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On the revealing of optical soliton solutions of the stochastic perturbed nonlinear Schrödinger equation in the presence of the Kerr law of self-phase modulation

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Abstract: In this study, an attempt to obtain optical soliton solutions of stochastic perturbed (1+1)-dimensional nonlinear Schrödinger equation having the Kerr law nonlinearity is presented. With the help of complex wave transform, the investigated nonlinear partial differential equation was converted into the nonlinear ordinary differential form. Analytical stochastic optical soliton solutions of the ordinary differential form were derived by applying the Kudryashov auxiliary equation method. By replacing the wave transformation and the appropriate solution set parameters, the main equation is provided. Graphic presentations were made for better interpretation of the results.

Keywords: Stochastic Schrödinger Equation; Optical soliton solutions; Brownian motion; Noise intensity

1 Introduction

Nonlinear Schrödinger equation (NLSE) has become one of the most studied equation in scientific investigations. Many physical phenomena can be modeled with NLSE [1,2,3,4,5]. In addition, optical-based phenomena of physics can also be modeled with NLSE [6,7,8,9,10,11]. As an additional ring, data transmission is one of the major pillars that gains great importance, and accordingly, it is possible to see many studies on nonlinear optics.

The stochastic perturbed nonlinear Schrödinger equation with Kerr law nonlinearity in the presence of chromatic dispersion is presented as:

$$i\frac{\partial \psi}{\partial t}+a\frac{\partial^2 \psi}{\partial x^2}+b|\psi|^2\psi=i\left(\tau\frac{\partial \psi}{\partial x}+\alpha\frac{\partial(|\psi|^2\psi)}{\partial x}+\beta\frac{\partial|\psi|^2}{\partial x}\psi\right)-\sigma\frac{\partial W(t)}{\partial t}\psi, \tag{1}$$

where $\psi=\psi(x,t)$ and σ is the coefficient of white noise (also σ is called as noise intensity or noise strength and $\sigma \geq 0$) and $W(t)$ represents the Wiener process, a represents group-velocity dispersion, b represents Kerr-law nonlinearity, τ represents inter-modal dispersion, α represents selfs-steepening dispersion and β is for nonlinear dispersion term.

There have been many studies recently on Schrödinger equation with Kerr law nonlinearity, a variant of which is studied in this article. Some of those; obtaining solutions via the generalized logistic equation method [12], via new extended direct algebraic method [13], abundant solutions [14], modified exponential function method and sinh-Gordon method [15], modified Kudryashov method [16], extended Jacobi elliptic function method [17], generated exponential rational function method [18], V-expansion method [19], Painlevé test [20] and bifurcation analysis [21].

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In this work, we have studied the stochastic perturbed nonlinear Schrödinger equation with Kerr law nonlinearity in the presence of chromatic dispersion. We have used the Kudryashov auxiliary equation method (KAEM) to obtain an optical soliton solution.

The organization scheme of the paper is as follows: In 2 we have obtained the nonlinear ordinary differential equation (NODE) form of the model in 1. In 3 we have implemented the KAEM on the model. In 4 we have presented 3D and 2D plots of the obtained solution. Lastly, we have presented the Conclusion in 5.

2 Revealing NODE form of the model

Suppose the complex wave transformation as:

$$\psi(x,t) = \Psi(\eta)e^{i\Omega}, \;\; \eta = x - \nu t, \;\; \Omega = -kx + \omega t + \sigma W(t) + \theta_0, \tag{2}$$

where ν is velocity, k,ω are wave number and frequency and lastly ψ_0 is phase constant. Substitute the 2 into 1, then separate the imaginary and real parts of the obtained equation, respectively as follows;

$$(2\beta + 3\alpha)\Psi^2\Psi' + (2ak + \nu + \tau)\Psi' = 0, \tag{3}$$

$$(k\alpha - b)\Psi^3 + \left(ak^2 + k\tau + \omega\right)\Psi - a\Psi'' = 0. \tag{4}$$

3 gives the following constraints:

$$\beta = \frac{-3\alpha}{2}, \quad \nu = -2ak - \tau. \tag{5}$$

In 4, taking into account the highest order linear and nonlinear terms Ψ'', Ψ^3 and utilizing the homogeneous balance rule [22], we gain $N = 1$, which N is a positive integer balance number.

