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Artificial neural network modeling and control for dynamical and statistical characteristics of photonic quantum memristor

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Abstract: Quantum memristor is a quantum device that accounts for the memory, with the decoherence mechanism controlled by a feedback algorithm [1]. Last year, a variety of prototypes for quantum memristors has been proposed: superconducting circuits and platforms, different realizations of Josephson junctions, photonic systems [2]. Nevertheless, still there is a lack of theoretical methods modeling efficient exploitation of such devices. A new statistical physics technique based on Artificial Neural Networks (ANNs) has been developed recently in [3]. Such ANNs are well–trained with the data collected from the '*Ab Initio*' experiment or numerical simulations to mimic the microscopic statistical states, and then to extrapolate the results for evaluation macroscopic states of the quantum system, its phase structures, and thermodynamic characteristics. Here we study a novel procedure based on the small–scale networks of Hodgkin-Huxley neurons [4] to model statistical micro–states and to control over dynamical characteristics (particularly, the reflectivity and the purity) of photonic quantum memristor [2]. We compare the pros and cons of a few alternative control algorithms: Fradkov's speed gradient [5] and Kolesnikov's target attractor feedback [6]. We discuss also possible applications of our approach to memristor–based reservoir computing.

Keywords: Artificial Neural Networks, Quantum Dynamical Systems, Feedback Control.

1 Introduction

Quantum memristor is a quantum device that accounts for the memory, with the decoherence mechanism controlled by a feedback algorithm [1]. Last year, a variety of prototypes for quantum memristors has been proposed: superconducting circuits and platforms, different realizations of Josephson junctions, photonic systems [2]. Nevertheless, still there is a lack of theoretical methods modeling efficient exploitation of such devices. A new statistical physics technique based on Artificial Neural Networks (ANNs) has been developed recently in [3]. Such ANNs are well–trained with the data collected from the '*Ab Initio*' experiment or numerical simulations to mimic the microscopic statistical states, and then to extrapolate the results for evaluation macroscopic states of the quantum system, its phase structures, and thermodynamic characteristics.

Here we study a novel procedure based on the small–scale networks of Hodgkin-Huxley (HH) neurons [4] to model statistical micro–states and to control over dynamical characteristics (particularly, the purity) of photonic quantum memristor [2]. We compare the pros and cons of a few alternative control algorithms: Fradkov's speed gradient [5] and Kolesnikov's target attractor feedback [6]. We discuss also possible applications of our approach to memristor–based reservoir computing.

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2 Mathematical model for photonic quantum memristor

Photonic Quantum Memristor (PQM) has been proposed as a principle scheme in [7] for a tunable beam splitter [8], and then developed in details in [2], see Fig. 1.

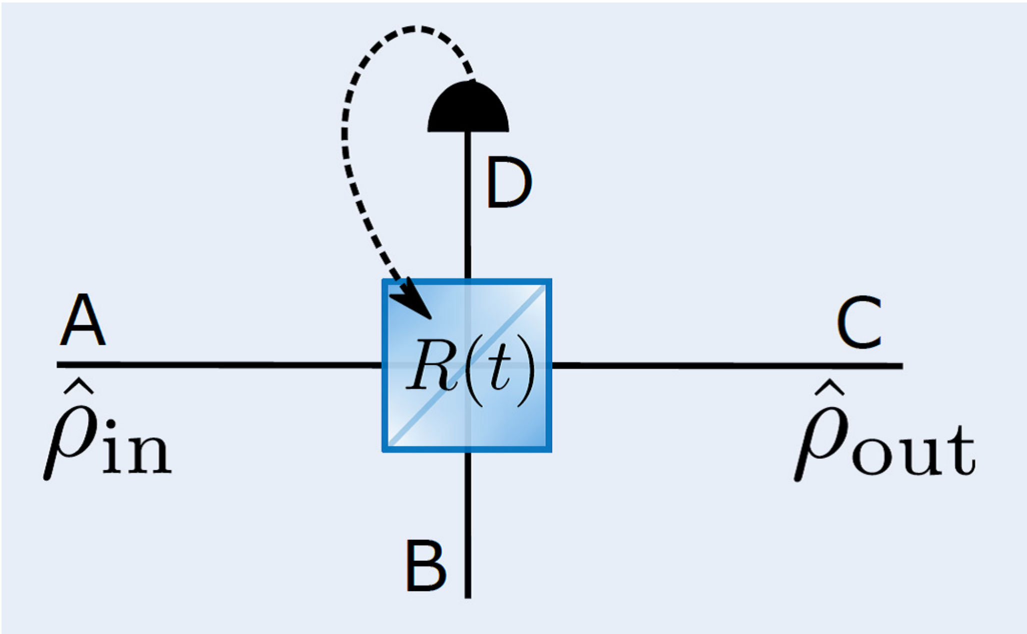

Fig. 1: Figure 1. Photonic Quantum Memristor principle scheme for a tunable beam splitter with active feedback [2].

