

Application of Standard and Exponential Grey Forecasting Models on Turkey's Education Expenditures

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Abstract: Grey System Theory predicts the behaviour of unknown systems using a small amount of data. Grey System Theory is an interdisciplinary science field and has been successfully applied to many fields. In this study, the Grey System Theory is used to estimate Turkey's education expenditures. Turkey's education expenditure data were taken from the Turkish Statistical Institute for the years of 2011-2020. It has been estimated for the years of 2021-2030 using Standard Grey Model (GM (1,1)) and Exponential Grey Model (EXGM (1,1)). The percentage relative error (RPE) between the actual and the predicted values. Also, the mean absolute percentage error (MAPE) were determined using the actual and the predicted data from 2011-2020.

Consequently, the MAPE values were calculated as 3,32 % and 3,09 % for GM (1,1) and EXGM (1,1) models, respectively. R^2 value which shows the correlation between the actual and predicted values was determined as 0,9845 and 0,9846 for GM (1,1) and EXGM (1,1), respectively. It has been determined that the estimation precision of the EXGM (1,1) method is higher according to calculated errors and R^2 values. Accordingly, education expenditures were estimated for the years 2021-2030. Hereby, it is predicted that our country's education expenditures will increase exponentially in the next 10 years.

Keywords: Grey System Theory, Exponential Grey Model, Differential Equations, Least Squares Method, Education Expenditures.

1 Introduction

Time series estimation refers to the process of estimating the future values of a system using past and present data [1]. Many linear statistical models have been developed and applied for time series estimation [2-8]. However, estimates of future values are limited due to the large amount of data required. Therefore, Grey System Theory, which is a method to predict the behaviour of unknown systems with few data, was developed by Deng [9]. Many different Grey Models have been developed over time. One of them is the Exponential Grey Model (EXGM (1,1)) [10]. Prediction accuracy is improved by adding terms such as (e^{-t}) to the exponential whitening differential equation, with a decreasing term.

Grey estimation theory is an interdisciplinary scientific field and has been applied to many systems with unknown data. Grey estimation models have been successfully applied to various fields such as industry, science and technology, economy, energy, tax, health, natural phenomena, tourist income, industrial economy, environment system, oil production, electricity consumption [11-24]. Grey models mostly yield effective results in estimating exponentially increasing number sequences.

In this study, Turkey's education expenditure data for the years of 2011-2020 were taken from the Turkish Statistical Institute. It has been estimated using the Standard Grey Model (GM (1,1)) and Exponential Grey Model EXGM (1,1) for the years of 2021-2030. Using data of the 2011-2020 years, the percentage relative error (RPE) and mean percentage

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relative error (MAPE) between the actual and the predicted value were determined. The correlation between the actual and the predicted value was examined. Error and correlation results of both models were compared.

2 Theory and Methods

2.1 The Standard GM (1,1) Model

In GM (1,1) model, an accumulating generation operator (AGO) is applied to the data, initially. Here, (1,1) represents the degree of the equation governing the model and the number of variables, respectively. For this reason, GM (1, 1) type of grey model is the most widely used in the literature, pronounced as “Grey model first order with one variable”. Then, the governing differential equation of the model is solved in order to obtain the predicted value of the system. Finally, the estimated value of original data is obtained using the inverse accumulating generation operator (IAGO). The standard GM (1,1) modelling process is given as follows.

Step 1. A data sequence, $X^{(0)}$, is created with the initial data.

$$X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)), \quad (1)$$

where n is the number of raw data.

Step 2. An accumulating sequence $X^{(1)}$ is created as,

$$X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)), \quad (2)$$

where,

$$X^{(1)}(k) = \sum_{i=1}^k [x^{(0)}(i)], \quad k = 2, 3, \dots, n. \quad (3)$$

Step 3. The first-order mean value operator $Z^{(1)}$ is created.

$$Z^{(1)} = \{z^{(1)}(1), z^{(1)}(2), \dots, z^{(1)}(n)\}, \quad (4)$$

where,

$$Z^{(1)}(k) = \frac{x^{(1)}(k) + x^{(1)}(k-1)}{2}, \quad k = 2, \dots, n. \quad (5)$$

Step 4. The winterization differential equation is set up as follows;

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b. \quad (6)$$

The winterization equation is solved and the estimation value, $\hat{X}^{(1)}$, can be evaluated as follow;

$$\hat{x}^{(1)}(k) = \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-a(k-1)} + \frac{b}{a}, \quad (7)$$

for $= 2, 3, \dots, n$. From here, the estimation values are created with the following formula.

$$\begin{aligned} \hat{x}^{(0)}(1) &= x^{(0)}(1), \\ \hat{x}^{(0)}(k) &= \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1), \end{aligned} \tag{8}$$

for $k = 2, 3, \dots, n$.

