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Dynamics and stability results for novel coronavirus (COVID-19) model via Caputo-Fabrizio fractional derivative

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Abstract: Recently, many mathematical models have been studied to better understand the coronavirus infection. Most of these models are based on classical integer-order derivatives which cannot capture the fading memory and crossover behavior found in many biological phenomena. Therefore, the aim of this paper is to establish the existence and uniqueness of solutions to novel coronavirus (COVID-19) model including Caputo-Fabrizio (CF)-fractional derivative. We derive existence and uniqueness results with the help of properties of CF-fractional calculus, fixed-point theorem and iterative method. Finally, the model is proved to have a disease-free and an endemic equilibrium point.

Keywords: Coronavirus; CF-fractional derivative; Non-singularity; Existence; Fixed point.
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1 Introduction

A novel coronavirus has been known to cause respiratory infection, for example an unusual pneumonia, in humans[13]. This illness, with the temporary name ’COVID-19 sensitive respiratory infection (COVID-19), is first identified in the winter month of December 2019 in a city of 11 million people - Wuhan in Hubei Province, China [32]. The COVID-19 is considered to be zoonotic in origin, from bats to intermediate host to humans in [33]; and its beginning is geographically connected, but with indecision, with the Huanan Seafood Market in Wuhan in [12]. Human-to-human transmission of COVID-19 has been established, such as through respiratory droplets [10] and there is also a suspicion of asymptomatic infection. To control the epidemics, the government of China has ordered cancellation of huge events for the Chinese New Year celebration, and the lockdown of Wuhan and other cities. The infection has been exported to other parts of China and to other countries, generally via travel-related activities [29]. The ways in which infections spread are a concern that we all have a risk in-research that helps further our understanding of infectious diseases can influence each of our lives. One distinct community of researchers working on understanding infectious disease dynamics is the mathematical modelling community, consisting of scientists from many different disciplines coming together to tackle a common problem through the use of mathematical models and computer simulations.

At the present time, the mathematical models involving fractional order derivative were given clear significance because they are more accurate and realistic as compared to the classical order models [18,24,25]. Inspired by the advancement of fractional calculus, many researchers have focused to study the solutions of nonlinear differential equations with the

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fractional operator by developing quite a few analytical or numerical techniques to find approximate solutions [5,14,28]. These differential equations involve several fractional differential operators like Riemann-Liouville, Caputo, Hilfer etc. [3,16,31].

Recently a new fractional derivative without any singularity in its kernel is proposed in [27,21]. The kernel of the new fractional derivative has the form of an exponential function. On the other hand, these operators have a power law kernel and have limitations in modeling physical problems. To conquer this difficulty, recently an alternate fractional differential operator having a kernel with exponential decay has been introduced by Caputo and Fabrizio [9] This new approach of fractional derivative is known as the CF- operator which has attracted many research scholars due to the fact that it has a non-singular kernel. Also the CF-operator is most suitable for modeling some class of real-world problem which follows the exponential decay law. With the course of time, developing a mathematical model using the CF fractional order derivative became a notable field of research. In recent times, several mathematicians were busy in development and simulation of differential equations of CF-fractional derivative. One can read the articles of the aforementioned derivative to see further characteristics and applications [4,7,15,26]. However, the CF-fractional derivative gives less noise than the power law one while the Atangana-Baleanu fractional derivative provides an excellent description. In [20], M. A. Khan and A. Atagana introduced the mathematical modelling and dynamics of a novel corona virus (2019-nCov) with fractional order derivative. The fractional model was solved numerically by the authors. For more results about Covid-2019, see [1,2,8,17].

In the present paper, we apply the CF- fractional derivativewith an expoential decay kernel to a novel corona virus model. The existence and uniqueness of the solution of the fractional model are estabilshed using fixed-point theory and an iterative method. The paper is structured as follows: The definition of the CF-fractional derivative and some its important properties are given in Sec. 2. The fractional model for novel coronavirus is described in Sec. 3. In Sec. 4, the existence and uniqueness of the solutions of the model are discussed. In Sec. 5, we determine the equilibrium points of the model and give conditions for local asymptotic stability. Lastly, some conclusion are presented in Sec. 6.

2 Prerequisites

In this section, we review the definitions and properties for the CF- fractional operators involved in this paper.

Let $H^1(a,b) = \{f|L^2(a,b) \quad \text{and} \quad f^1 \in (a,b)\}$, where $L^2(a,b)$ is the space of square integrable on the interval (a,b).

Definition 1. *Let $f \in H^1(a,b)$ and $\rho \in (0,1)$. Then the CF-fractional derivative [9] is defined as*

$$^{CF}\mathscr{D}_t^{\rho}(f(t)) = \frac{\mathscr{M}(\rho)}{1-\rho}\int_a^t f^{'}(x)exp\left[-\rho\frac{t-x}{1-\rho}\right]dx, \tag{1}$$

where $\mathscr{M}(\rho)$ is a normalization function such that $\mathscr{M}(0) = \mathscr{M}(1) = 1$. However, if $f \in H^1(a,b)$, then the derivative is defined as

$$^{CF}\mathscr{D}_t^{\rho}(f(t)) = \frac{\rho\mathscr{M}(\rho)}{1-\rho}\int_a^t (f(t)-f(x))\,exp\left[-\frac{t-x}{1-\rho}\right]dx. \tag{2}$$

Remark.[9] If we let $\sigma = \frac{1-\rho}{\rho} \in (0,\infty)$, then $\rho \in \frac{1}{1+\sigma} \in (0,1)$. In consequence, Eq. (2) can be reduced to

$$^{CF}\mathscr{D}_t^{\rho}(f(t)) = \frac{\mathscr{N}(\sigma)}{\sigma}\int_a^t f^{'}(x)exp\left[-\frac{t-x}{\sigma}\right]dx, \tag{3}$$

where $\mathcal{N}(\sigma)$ is the normalization term corresponding to $\mathcal{M}(\rho)$ such that $\mathcal{N}(0) = \mathcal{N}(\infty) = 1$.

