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The soliton solutions of (2+1)-dimensional nonlinear two-coupled Maccari equation with complex structure via new Kudryashov scheme

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Abstract: In this study, we tackle with the (2+1)-dimensional complex nonlinear two-coupled Maccari equation which is a kind of Schrodinger equation and presented by Attilio Maccari in 1996. The Maccari equations has an importance to define the motion of isolated waves concentrated in a tiny region of space in a variety of fields, including hydrodynamics, plasma physics, quantum field theory, nonlinear optics, the sonic Langmuir solitons and so on. We utilized the popular method namely new Kudryashov scheme, we obtained the soliton solutions, graphical representation and made its physical interpretation.

Keywords: Two Coupled-Maccari system, new Kudryashov scheme, soliton solutions.

1 Introduction

In recent years, due to the importance of nonlinear partial differential equations (NLPDEs) in modeling many physical phenomena such as physics, chemistry, biology, genetics, plasma physics, optic, optical physics, ocean engineering, telecommunication, wave propagation, heat conduction, statistics, economy, health science, optoelectronics so on, many studies have been hold on the solution of nonlinear equations and systems of equations [1,2,3,4,5,6,7,8,9,10,11].

They have been developed various methods and applied to solve to NLPDEs by many researchers. The tanh-coth method [12], exp-function method [13], F-expansion method [14], variational iteration method [15], extended rational sin-cosine and sinh-cosh method [16], simplest equation method [17], trial equation method [18], sine-Gordon method [19] and Kudryashov method [20], etc.

In this paper we investigate the nonlinear complex two coupled Maccari's system [21,22,23,24] via recently presented the new Kudryashov method [25,26,27,28].

Consider the two coupled Maccari's system is given,

$$\begin{cases} iX_t + X_{xx} + RX = 0, \\ iY_t + Y_{xx} + RY = 0, \\ R_t + R_y + \left(|X+Y|^2\right)_x = 0, \end{cases} \quad , i = \sqrt{-1}\,. \tag{1}$$

eq. (1) and it is frequently used to describe the motion of the isolated waves localized in a very small part of space, in many fields such as hydrodynamic, plasma physics, nonlinear optics, quantum field theory, and also for the sonic Langmuir solitons.

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The rest of the article is organized as follows: Nonlinear ordinary differential equation (NODE) and the application of the new Kudryashov method to the two coupled Maccari's system is given in section 2. Numerical results and graphical representations are given in section 3 and conclusion is given in the last part.

2 Mathematical analysis and obtaining the NODE form of the eq.(1).

In order to investigate solutions of eq.(1), we define the following complex wave transformations;

$$X = X(x,y,t) = e^{i*\theta}M(x,y,t) \tag{2a}$$
$$Y = Y(x,y,t) = e^{i*\theta}N(x,y,t) \tag{2b}$$
$$R = R(x,y,t) \tag{2c}$$

$$\theta = \alpha x + \beta y + \Omega t \tag{3}$$

where $X(x,y,t), Y(x,y,t)$ are complex and $R(x,y,t)$ is a real scaler field. θ is the phase component, α, β, ω are real constants to be found. x, y are the spatial independent variables and t represents the temporal variable.

Inserting the eq.(2) and eq.(3) into eq.(1) produces,

$$i\left(M_t + 2\alpha M_x\right) + M_{xx} - \left(\alpha^2 + \Omega\right)M + MR = 0 \tag{4a}$$
$$i\left(N_t + 2\alpha N_x\right) + N_{xx} - \left(\alpha^2 + \Omega\right)N + NR = 0 \tag{4b}$$
$$R_t + R_y + \left(|M+N|^2\right)_x = 0, \;\; i = \sqrt{-1} \tag{4c}$$

Define the following the transformation and substitute into eq.(4) yields,

$$M = M(\xi) \;\;,\;\; N = N(\xi) \;\;,\;\; R = R(\xi) \tag{5}$$

$$\xi = x + y + \omega t, \qquad \omega \in \mathbb{R} \tag{6}$$

$$M'' + i(2\alpha + \omega)M' - (\alpha^2 + \Omega)M + MR = 0, \tag{7a}$$
$$N'' + i(2\alpha + \omega)N' - (\alpha^2 + \Omega)N + NR = 0, \tag{7b}$$
$$(\omega + 1)R' + 2(M+N)(M' + N') = 0. \tag{7c}$$