Step 5. From equation(6);

$$\int_{k-1}^k \frac{dx^{(1)}}{dt} dt + a \int_{k-1}^k x^{(1)} dt = \int_{k-1}^k b dt, \tag{9}$$

According to the Newton-Leibniz formula, the first integral of Eq. (9) can be expressed as

$$\int_{k-1}^k \frac{dx^{(1)}}{dt} dt = x^{(1)}(k) - x^{(1)}(k-1).$$

It is clear that the integration term $\int_{k-1}^k x^{(1)} dt$ denotes the area between t -axis and the curve $x^{(1)}(t)$ in the interval $[k-1, k]$. Then, using the generalized trapezoid formula the second integral of Eq. (9) can be obtained as

$$\int_{k-1}^k x^{(1)} dt = \frac{1}{2} (x^{(1)}(k) + x^{(1)}(k-1)),$$

and the right side of Eq. (9) is equal to

$$\int_{k-1}^k b dt = b.$$

Hence, the Eq. (9) can be written as

$$x^{(1)}(k) - x^{(1)}(k-1) + \frac{a}{2} (x^{(1)}(k) + x^{(1)}(k-1)) = b, \tag{10}$$

and it can be written as,

$$x^{(0)}(k) + az^{(1)}(k) = b. \tag{11}$$

Equation (11) is the basic form of the GM (1,1) model. From this equation, a and b coefficients are determined using the least squares method. Where, k is the time point, a and b are the enhancement and advancement coefficients, respectively [25].

$$\begin{aligned} x^{(0)}(2) + az^{(1)}(2) &= b \\ x^{(0)}(3) + az^{(1)}(3) &= b \\ &\vdots \\ x^{(0)}(n) + az^{(1)}(n) &= b \end{aligned} \tag{12}$$

From this system, it is clear that,

$$Y = B\hat{a},$$

where

$$B = \begin{pmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \\ -z^{(1)}(n) & 1 \end{pmatrix}, \quad (13)$$

$$Y = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(1)}(n) \end{pmatrix}, \quad \hat{a} = \begin{pmatrix} a \\ b \end{pmatrix}. \quad (14)$$

In here, the purpose is to determine a and b coefficients. According to least squares method, if both sides of $Y = B\hat{a}$ are multiplied by B^T ,

$$B^T Y = B^T B \hat{a}, \quad (15)$$

is obtained. From here,

$$\hat{a} = (B^T B)^{-1} B^T Y. \quad (16)$$

Equation (16) is obtained. Matrix multiplication algorithm and least squares method are used to calculate the parameters of this model. The proof of the Eq. (16) was given in the next section.

3 EXGM (1,1) Model

EXGM (1, 1) model is a new grey estimation model [10]. The raw data vary exponentially. If the exponential variation of the raw data sequence is split, the amount of grey action is time dependent. This change is exponential over time. The standard GM (1, 1) treats the amount of grey action as a constant, and its effect falls short of predictive accuracy. Therefore, the estimated error produced by the model increases with time. The EXGM (1,1) model treats the amount of grey effect as an exponential function of time and a constant.

Definition 1. *The governing differential equation of the EXGM (1,1) is*

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b + ce^{-t}. \quad (17)$$

Equation (17) is called the whitening equation of the EXGM (1,1) model (19). The sequence Eq. (1) increases monotonically and the solution of linear equations contains increasing exponential functions. Also, the first order derivative in the linear equation can be written as differential equation.

The grey derivative of the first order grey differential equation is represented as follow:

$$\frac{dx^{(1)}(t)}{dt} = \lim_{\Delta t \rightarrow 0} \frac{x^{(1)}(t + \Delta t) - x^{(1)}(t)}{\Delta t}. \quad (18)$$

Where, t , represents the increment of the parameter t , which can be time, location or other usable parameter and it is considered constant [26]. Therefore, this increase can be made in unit quantity, on the other hand $x^{(1)}(t + \Delta t) - x^{(1)}(t)$ is

the difference data between consecutive points in the data sequence. Hence, it is clear that,

$$\frac{dx^{(1)}(t)}{dt} \approx x^{(1)}(k+1) - x^{(1)}(k) = x^{(0)}(k). \tag{19}$$

Theorem 1.

$$x^{(0)}(k) + az^{(1)}k = b + c(e-1)e^{-k}, \tag{20}$$

Equation (20) is called as basic difference equation of EXGM (1,1) model. Where, $z^{(1)}k$ is given from Eq. (5).