Remark.[9] We have the following property:

$$\lim_{\sigma \to 0} \frac{1}{\sigma} exp\left[-\frac{t-x}{\sigma}\right] = \delta(x-t), \tag{4}$$

where $\delta(x-t)$ is the Dirac delta function.

The above CF-fractional derivative was later modified (see [23]) as

$$^{CF}\mathscr{D}_t^{\rho}(f(t)) = \frac{(2-\rho)\mathcal{M}(\rho)}{2(1-\rho)} \int_a^t f'(x) exp\left[-\rho\frac{t-x}{1-\rho}\right] dx. \tag{5}$$

The fractional integral corresponding to the derivative in Eq. (5) was defined in [23] as follows:

Definition 2. *Let* $0 < \rho < 1$. *The fractional integral of order* ρ *of a function* f *is defined by*

$$^{CF}\mathscr{I}_t^{\rho}(f(t)) = \frac{2(1-\rho)}{(2-\rho)\mathcal{M}(\rho)} f(t) + \frac{2\rho}{(2-\rho)\mathcal{M}(\rho)} \int_0^t f(x)dx, \quad t \geq 0. \tag{6}$$

Remark.[23] From the definition in Eq. (6), the fractional integral of CF type of the function f of order $0 < \rho < 1$ is a mean between the function f and its integral of order one, i.e.,

$$\frac{2(1-\rho)}{(2-\rho)\mathcal{M}(\rho)} + \frac{2\rho}{(2-\rho)\mathcal{M}(\rho)} = 1, \tag{7}$$

and therefore $\mathcal{M}(\rho) = \frac{2}{2-\rho}$, $0 < \rho < 1$.

Using $\mathcal{M}(\rho) = \frac{2}{2-\rho}$, the new CF-derivative and its integral as follows:

Definition 3. *[23] let* $0 < \rho < 1$. *The fractional CF-derivative of order* ρ *of a function* f *is given by*

$$^{CF}\mathscr{D}_t^{\rho}(f(t)) = \frac{1}{1-\rho} \int_a^t f'(x) exp\left[-\rho\frac{t-x}{1-\rho}\right] dx, \quad t \geq 0, \tag{8}$$

and its fractional integral is defined as

$$^{CF}\mathscr{I}_t^{\rho}(f(t)) = (1-\rho)f(t) + \rho \int_0^t f(x)dx, \quad t \geq 0. \tag{9}$$

3 CF-fractional model for novel coronavirus (COVID-19)

In this section, we consider the coronavirus epidemic model proposed by M. A. Khan and A. Atangana in [20]. In this model, it is assumed that the total population of people is denoted by N_p which is classified further into five subgroups such as S_p, E_p, I_p, A_p and R_p which represent respectively, the susceptible, exposed, infected (symptomatic), asymptotically infected and the recovered or the removed people. The evolutionary dynamics of the bats, host, people and the seafood markset (reservoir) is described through the nonlinear differential equations. Hence, the total population is $N_p(t) = S_p + E_p + I_p + A_p + R_p$ and the original integer-order model adopted from [20] can be written as

$$\begin{cases} \frac{dS_p}{dt} = \Pi_p - \mu_p S_p - \frac{\eta_p S_p (I_p + \psi A_p)}{N_p} - \eta_w S_P M, \\ \frac{dE_p}{dt} = \frac{\eta_p S_p (I_p + \psi A_p)}{N_p} + \eta_w S_p M - (1-\theta)\omega_p E_p - \theta_p \rho_p E_p - \mu_p E_p, \\ \frac{dI_p}{dt} = (1-\theta_p)\omega_p E_p - (\tau_p + \mu_p) I_p, \\ \frac{dA_p}{dt} = \theta_p \rho_p E_p - (\tau_{ap} + \mu_p) A_p, \\ \frac{dR_p}{dt} = \tau_p I_p + \tau_{ap} A_p - \mu_p R_p, \\ \frac{dM_p}{dt} = Q_p I_p + \overline{\omega}_p A_p - \pi M. \end{cases} \tag{10}$$

All parameters in the model are assumed to be positive constants and the definitions are as follows. The birth and natural death rate of the people is given by the parameters Π_p and μ_p respectively. The susceptible people S_p will be infected through sufficient contacts with the infected people I_p through the term given by $\eta_p S_p I_p$ where the η_p is the disease transmission coefficient. The transmission among the asymptomatically infected people with health people could take place at form $\psi \eta_p S_p A_p$, where ψ the transmissibility multiple will exists and hence vanish, and if $\psi = 1$, then the same will take place like I_p and $\psi \in [0,1]$, when $\psi = 0$, no transmisibility multiple will exists and hence vanish, and if $\psi = 1$, then the same will take place like I_p infection. The parameter θ_p is the proportion of asymptomatic infection. The parameters ω_p and ρ_p respectively represent the transmission rate after completing the incubation period and becomes infected, joining the class I_p and A_p. The people in the symptomatic class I_p and asymptomatic classAp joining these class R_p with the removal or recovery rate respectively by τ_p and τ_{ap}. The class M which is denoted be the reservoir or the seafood place or market. The susceptible people infected after the interaction with M, given by $\eta_w M S_p$, where η_w the disease transmission coefficient from M to S_p. The host visiting the seafood market by purchasing the items (retail purchase) shown by b with bMI_h/N_h. The parameters Q_p and $\overline{w}_p$ of the infected symptomatic and asymptomatically infected respectively contributing the virus into the seafood market M. The removing rate of the virus from the seafood market M is given by the rate π.

