

Comparison of Rough Sets and Local Rough Sets in Data Analysis

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Abstract: As it is well known the rough set is a beneficial method for rough data uncertainty analysis. However, this is a time-consuming task for many big data sets. So we utilized the concept of local rough sets in data analysis of children addicted to social media to handle big data efficiently and give some of the properties. With the results, we proved that local rough sets gave more concrete and clear information than rough sets in data analysis.

Keywords: Rough sets, Local rough sets, rough set classification.

1 Introduction

The rough set theory, improved by Pawlak [1], has a method of handling the uncertainty and incomprehensibility of imprecise information. In rough set theory, classification of objects is established on equivalence classes. This theory aims to characterize any given subset of a universal set by upper and lower approximation. Theory of rough sets have been used in solving various problems such as the definition of sets that cannot be determined with the help of existing information, and reasoning based on incomplete information [1].

In this theory, the subsets of the universal set formed by the objects in the rough set are taken and analyzed by localizing them according to the indistinguishability (equivalence) relations. In this way, it can be determined more clearly whether an element belongs to that set or not than the rough set.

The theory has been implemented for feature selection [2-4], pattern familiarization [5, 6], uncertainty reasoning [7], granular computing [8-10], data mining and information exploration [11-13]. Over the past years, it had a tremendous effect on uncertainty administration and uncertainty reasoning. Moreover, in recent years, it has been joined with some mathematical theories such as algebra and topology [14-35]. Local rough set theory is utilized in the same fields of study as rough set theory, such as artificial intelligence, medicine, machine learning, data mining, incomplete information reasoning and training data. In fact, it provides a higher success rate in revealing information.

In this paper we also define four classes of local rough sets. Then as an application, we examined the social media addiction of secondary school students in terms of rough set and local rough set, and we found with numerical data that the information obtained in the local rough set was more concrete and understandable.

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2 Material method

2.1 Rough sets

Let U be a non-empty finite set of objects. A subset R of the product set $U \times U$ corresponds to a relation on U . If R is an equivalence relation over U , then the (U, R) pair is called the approximation space [36].

Let U be a non-empty finite universal set of objects and R be an equivalence relation over U . For $x \in U$, the set $R(x) = [x]_R = \{ y \in U : xRy \}$ is defined as the equivalence class of x .

Let (U, R) be the approximation space and $\emptyset \neq X \subseteq U$. The sets

$$\underline{R}(X) = \{ x : [x]_R \subseteq X \},$$

$$\overline{R}(X) = \{ x : [x]_R \cap X \neq \emptyset \}$$

are called lower and upper approximation of the set X , respectively. Namely, the lower approximation of the set X consists of a combination of equivalence classes completely covered by the set X . The upper approximation of the set X consists of elements of equivalence classes whose intersection with X is non-empty [37].

The upper and lower approaches of the $X \subseteq U$ divide U into three regions: Positive $POS(X)$, negative $NEG(X)$, and boundary region $BndR(X)$ [1]. These are defined as follows

$$POS(X) = \underline{R}(X)$$

$$NEG(X) = U - \overline{R}(X)$$

$$BndR(X) = \overline{R}(X) - \underline{R}(X).$$

The set $Edg(X) = X - \underline{R}(X)$ is named the inner boundary region of the set X . The set $\overline{Edg}(X) = \overline{R}(X) - X$ is named the outer boundary region of the set X .

The boundary region of the set X consists of the elements of the inner and outer boundary region of the set X [1]. So, $BndR(X) = Edg(X) \cup \overline{Edg}(X)$.

If $BndR(X) = \overline{R}(X) - \underline{R}(X) \neq \emptyset$, then the set X is named rough set.

Let $X \neq \emptyset$ be a set and $|X|$ be the cardinality (number of elements) of the set X . Accuracy of approximation (measure of completeness) in the rough set X in the approximation space (U, R) is

$$\alpha_R(X) = \frac{|\underline{R}(X)|}{|\overline{R}(X)|} \quad (1)$$

If $\alpha_R(X) = 1$ then $\underline{R}(X) = \overline{R}(X)$. Therefore, the set X is named a crisp set according to the R relation. If $\alpha_R(X) < 1$, the set X is named a rough set according to the R relation [1].