Proof. If the integrated of the both sides of whitening equation of differential equation (17) within the range of $[k-1, k]$,

$$\int_{k-1}^k \frac{dx^{(1)}(t)}{dt} dt + a \int_{k-1}^k x^{(1)}(t) dt = \int_{k-1}^k (b + ce^{-t}) dt, \tag{21}$$

and so,

$$x^{(1)}(k) - x^{(1)}(k-1) + a \int_{k-1}^k x^{(1)}(t) dt = b + c(e-1)e^{-k}. \tag{22}$$

Eq. (19) and Eq. (22) can be written as

$$x^{(0)}(k) + az^{(1)}k = b + c(e-1)e^{-k}. \tag{23}$$

The general solution of the linear equation (17) can be obtained as follows.

$$x^{(1)}(k) = \frac{b}{a} + \frac{c}{a-1}e^{-1} + de^{-at}. \tag{24}$$

Where, d is the integral constant. Using the initial condition $x^{(1)}(1) = x^{(0)}(1)$, d constant is evaluated as follows:

$$d = \left(x^{(0)}(1) - \frac{b}{a} - \frac{c}{a-1}e^{-1} \right) e^a. \tag{25}$$

Therefore, the grey prediction model is obtained from equation (24) with the following equation.

$$\hat{x}^{(1)}(k) = \left(x^{(0)}(1) - \frac{b}{a} - \frac{c}{a-1}e^{-1} \right) e^{a(1-t)} + \frac{b}{a} + \frac{c}{a-1}e^{-t}. \tag{26}$$

Using the equation (26), values of the $\hat{x}^{(1)}(k)$ series are evaluated and the estimated values of the original series $\hat{x}^{(0)}(k)$ can be obtained as,

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1), \tag{27}$$

where $= 2, 3, \dots, n$. Linear equation system (20) can be written as follows.

$$\begin{aligned} x^{(0)}(2) &= -az^{(1)}(2) + b + c(e-1)e^{-2} \\ x^{(0)}(3) &= -az^{(1)}(3) + b + c(e-1)e^{-3} \\ &\vdots \\ x^{(0)}(n) &= -az^{(1)}(n) + b + c(e-1)e^{-n} \end{aligned} \tag{28}$$

or

$$Y = B\hat{a}, \tag{29}$$

where

$$B = \begin{pmatrix} -z^{(1)}(2) & 1 & (e-1)e^{-2} \\ -z^{(1)}(3) & 1 & (e-1)e^{-3} \\ \vdots & \vdots & \vdots \\ -z^{(1)}(n) & 1 & (e-1)e^{-n} \end{pmatrix}, Y = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{pmatrix}, \hat{a} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}. \quad (30)$$

Where n is the number of the samples used to create the model. The parameters (a , b and c) can be easily determined by using the least squared procedure as follows.

For the estimated value of the parameter sequence \hat{a} , the $x^{(0)}(k)$ on the left side of the equation (28) is replaced with $-az^{(1)}(k2) + b + c(e-1)e^{-k}$. Therefore the error sequence $\varepsilon = Y - B\hat{a}$ is obtained. Here,

$$\varepsilon = [\varepsilon_2, \varepsilon_3, \dots, \varepsilon_n]^T$$

and ε_k are represent the error for each equation in the system (28) for $k = 2, 3, \dots, n$.