To obtain CF-fractional derivative model, we replace the first-order time derivatives of the left-hand side of (10) by the CF-fractional derivative defined in Eq. (5). The proposed new CF-fractional model for novel corona virus (COVID-19) can therefore be written as follows:

$$\begin{cases} {}^{CF}\mathscr{D}_t^{\rho_1} S_p = \Pi_p - \mu_p S_p - \frac{\eta_p S_p (I_p + \psi A_p)}{N_p} - \eta_w S_P M, \\ {}^{CF}\mathscr{D}_t^{\rho_2} E_p = \frac{\eta_p S_p (I_p + \psi A_p)}{N_p} + \eta_w S_p M - (1-\theta)\omega_p E_p - \theta_p \rho_p E_p - \mu_p E_p, \\ {}^{CF}\mathscr{D}_t^{\rho_3} I_p = (1-\theta_p)\omega_p E_p - (\tau_p + \mu_p) I_p, \\ {}^{CF}\mathscr{D}_t^{\rho_4} A_p = \theta_p \rho_p E_p - (\tau_{ap} + \mu_p) A_p, \\ {}^{CF}\mathscr{D}_t^{\rho_5} R_p = \tau_p I_p + \tau_{ap} A_p - \mu_p R_p, \\ {}^{CF}\mathscr{D}_t^{\rho_6} M = Q_p I_p + \overline{\omega}_p A_p - \pi M. \end{cases} \tag{11}$$

with initial conditions

$$S_p(0) = S_0, \quad E_p(0) = E_0, \quad I_p(0) = I_0, \quad A_p(0) = A_0, \quad R_p(0) = R_0, \quad M(0) = M_0. \tag{12}$$

In the theoretical treatment, we will assume that the fractional orders $(0 < \rho_i < 1, i = 1,2,3,\ldots 6)$ for each of six groups can be different.

4 Existence and uniqueness of solutions of the model

In this section, we investigate the existence and uniqueness of the solutions of the CF-fractional model for novel corona virus in Eq. (11) with initial conditions (12). Using fixed point theory (see, e.g., [22,19]), we can prove existence of solutions for the model as follows.

$$\begin{cases} S_p(t)-S(0)= {}^{CF}\mathscr{I}_t^{\rho_1}\left[\Pi_p-\mu_pS_p-\frac{\eta_pS_p(I_p+\psi A_p)}{N_p}-\eta_wS_PM\right] \\ E_p(t)-E(0)= {}^{CF}\mathscr{I}_t^{\rho_2}\left[\frac{\eta_pS_p(I_p+\psi A_p)}{N_p}+\eta_wS_pM-(1-\theta)\omega_pE_p-\theta_p\rho_pE_p-\mu_pE_p\right] \\ I_p(t)-I(0)= {}^{CF}\mathscr{I}_t^{\rho_3}\left[(1-\theta_p)\omega_pE_p-(\tau_p+\mu_p)I_p\right] \\ A_p(t)-A(0)= {}^{CF}\mathscr{I}_t^{\rho_4}\left[\theta_p\rho_pE_p-(\tau_{ap}+\mu_p)A_p\right] \\ R_p(t)-R(0)= {}^{CF}\mathscr{I}_t^{\rho_5}\left[\tau_pI_p+\tau_{ap}A_p-\mu_pR_p\right] \\ M(t)-M(0)= {}^{CF}\mathscr{I}_t^{\rho_6}\left[Q_pI_p+\overline{\omega}_pA_p-\pi M\right]. \end{cases} \tag{13}$$

Then, for sake of brevity, we define the following kernels:

$$\begin{cases} G_1(t,S_p)=\Pi_p-\mu_pS_p-\frac{\eta_pS_p(I_p+\psi A_p)}{N_p}-\eta_wS_PM \\ G_2(t,E_p)=\frac{\eta_pS_p(I_p+\psi A_p)}{N_p}+\eta_wS_pM-(1-\theta)\omega_pE_p-\theta_p\rho_pE_p-\mu_pE_p \\ G_3(t,I_p)=(1-\theta_p)\omega_pE_p-(\tau_p+\mu_p)I_p \\ G_4(t,A_p)=\theta_p\rho_pE_p-(\tau_{ap}+\mu_p)A_p \\ G_5(t,R_p)=\tau_pI_p+\tau_{ap}A_p-\mu_pR_p \\ G_6(t,M)=Q_pI_p+\overline{\omega}_pA_p-\pi M, \end{cases} \tag{14}$$

and the functions

$$\Omega(\rho)=\frac{2(1-\rho)}{(2-\rho)\mathscr{M}(\rho)} \quad \text{and} \quad \omega(\rho)=\frac{2\rho}{(2-\rho)\mathscr{M}(\rho)}. \tag{15}$$