Utilizing the accuracy of approximation, the uncertainty measure of the cluster can also be calculated. The uncertainty measure of the set is

$$\rho_R(X) = 1 - \alpha_R(X). \quad (2)$$

Let $X \neq \emptyset$ be a set and $|X|$ be the cardinality (number of elements) of the set X . An approximate membership function on a rough set X is defined

$$\mu_X^R(x) = \frac{|X \cap [x]_R|}{|[x]_R|} \tag{3}$$

The approximate membership function is provided as follows

$$\mu_X^R(x) = 0, \text{ if } X \cap [x]_R = \emptyset \tag{4}$$

$$0 < \mu_X^R(x) < 1, \text{ if } X \cap [x]_R \neq \emptyset \tag{5}$$

$$\mu_X^R(x) = 1, \text{ if } [x]_R \subseteq X. \tag{6}$$

Lower, upper approaches, and boundary regions of a rough set X are defined by approximate membership function as follows;

$$\underline{R}(X) = \{x \in U : \mu_X^R(x) = 1\}$$

$$\overline{R}(X) = \{x \in U : \mu_X^R(x) > 0\}$$

$$BndR(X) = \overline{R}(X) - \underline{R}(X) = \{x \in U : 0 < \mu_X^R(x) < 1\}.$$

The membership functions satisfy the following properties [38].

- (1) $\mu_X^R(x) = 1 \Leftrightarrow x \in \underline{R}(X)$
- (2) $\mu_X^R(x) = 0 \Leftrightarrow x \in U - \overline{R}(X)$
- (3) $0 < \mu_X^R(x) < 1 \Leftrightarrow x \in BndR(X)$
- (4) $\mu_{U-X}^R(x) = 1 - \mu_X^R(x), x \in U$
- (5) $\mu_{X \cup Y}^R(x) \geq \max\{\mu_X^R(x), \mu_Y^R(x)\}, x \in U$
- (6) $\mu_{X \cap Y}^R(x) \leq \min\{\mu_X^R(x), \mu_Y^R(x)\}, x \in U$

2.2 Classification of rough sets

We can classify rough sets into four categories [1].

- (1) If $\underline{R}(X) \neq \emptyset$ and $\overline{R}(X) \neq U$, X will be named roughly R -definable. If X is roughly R -definable, Namely, we can determine for some elements of U whether they belong to X or $-X$, using R .
- (2) If $\underline{R}(X) = \emptyset$ and $\overline{R}(X) \neq U$, X will be named internally R -indefinable. If X is internally R -indefinable, Namely, we can determine whether some elements of U belong to $-X$, but we can't determine for any element of U , whether it belongs to X or not, using R .
- (3) If $\underline{R}(X) \neq \emptyset$ and $\overline{R}(X) = U$, X will be named externally R -indefinable. If X is externally R -indefinable, Namely, we can determine for some elements of U whether they U belong to X , but we can't determine for any element of U whether it belongs to $-X$ or not, using R .
- (4) If $\underline{R}(X) = \emptyset$ and $\overline{R}(X) = U$, X will be named totally R -indefinable. If X is totally R -indefinable, we can't determine for any element of U whether it belongs to X or $-X$, using R .

3 The research findings and discussion

3.1 Local rough set

It is well as known that the rough set is a beneficial utensil for rough data uncertainty analysis. But this is a time-consuming task for many big data sets. we introduce a local rough set to handle big data efficiently and effectively.

The idea of local rough set, introduced by the third author, occurs in his student’s thesis [39]. Here, we explore this concept and give some of the properties.

Let U be a set of a finite number of objects such that for $i, U_i \subseteq U, U = \cup U_i$.

Definition 1. A family $\{(U_i, R_i) \mid i \in I, R_i \subseteq U_i \times U_i \text{ equivalence relation. is named a local approximation space (LAS) if for any indices } i \text{ and } j, U_i \cap U_j \text{ is covered by set } W \text{ such that } R_i|_W = R_j|_W. \text{ This family is denoted by } (U, R)_I.$

Lemma 1. Every approximation space (U, R) defines a local approximation space $(U, R)_I$

Proof. Let U be finite set, $U_i \subseteq U$ and $U = \cup U_i$. Since (U, R) is an approximation space, there is an equivalence relation $R \subseteq U \times U$. For $i, U_i \subseteq U, R_i = R \cap (U_i \times U_i)$ defines an equivalence relation on U_i . For i, j , let $W = U_i \cap U_j$, then it is clearly seen $R_i|_W = R_j|_W$. So we obtain a local approximation space.

This local approximation space $\{(U_i, R|_{U_i} = R_i) : i \in I, R_i \subseteq U_i \times U_i \text{ equivalence relation}\}$ is denoted by $(U, R)_{loc}$.