In. eq.(7) the prime represents the derivative with respect to ξ . Integrating eq.(7.c) with respect to ξ and taking the integration constant as zero yields,

$$R = -\frac{(M+N)^2}{\omega + 1} \tag{8}$$

Substituting eq.(8) into eqs.(7.a) and eq.(7.b) we obtain

$$M'' + i(2\alpha + \omega)M' - (\omega^2 + \Omega)M - M\frac{(M+N)^2}{\omega + 1} = 0, \tag{9a}$$

$$N'' + i(2\omega + \omega)N' - (\omega^2 + \Omega)N - N\frac{(M+N)^2}{\omega + 1} = 0, \;\; i = \sqrt{-1}. \tag{9b}$$

In order to solve the eqs.(9.a) and eq(9.b) let we write the simple equation,

$$N = kM, \tag{10}$$

where k is an arbitrary real value. Substituting eq. (10) into eqs.(8) and eq.(9) we get,

$$M'' + i(2\alpha + \omega)M' - (\alpha^2 + \Omega)M - \frac{(k+1)^2}{\omega + 1}M^3 = 0, \tag{11a}$$

$$N'' + i(2\alpha + \omega)N' - (\alpha^2 + \Omega)N - \frac{(k+1)^2}{k^2(\omega + 1)}N^3 = 0, \;\; i = \sqrt{-1}. \tag{11b}$$

Decomposing the eq.(11.a) into imaginary and real part, then equating to zero we get;

$$2\alpha + \omega = 0, \tag{12}$$

$$M'' - (\alpha^2 + \Omega)M - \frac{(k+1)^2}{\omega + 1}M^3 = 0. \tag{13}$$

Eq.(12) yields

$$\omega = -2\alpha, \tag{14}$$

and one can rewrite eq.(13) in the form as follow,

$$(k+1)^2(M(\xi))^3 + (\omega + 1)\left(\alpha^2 + \Omega\right)M(\xi) - (\omega + 1)\frac{\mathrm{d}^2M(\xi)}{\mathrm{d}\xi^2} = 0. \tag{15}$$

Eq.(15) is the NODE form of the eq.(1) and eq.(14) is the constraint equation. According to homogeneous balance principle, if we consider the terms M^3 and M'' in eq.(15) we get m=1 which is the balancing constant.

2.1 Implementation of the new Kudryashov method

According to the new Kudryashov method [25,26,27,28] we propose the solution of eq.(15) in the following form,

$$M(\xi) = \sum_{i=0}^{m} A_i \kappa^i(\xi), \;\; A_m \neq 0. \tag{16}$$

If we consider that the balancing constant m=1 then eq.(16) takes the form,

$$M(\xi) = A_0 + A_1\kappa(\xi), \;\; A_1 \neq 0, \tag{17}$$

where A_0, A_1 are unknown constants to be found and $\kappa(\xi)$ satisfies the following Riccati equation,

$$\left[\frac{d\kappa(\xi)}{d\xi}\right]^2 = \delta^2\kappa^2(\xi)\left[1 - \eta\kappa^2(\xi)\right]. \tag{18}$$

The solution of eq.(18) is given in the following exponential or hyperbolic form,

$$\kappa(\xi) = \mp\frac{2L}{L^2 e^{\eta\xi} + \eta e^{-\eta\xi}}, \tag{19}$$

$$\kappa(\xi) = \mp\frac{2L}{(L^2 - \eta)\sinh(\delta\xi) + (L^2 + \eta)\cosh(\delta\xi)}, \tag{20}$$

where L, δ, η are non-zero arbitrary real constant to be found later.

Step 2: Substituting eq.(17) along with eq.(18) into eq.(15) gives a polynomial in powers of $\kappa(\xi)$. Collecting the coefficients of $\kappa(\xi)$ with the same power and setting each of them to zero, we get a set of algebraic equation system as follow.