In proving the following theorems, we will assume that S_p,E_p,I_p,A_p,R_p and M are nonnegative bounded functions, i.e., $\|S_p(t)\|\le\theta_1$, $\|E_p(t)\|\le\theta_2$, $\|I_p(t)\|\le\theta_3$, $\|A_p(t)\|\le\theta_4$, $\|R_p(t)\|\le\theta_5$, $\|M(t)\|\le\theta_6$, where $\theta_i, i=1,2,...,6$ are some positive constants. Denote

$$\begin{cases} \gamma_1=\mu+\frac{\eta_p(\theta_3+\psi_p\theta_4)}{N_p}+\eta_w\theta_6 \\ \gamma_2=(1-\theta_p)\omega_p+\theta_p\rho_p+\mu_p \\ \gamma_3=\tau_p+\mu_p \\ \gamma_4=\tau_{ap}+\mu_p \\ \gamma_5=\mu_p \\ \gamma_6=\Pi. \end{cases} \tag{16}$$

Applying the definition of the CF-fractional integral in Eq. (6) to Eq. (13), we obtain

$$\begin{cases} S_p(t) - S(0) = \Omega(\rho_1) G_1(t, S_p) + \omega(\rho_1) \int_0^t G_1(y, S_p) dy, \\ E_p(t) - E(0) = \Omega(\rho_2) G_2(t, E_p) + \omega(\rho_2) \int_0^t G_2(y, E_p) dy, \\ I_p(t) - I(0) = \Omega(\rho_3) G_3(t, I_p) + \omega(\rho_3) \int_0^t G_3(y, I_p) dy, \\ A_p(t) - A(0) = \Omega(\rho_4) G_4(t, A_p) + \omega(\rho_4) \int_0^t G_4(y, A_p) dy, \\ R_p(t) - R(0) = \Omega(\rho_5) G_5(t, R_p) + \omega(\rho_5) \int_0^t G_5(y, R_p) dy, \\ M(t) - M(0) = \Omega(\rho_6) G_6(t, M) + \omega(\rho_6) \int_0^t G_6(y, M) dy. \end{cases} \tag{17}$$

Theorem 1. *If the following inequality holds*

$$0 \leq \nu = \max\{\gamma_i, i = 1, 2, ...6\} < 1, \tag{18}$$

then the kernels $G_i, i = 1, 2, ..., 6$ satisfy Lipschitz conditions and are contraction mappings.

Proof. We consider the kernel G_1. Let S_p and S_{p1} be any two functions, then we have

$$\begin{aligned} \left\| G_1(t, S_p) - G_1(t, S_{p1}) \right\| &= \left(\mu + \frac{\eta_p(\theta_3 + \psi_p \theta_4)}{N_p} + \eta_w \theta_6 \right) \left\| S_p(t) - S_{p1}(t) \right\| \\ &= \gamma_1 \left\| S_p(t) - S_{p1}(t) \right\|, \end{aligned} \tag{19}$$

where γ_1 is defined in Eq. (16). Similar results for the kernels $G_i, i = 2, ..., 6$ can be obtained using $\{E_p, E_{p1}\}$, $\{I_p, I_{p1}\}$, $\{A_p, A_{p1}\}$, $\{R_p, R_{p1}\}$ and $\{M, M_1\}$, respectively, as follows:

$$\begin{aligned} \left\| G_2(t, E_p) - G_2(t, E_{p1}) \right\| &= \gamma_2 \left\| E_p(t) - E_{p1}(t) \right\|, \\ \left\| G_3(t, I_p) - G_3(t, I_{p1}) \right\| &= \gamma_3 \left\| I_p(t) - I_{p1}(t) \right\|, \\ \left\| G_4(t, A_p) - G_4(t, A_{p1}) \right\| &= \gamma_4 \left\| A_p(t) - A_{p1}(t) \right\|, \\ \left\| G_5(t, R_p) - G_5(t, R_{p1}) \right\| &= \gamma_5 \left\| R_p(t) - R_{p1}(t) \right\|, \\ \left\| G_6(t, M) - G_2(t, M_1) \right\| &= \gamma_6 \left\| M(t) - M_1(t) \right\|, \end{aligned}$$

where $\gamma_i, i = 2, ..., 6$ are defined in Eq. (16). Therefore, the Lipschitz conditions are satisfied for $G_i, i = 1, 2, ..., 6$. In addition, since $0 \leq \nu = \max\{\gamma_i, i = 1, 2, ...6\} < 1$, the kernels are contractions.

From Eq. (17), the state variable can be displayed in terms of the kernels as follows:

$$\begin{cases} S_p(t) = S(0) + \Omega(\rho_1) G_1(t, S_p) + \omega(\rho_1) \int_0^t G_1(y, S_p) dy, \\ E_p(t) = E(0) + \Omega(\rho_2) G_2(t, E_p) + \omega(\rho_2) \int_0^t G_2(y, E_p) dy, \\ I_p(t) = I(0) + \Omega(\rho_3) G_3(t, I_p) + \omega(\rho_3) \int_0^t G_3(y, I_p) dy, \\ A_p(t) = A(0) + \Omega(\rho_4) G_4(t, A_p) + \omega(\rho_4) \int_0^t G_4(y, A_p) dy, \\ R_p(t) = R(0) + \Omega(\rho_5) G_5(t, R_p) + \omega(\rho_5) \int_0^t G_5(y, R_p) dy, \\ M(t) = M(0) + \Omega(\rho_6) G_6(t, M) + \omega(\rho_6) \int_0^t G_6(y, M) dy. \end{cases} \tag{20}$$

