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On Feng Qi-type integral inequalities for local fractional integrals

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Abstract: In this paper, we establish the generalized Qi-type inequality involving local fractional integrals on fractal sets $\mathbb{R}^{\alpha}$ $(0<\alpha<1)$ of real line numbers. Some applications for special means of fractal sets $\mathbb{R}^{\alpha}$ are also given. The results presented here would provide extensions of those given in earlier works.

Keywords: Local fractional integrals, Generalized Qi inequality, Integral inequalities.

1 Introduction

In recent years, there have been many useful developments in the theory of inequalities, which is one of the important areas of mathematics. One of these fields is fractional integral inequalities. The reason why fractional integral inequalities are so important is that they are new and open to development. Integral inequalities have been frequently employed in the theory of applied sciences, differential equations, and functional analysis. In the last two decades, they have been the focus of attention in [3]-[7]. Recently, especially Qi inequality, one of the integral inequalities, has been studied by many authors. Recall the famous integral inequality of Feng Qi type:

Theorem 1. *(Proposition 1.1, [7]). Let $f(x)$ be differentiable on (a,b) and $f(a)=0$. If $f'(x)\geq 1$ for $x\in(a,b)$, then*

$$\left(\int_a^b [f(t)]^3\,dt\right)\geq\left(\int_a^b f(t)\,dt\right)^2. \tag{1}$$

If $0\leq f'(x)\leq 1$, then the inequality (1) reverses.

Theorem 2. *(Proposition 1.3, [7]). Let n be a positive integer. Suppose $f(x)$ has continuous derivative of the $n-$th order on the interval $[a,b]$ such that $f^{(i)}(a)\geq 0$, where $0\leq i\leq n-1$, $f^{(n)}(x)\geq n!$, then*

$$\left(\int_a^b [f(t)]^{n+2}\,dt\right)\geq\left(\int_a^b f(t)\,dt\right)^{n+1}. \tag{2}$$

In [6], Ngô et al. gave the following inequality which is one of the open problem's solution. Let $f(x)$ be differentiable on (a,b) and $f(a)=0$. If $f'(x)\geq 1$ for $x\in(a,b)$, then where $f\in C^n(a,b)$, $f^{(i)}\geq$, $0\leq i\leq n$, $f^{(n)}\geq n!$, $n\in\mathbb{Z}$.

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Theorem 3. *Let $f \in C[0,1]$ and $f(x) \geq 0$ for every $x \in [0,1]$. If*

$$\int_x^1 f(t)\,dt \geq \frac{1-x^2}{2} \tag{3}$$

then, for every $n \in \mathbb{N}$

$$\int_0^1 f^{n+1}(t)\,dt \geq \int_0^1 t^n f(t)\,dt \tag{4}$$

holds.

2 Preliminaries

Definition 1. *[11] A non-differentiable function $f : R \to R^{\alpha}$, $x \to f(x)$ is called to be local fractional continuous at x_0, if for any $\varepsilon > 0$, there exists $\delta > 0$, such that*

$$|f(x) - f(x_0)| < \varepsilon^{\alpha}$$

holds for $|x - x_0| < \delta$, where $\varepsilon, \delta \in R$. If $f(x)$ is local continuous on the interval (a,b), we denote $f(x) \in C_{\alpha}(a,b)$.

Definition 2. *[11]The local fractional derivative of $f(x)$ of order α at $x = x_0$ is defined by*

$$f^{(\alpha)}(x_0) = \left.\frac{d^{\alpha} f(x)}{dx^{\alpha}}\right|_{x=x_0} = \lim_{x \to x_0} \frac{\Delta^{\alpha}(f(x) - f(x_0))}{(x - x_0)^{\alpha}},$$

where $\Delta^{\alpha}(f(x) - f(x_0)) \cong \Gamma(\alpha+1)(f(x) - f(x_0))$.