3 Application of the KAEM

According to the KAEM [23,24], we propose the following solution for 4:

$$\Psi(\eta) = \kappa_0 + \kappa_1 R(\eta), \tag{6}$$

where $\kappa_1 \neq 0$ and $R(\eta)$ fulfills the following differential equation;

$$\frac{dR(\eta)}{d\eta} = \sqrt{\delta^2 R(\eta)^2 \left(1 - R(\eta) - \chi R(\eta)^2\right)}, \tag{7}$$

where δ and χ are real coefficients. Substituting the 6 into the 4 by considering 7, collecting the coefficient of $R^i(\eta)$, then equating the coefficients to zero, we get an algebraic system as follows;

$$\begin{aligned}
&R(\eta)^0 : \left((k\alpha - b)\kappa_0^2 + ak^2 + k\tau + \omega\right)\kappa_0 = 0,\\
&R(\eta)^1 : \kappa_1\left(3(k\alpha - b)\kappa_0^2 + ak^2 - a\delta^2 + k\tau + \omega\right) = 0,\\
&R(\eta)^2 : 3\left(-2\kappa_0(-k\alpha + b)\kappa_1 + a\delta^2\right)\kappa_1 = 0,\\
&R(\eta)^3 : (k\alpha - b)\kappa_1^3 + 2a\chi\delta^2\kappa_1 = 0,
\end{aligned} \tag{8}$$

where $R(\eta)$ is the following function:

$$R(\eta) = \frac{4}{(4\chi + 1)\mathrm{e}^{\delta\eta} + 2 + \mathrm{e}^{-\delta\eta}}. \tag{9}$$

Solving the obtained algebraic system in 8, we obtain the following solution set:

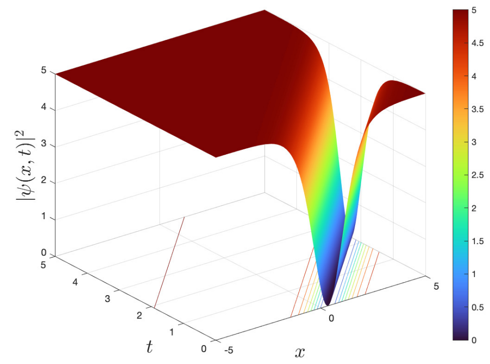
$$\textbf{SET}:\left\{\chi=-\frac{1}{4},\omega=-\frac{1}{2}a\delta^2-ak^2-k\tau,\kappa_0=\frac{\sqrt{2(k\alpha-b)a}\,\delta}{2(k\alpha-b)},\kappa_1=-\frac{a\delta}{\sqrt{2(k\alpha-b)a}}\right\}. \tag{10}$$

Substituting the 10 into the 6 with 9, gives the following solution:

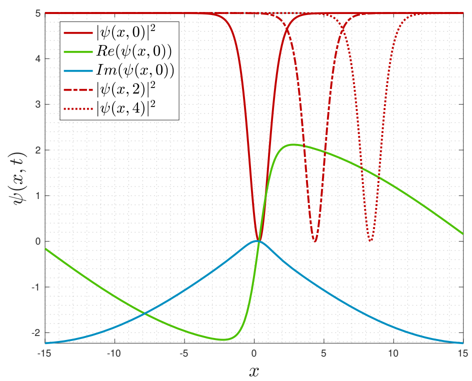
$$\psi(x,t)=\left(\frac{\sqrt{2}\,\delta a\left(\mathrm{e}^{\delta(vt-x)}-2\right)}{\sqrt{(b-k\alpha)a}\,\left(4+2\mathrm{e}^{\delta(vt-x)}\right)}\right)e^{i\left(-kx+\left(-\frac{1}{2}a\delta^2-ak^2-k\tau\right)t+\sigma W(t)+\theta_0\right)}. \tag{11}$$

4 Result and Discussion

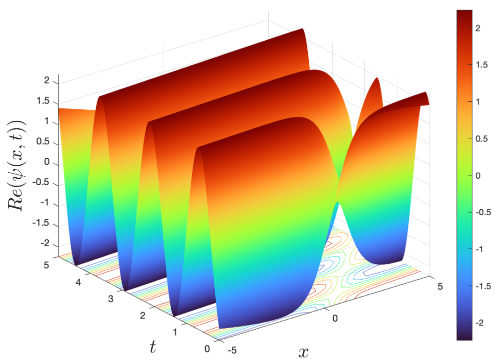
In 1, we have presented the 2D and 3D plots of the obtained solution $\psi(x,t)$ in 11. The figures were obtained for $\alpha=2, b=1, k=0.1, a=\delta=-2, v=2, \tau=-1$ and $\theta_0=0$. 1a represents 3D view of $|\psi(x,t)|^2$ when $\sigma=0$. **??** represent 3D views of real and imaginary parts of the solution $\psi(x,t)$ respectively. Lastly, 1c simulates 2D view of the modulus, real and imaginary parts of the solution. According to this figure, the motion is on the positive-x axis. This situation is depicted via dashed and dotted lines in 1c (red lines).