Let $(U, R)_I$ be a local approximation space. For $x \in U_i$, equivalence class of x according to the relation R_i on U_i is defined as follows

$$R_i(x) = [x]_i = \{y \in U : xR_i y\}.$$

For $\emptyset \neq X \subseteq U$ and the pair $(U_i, R_i) \in (U, R)_I$, then the local lower and local upper approximation of the set X are defined as follows, respectively

$$\underline{R}_I(X) = \{x \in U : [x]_i \subseteq X, \forall i \in I\}$$

$$\overline{R}_I(X) = \{x \in U : [x]_i \cap X \neq \emptyset, \forall i \in I\}.$$

Definition 2. Let $\underline{R}_I(X), \overline{R}_I(X)$ be local lower and local upper approximation of a set X . The set X is named local rough set if $\overline{R}_I(X) - \underline{R}_I(X) \neq \emptyset$.

The local lower and local upper approximations of a set X are shown in Figure 1. (a) and (b), respectively.

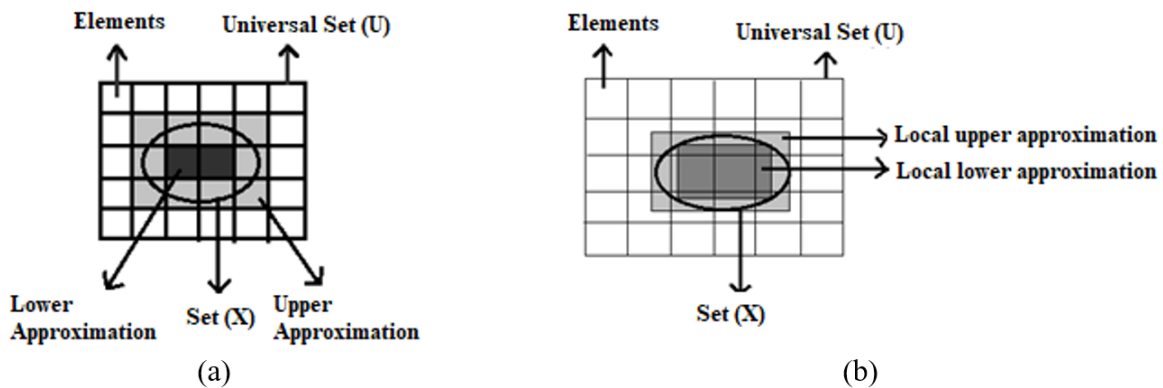


Fig. 1: (a) For rough set. (b) For local rough set.

Lemma 2. Let (U, R) be approximation space and $(U, R)_I$ be a local approximation space. For $X \subseteq U$, let $(\underline{R}(X), \overline{R}(X))$ and $(\underline{R}_I(X), \overline{R}_I(X))$ denote lower - upper approximation spaces and local lower - local upper approximation spaces, respectively. Then, the followings are satisfied.

- (i) $\underline{R}(X) \subseteq \underline{R}_l(X)$
- (ii) $\overline{R}_l(X) \subseteq \overline{R}(X)$.

Proof. Easily seen from the definitions.

Local lower and local upper approximations of a set $X \subseteq U$ divides U into three regions: Positive $POS_l(X)$, negative $NEG_l(X)$, and boundary region $BndR_l(X)$.

$$POS_l(X) = \underline{R}_l(X)$$

$$NEG_l(X) = U - \overline{R}_l(X)$$

$$BndR_l(X) = \overline{R}_l(X) - \underline{R}_l(X)$$

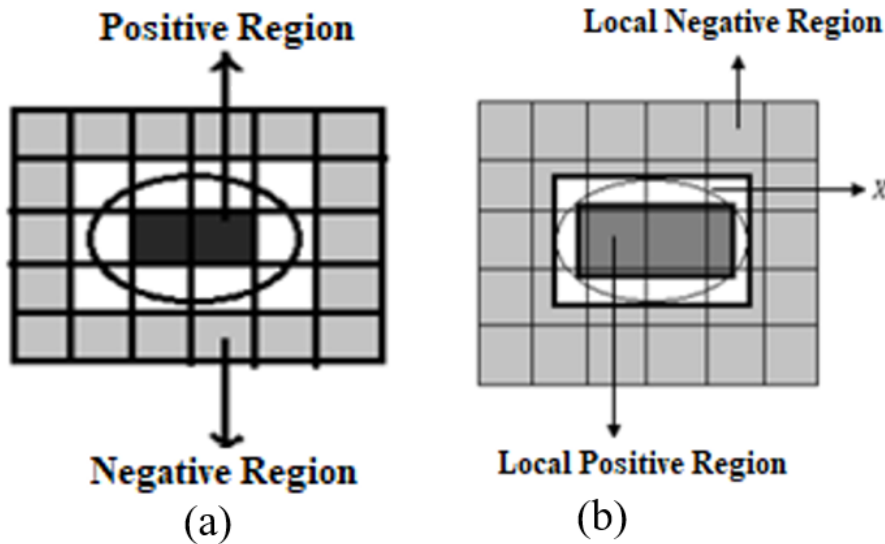


Fig. 2: (a) for rough set (b) for Local Rough Sets

Results: Let $X \subseteq U$ be a set. Let $POS(X)$, $NEG(X)$, $BndR(X)$ be the positive, the negative and the boundary region for approximation space (U, R) , respectively. Let $POS_l(X)$, $NEG_l(X)$, $BndR_l(X)$ be the local positive, the local negative and local boundary region for the local approximation space $(U, R)_l$, respectively. The following properties are provided.