If there exists $f^{(k+1)\alpha}(x) = \overbrace{D_x^{\alpha} ... D_x^{\alpha}}^{k+1 \text{ times}} f(x)$ for any $x \in I \subseteq R$, then we denoted $f \in D_{(k+1)\alpha}(I)$, where $k = 0, 1, 2, ...$

Definition 3. *[11] Let $f(x) \in C_{\alpha}[a,b]$. Then the local fractional integral is defined by,*

$${}_aI_b^{\alpha} f(x) = \frac{1}{\Gamma(\alpha+1)} \int_a^b f(t)(dt)^{\alpha} = \frac{1}{\Gamma(\alpha+1)} \lim_{\Delta t \to 0} \sum_{j=0}^{N-1} f(t_j)(\Delta t_j)^{\alpha},$$

with $\Delta t_j = t_{j+1} - t_j$ and $\Delta t = \max\{\Delta t_1, \Delta t_2, ..., \Delta t_{N-1}\}$, where $[t_j, t_{j+1}]$, $j = 0, ..., N-1$ and $a = t_0 < t_1 < ... < t_{N-1} < t_N = b$ is partition of interval $[a,b]$. Here, it follows that ${}_aI_b^{\alpha} f(x) = 0$ if $a = b$ and ${}_aI_b^{\alpha} f(x) = -{}_bI_a^{\alpha} f(x)$ if $a < b$. If for any $x \in [a,b]$, there exists ${}_aI_x^{\alpha} f(x)$, then we denoted by $f(x) \in I_x^{\alpha}[a,b]$.

Lemma 1. *[11].*

(1) *(Local fractional integration is anti-differentiation) Suppose that $f(x) = g^{(\alpha)}(x) \in C_{\alpha}[a,b]$, then we have*

$${}_aI_b^{\alpha} f(x) = g(b) - g(a).$$

(2) *(Local fractional integration by parts) Suppose that $f(x), g(x) \in D_{\alpha}[a,b]$ and $f^{(\alpha)}(x)$, $g^{(\alpha)}(x) \in C_{\alpha}[a,b]$, then we have*

$${}_aI_b^{\alpha} f(x) g^{(\alpha)}(x) = f(x)g(x)|_a^b - {}_aI_b^{\alpha} f^{(\alpha)}(x) g(x).$$

Lemma 2. *[11].*

$$\frac{d^{\alpha}x^{k\alpha}}{dx^{\alpha}}=\frac{\Gamma(1+k\alpha)}{\Gamma(1+(k-1)\alpha)}x^{(k-1)\alpha};$$
$$\frac{1}{\Gamma(\alpha+1)}\int_a^b x^{k\alpha}(dx)^{\alpha}=\frac{\Gamma(1+k\alpha)}{\Gamma(1+(k+1)\alpha)}\left(b^{(k+1)\alpha}-a^{(k+1)\alpha}\right),k\in R.$$

The interested reader is referred to [8]-[15] for local fractional theory.

The aim of the paper is to establish some generalized Qi-inequality involving local fractional integrals.

3 Main results

We start the following important inequality for local fractional integrals:

Theorem 4. *Let $f(x)\in C_{\alpha}[a,b]$ and $f(a)=0$. If $0^{\alpha}\leq\frac{[f'(x)]^{\alpha}}{\Gamma(\alpha+1)}\leq 1^{\alpha}$, then*

$$\frac{1}{\Gamma(\alpha+1)}\int_a^t [f(x)]^{3\alpha}(dx)^{\alpha}\leq\left(\int_a^t f(x)dx\right)^{2\alpha}. \tag{5}$$

Proof. Let

$$F(t)=\left(\int_a^t f(x)dx\right)^{2\alpha}-\frac{1}{\Gamma(\alpha+1)}\int_a^t [f(x)]^{3\alpha}(dx)^{\alpha}.$$