PQM contains the input part (the mode A) and two output parts (the modes C and D). The reflectivity $R(t)$ can be updated based on the measurement made at the mode D.
The input state $|\psi_{\text{in}}(t)>$ of the PQM is taken as a superposition of a vacuum $|0>_A$ and a single photon $|1>_{\text{A}}$ states in the mode A:

$$|\psi_{\text{in}}(t)>=\alpha(t)|0>_{\text{A}}+\beta(t)|1>_{\text{A}}\ , \tag{1}$$

with the probability normalization over the complex α and β: $|\alpha(t)|^2+|\beta(t)|^2=1$.
The output mode C depends on the detection of the photon in D: if the photon is detected in D, then $|\psi_{\text{out,C}}(t)>=|0>_{\text{C}}$, otherwise:

$$|\psi_{\text{out,C}}(t)>=\frac{\alpha(t)}{\sqrt{N}}|0>_{\text{C}}+\frac{\beta(t)\sqrt{1-R(t)}}{\sqrt{N}}|1>_{\text{C}}\ , \tag{2}$$

where N is the normalization factor, and the reflectivity $R(t)$ is dynamically controlled, see next Section.
Finally, the output state can be expressed as [2]:

$$\rho_{\text{out,C}}(t)=|\beta(t)|^2R(t)\cdot|0>_{\text{C}}<0|_C++\left[1-|\beta(t)|^2R(t)\right]\cdot|\psi_{\text{out,C}}><\psi_{\text{out,C}}|\ ,$$

with the *purity*:

$$P_{\text{out,C}}(t)=\text{Tr}\left(\rho^2_{\text{out,C}}(t)\right)=1-2|\beta(t)|^4R(t)\left(1-R(t)\right)\ . \tag{3}$$

The purity (3) indicates how close the outcome C is to a pure state [9]: it is equal to 1 for pure states, otherwise, for a mixed state it is less than 1. Purity is related to the linear entropy $S_L=1-P$, which is the lower approximation to the von Neumann entropy [10].

3 Feedback control over reflectivity and purity

The expectation value at mode C is given by [2]:

$$< n_{\text{out}}(t) >= [1 - R(t)] < n_{\text{in}}(t) > , \tag{4}$$

and

$$\frac{dR(t)}{dt} =< n_{\text{in}} > -\frac{1}{2} < n >_{\max} , \tag{5}$$

where $< n_{\text{in}} >= |\beta(t)|^2 = u(t)$. Here $u(t)$ stands for the control signal.
To drive the reflectivity $R(t)$, we can apply different feedback algorithms for $u(t)$. In the speed gradient approach [5] we define the non-negative goal function:

$$G = \frac{1}{2}(R - R_\star)^2 , \tag{6}$$

where $R_\star$ is a desired level of the reflectivity stabilization. Then we compute the gradient for the signal in the control domain:

$$u(t) = -\Gamma \nabla_u \left(\frac{dG}{dt}\right) + u_0 = -\Gamma \frac{\partial}{\partial u}\left(\frac{dG}{dt}\right) + u_0 , \tag{7}$$

with a positive constant Γ and a constant control component u_0.
Substitution of (6) to (7) leads to:

$$u(t) = -\Gamma(R - R_\star) + u_0 , \tag{8}$$

and by (5), correspondingly,

$$\frac{dR}{dt} = -\Gamma(R - R_\star) + u_0 - \frac{1}{2} < n >_{\max} . \tag{9}$$

For the successful stabilization we need to fix $u_0 =< n >_{\max} /2$:

$$u(t) = -\Gamma\,(R(t) - R_\star) . \tag{10}$$

The alternative approach deals with the creation in the dynamical system an artificial target attractor [6]:

$$\psi(t) = R(t) - R_\star , \tag{11}$$

with the exponentially fast converging trajectories in its neighborhood:

$$\frac{d\psi}{dt} = -\Gamma\psi . \tag{12}$$

It is easy to check that in our model, (11)-(12) lead to the same result (9)-(10) , with the solution:

$$R(t) = R(0)e^{-\Gamma t} + R_\star(1 - e^{-\Gamma t}). \tag{13}$$

Thus, there is no principal difference for Fradkov's and Kolesnikov's control algorithms in our model.
Then via the control over the reflectivity $R(t)$ we get by (8) and (13) the control over the purity (3).

4 Artificial neural network for modeling PQR

Now we can control the reflectivity $R(t)$ with one of the alternative feedback algorithms, and we will apply it for modeling PQR quantum statistics with an ANN. Here we use the Hodgkin-Huxley neurons [4].