- (i) $POS(X) \subseteq POS_l(X)$
- (ii) $NEG(X) \subseteq NEG_l(X)$
- (iii) $BndR_l(X) \subseteq BndR(X)$

Let the $(U, R)_l$ be the local approximation space and $X \subseteq U$. The set $\underline{Edg}_l(X) = X - \underline{R}_l(X)$ is named the local inner boundary region of the X . The set $\overline{Edg}_l(X) = \overline{R}_l(X) - X$ is named the local outer boundary region of the X .

The local boundary region of the set X consists of the elements of the local inner boundary region of the set X and the local outer boundary region of the set X . So, $BndR_l(X) = \underline{Edg}_l(X) \cup \overline{Edg}_l(X)$

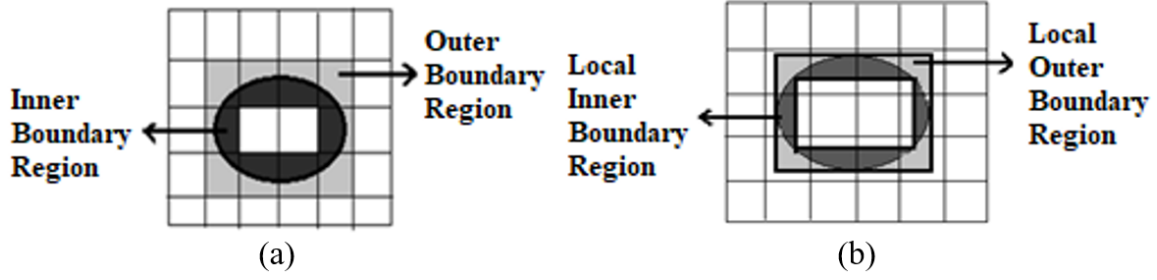


Fig. 3: (a) For rough sets. (b) For Local Rough Sets.

Theorem 1. Let $\emptyset \neq X, Y \subseteq U$. The local lower and local upper approximations of X, Y sets satisfy the following properties [39].

- (1) $\underline{R}_l(X) \subseteq X \subseteq \overline{R}_l(X)$
- (2) $\underline{R}_l(\emptyset) = \overline{R}_l(\emptyset) = \emptyset$
- (3) $\underline{R}_l(U) = \overline{R}_l(U) = U$
- (4) $\overline{R}_l(X \cup Y) = \overline{R}_l(X) \cup \overline{R}_l(Y)$
- (5) $\underline{R}_l(X \cap Y) = \underline{R}_l(X) \cap \underline{R}_l(Y)$
- (6) $X \subseteq Y$ ise $\underline{R}_l(X) \subseteq \underline{R}_l(Y)$
- (7) $X \subseteq Y$ ise $\overline{R}_l(X) \subseteq \overline{R}_l(Y)$
- (8) $\underline{R}_l(X \cup Y) \supseteq \underline{R}_l(X) \cup \underline{R}_l(Y)$
- (9) $\overline{R}_l(X \cap Y) \subseteq \overline{R}_l(X) \cap \overline{R}_l(Y)$
- (10) $\underline{R}_l(-X) = -\overline{R}_l(X)$
- (11) $\overline{R}_l(-X) = -\underline{R}_l(X)$
- (12) $\underline{R}_l \underline{R}_l(X) = \overline{R}_l \underline{R}_l(X) = \underline{R}_l(X)$
- (13) $\overline{R}_l \overline{R}_l(X) = \underline{R}_l \overline{R}_l(X) = \overline{R}_l(X)$.

Let $X \neq \emptyset$ and $(U, R)_l$ be the local approximation space. Accuracy of approximation in a local rough set X (measure of completeness) is defined as follows

$$\alpha_{R_l}(X) = \frac{|R_l(X)|}{|\overline{R}_l(X)|}. \tag{7}$$

Conclusion: The accuracy of the approximation in the local rough set may be greater than or equal to the accuracy of the approximation in the rough set.

$$\alpha_{R_l}(X) \geq \alpha_R(X). \tag{8}$$

Let $X \neq \emptyset$ be a set. Approximate membership function on a local rough set X in the local approximation space $(U, R)_l = \{(U_i, R_i) : \forall i \in I\}$ is represented by the ratio of the number of elements of the intersection of the set X and the equivalence classes $[x]_i$ to the number of elements of the equivalence class $[x]_i$

$$\mu_X^{R_l}(x) = \frac{|X \cap [x]_i|}{|[x]_i|}. \tag{9}$$

The membership function describes the degree to which element x belongs to the set X . For this approximate membership function, the following properties are true.