Simple computation yields

$$F^{(\alpha)}(t)=\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)}\left(\int_a^t f(x)dx\right)^{\alpha}[f(t)]^{\alpha}-[f(t)]^{3\alpha}=\left[\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)}\left(\int_a^t f(x)dx\right)^{\alpha}-[f(t)]^{2\alpha}\right][f(t)]^{\alpha}=G(t)[f(t)]^{\alpha}.$$
$$G^{(\alpha)}(t)=\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)}\Gamma(1+\alpha)[f(t)]^{\alpha}-\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)}[f(t)]^{\alpha}\left[f'(t)\right]^{\alpha}=\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)}\left[\Gamma(1+\alpha)-\left[f'(t)\right]^{\alpha}\right][f(t)]^{\alpha}.$$

Since $[f'(t)]^{\alpha}\geq 0$ and $f(a)=0$, thus $f(t)$ is increasing and $f(t)\geq 0$. When $0^{\alpha}\leq\frac{[f'(x)]^{\alpha}}{\Gamma(\alpha+1)}\leq 1^{\alpha}$, we have $G^{(\alpha)}(t)\geq 0$, $G(t)$ increases and $G(t)\geq 0$ because of $G(a)=0$, hence $F^{(\alpha)}(t)=G(t)[f(t)]^{\alpha}$, $F^{(\alpha)}(t)$ is increasing. Since $F(a)=0$, we have $F(t)>0$, and $F(b)>0$. Therefore, the inequality (5) holds.

Remark. If we choose $\alpha=1$ in (5), then we have Proposition 1.1 in [7].

Theorem 5. *Let $f(x)\in C_{\alpha}[a,b]$, $f^{(i)}(a)\geq 0$ and $f^{(n)}(x)\geq\Gamma(1+n\alpha)$, where $0^{\alpha}\leq\frac{[f'(x)]^{\alpha}}{\Gamma(\alpha+1)}\leq 1^{\alpha}$, then*

$$\frac{1}{\Gamma(\alpha+1)}\int_a^t [f(x)]^{(n+2)\alpha}(dx)^{\alpha}\geq\left(\int_a^t f(x)dx\right)^{(n+1)\alpha}. \tag{6}$$

Proof. Let

$$H(t)=\frac{1}{\Gamma(\alpha+1)}\int_a^t [f(x)]^{(n+2)\alpha}(dx)^{\alpha}-\left(\int_a^t f(x)dx\right)^{(n+1)\alpha}.$$

Simple computation yields

$$
\begin{aligned}
H^{(\alpha)}(t) &= [f(t)]^{(n+2)\alpha} - \frac{\Gamma(1+(n+1)\alpha)}{\Gamma(1+n\alpha)}\left(\int_a^t f(x)dx\right)^{n\alpha}[f(t)]^{\alpha} \\
&= \left[[f(t)]^{(n+1)\alpha} - \frac{\Gamma(1+(n+1)\alpha)}{\Gamma(1+n\alpha)}\left(\int_a^t f(x)dx\right)^{n\alpha}\right][f(t)]^{\alpha} \\
&= h_1(t)[f(t)]^{\alpha}.
\end{aligned}
$$

Direct calculation produces

$$
\begin{aligned}
h_1^{(\alpha)}(t) &= \frac{\Gamma(1+(n+1)\alpha)}{\Gamma(1+n\alpha)}[f(t)]^{n\alpha}\left[f'(t)\right]^{\alpha} - \frac{\Gamma(1+(n+1)\alpha)}{\Gamma(1+(n-1)\alpha)}\left(\int_a^t f(x)dx\right)^{(n-1)\alpha}[f(t)]^{\alpha} \\
&= \frac{\Gamma(1+(n+1)\alpha)}{\Gamma(1+n\alpha)}\left[[f(t)]^{(n-1)\alpha}\left[f'(t)\right]^{\alpha} - \frac{\Gamma(1+n\alpha)}{\Gamma(1+(n-1)\alpha)}\left(\int_a^t f(x)dx\right)^{(n-1)\alpha}\right][f(t)]^{\alpha} \\
&= \frac{\Gamma(1+(n+1)\alpha)}{\Gamma(1+n\alpha)}h_2(t)[f(t)]^{\alpha},
\end{aligned}
$$