Using (20), we now introduce the following recursive formulas:

$$\begin{cases} S_{pn}(t)=\Omega(\rho_1)G_1(t,S_{p(n-1)})+\omega(\rho_1)\int_0^t G_1(y,S_{p(n-1)})dy,\\ E_{pn}(t)=\Omega(\rho_2)G_2(t,E_{p(n-1)})+\omega(\rho_2)\int_0^t G_2(y,E_{p(n-1)})dy,\\ I_{pn}(t)=\Omega(\rho_3)G_3(t,I_{p(n-1)})+\omega(\rho_3)\int_0^t G_3(y,I_{p(n-1)})dy,\\ A_{pn}(t)=\Omega(\rho_4)G_4(t,A_{p(n-1)})+\omega(\rho_4)\int_0^t G_4(y,A_{p(n-1)})dy,\\ R_{pn}(t)=\Omega(\rho_5)G_5(t,R_{p(n-1)})+\omega(\rho_5)\int_0^t G_5(y,R_{p(n-1)})dy,\\ M_{n}(t)=\Omega(\rho_6)G_6(t,M_{(n-1)})+\omega(\rho_6)\int_0^t G_6(y,M_{(n-1)})dy. \end{cases}$$

The initial components of the above recursive formulas are determined by the given initial conditions as follows:

$$\begin{cases} S_0(t)=S_p(0), \quad E_0(t)=E_p(0), \quad I_0(t)=I_p(0),\\ A_0(t)=A_p(0), \quad R_0(t)=R_p(0), \quad M_0(t)=M(0). \end{cases} \tag{21}$$

The difference between the consecutive terms for the recursive formulas can be written as

$$\begin{cases} \phi_{pn}(t) &= S_{pn}(t)-S_{p(n-1)}(t)\\ &= \Omega(\rho_1)\left(G_1(t,S_{p(n-1)})-G_1(t,S_{p(n-2)})\right)\\ &\quad +\omega(\rho_1)\int_0^t \left(G_1(y,S_{p(n-1)})-G_1(y,S_{p(n-2)})\right)dy\\ \psi_{pn}(t) &= E_{pn}(t)-E_{p(n-1)}(t)\\ &= \Omega(\rho_2)\left(G_2(t,E_{p(n-1)})-G_2(t,E_{p(n-2)})\right)\\ &\quad +\omega(\rho_2)\int_0^t \left(G_2(y,E_{p(n-1)})-G_2(y,E_{p(n-2)})\right)dy\\ \chi_{pn}(t) &= I_{pn}(t)-I_{p(n-1)}(t)\\ &= \Omega(\rho_3)\left(G_3(t,I_{p(n-1)})-G_3(t,I_{p(n-2)})\right)\\ &\quad +\omega(\rho_3)\int_0^t \left(G_3(y,I_{p(n-1)})-G_3(y,I_{p(n-2)})\right)dy\\ \kappa_{pn}(t) &= A_{pn}(t)-A_{p(n-1)}(t)\\ &= \Omega(\rho_4)\left(G_4(t,A_{p(n-1)})-G_4(t,I_{p(n-2)})\right)\\ &\quad +\omega(\rho_4)\int_0^t \left(G_4(y,A_{p(n-1)})-G_4(y,A_{p(n-2)})\right)dy\\ \varphi_{pn}(t) &= R_{pn}(t)-R_{p(n-1)}(t)\\ &= \Omega(\rho_5)\left(G_5(t,R_{p(n-1)})-G_5(t,I_{p(n-2)})\right)\\ &\quad +\omega(\rho_5)\int_0^t \left(G_5(y,R_{p(n-1)})-G_5(y,R_{p(n-2)})\right)dy\\ \lambda_{n}(t) &= M_{n}(t)-M_{(n-1)}(t)\\ &= \Omega(\rho_6)\left(G_6(t,M_{(n-1)})-G_6(t,M_{(n-2)})\right)\\ &\quad +\omega(\rho_6)\int_0^t \left(G_6(y,M_{(n-1)})-G_6(y,M_{(n-2)})\right)dy. \end{cases} \tag{22}$$

Note that:

$$\begin{cases} S_{pn}(t) = \sum_{i=1}^{n} \phi_{pi}(t) \\ E_{pn}(t) = \sum_{i=1}^{n} \psi_{pi}(t) \\ I_{pn}(t) = \sum_{i=1}^{n} \chi_{pi}(t) \\ A_{pn}(t) = \sum_{i=1}^{n} \kappa_{pi}(t) \\ R_{pn}(t) = \sum_{i=1}^{n} \varphi_{pi}(t) \\ M_{n}(t) = \sum_{i=1}^{n} \lambda_{i}(t). \end{cases} \tag{23}$$

Next, we formulate the recursive inequalities for the differences $\phi_{pn}(t), \psi_{pn}(t), \chi_{pn}(t), \kappa_{pn}(t), \varphi_{pn}(t)$ and λ_n as follows:

$$\begin{aligned} \left\|\phi_{pn}(t)\right\| &= \left\|S_{pn}(t) - S_{p(n-1)}(t)\right\| \\ &= \left\|\Omega(\rho_1)\left(G_1(t, S_{p(n-1)}) - G_1(t, S_{p(n-2)})\right)\right\| \\ &\quad + \left\|\omega(\rho_1)\int_0^t \left(G_1(y, S_{p(n-1)}) - G_1(y, S_{p(n-2)})\right) dy\right\|. \end{aligned} \tag{24}$$