$$
\begin{aligned}
&\kappa^0(\zeta): A_0{}^3k^2 - 2A_0\alpha^3 + 2A_0{}^3k - 2A_0\Omega\alpha + A_0\alpha^2 + A_0{}^3 + A_0\Omega = 0\\
&\kappa^1(\zeta): 3k^2A_0{}^2A_1 - 2\alpha^3A_1 + 2\alpha\delta^2A_1 + 6kA_0{}^2A_1 - 2\Omega\alpha A_1 + \alpha^2A_1 - A_1\delta^2 + 3A_0{}^2A_1 + \Omega A_1 = 0\\
&\kappa^2(\zeta): 3k^2A_0A_1{}^2 + 6kA_0A_1{}^2 + 3A_0A_1{}^2 = 0\\
&\kappa^3(\zeta): -4\alpha\delta^2\eta A_1 + k^2A_1{}^3 + 2A_1\delta^2\eta + 2kA_1{}^3 + A_1{}^3 = 0
\end{aligned}
\tag{21}
$$

Step 3: Solution of the eq.(21), gives the following solution sets

$$
\begin{aligned}
&Set_{1,2}: \left\{\alpha = \frac{1}{\eta}\sqrt{-\eta^2\left(-\delta^2+\Omega\right)}, A_0 = 0, A_1 = \mp\frac{\delta\sqrt{-2\eta+4\sqrt{-\eta^2\left(-\delta^2+\Omega\right)}}}{1+k}\right\},\\
&Set_{3,4}: \left\{\alpha = -\frac{1}{\eta}\sqrt{-\eta^2\left(-\delta^2+\Omega\right)}, A_0 = 0, A_1 = \mp\frac{\delta\sqrt{-2\eta-4\sqrt{-\eta^2\left(-\delta^2+\Omega\right)}}}{1+k}\right\},\\
&Set_{5,6}: \left\{\Omega = -\alpha^2+\delta^2, A_0 = 0, A_1 = \mp\frac{\delta\sqrt{2\eta\left(2\alpha-1\right)}}{1+k}\right\},\\
&Set_{7}: \left\{\begin{aligned}&\Omega = -\frac{1}{16\delta^4\eta^2}\left((1+k)^2A_1{}^2 - 2\eta\left(2\delta-1\right)\delta^2\right)\left((1+k)^2A_1{}^2 + 42\eta\delta^2\left(2\delta+1\right)\right),\\ &\alpha = \frac{(1+k)^2A_1{}^2+2\delta^2\eta}{4\delta^2\eta}, A_0 = 0, A_1 = A_1\end{aligned}\right\},\\
&Set_{8,9}: \left\{\delta = \sqrt{\alpha^2+\Omega}, A_0 = 0, A_1 = \mp\frac{\sqrt{2\eta\left(2\alpha-1\right)\left(\alpha^2+\Omega\right)}}{1+k}\right\},\\
&Set_{10,11}: \left\{\delta = -\sqrt{\alpha^2+\Omega}, A_0 = 0, A_1 = \mp\frac{\sqrt{2\eta\left(2\alpha-1\right)\left(\alpha^2+\Omega\right)}}{1+k}\right\},\\
&Set_{12,13}: \left\{\delta = \mp\sqrt{\alpha^2+\Omega}, \eta = \frac{(1+k)^2A_1{}^2}{\left(4\alpha-2\right)\left(\alpha^2+\Omega\right)}, A_0 = 0, A_1 = A_1\right\}.
\end{aligned}
\tag{22}
$$

Step 4: First, select any set from eq.(22), then insert into eq.(17) and eq.(19) by considering the eqs.(2), eq.(3), eq.(5), eq.(6), eq.(8) and eq.(10). The result is the solution of two coupled Maccari's system in eq.(1).

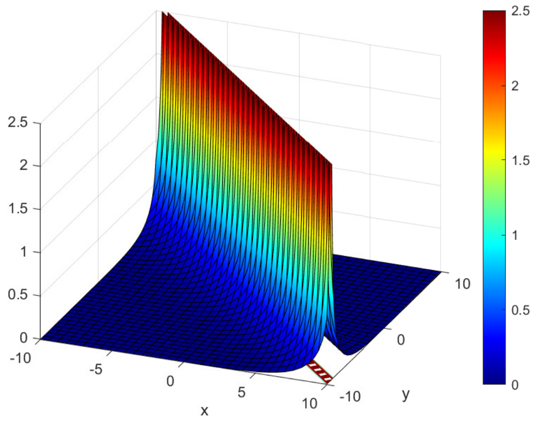
3 Numerical results and graphical presentations

If we select Set_3 and follow the algorithm in Step 4, we write the solution functions for eq.(1) as follows.