Notice, $E(\hat{a})$ is defined as the sum of ses of errors, which yields

$$E(\hat{a}) = \int_{k=2}^n \varepsilon_k^2 = \varepsilon^T \varepsilon = (Y - B\hat{a})^T (Y - B\hat{a}) = Y^T Y - 2\hat{a}^T B^T Y + \hat{a}^T B^T B \hat{a}.$$

The parameter vector $\hat{a} = [a, b, c]^T$ that minimize $E(\hat{a})$ satisfy

$$\frac{\partial E}{\partial \hat{a}} = -2B^T Y + 2B^T B \hat{a} = 0.$$

Therefore it can be written as

$$\hat{a} = [a, b, c]^T = (B^T B)^{-1} B^T Y. \quad (31)$$

4 Results and Discussion

Turkey's education expenditures between years of 2011-2020 were taken from the Turkish Statistical Institute and estimated using the GM (1,1) and EXGM (1,1) models. The actual and the predicted values were compared for the years 2011-2020. Comparison between the actual and predicted values was made using the relative percentage error (RPE) and the mean relative percentage error (MAPE) formulas given in equation (32) and equation (33), respectively. Due to the determined error rates being less than 10%, the prediction has been made for the years 2021-2030 [27].

$$RPE(k) = \left| \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \right| * 100 \quad (32)$$

$$MAPE = \frac{1}{n} \sum_{k=1}^n RPE(k) \quad (33)$$

For the GM(1,1) model, we obtain the parameters a and b using the least squares solution as

$$a = -0.1445$$

$$b = 65870.01044.$$

By substituting the parameters into the response function Eq. (7) we have

$$\hat{x}^{(1)}(k) = \left(x^{(0)}(1) + 455847.8231\right) e^{0.1445(k-1)} - 455847.8231.$$

In addition, we obtain the parameters of the EXGM(1,1) model using the least squares procedure as follow,

$$\begin{aligned} a &= -0.1425 \\ b &= 67761.4442 \\ c &= -16604.0111. \end{aligned}$$

By substituting the parameters into the response function Eq. (26) we have

$$\hat{x}^{(1)}(k) = \left(x^{(0)}(1) + 475518.9067 - 14533.0513e^{-1}\right) e^{-0.1425(1-t)} - 475518.9067 + 14533.0513e^{-t}.$$

The actual values of Turkey’s education expenditures between 2011-2020, the estimated values calculated with the GM (1,1) and EXGM (1,1) models, and MAPE values of the relevant models are given in Table 1. The forecasting values of Turkey’s education expenditures between 2021-2030 are given in Table 2. The MAPE values were calculated as 3,32 %

Years	Actual Value (TL)	GM (1,1) Prediction Value (TL)	GM (1,1) RPE	EXGM (1,1) Prediction Value (TL)	EXGM (1,1) RPE
2011	65807	65807	0	65807	0
2012	78046	81093	3,90	78707	0,85
2013	96706	93697	3,11	93415	3,40
2014	108811	108260	0,51	108698	0,10
2015	122496	125087	2,12	125705	2,62
2016	143039	144528	1,04	145089	1,43
2017	157120	166991	6,28	167359	6,52
2018	206120	192946	6,39	193008	6,36
2019	235473	222935	5,32	222575	5,48
2020	246419	257584	4,53	256666	4,16
		MAPE	3,32 %	MAPE	%3,09

Table 1: The actual values of Turkey’s education expenditures, the estimation values and error values obtained by the GM(1.1) and EXGM(1.1) models

and 3,09% for GM (1,1) and EXGM (1,1) model, respectively. The graphs of the estimation results are given in Figure 1 and Figure 2. Error rates are low for both models. The low values of the error rates indicate that the prediction accuracy of the models is high. As can be seen from Figure 1 and Figure 2, there is a high agreement between actual and estimated values for the education expenditures estimated by the GM (1,1) and EXGM (1,1) models between the years of 2010-2020. It is observed that education expenditures increased exponentially between the years of 2021-2030.

Figure 3 and 4 show the correlation between the actual and estimated values of education expenditures between 2011 and 2020 for GM (1,1) and EXGM (1,1) models. As can be seen from Figure 3 and 4, there is a good correlation between the actual and predicted values of education expenditures between the years of 2011 and 2020. R² value which shows the correlation between the actual and the predicted values were determined as 0,9974 and 0,9985 for GM (1,1) and EXGM (1,1), respectively. The precision of the estimation is demonstrated with the calculated error and R² values. Due to the R² value is higher for the EXGM (1,1) model, it can be said that the prediction accuracy of the EXGM (1,1) model is higher

Years	GM (1,1) Predictions (TL)	EXGM (1,1) Predictions (TL)
2021	297619	295976
2022	343876	341306
2023	397323	393578
2024	459076	453856
2025	530428	523366
2026	612869	603522
2027	708124	695953
2028	818184	802541
2029	945350	925453
2030	1092280	1067189