- If $X \cap [x]_i = \emptyset$ then $\mu_X^{R_l}(x) = 0$,
- If $X \cap [x]_i \neq \emptyset$ then $0 < \mu_X^{R_l}(x) < 1$,
- If $[x]_i \subset X$ then $\mu_X^{R_l}(x) = 1$.

Approaches and boundary regions of a set X are defined by the approximate membership function.

$$\underline{R}_l(X) = \{x \in U : \mu_X^{R_l}(x) = 1\}$$

$$\overline{R}_l(X) = \{x \in U : \mu_X^{R_l}(x) > 0\}$$

$$BndR_l(X) = \overline{R}_l(X) - \underline{R}_l(X) = \{x \in U : 0 < \mu_X^{R_l}(x) < 1\}$$

Membership functions satisfy the following properties.

- (1) $\mu_X^{R_l}(x) = 1 \Leftrightarrow x \in \underline{R}_l(X)$
- (2) $\mu_X^{R_l}(x) = 0 \Leftrightarrow x \in U - \overline{R}_l(X)$
- (3) $0 < \mu_X^{R_l}(x) < 1 \Leftrightarrow x \in BndR_l(X)$
- (4) $\mu_{U-X}^{R_l}(x) = 1 - \mu_X^{R_l}(x), x \in U$
- (5) $\mu_{X \cup Y}^{R_l}(x) \geq \max\{\mu_X^{R_l}(x), \mu_Y^{R_l}(x)\}, x \in U$
- (6) $\mu_{X \cap Y}^{R_l}(x) \leq \min\{\mu_X^{R_l}(x), \mu_Y^{R_l}(x)\}, x \in U.$

3.2 Classification of local rough sets

We can classify local rough sets into the following four categories.

- (1) Let $\underline{R}_l(X) \neq \emptyset$ and $\overline{R}_l(X) \neq U$, X will be named local roughly R_l -definable. If X is local roughly R_l -definable, Namely, we can determine for some elements of U whether they belong to X or -X, using R_l .

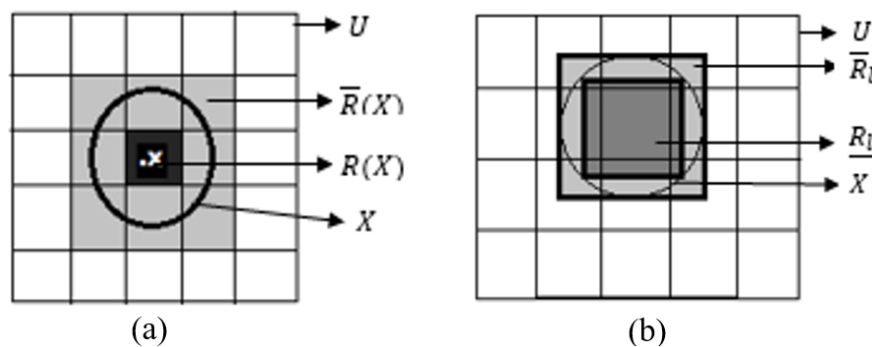


Fig. 4: (a) Rough R-definable set. (b) Local rough R_l -definable set.

- (2) Let $\underline{R}_l(X) = \emptyset$ and $\overline{R}_l(X) \neq U$, X will be named local internally R_l -indefinable. If X is local internally R_l -indefinable, Namely, we can determine whether some elements of U belong to -X, but we can't determine for any element of U, whether it belongs to X or not, using R_l .

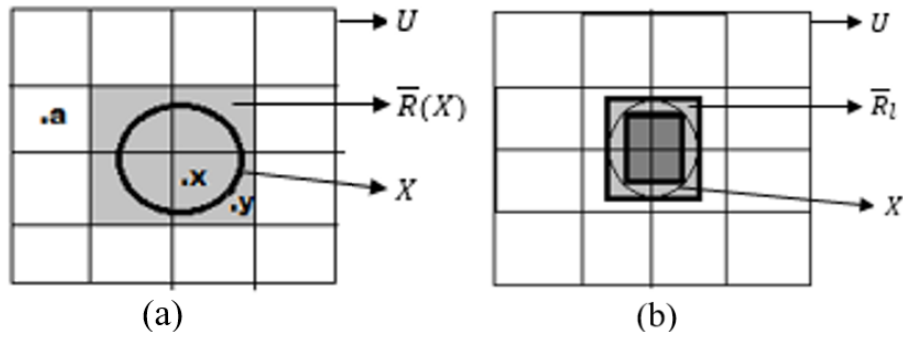


Fig. 5: (a) Internally R-indefinable set. (b) Local internally R_l -indefinable set.