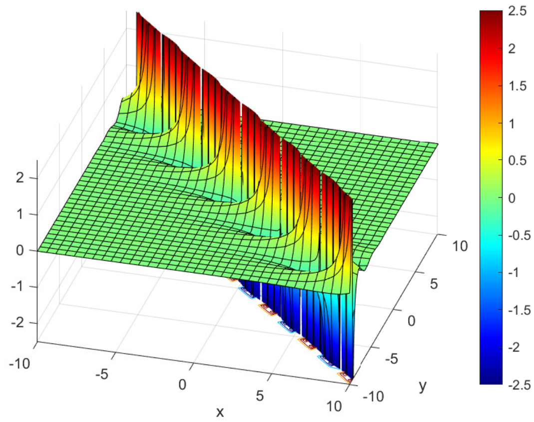
$$
\begin{aligned}
&X(x,y,t) = \mathrm{e}^{i*\theta}M(x,y,t) = \mathrm{e}^{i\left(\Omega t - \frac{\sqrt{-\eta^2\left(-\delta^2+\Omega\right)}x}{\eta} + \beta y\right)}\frac{\sqrt{-8\eta-16\sqrt{-\eta^2\left(-\delta^2+\Omega\right)}}\delta L}{(1+k)\left(L^2\mathrm{e}^{\delta\left(2\frac{\sqrt{-\eta^2\left(-\delta^2+\Omega\right)}t}{\eta}+x+y\right)} + \eta\mathrm{e}^{-\delta\left(2\frac{\sqrt{-\eta^2\left(-\delta^2+\Omega\right)}t}{\eta}+x+y\right)}\right)},\\
&Y(x,y,t) = k.\mathrm{e}^{i*\theta}M(x,y,t),\quad R(x,y,t) = -\frac{\left((1+k)M(x,y,t)\right)^2}{\omega+1}.
\end{aligned}
\tag{23}
$$

If we select Set_1 and follow the algorithm in Step 4, we write the solution functions for eq.(1) as follows.

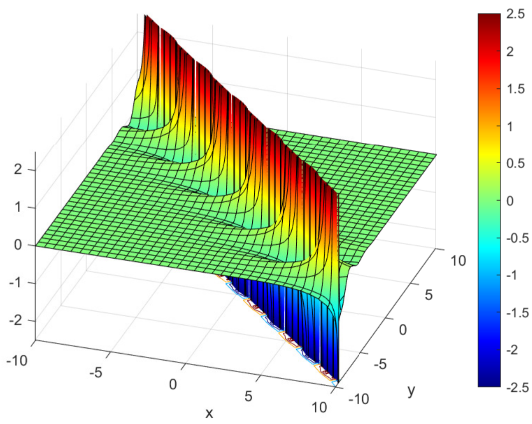
$$
\begin{aligned}
X(x,y,t) &= \mathrm{e}^{i*\theta} M(x,y,t) = -2\frac{\mathrm{e}^{i(\Omega t+\alpha x+\beta y)}\sqrt{2}\sqrt{\eta\,(2\alpha-1)\,(\alpha^2+\Omega)}L}{(1+k)\left(L^2\mathrm{e}^{-\sqrt{\alpha^2+\Omega}(-2\alpha t+x+y)}+\eta\mathrm{e}^{\sqrt{\alpha^2+\Omega}(-2\alpha t+x+y)}\right)},\\
Y(x,y,t) &= k.\mathrm{e}^{i*\theta} M(x,y,t),\quad R(x,y,t) = -\frac{((1+k)M(x,y,t))^2}{\omega+1}.
\end{aligned}
\tag{24}
$$



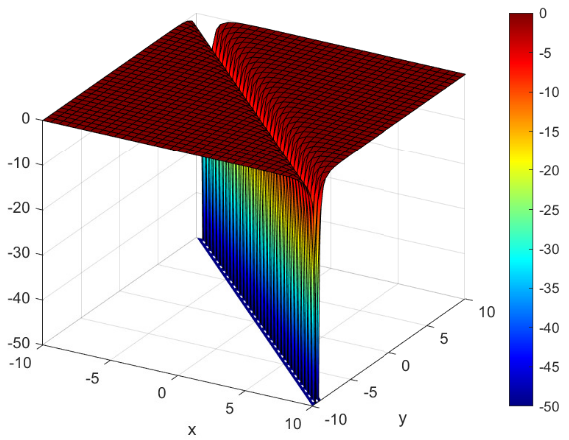
(a) (a)



(b) (b)