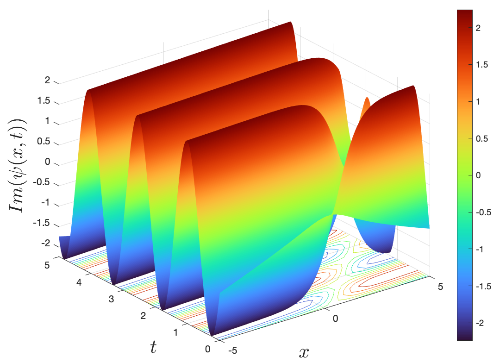
(a) $|\psi(x,t)|^2$ in 3D when $\sigma=0$.



(c) $|\psi(x,t)|^2, Re(\psi(x,1)), Im(\psi(x,1))$.



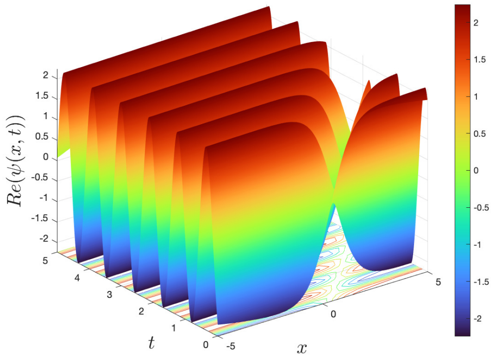
(b) $Re(\psi(x,t))$ in 3D when $\sigma=0$.



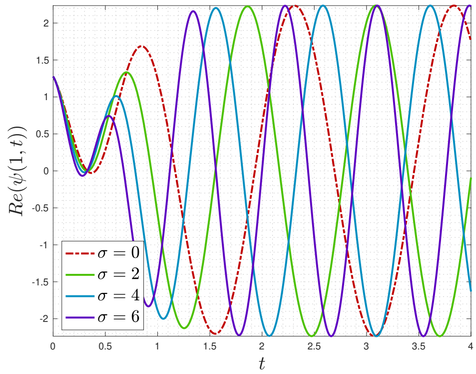
(d) $Im(\psi(x,t))$ in 3D when $\sigma=0$.

Fig. 1: Projections of $\psi(x,t)$ in 11 for $\alpha=v=2, b=1, k=0.1, a=\delta=-2, \tau=-1, \theta_0=\sigma=0$.

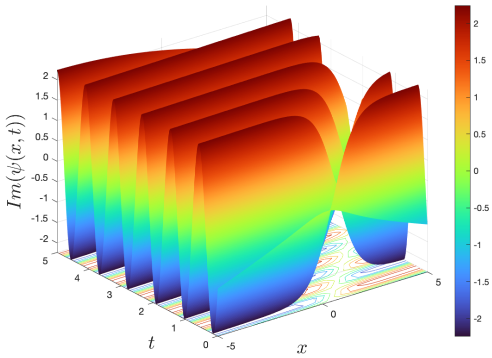
In 2, we have presented 2D and 3D views of the solution in 11 via different σ values to show the effect of the σ parameter on wave behavior. **??** show the 3D views of real and imaginary parts of $\psi(x,t)$ when $\sigma = 6$. According to these figures, it is clearly seen that when σ increased the fluctuations in real and imaginary parts increased. **??** show the 2D views of real and imaginary parts of $\psi(x,t)$ when $x = 1$ and $\sigma = 0,2,4,6$. According to these 2D plots, again, the fluctuations in real and imaginary parts were increased owing to an increase in σ.



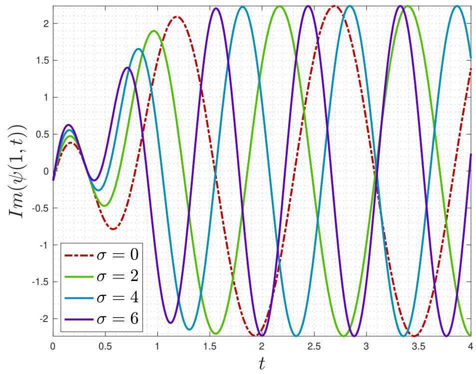
(a) $Re(\psi(x,t))$ in 3D when $\sigma = 6$.



(c) $Re(\psi(1,t))$ for various σ.



(b) $Im(\psi(x,t))$ in 3D when $\sigma = 6$.



(d) $Im(\psi(1,t))$ for various σ.

Fig. 2: Projections of $\psi(x,t)$ in 11 for $\alpha = v = 2, b = 1, k = 0.1, a = \delta = -2, \tau = -1, \theta_0 = 0$ and $\sigma = 6$ (**??**) and $\sigma = 0,2,4,6$ (**??**).

5 Conclusion

In this study, we have studied stochastic perturbed nonlinear Schrödinger equation with Kerr law nonlinearity. The optical soliton solution of the model was obtained via the Kudryashov auxiliary equation method. We have obtained the dark soliton solution as a result of the implementation. Also, we have placed the 2D and 3D plots of the obtained solution. In addition, we have presented the 2D plots to show σ white noise effect on the real and imaginary parts of the solution.

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