Then, since the kernel G_1 satisfies the Lipschitz condition with Lipschitz constant γ_1, we have

$$\begin{aligned} \left\|S_{pn}(t) - S_{p(n-1)}(t)\right\| \leq \Omega(\rho_1)\gamma_1 \left\|S_{p(n-1)} - S_{p(n-2)}\right\| \\ + \omega(\rho_1)\gamma_1 \int_0^t \left\|S_{p(n-1)} - S_{p(n-2)}\right\| dy. \end{aligned}$$

Thus, we obtain

$$\left\|\phi_{pn}(t)\right\| \leq \Omega(\rho_1)\gamma_1 \left\|\phi_{p(n-1)}(t)\right\| + \omega(\rho_1)\gamma_1 \int_0^t \left\|\phi_{p(n-1)}(y)\right\| dy. \tag{25}$$

In a similar manner, we can obtain the following results:

$$\begin{cases} \left\|\psi_{pn}(t)\right\| \leq \Omega(\rho_2)\gamma_2 \left\|\psi_{p(n-1)}(t)\right\| + \omega(\rho_2)\gamma_2 \int_0^t \left\|\psi_{p(n-1)}(y)\right\| dy \\ \left\|\chi_{pn}(t)\right\| \leq \Omega(\rho_3)\gamma_3 \left\|\chi_{p(n-1)}(t)\right\| + \omega(\rho_3)\gamma_3 \int_0^t \left\|\chi_{p(n-1)}(y)\right\| dy \\ \left\|\kappa_{pn}(t)\right\| \leq \Omega(\rho_4)\gamma_4 \left\|\kappa_{p(n-1)}(t)\right\| + \omega(\rho_4)\gamma_4 \int_0^t \left\|\kappa_{p(n-1)}(y)\right\| dy \\ \left\|\varphi_{pn}(t)\right\| \leq \Omega(\rho_5)\gamma_5 \left\|\varphi_{p(n-1)}(t)\right\| + \omega(\rho_5)\gamma_5 \int_0^t \left\|\varphi_{p(n-1)}(y)\right\| dy \\ \left\|\lambda_{n}(t)\right\| \leq \Omega(\rho_6)\gamma_6 \left\|\lambda_{(n-1)}(t)\right\| + \omega(\rho_6)\gamma_6 \int_0^t \left\|\lambda_{(n-1)}(y)\right\| dy. \end{cases} \tag{26}$$

Theorem 2. *If there exists a time $t_0 > 0$ such that the following inequality hold:*

$$\Omega(\rho_i)\gamma_i + \omega(\rho_i)\gamma_i t_0 < 1, \quad for \quad i = 1, 2, ..., 6, \tag{27}$$

then a system of solutions exists for the CF-fractional novel corona virus model (11)-(12).

Proof. Since the functions $S_p(t), E_p(t), I_p(t), A_p(t)$, $R_p(t)$ and $M(t)$ are assumed to be bounded and each of the kernels satisfies a Lipschitz condition, the following relations can be obtained using Eqs. (25)-(26) recursively:

$$\begin{cases} \left\|\phi_{pn}(t)\right\| \leq \left\|S_p(0)\right\| \left[\Omega(\rho_1)\gamma_1 + \omega(\rho_1)\gamma_1 t\right]^n \\ \left\|\psi_{pn}(t)\right\| \leq \left\|E_p(0)\right\| \left[\Omega(\rho_2)\gamma_2 + \omega(\rho_2)\gamma_2 t\right]^n \\ \left\|\chi_{pn}(t)\right\| \leq \left\|I_p(0)\right\| \left[\Omega(\rho_3)\gamma_3 + \omega(\rho_3)\gamma_3 t\right]^n \\ \left\|\kappa_{pn}(t)\right\| \leq \left\|A_p(0)\right\| \left[\Omega(\rho_4)\gamma_4 + \omega(\rho_4)\gamma_4 t\right]^n \\ \left\|\varphi_{pn}(t)\right\| \leq \left\|R_p(0)\right\| \left[\Omega(\rho_5)\gamma_5 + \omega(\rho_5)\gamma_5 t\right]^n \\ \left\|\lambda_n(t)\right\| \leq \left\|M(0)\right\| \left[\Omega(\rho_6)\gamma_6 + \omega(\rho_6)\gamma_6 t\right]^n . \end{cases} \tag{28}$$

Equation (28) shows that existence and smoothness of the functions defined in Eq. (23).

To complete the proof, we prove that the functions $S_{pn}(t), E_{pn}(t), I_{pn}(t), A_{pn}(t)$, $R_{pn}(t)$ and $M_n(t)$ converge to a system of solutions of (11)-(12). We define B_{pn}, C_{pn}, D_{pn}, F_{pn}, H_{pn} and J_n as remainder terms after n iterations, i.e.,

$$\begin{cases} S_p(t) - S_p(0) = S_{pn}(t) - B_{pn}(t) \\ E_p(t) - E_p(0) = E_{pn}(t) - C_{pn}(t) \\ I_p(t) - I_p(0) = I_{pn}(t) - D_{pn}(t) \\ A_p(t) - A_p(0) = A_{pn}(t) - F_{pn}(t) \\ R_p(t) - R_p(0) = R_{pn}(t) - H_{pn}(t) \\ M(t) - M(0) = M_n(t) - J_n(t). \end{cases} \tag{29}$$