(3) Let $R_l(X) \neq \emptyset$ and $\bar{R}_l(X) = U$, X will be named local externally R_l -indefinable. Namely, we can determine for some elements of U whether they U belong to X , but we can't determine for any element of U whether it belongs to $-X$ or not, using R_l .

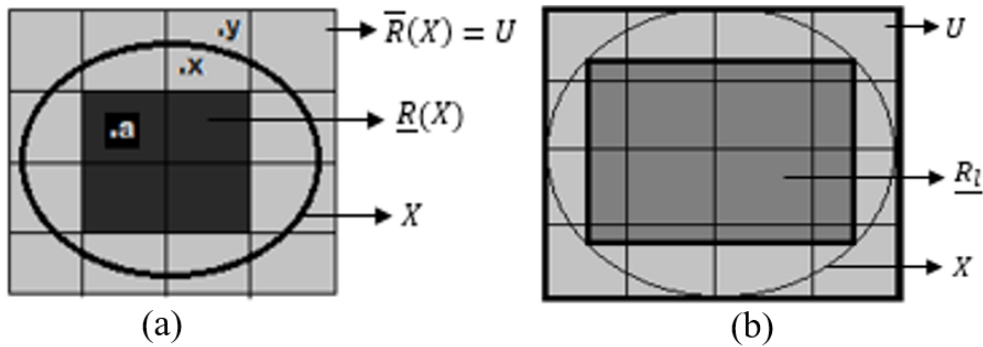


Fig. 6: (a) Externally R-indefinable set. (b) Local externally R_l -indefinable set.

(4) Let $R_l(X) = \emptyset$ and $\bar{R}_l(X) = U$, X will be named local totally R_l -indefinable. If X is local totally R_l -indefinable, we can't determine for any element of U whether it belongs to X or $-X$, using R_l .

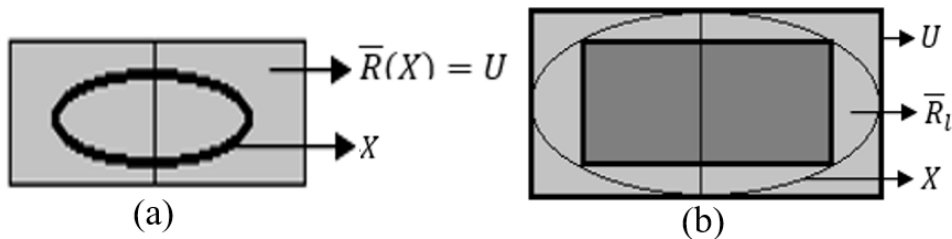


Fig. 7: (a) Totally R-indefinable set. (b) Local totally R_l -indefinable set.

3.3 Comparative example of rough and local rough sets

In this section, we join the Pawlak’s rough set with the local equivalence relation into the same rough set model, as an exemplary kind of global rough sets. We know from the general rough set definition that both concept approximation and attribute reduction are computationally time-consuming for many data sets. The objective of this study is to propose a new and general rough set framework computed locally, providing both efficiency and accuracy for encourage effective and efficient applications of rough sets.

Three different features adapted from the “social media attitude scale” questionnaire were applied face to face to a total of 10 randomly selected students from each of the 5th, 6th, 7th and 8th grade levels. The collected data was analyzed in terms of both rough sets and local rough sets and the two results were compared.

Questions	I strongly disagree (0)	I disagree (1)	I'm undecided (2)	I agree (3)	Absolutely I agree (4)
(S1) The reason I spend so much time on social media is that I don't get enough attention from friends or family.	(...)	(...)	(...)	(...)	(...)
(S2) I like that my posts are seen by everyone.	(...)	(...)	(...)	(...)	(...)
(S3) I think my reputation has increased thanks to social media.	(...)	(...)	(...)	(...)	(...)

Table 1: Social media attitude scale questions and answer options.

The information table created from the answers given by 10 secondary school students to the applied questions are given Table 2.