Table 2: The prediction values of Turkey's education expenditures from 2021 to 2030 by the GM(1.1) and EXGM(1.1) models

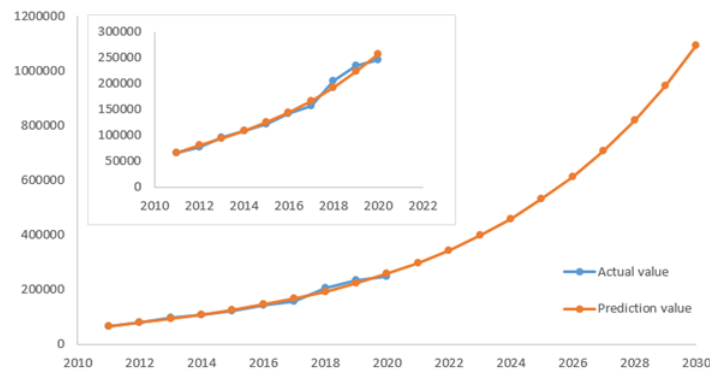


Fig. 1: Estimated values of Turkey's education expenditures between the years 2011-2030 obtained by the GM (1, 1) model.

than the GM (1,1) model. Nowadays, education expenditure is one of the indicators of country's development. There is a parallelism between education level and education expenditures. The increase in education level leads to economic growth, increasing of personal income level and income distribution. For this reason, education expenditures are increasing in both developed and developing countries. In our country, education expenditures are the sum of the expenditures made by the Ministry of National Education, Higher Education Institution, universities and educational institutions in the private sector.

In this study, education expenditures of our country were estimated using the GM (1, 1) and EXGM (1, 1) models for the years of 2021-2030. Also, precisions of the methods were compared. According to error and correlation results, the prediction accuracy of the EXGM (1,1) model was found to be higher than the GM (1,1) model.

When Strategic Plan of the Ministry of National Education is examined, it can be seen that innovative practices that will increase the quality of education and schools are supported by the ministry. Also, some investments are made in technology, the structure of vocational and technical education schools is strengthened, and convenient places are provided for the training of gifted students by the Ministry of National Education. It is emphasized that these investments should be increased in the Strategic Plan of the Ministry of National Education. In this case, it is understood that the resources allocated to education services will increase day by day. It can be predicted that the rate of increase in education expenditures is low until 2020, but the rate of increase will be higher after than 2020. It can be said that the

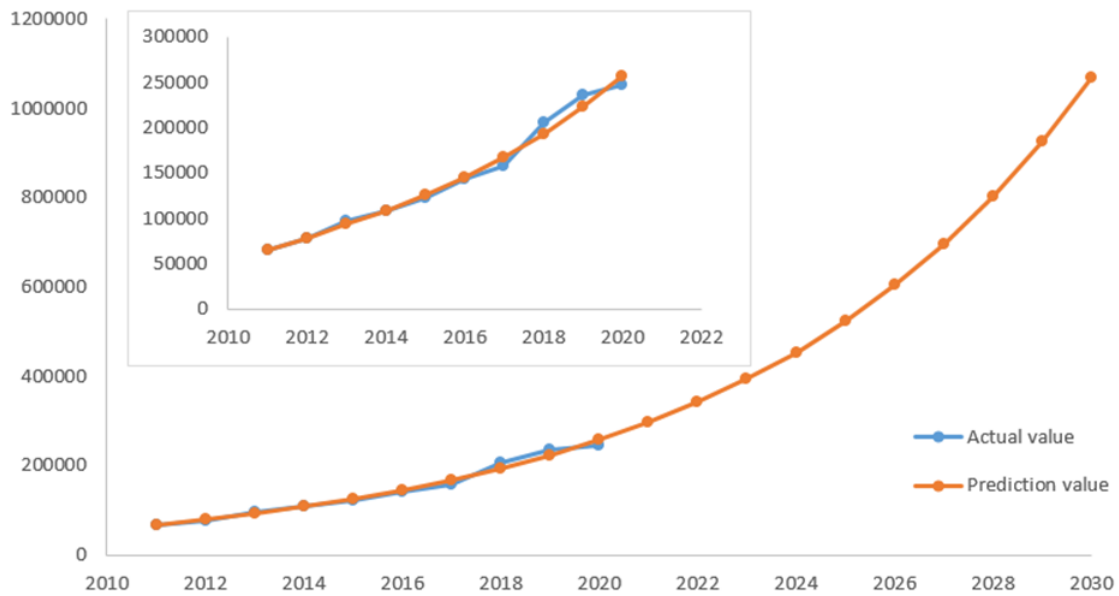


Fig. 2: Estimated values of Turkey’s education expenditures between the years 2011-2030 obtained by the EXGM (1,1) model.

