 49, 50, 61
Geophysical data analysis	7	Mathematical neuroscience and biology	138
Golden ratio	98	Mathematics Education	64
Greenhouse tomato	12	Maximal L^p -regularity	88
Growth-optimal portfolios	103	Maximal Lyapunov Characteristic Exponent	118

Maxwellian polarization force	21	Participating media	99
Mechanical Strength	65	Pedagogy-Aware AI	64
Medical Imaging	96	Permeability	65
Melissopalynology	26	Perturbation	123, 124, 126
Memory effects	109	phase transitions	47
Memory-Driven Flows	94	Phenolic compounds	26
Merton financial model	67	Physics-Informed Neural Networks for Anisotropic Compact Star Modeling	89
Merton jump-diffusion model	71	Pitt's inequality	30
MHD	51	Planar polynomial differential system	6
microfluidics	15	Plasma sheath	22
Minkowski space	30	Poles	53
Mixed convection	51	Polynomials and rational functions of several complex variables	36
Mobile Gateway	39	Population persistence	88
modeling	37	Porosity	65
molecular graph	18	Pozzolana	52
Moving interfaces	88	Practical error analysis	62
multi-prime power RSA	68	Precision medicine	138
Multi-server queues	33	Predictive bias	138
Multivariate analysis	13	primal simplex method	41
Nanofluid	51, 61	Q-Learning	39
Nazarov inequality	30	QoS	39
Neural fractional differential equations	115	quadratic programming	41, 55
Neural Networks	20, 138	Quality of service	33
neural ordinary differential equations	114	Quanser 2-DOF helicopter	17
Neuro imaging	138	Quantum error correction	32
Neurocomputing	138	Quaternion algebra	105
New Kudryashov Method	133	Queue-Learning	33
Nipah virus	109	Radiative heat transfer	99
Noise effect	123	rainfall-runoff modelling	115
non-interactive protocol	68	Ranau earthquake	7
Non-linear PDEs	20	Randic index	18
non-local Greeks	60	Rational functions	53
Non-Normal Cyclic Subgroups	28	Reaction-diffusion	88
nonabelian tensor square	24	recurrence relation	100
Nonlinear Control Systems	50	Residue class rings	32
Nonlinear dynamical systems	87	Resonant Maccari system	131
Nonlinear Equations	79	Retrial Queues	34
Nonlinear optics	131, 133	Reversibility Error	118
Nonlinear Systems	49	Richards Growth Model	75
Nonlocal law	124	Ride Comfort	16
Nonlocal Transport	94	Riemannian geometry	47
nonsmooth basis	41	Road accidents	37
Nuclear medicine	138	road risk factors	37
Numerical Application	79	Runway Optimization	34
numerical simulation	85	s-Holder Inequality	90
Numerical simulations	88	Self-organizing maps	87
Open cavity	51	Self-steepening	124
Operational matrix	112	Semi-Active Suspension	16
Optic solitons	124	Semi-explicit three-level scheme	62
Optical fiber	123	sensitivity analysis	85
optimal control	85	Simulation	97
Optimal Leverage	103	single-particle tracking	142
Optimality equivalence	55	Sixth-order nonlinear Schrödinger equation	133
Orlicz-Morrey space	92	Smaller Alignment Index	118
oscillatory difference equations	113	Sol_3	56
Parameter distribution map	137	Soliton behavior	130
Parameter effects	106, 130		
Parameter estimation	109		

Soliton pulse	106	Transport Optimization	46
Soliton solution	126	Triangular distribution	97
Spatial-temporal variation	13	Type 1 diabetes	19
Spatiotemporal brain neural dynamics and stochastic networks' complexity	138	Type 1 fuzzy system	17
Spectral-collocation method	111	Type 2 fuzzy system	17
Spherical data	7	Type-2 Fuzzy Logic Controller	50
stability analysis	81	Ulam-Hyers stability	5
Starlike function	74	Uncertainty Handling	50
Statistical analysis	12	Uncertainty quantification	109
Statistical boundedness	102	Unified Riccati equation expansion scheme	126
Statistical convergence	102	Uniform distribution	97
Stem-cell proliferation	81	user behavior	37
Stochastic	123	Variable speed of sound and density	62
Stochastic decision problems	103	Vehicle Routes	43
Stochastic modeling	71	Vehicle Routing Problem	46
Stochastic models: economic and financial models	103	Virtual Model	16
Stochastic Processes	34	Volterra integro-differential equations	116
Sunspot prediction	113	Von Mises–Fisher distribution	7
Support Vector Machines	96	W-shaped soliton	133
Support vector regression	41	Weakly singular kernel	111
t-Symmetric points	74	well-posedness	85, 108
Temporal Fusion Transformer	75	White noise	141
Thermal radiation	61	Working vacations	33
Third-kind FIDE	112	Wu-Baleanu map	142
Three-Tank Process	49	York cavity formalism	47
time-series forecasting	113	Young function	92
Tolman–Kuchowicz Metric	95	ZA statistic	7
Transformations	36	Zagreb Indices	28
Transient radiation	99	Zeros	53
		Zoonotic spillover	109

ICAAMM 2026

13th International Conference on Applied Analysis and Mathematical Modeling

27–31 July 2026, Istanbul, Türkiye

ICAAMM 2026

About the Conference

The 13th International Conference on Applied Analysis and Mathematical Modeling (ICAAMM 2026) brings together researchers, scholars, and professionals working in applied analysis, mathematical modeling, nonlinear science, computational methods, artificial intelligence, and interdisciplinary applications. Hosted by Biruni University in Istanbul, Türkiye, ICAAMM 2026 provides an international platform for scientific exchange, collaboration, and innovation.



27–31 July 2026



Biruni University,
Istanbul, Türkiye

$$\int_{\Omega} u(x) dx$$

$$\frac{\partial u}{\partial t} = \mathcal{L}(u)$$

$$\|u\|^2 \leq C$$

*Where
Mathematics
Bridges Horizons*

ISBN: 978-605-69181-9-3