$$
\begin{aligned}
h_2^{(\alpha)}(t) &= \frac{\Gamma(1+(n-1)\alpha)}{\Gamma(1+(n-2)\alpha)}[f(t)]^{(n-2)\alpha}\left[f'(t)\right]^{2\alpha} + \Gamma(1+\alpha)[f(t)]^{(n-1)\alpha}\left[f''(t)\right]^{\alpha} \\
&\quad - \frac{\Gamma(1+n\alpha)}{\Gamma(1+(n-1)\alpha)}\frac{\Gamma(1+(n-1)\alpha)}{\Gamma(1+(n-2)\alpha)}\left(\int_a^t f(x)dx\right)^{(n-2)\alpha}[f(t)]^{\alpha} \\
&= h_3(t)[f(t)]^{\alpha},
\end{aligned}
$$

and

$$
\begin{aligned}
h_3^{(\alpha)}(t) &= \frac{\Gamma(1+(n-1)\alpha)}{\Gamma(1+(n-2)\alpha)}\frac{\Gamma(1+(n-3)\alpha)}{\Gamma(1+(n-4)\alpha)}[f(t)]^{(n-4)\alpha}\left[f'(t)\right]^{3\alpha} \\
&\quad + \frac{\Gamma(1+(n-1)\alpha)}{\Gamma(1+(n-2)\alpha)}\frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)}[f(t)]^{(n-3)\alpha}\left[f'(t)\right]^{\alpha}\left[f''(t)\right]^{\alpha} \\
&\quad + \Gamma(1+\alpha)\frac{\Gamma(1+(n-2)\alpha)}{\Gamma(1+(n-3)\alpha)}[f(t)]^{(n-3)\alpha}\left[f'(t)\right]^{\alpha}\left[f''(t)\right]^{\alpha} \\
&\quad + \Gamma^2(1+\alpha)[f(t)]^{(n-2)\alpha}\left[f'''(t)\right]^{\alpha} \\
&\quad - \frac{\Gamma(1+n\alpha)}{\Gamma(1+(n-3)\alpha)}\left(\int_a^t f(x)dx\right)^{(n-3)\alpha}[f(t)]^{\alpha}
\end{aligned}
$$

$$= \left\{ \Gamma^2 (1+\alpha) [f(t)]^{(n-3)\alpha} \left[f'''(t)\right]^{\alpha} + \frac{\Gamma(1+(n-1)\alpha)}{\Gamma(1+(n-2)\alpha)} \frac{\Gamma(1+(n-3)\alpha)}{\Gamma(1+(n-4)\alpha)} [f(t)]^{(n-5)\alpha} \left[f'(t)\right]^{3\alpha} \right.$$

$$+ \frac{\Gamma(1+(n-1)\alpha)}{\Gamma(1+(n-2)\alpha)} \frac{\Gamma(1+2\alpha)}{\Gamma(1+\alpha)} [f(t)]^{(n-4)\alpha} \left[f'(t)\right]^{\alpha} \left[f''(t)\right]^{\alpha}$$

$$+ \Gamma(1+\alpha) \frac{\Gamma(1+(n-2)\alpha)}{\Gamma(1+(n-3)\alpha)} [f(t)]^{(n-4)\alpha} \left[f'(t)\right]^{\alpha} \left[f''(t)\right]^{\alpha}$$

$$\left. - \frac{\Gamma(1+n\alpha)}{\Gamma(1+(n-3)\alpha)} \left(\int_a^t f(x)dx \right)^{(n-3)\alpha} \right\} [f(t)]^{\alpha}$$

$$= h_4(t) [f(t)]^{\alpha}.$$

By induction, we obtain

$$h_i^{(\alpha)}(t) = \left\{ [\Gamma(1+\alpha)]^{(i-1)} [f(t)]^{(n-i)\alpha} \left[f^{(i)}(t)\right]^{\alpha} + P_i(t) \right.$$