STUDENTS	S1	S2	S3
α	(3)	(4)	(3)
β	(1)	(3)	(3)
γ	(3)	(3)	(3)
δ	(0)	(3)	(3)
ϵ	(1)	(3)	(3)
ζ	(4)	(4)	(4)
η	(3)	(4)	(3)
θ	(1)	(4)	(4)
λ	(3)	(4)	(3)
μ	(3)	(3)	(3)

Table 2: Information Table.

Let’s interpret the data according to the rough set theory by looking at Table1.

Let $U = \{\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta, \theta, \lambda, \mu\}$ be the set of objects, let $A = \{S1, S2, S3\}$ be the set of features, let $V = \{0, 1, 2, 3, 4\}$ be the value sets and let $R(\alpha) = \{\beta : \beta, \text{ has the same dimensions as } \alpha\}$ be an equivalence relation over U . With the help of equivalence classes, the indistinguishability relations for the set of social media addicted students $X = \{\alpha, \zeta, \eta, \lambda\}$ are given in the Table 3 separately according to their subsets.

Subsets of Questions	Indistinguishability Relations
$U/\{S1, S2, S3\}$	$\{\{\alpha, \eta, \lambda\}, \{\gamma, \mu\}, \{\delta\}, \{\beta, \varepsilon\}, \{\zeta\}, \{\theta\}\}$
$U/\{S1\}$	$\{\{\delta\}, \{\beta, \varepsilon, \theta\}, \{\alpha, \gamma, \eta, \lambda, \mu\}, \{\zeta\}\}$
$U/\{S2\}$	$\{\{\beta, \gamma, \delta, \varepsilon, \mu\}, \{\alpha, \zeta, \eta, \theta, \lambda\}\}$
$U/\{S3\}$	$\{\{\alpha, \beta, \gamma, \delta, \varepsilon, \eta, \lambda, \mu\}, \{\zeta, \theta\}\}$
$U/\{S1, S2\}$	$\{\{\alpha, \xi_7, \lambda\}, \{\beta, \varepsilon\}, \{\gamma, \mu\}, \{\delta\}, \{\zeta\}, \{\theta\}\}$
$U/\{S1, S3\}$	$\{\{\alpha, \gamma, \eta, \lambda, \mu\}, \{\beta, \varepsilon\}, \{\delta\}, \{\zeta\}, \{\theta\}\}$
$U/\{S2, S3\}$	$\{\{\alpha, \eta, \lambda\}, \{\beta, \gamma, \delta, \varepsilon, \mu\}, \{\zeta, \theta\}\}$

Table 3: The indistinguishably relations.

The lower-upper approaches, boundary set and completeness measure of each subset of the questions are given in the Table 4.

Subsets of Questions	$\underline{R}(X)$	$\overline{R}(X)$	$BndR(X)$	$\alpha_R(X) = \frac{ R(X) }{ \overline{R}(X) }$
$\{S1, S2, S3\}$	$\{\alpha, \zeta, \eta, \lambda\}$	$\{\alpha, \zeta, \eta, \lambda\}$	\emptyset	1
$\{S1\}$	$\{\zeta\}$	$\alpha, \gamma, \zeta, \eta, \lambda, \mu$	$\alpha, \gamma, \eta, \lambda, \mu$	1/6
$\{S2\}$	\emptyset	$\{\alpha, \zeta, \eta, \theta, \lambda\}$	$\{\alpha, \zeta, \eta, \theta, \lambda\}$	0
$\{S3\}$	\emptyset	$\{\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta, \theta, \lambda, \mu\}$	$\{\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta, \theta, \lambda, \mu\}$	0
$\{S1, S2\}$	$\{\alpha, \zeta, \eta, \lambda\}$	$\{\alpha, \zeta, \eta, \lambda\}$	\emptyset	1
$\{S1, S3\}$	$\{\zeta\}$	$\{\alpha, \gamma, \zeta, \eta, \lambda, \mu\}$	$\{\alpha, \gamma, \eta, \lambda, \mu\}$	1/6
$\{S2, S3\}$	$\{\alpha, \eta, \lambda\}$	$\{\alpha, \zeta, \eta, \theta, \lambda\}$	$\{\zeta, \theta\}$	3/5

Table 4

Sometimes the table includes data that isn't required in the information system. This increases the workload in decision making and causes unnecessary time loss. The same result can be achieved without causing loss of information. The method named sorting or elimination is very useful. If the equivalence relation obtained by removing an element from the set of properties remains the same as the original relation, that property is dispensable. Otherwise, that feature is called indispensable. An efficient decision-making process is created with the chart, which is simplified by removing the dispensable feature or features from the information system.

$\{S1, S2\}$ subset obtained from the $A = \{ S1, S2, S3\}$ feature set of the social media addicted students in our sample was determined as an indispensable feature. It is concluded that students who are social media addicts can be identified by their answers to S1 and S2 characteristics.