Then, using the triangle inequality and the Lipschitz condition for G_1, we have

$$\begin{aligned} \left\|B_{pn}(t)\right\| &= \left\|\Omega(\rho_1)\left(G_1(t,S_p) - G_1(t,S_{p(n-1)})\right) + \omega(\rho_1)\int_0^t \left(G_1(t,S_p) - G_1(t,S_{p(n-1)})\right) dy\right\| \\ &= \Omega(\rho_1)\left\|\left(G_1(t,S_p) - G_1(t,S_{p(n-1)})\right)\right\| + \omega(\rho_1)\int_0^t \left\|\left(G_1(t,S_p) - G_1(t,S_{p(n-1)})\right) dy\right\| \\ &\leq \Omega(\rho_1)\gamma_1 \left\|S_p - S_{p(n-1)}\right\| + \omega(\rho_1)\gamma_1 \left\|S_p - S_{p(n-1)}\right\| t. \end{aligned}$$

Applying the above process recursively, we obtain

$$\left\|B_{pn}(t)\right\| \leq \left[\Omega(\rho_1)\gamma_1 + \omega(\rho_1)\gamma_1 t\right]^{n+1} \theta_1. \tag{30}$$

Then at t_0, we obtain

$$\left\|B_{pn}(t)\right\| \leq \left[\Omega(\rho_1)\gamma_1 + \omega(\rho_1)\gamma_1 t_0\right]^{n+1} \theta_1. \tag{31}$$

Taking the limit on Eq. (31) as $n \to \infty$ and then using condition (27), we obtain $\|B_n(t)\| \to 0$. Using the same process as described above, we have the following relations:

$$\|C_{pn}(t)\| \leq [\Omega(\rho_2)\gamma_2 + \omega(\rho_2)\gamma_2 t_0]^{n+1}\, \theta_2, \tag{32}$$
$$\|D_{pn}(t)\| \leq [\Omega(\rho_3)\gamma_3 + \omega(\rho_3)\gamma_3 t_0]^{n+1}\, \theta_3, \tag{33}$$
$$\|F_{pn}(t)\| \leq [\Omega(\rho_4)\gamma_4 + \omega(\rho_4)\gamma_4 t_0]^{n+1}\, \theta_4, \tag{34}$$
$$\|H_{pn}(t)\| \leq [\Omega(\rho_5)\gamma_5 + \omega(\rho_5)\gamma_5 t_0]^{n+1}\, \theta_5, \tag{35}$$
$$\|J_n(t)\| \leq [\Omega(\rho_6)\gamma_6 + \omega(\rho_6)\gamma_6 t_0]^{n+1}\, \theta_6. \tag{36}$$

Similarly, taking the limit on Eq. (32)-(36) as $n \to \infty$ and then using condition (27), we have $\|C_{pn}(t)\| \to 0$, $\|D_{pn}(t)\| \to 0$, $\|F_{pn}(t)\| \to 0$, $\|H_{pn}(t)\| \to 0$ and $\|J_{pn}(t)\| \to 0$. Therefore, the existence of the system of solutions of system (11)-(12) is proved.

We now give conditions for the system of solutions to be unique.

Theorem 3. *System* (11) *along with the initial conditions* (12) *has a unique system of solutions if the following conditions hold:*

$$(1 - \Omega(\rho_i)\gamma_i - \omega(\rho_i\gamma_i t)) > 0, \quad \textit{for} \quad i = 1, 2, \ldots, 6. \tag{37}$$

Proof. Assume that $\{S_{p1}(t), E_{p1}(t), I_{p1}(t), A_{p1}(t), R_{p1}(t), M_1(t)\}$ is another set of solution of model (11)-(12) in additiion to the solution set $\{S_p(t), E_p(t), I_p(t), A_p(t), R_p(t), M(t)\}$ proved to exist in Theorems 1 and 2. Then

$$\begin{aligned} S_p(t) - S_{p1}(t) &= \Omega(\rho_1)\left(G_1(t, S_p) - G_1(t, S_{p1})\right) \\ &\quad + \omega(\rho_1)\int_0^t \left(G_1(y, S_p) - G_1(y, S_{p1})\right) dy. \end{aligned} \tag{38}$$

Taking the norm on both sides of Eq.(38) and using the triangle inequality, we obtain

$$\begin{aligned} \|S_p(t) - S_{p1}(t)\| &= \Omega(\rho_1)\left\|(G_1(t, S_p) - G_1(t, S_{p1}))\right\| \\ &\quad + \omega(\rho_1)\int_0^t \left\|(G_1(y, S_p) - G_1(y, S_{p1}))\right\| dy. \end{aligned} \tag{39}$$

Using the Lipschitz condition for kernel G_1, we find

$$\|S_p(t) - S_{p1}(t)\| = \Omega(\rho_1)\gamma_1 \|S_p(t) - S_{p1}(t)\| + \omega(\rho_1)\gamma_1 t \|S_p(t) - S_{p1}(t)\|. \tag{40}$$

Then, rearranging Eq. (40), we obtain

$$\|S_p(t) - S_{p1}(t)\| (1 - \Omega(\rho_1) - \omega(\rho_1)t) \geq 0. \tag{41}$$

Finally, applying condition (37) for $i = 1$ to Eq. (42), we obtain

$$\|S_p(t) - S_{p1}\| = 0, \tag{42}$$

and therefore $S_p(t) = S_{p1}(t)$.