$$\left. - \frac{\Gamma(1+n\alpha)}{\Gamma(1+(n-i)\alpha)} \left(\int_a^t f(x)dx \right)^{(n-i)\alpha} \right\} [f(t)]^{\alpha}$$

$$= h_{i+1}(t) [f(t)]^{\alpha}.$$

where $2 < i < n$ and

$$P_2(t) = \frac{\Gamma(1+(n-1)\alpha)}{\Gamma(1+(n-2)\alpha)} [f(t)]^{(n-3)\alpha} \left[f'(t)\right]^{2\alpha},$$

$$P_{i+1}(t) [f(t)]^{\alpha} = P_i^{(\alpha)}(t) + \Gamma(1+\alpha) \frac{\Gamma(1+(n-i)\alpha)}{\Gamma(1+(n-(i+1))\alpha)} [f(t)]^{(n-(i+1))\alpha} \left[f^{(i)}(t)\right]^{\alpha} \left[f'(t)\right]^{\alpha}.$$

From $f^{(n)}(x) \geq \Gamma(1+n\alpha)$ and $f^{(i)}(a) \geq 0$ for $0 \leq i \leq n-1$, it follows that $f^{(i)}(t) \geq 0$ and are increasing for $0 \leq i \leq n-1$.

Therefore, we obtain $p_k^{(\alpha)}(t) \geq 0$ and $p_{k+1}(t) \geq 0$, then $p_{k-1}^{(\alpha)}(t) \geq 0$ and $p_k(t) \geq 0$ are increasing for $2 \leq k \leq n$. Straightforward computation yields

$$h_{n+1}(t) = [\Gamma(1+\alpha)]^{(n-1)} \left[f^{(n)}(t)\right]^{\alpha} + p_i(t) - \Gamma(1+n\alpha).$$

Considering $f^{(n)}(x) \geq \Gamma(1+n\alpha)$, we get $h_{n+1}(t) \geq 0$, and $h_n^{(\alpha)}(t) \geq 0$, then $h_n(t)$ increases.

By our definitions of $h_i(t)$, we have, for $1 \leq i \leq n-1$,

$$h_{n+1}(a) = [f(a)]^{(n-i)\alpha} \left[f^{(i)}(a)\right]^{\alpha} + p_i(a) \geq 0.$$

Therefore, using induction on i, we obtain $h_i^{(\alpha)}(t) \geq 0$, $h_i(t) \geq 0$, and $h_i(t)$ are increasing for $1 \leq i \leq n$. Then $H_i^{(\alpha)}(t) \geq 0$ and increases, and $H_i(t) \geq 0$. The inequality (6) follows from $H(b) \geq 0$. Thus, Theorem 5 is proved.

Remark. If we choose $\alpha = 1$ in (6), then we have Proposition 1.3 in [7].

Theorem 6. *Let* $f(x) \in C_\alpha[0,1]$ *satisfying*

$$\int_t^1 f(x)(dx)^\alpha \geq \frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}\left(1^{2\alpha}-t^{2\alpha}\right),\ \forall t \in [0,1], \tag{7}$$

then

$$\frac{1}{\Gamma(1+\alpha)}\int_0^1 t^\alpha f(t)(dt)^\alpha \geq \left[\frac{1}{\Gamma(1+2\alpha)}-\frac{\Gamma(1+\alpha)}{\Gamma(1+3\alpha)}\right]. \tag{8}$$

Proof. Let

$$\int_0^1 \left(\int_t^1 f(x)(dx)^\alpha\right)(dt)^\alpha.$$

By using our assumption we have

$$\begin{aligned}\int_0^1 \left(\int_t^1 f(x)(dx)^\alpha\right)(dt)^\alpha &\geq \frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}\int_0^1 \left(1^{2\alpha}-t^{2\alpha}\right)(dt)^\alpha \\ &= \frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}\left[\frac{1}{\Gamma(1+\alpha)}-\frac{\Gamma(1+2\alpha)}{\Gamma(1+3\alpha)}\right] \\ &= \left[\frac{1}{\Gamma(1+2\alpha)}-\frac{\Gamma(1+\alpha)}{\Gamma(1+3\alpha)}\right].\end{aligned}$$