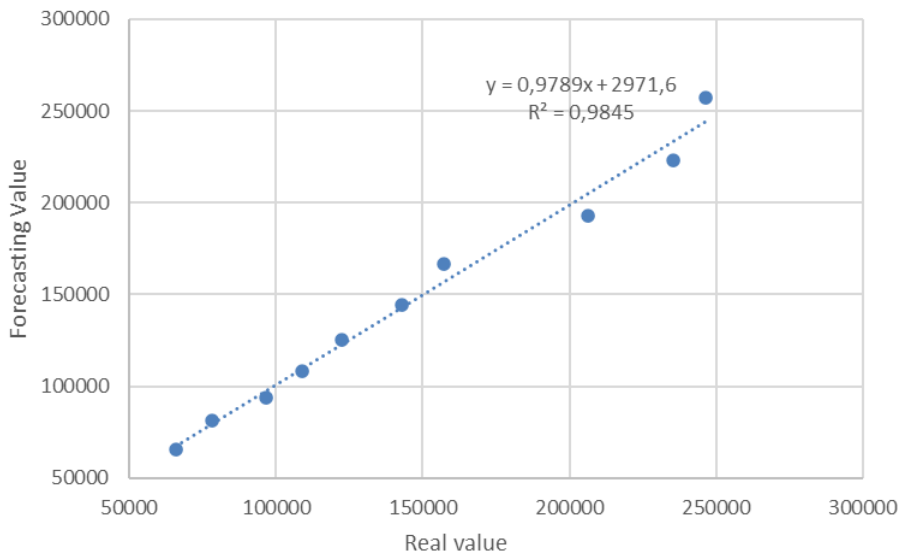


Fig. 3: Correlation between the estimated and actual values of education expenditures for 2011-2020 for GM (1,1) model.

exponentially increasing estimation values for the next 10 years obtained from this study are in line with these explanations.

Competing interests

The authors declare that they have no competing interests.

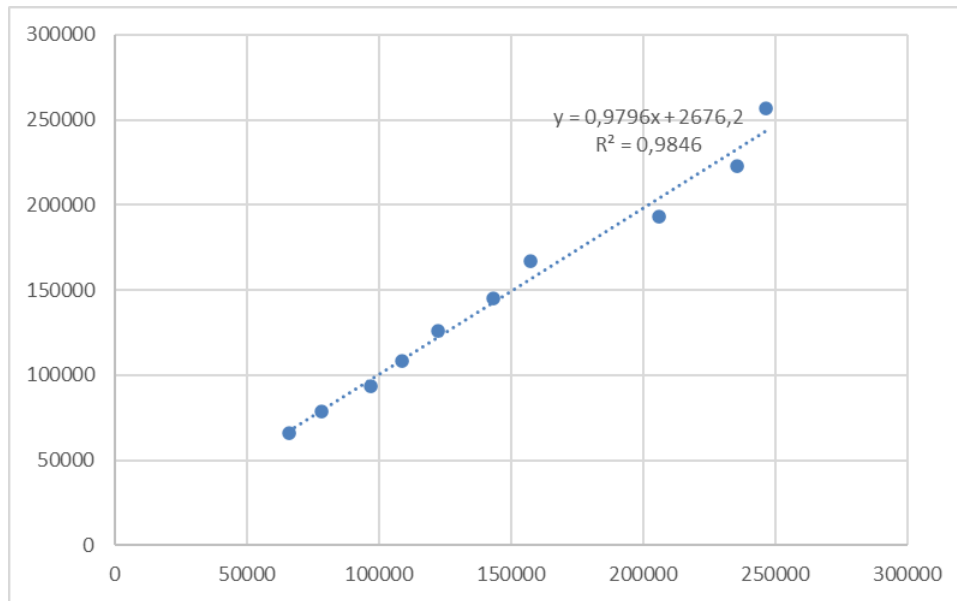


Fig. 4: Correlation between the estimated and actual values of education expenditures for 2011-2020 for EXGM (1, 1) model.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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