A single HH neuron is represented with the system of four non-linear ordinary differential equations [4]:

$$
\begin{aligned}
C_M \frac{dv}{dt} &= -g_{Na} m^3 h (v - E_{Na}) - g_K n^4 (v - E_K) - \\
&\quad - g_{Cl}(v - E_{Cl}) + I(t) \ , \\
\frac{dm}{dt} &= \alpha_m(v) \cdot (1 - m) + \beta_m(v) \cdot m \ ; \\
\frac{dn}{dt} &= \alpha_n(v) \cdot (1 - n) + \beta_n(v) \cdot n \ ; \\
\frac{dh}{dt} &= \alpha_h(v) \cdot (1 - h) + \beta_h(v) \cdot h \ ;
\end{aligned}
$$

Here $v(t)$ stands for the axon membrane action potential, $m(t)$, $n(t)$, $h(t)$ are the membrane gate variables, and the control signal $I(t)$ is represented by the sum of external and synaptic currents entering the cell.

The nonlinear functions of the action potential v are given by:

$$
\begin{aligned}
\alpha_m(v) &= \frac{0.1 \cdot (25 - v)}{\exp\left[\frac{25-v}{10}\right] - 1} \ ; \\
\beta_m(v) &= 4 \cdot \exp\left[-\frac{v}{18}\right] \ ; \\
\alpha_n(v) &= \frac{0.01 \cdot (10 - v)}{\exp\left[\frac{10-v}{10}\right] - 1} \ ; \\
\beta_n(v) &= 0.125 \cdot \exp\left[-\frac{v}{80}\right] \ ; \\
\alpha_h(v) &= 0.07 \cdot \exp\left[-\frac{v}{20}\right] \ ; \\
\beta_h(v) &= \frac{1}{\exp\left[\frac{30-v}{10}\right] + 1} \ .
\end{aligned}
$$

The set of constants includes the axon equilibrium potentials E_{Na} (at which the net flow of Na ions is zero), E_K (at which the net flow of K ions is zero), E_{Cl} (at which leakage is zero), the membrane capacitance C_M and the conductivities g_{Na} for the sodium channel, g_K for the potassium channel, and g_{Cl} for the leakage channel.

The important feature of the HH neuron (14)-(14) is the existence of a threshold: only for a certain level of the control current $I(t)$ this dynamical system produces a spike, below the threshold it keeps the neuron to stay in rest. Based on that, let's define the current $I(t)$ driving action potential $v(t)$ of the HH neuron in the form:

$$
I(t) = \frac{1}{2}\left[1 - (-1)^{\theta(1-R(t)+\Delta) - \theta(1-R(t)-\Delta)}\right] \cdot I_t \ . \tag{14}
$$

Here $\theta(x)$ is the step function, which is equal to 0 for all negative x and 1 for $x \geq 0$, and Δ is a small positive constant that defines the allowable error for the definition of $R(t)$. I_t stands for the minimum threshold level of the current which generates a single spike. The coefficient in (14) becomes 1 only in the domain $[1 - R(t) - \Delta, 1 - R(t) + \Delta]$ (otherwise it is 0). For this domain, the HH generates a spike.

Thus, the spiking component of the HH neuron reflects the coefficient in (4). In the place of HH cells, one can use any alternative neurons with similar threshold properties. A single HH neuron can check if the reflectivity belongs to a certain narrow interval with the precision Δ and produce a spike in this case. The small-scale ANN with a few parallel HH elements may serve for the trainable readout in the process of quantum reservoir computing.

Thus, the ANN with a single non-linear layer may reflect the statistics of the microscopic states for our quantum system.

5 PQR-based quantum reservoir computing: possibilities and perspectives

In a classical neural approach, the input information is fed to the first layer of the multi-layer ANN, and then the network can be trained to transfer the algorithm to the last layer neurons corresponding to the correct classification class. In so-called '*reservoir computing*' the input information is mapped to a nonlinear, high-dimensional space, whose output is interpreted by a simple linear readout network [2], sometimes using a linear weight of time-delay states [11]. Only such the readout network is trained, which requires minimal resources.

The input information is encoded on quantum states of a few photons [2]. A fixed matrix of beam splitters with random reflectivity distributes the transferred information across all optical modes, which are fed to a few PQMs embedded inside the quantum reservoir. The outputs of the memristors are scrambled again before reaching photon counters.

The output pattern must be interpreted by a trainable readout network. Usage of a non-linear element with threshold properties can drastically decrease its complexity and the time of training, and improve its reliability. The details of the adaptation of HH neurons to PQR-based quantum reservoir computing will be a matter of our further research.

6 Conclusions

Small-scale ANNs based on the neurons with threshold properties can be used successfully to model microscopic states of photonic quantum memristors. We preferred to use Hodgkin–Huxley elements keeping in our mind the perspective of modeling more flexible statistical characteristics of PQM using the extremely reach set of HH thresholds producing different spiking regimes [12]. Alternatively, other neurons with the stimulating threshold (Izhikevich, FitzHugh–Nagumo, Morris–Lecar) can be chosen for a similar algorithm.

The control over dynamical characteristics of photonic quantum memristor to drive its microscopic states can be done with one of alternative feedback algorithmic approaches: gradient methods (like Fradkov's control) or target attractor methods (like Kolesnikov's control).

The novel algorithmic approach proposed here can help to increase the efficiency of using PQR, for instance, in quantum reservoir computing.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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