On the other hand, integrating by parts, we also get

$$\begin{aligned}\int_0^1 \left(\int_t^1 f(x)(dx)^\alpha\right)(dt)^\alpha &= \frac{t^\alpha}{\Gamma(1+\alpha)}\left(\int_t^1 f(x)(dx)^\alpha\right)\Bigg|_0^1 + \frac{1}{\Gamma(1+\alpha)}\int_0^1 t^\alpha f(t)(dt)^\alpha \\ &= \frac{1}{\Gamma(1+\alpha)}\int_0^1 t^\alpha f(t)(dt)^\alpha\end{aligned}$$

Thus,

$$\frac{1}{\Gamma(1+\alpha)}\int_0^1 t^\alpha f(t)(dt)^\alpha \geq \left[\frac{1}{\Gamma(1+2\alpha)}-\frac{\Gamma(1+\alpha)}{\Gamma(1+3\alpha)}\right].$$

The proof is completed.

Theorem 7. *If (7) holds then we have,*

$$\int_0^1 t^{(n+1)\alpha} f(t)(dt)^\alpha \geq \frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}\left[1-\frac{\Gamma(1+(n+1)\alpha)\Gamma(1+(n+2)\alpha)}{\Gamma(1+n\alpha)\Gamma(1+(n+3)\alpha)}\right]. \tag{9}$$

Proof. Let

$$\int_0^1 t^{n\alpha}\left(\int_t^1 f(x)(dx)^\alpha\right)(dt)^\alpha.$$

Integrating by parts, we have

$$\begin{aligned}\int_0^1 t^{n\alpha}\left(\int_t^1 f(x)\,(dx)^{\alpha}\right)(dt)^{\alpha} &= t^{(n+1)\alpha}\frac{\Gamma(1+n\alpha)}{\Gamma(1+(n+1)\alpha)}\left(\int_t^1 f(x)\,(dx)^{\alpha}\right)\Bigg|_0^1 \\ &\quad+\frac{\Gamma(1+n\alpha)}{\Gamma(1+(n+1)\alpha)}\int_0^1 t^{(n+1)\alpha} f(t)\,(dt)^{\alpha} \\ &= \frac{\Gamma(1+n\alpha)}{\Gamma(1+(n+1)\alpha)}\int_0^1 t^{(n+1)\alpha} f(t)\,(dt)^{\alpha}\end{aligned}$$

On the other hand, by using our assumption we have

$$\begin{aligned}\int_0^1 t^{n\alpha}\left(\int_t^1 f(x)\,(dx)^{\alpha}\right)(dt)^{\alpha} &\geq \frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}\int_0^1 t^{n\alpha}\left(1^{2\alpha}-t^{2\alpha}\right)(dt)^{\alpha} \\ &= \frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}\left[\frac{\Gamma(1+n\alpha)}{\Gamma(1+(n+1)\alpha)}-\frac{\Gamma(1+(n+2)\alpha)}{\Gamma(1+(n+3)\alpha)}\right]\end{aligned}$$

Thus,

$$\int_0^1 t^{(n+1)\alpha} f(t)\,(dt)^{\alpha} \geq \frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)}\left[1-\frac{\Gamma(1+(n+1)\alpha)\Gamma(1+(n+2)\alpha)}{\Gamma(1+n\alpha)\Gamma(1+(n+3)\alpha)}\right].$$

The proof is completed.

Remark. If we choose $\alpha = 1$ in (9), then we have Lemma 1.3 in [6].

4 Conclusion

In the present paper, we establish the generalized Qi-type inequality involving local fractional integrals on fractal sets $\mathbb{R}^{\alpha}$ $(0<\alpha<1)$ of real line numbers. Some applications for special means of fractal sets $\mathbb{R}^{\alpha}$ are also given. Some special cases are also discussed.

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