Applying a similar procedure to each of the following pairs $\{E_p, E_{p1}\}$, $\{I_p, I_{p1}\}$, $\{A_p, A_{p1}\}$, $\{R_p, R_{p1}\}$ and $\{M, M_1\}$ with inequality (37) for $i = 2, \ldots 6$, respectively, we obtain

$$E_p(t) = E_{p1}(t), \quad I_p(t) = I_{p1}(t), \quad A_p(t) = A_{p1}(t), \quad R_p(t) = R_{p1}(t), \quad M(t) = M_1(t). \tag{43}$$

Thus, the uniqueness of the system of solutions of the CF-fractional order system is proved.

5 Stability analysis

We determine the equilibrium points of the fractional order system (11) by equating its right-hand side to zero. Solving the resulting algebric system, we obtain only two equilibrium points, namely, disease-free and an endemic equilibrium point which are already given in [20]. From [20], we have the disease-free equilibrium point given by

$$E^0 = \left(S_p^0, 0,0,0,0,0\right) = \left(\frac{\Pi_p}{\mu_p}, 0,0,0,0,0\right), \tag{44}$$

and the endemic equilibrium point, we denote it by E^* and $E^* = \left(S_p^*, E_p^*, I_p^*, A_p^*, R_p^*, M_p^*\right)$, given by

$$\begin{cases} S_p^* = \frac{\Pi_p}{\lambda+\mu_p} \\ E_p^* = \frac{\lambda S_p^*}{\theta_p\rho_p - \theta_p\omega_p + \mu_p + \omega_p} \\ I_p^* = \frac{E_p^*(1-\theta_p)\omega_p}{\mu_p+\tau_p} \\ A_p^* = \frac{E_p^*\theta_p\rho_p}{\tau_{ap}+\mu_p} \\ R_p^* = \frac{A_p^*\tau_{ap}+I_p^*\tau_p}{\mu_p} \\ M^* = \frac{A_p^*\overline{\omega}_p + I_p^* Q_p}{\pi}, \end{cases} \tag{45}$$

where the basic reproduction number R_0 which is the average number of infected contacts per infected individual. R_0 can be obtained using the next generation matrix method [6,?], is written as

$$R_0 = \frac{\theta_p\rho_p(\mu_p+\tau_p)\left(\pi\psi\eta_p + \Pi_p\overline{\omega}\eta_w\right) + (1-\theta_p)\omega_p(\tau_{ap}+\mu_p)\left(\pi\eta_p\mu_p + \Pi_p Q_p\eta_w\right)}{\pi\mu_p(\mu_p+\tau_p)(\tau_{ap}+\mu_p)\left(\theta_p(\rho_p-\omega_p)+\mu_p+\omega_p\right)}. \tag{46}$$

It can be noticed that the unique endemic equilibrium point E^* exists if $R_0 > 1$.

Consider the following fractional order linear system described by CF-fractional derivative:

$$^{CF}\mathscr{D}_t^{\rho} x(t) = Ax(t), \tag{47}$$

where $x(t) \in R^n$, $A \in R^{n\times n}$, and $0 < \rho < 1$.

Definition 4. *[11] The characteristic equation of system (47) is*

$$det\left(s\left(I-(1-\rho)A\right) - \rho A\right) = 0. \tag{48}$$

Theorem 4. *[11] If $(I-(1-\rho)A) - \rho A$ is invertible, then system (47) is asymptotically stable if and only if the real part of the roots to the characteristic equation of system (47) are negative.*

The linearization matrix of model (11) evaluated at the disease-free equilibrium point E^0 is

$$J(E^0) = \begin{bmatrix} -\mu_p & 0 & -\eta_p & -\psi\eta_p & 0 & -\frac{\eta_w\Lambda_p}{\mu_p} \\ 0 & -\mu_p-\theta_p\rho_p-(1-\theta_p)\omega_p & \eta_p & \psi\eta_p & 0 & \frac{\eta_w\Lambda_p}{\mu_p} \\ 0 & (1-\theta_p)\omega_p & \mu_p-\tau_p & 0 & 0 & 0 \\ 0 & \theta_p\rho_p & 0 & -\mu_p\tau_{ap} & 0 & 0 \\ 0 & 0 & \tau_p & \tau_{ap} & -\mu_p & 0 \\ 0 & 0 & Q_p & \overline{w}_p & 0 & -pi \end{bmatrix} \tag{49}$$

If model (11) has a commensurate order, i.e., $\rho_1 = \rho_2 = \rho_3 = \rho_4 = \rho_5 = \rho_6 = \rho \in (0,1)$, then the characteristic equation of the linearized system of model (11) at E^0 is

$$det\left(s\left(I-(1-\rho)J(E^0)\right)-\rho J(E^0)\right)=0. \tag{50}$$

Theorem 5. *The disease-free equilibrium point E^0 of model (11) with a commensurate order $\rho \in (0,1)$ is asymptotically stable if and only if real parts of the characteristic equation (50) are negative.*

Proof. The proof of the theorem is similar to the above Theorem 4. Hence, the proof is omitted (for detailed information, see [11]).

6 Conclusion

In this paper, a CF-fractional differential equation model for novel coronavirus (COVID-19) has been studied. This fractional model is based on the use of the non-singular expoentially decreasing kernels appearing in the CF-fractional derivative. Using fixed point theorey and an iterative method, the existence and uniqueness of solutions for the model have been investigated. We have determined the conditions for local asymptotic stability of the disease-free equilibrium point.

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Competing interest

The authors declare that they have no competing interests.

Authors' contribution

All of the authors contributed to the conception and development of this manuscript.

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