Now, let's interpret the same example for Local Rough Set Theory. $U = \{\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta, \theta, \lambda, \mu\}$ is the set of objects, $A = \{S1, S2, S3\}$ is the set of features, $V = \{ 0, 1, 2, 3, 4 \}$ is the value sets and for the equivalence relation $R(\alpha) = \{ \beta : \beta, \text{ has the same dimensions as } \alpha \}$.

In the information system given earlier, U is the finite set of objects, $U_i \subseteq U$ subset, let $R_i \subseteq U_i \times U_i$ equivalence relation and $U = \cup U_i$.

Let $v = \{(U_i, R_i) : i \in I, U_i \subseteq U \text{ finite set of objects, } R_i \subseteq U_i \times U_i \text{ equivalence relation}\}$ be the family local approximation space, $U_1 = \{\alpha, \varepsilon, \zeta, \theta\}$ be a subset and let (U_1, R_1) be the pair selected from the family of local approximation spaces.

The indistinguishability relations formed by the R_1 local equivalence relation on the set $U_1 = \{\alpha, \varepsilon, \zeta, \theta\}$ are shown in the Table 5.

Subsets of Questions	Indistinguishability Relation
$U_1/\{S1, S2, S3\}$	$\{\{\alpha\}, \{\delta\}, \{\varepsilon\}, \{\zeta\}, \{\theta\}\}$
$U_1/\{S1\}$	$\{\{\alpha\}, \{\delta\}, \{\varepsilon, \theta\}, \{\zeta\}\}$
$U_1/\{S2\}$	$\{\{\delta, \varepsilon\}, \{\alpha, \zeta, \theta\}\}$
$U_1/\{S3\}$	$\{\{\alpha, \delta, \varepsilon\}, \{\zeta, \theta\}\}$
$U_1/\{S1, S2\}$	$\{\{\alpha\}, \{\delta\}, \{\varepsilon\}, \{\zeta\}, \{\theta\}\}$
$U_1/\{S1, S3\}$	$\{\{\alpha\}, \{\delta\}, \{\varepsilon\}, \{\zeta\}, \{\theta\}\}$
$U_1/\{S2, S3\}$	$\{\{\alpha\}, \{\delta, \varepsilon\}, \{\zeta, \theta\}\}$

Table 5

For $X = \{\alpha, \zeta, \eta, \lambda\}$ in the U_1 set, the lower-upper approximations of each subset of the questions, the boundary set and the completeness measure are shown in the Table 6.

Subsets of Questions	$\underline{R}_l(X)$	$\overline{R}_l(X)$	$BndR_l(X)$	$\alpha_{R_l}(X) = \frac{ \underline{R}_l(X) }{ \overline{R}_l(X) }$
$\{S1, S2, S3\}$	$\{\alpha, \zeta\}$	$\{\alpha, \zeta\}$	\emptyset	1
$\{S1\}$	$\{\alpha, \zeta\}$	$\{\alpha, \zeta\}$	\emptyset	1
$\{S2\}$	\emptyset	$\{\alpha, \zeta, \theta\}$	$\{\alpha, \zeta, \theta\}$	0
$\{S3\}$	\emptyset	$\{\alpha, \delta, \varepsilon, \zeta, \theta\}$	$\{\alpha, \delta, \varepsilon, \zeta, \theta\}$	0
$\{S1, S2\}$	$\{\alpha, \zeta\}$	$\{\alpha, \zeta\}$	\emptyset	1
$\{S1, S3\}$	$\{\alpha, \zeta\}$	$\{\alpha, \zeta\}$	\emptyset	1
$\{S2, S3\}$	$\{\alpha\}$	$\{\zeta, \theta\}$	$\{\zeta, \theta\}$	1/2

Table 6

In local rough set theory, the $\{S1\}$ subset obtained from the $A = \{S1, S2, S3\}$ feature set of the social media addicted students in the U_1 cluster has been determined as an indispensable feature. It is concluded that students who are addicted to social media can be identified by their answers to the $\{S1\}$ feature.

3.3.1 Conclusion

According to the “Dependency level of social media use” research conducted with middle school students, it is seen that the precision of the approach in the local rough set (α_{RI}) may be greater or equal to the precision of the approach in the rough set (α_R).

3.4 Conclusion

In this study, the basic concepts of rough set and local rough set theories are given and compared. The characteristics that affect the use levels of children addicted to social media use were determined by reducing the rough and local rough set approaches. According to the results, it has been determined that the local rough set theory provides more concrete and clear information than the rough set theory. As a result, the local rough set theory can provide more reliable information in the usage areas of rough sets.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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