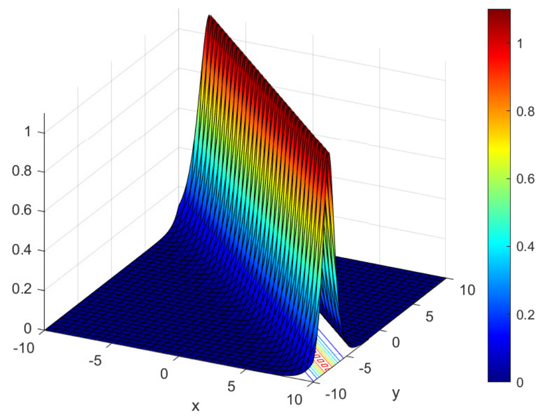


(c) (c)

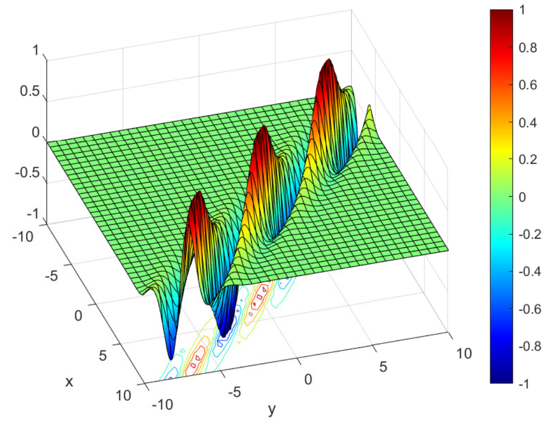


(d) (d)

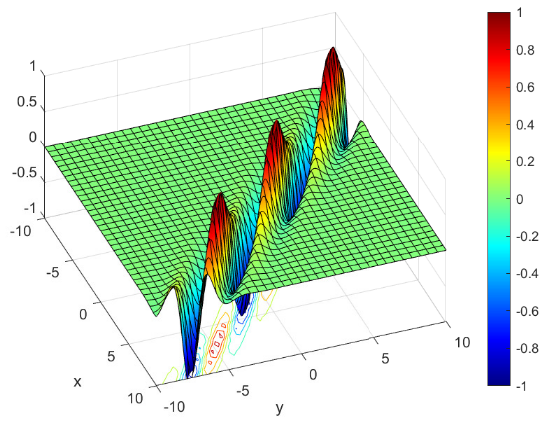
Fig. 1: The 3D modulus (a), real (b), imaginary (c) surface of $X(x,y,t), Y(x,y,t)$, and $R(x,y,t)$ (d) in eq.(23) for L=1,δ= 0.6, η= 1.2,k=1 ,β=2, Ω=0.3,t=1.



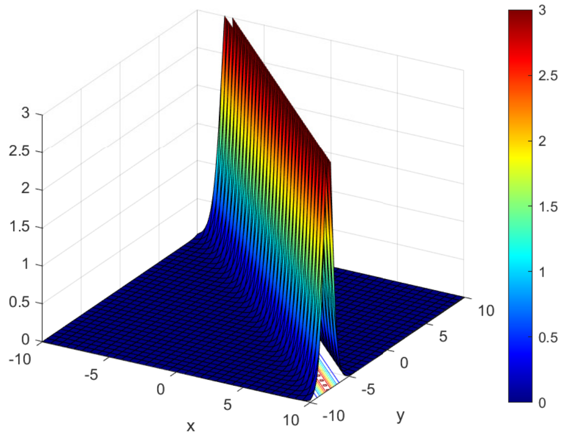
(a) (a)



(b) (b)



(c) (c)



(d) (d)

Fig. 2: The 3D modulus (a), real (b), imaginary (c) surface of $X(x,y,t), Y(x,y,t)$, and $R(x,y,t)$ (d) in eq.(24) for L=1,η= 1.2,k=1 , $\alpha = 1.2$, β=2.2, Ω=0.3,t=1.

4 Conclusion

We successfully depicted the complex two coupled Maccari's system which is very important equation to describe the motion of the isolated waves, localized in a small part of space, in many fields such as hydrodynamic, plasma physics, nonlinear optics. We utilized the new Kudryashov scheme, presented the physical features of the obtained results via the 3-dimensional graphs. The new Kudryashov scheme is quite basic and reliable method that might be applied to investigate various evolution equations solutions.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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