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A New Approximate Solution for the Differential Equations Systems of the Spherical Curves with Adomian Decomposition Method

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Abstract: Our purpose is to solve the system of differential equations of spherical curves in 3-dimensional euclidean space using a numerical method such as the Adomian Decomposition Method. In the different values of x, we compare the Adomian decomposition method solution with the exact solution. We demonstrate the obtained numerical results on tables and figures. Thus we prove the reliability of the proposed method with an example.

Keywords: Spherical curves, system of differential equations, approximate solution, Adomian decomposition method

1 Introduction

Curves are seen in some fields such as mechanics, kinematics and differential geometry [1]. The curve located on a sphere is called the spherical curve. The condition for a curve to be a spherical curve is usually given in the form

$$\frac{\tau}{\kappa}+\left[\frac{1}{\tau}\left(\frac{1}{\kappa}\right)'\right]'=0,$$

where κ is curvature function and τ is torsion function [1]. The system of differential equations characterizing a unit speed spherical curve in Euclidean 3-space is given as [1]

$$\begin{cases} \lambda'_1(x)=\kappa(x)\lambda_2(x)+1, \\ \lambda'_2(x)=-\kappa(x)\lambda_1(x)+\tau(x)\lambda_3(x), \quad a\le x\le b, \\ \lambda'_3(x)=\tau(x)\lambda_2(x). \end{cases} \tag{1}$$

$$\lambda_1(a)=A,\ \lambda_2(a)=B,\ \lambda_3(a)=C, \tag{2}$$

where a,b,A,B,C are real constants; $\kappa(x)=\frac{1}{\sqrt{1-x^2}}$ and $\tau(x)=\frac{-1}{\sqrt{1-x^2}}$.
We use the Adomian decomposition method (ADM) to approximate the system of equations (1)-(2). The ADM was first introduced by George Adomian in 1980 [2]. It is applied to stochastic and deterministic problems in biology, physics, and chemical. This method can easily solve a wide class of linear (nonlinear) of ordinary and partial differential equations, integral equations, integro-differential, difference, delay differential equations and class neural networks with time-varying lags etc. [4, 5, 11–17]. ADM method is powerful and effective. Many researchers have preferred ADM to obtain numerical solutions. Abbaoui and Cherruault evidenced the convergence of the Adomian method in 1994 [1].

This paper can be presented as follows: In Section 1, we present the introduction part. In Section 2, we implement ADM to solve the system of differential equations of spherical curves in 3-dimensional euclidean space. From here obtained results are shown on tables and figures. In Section 3, the convergence of ADM is proved. The study is completed with the

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conclusion section. We have seen that these problems can be solved by many methods such as Taylor matrix collocation method, hermite polynomial approach, Lucas collocation method, Taylor polynomial solutions [6–10]. Therefore, we had to show that ADM's solution process for the system of differential equations of spherical curves in 3-dimensional euclidean space could be easier and more reliable. The results demonstrate that ADM is as powerful as other methods for our proposed problem.

2 Implementation of ADM

In this section we consider the following system of differential equations of spherical curves in 3-dimensional euclidean space. We present a particular example that confirms the results obtained [1].

$$\begin{cases} \lambda'_1(x) = \frac{1}{\sqrt{1-x^2}}\lambda_2(x)+1, \\ \lambda'_2(x) = -\frac{1}{\sqrt{1-x^2}}\lambda_1(x) - \frac{1}{\sqrt{1-x^2}}\lambda_3(x), \\ \lambda'_3(x) = \frac{-1}{\sqrt{1-x^2}}\lambda_2(x), \end{cases} \tag{3}$$

$$\lambda_1(0) = A,\ \lambda_2(0) = B,\ \lambda_3(0) = C. \tag{4}$$

The approximate solutions of the above system of differential equations are obtained by using ADM as follows:

The equations (3) are rewritten in the form

$$\begin{aligned} L\lambda_1 &= \frac{1}{\sqrt{1-x^2}}\lambda_2(x)+1, \\ L\lambda_2 &= -\frac{1}{\sqrt{1-x^2}}\lambda_1(x) - \frac{1}{\sqrt{1-x^2}}\lambda_3(x), \\ L\lambda_3 &= -\frac{1}{\sqrt{1-x^2}}\lambda_2(x), \end{aligned} \tag{5}$$

where differential operator L is given by $L(.) = \frac{d}{dx}(.)$ and L is the highest order derivation. If the integral operator $L^{-1}(.) = \int_0^x(.)$ is applied to each term of equation (5), we obtain the following recurrence relation

$$\begin{aligned} (\lambda_1(x))_{k+1} &= \lambda_1(0) + L^{-1}\left(\frac{1}{\sqrt{1-x^2}}(\lambda_2(x))_k + 1\right), \\ (\lambda_2(x))_{k+1} &= \lambda_2(0) + L^{-1}\left(-\frac{1}{\sqrt{1-x^2}}(\lambda_1(x))_k - \frac{1}{\sqrt{1-x^2}}(\lambda_3(x))_k\right), \\ (\lambda_3(x))_{k+1} &= \lambda_3(0) + L^{-1}\left(-\frac{1}{\sqrt{1-x^2}}(\lambda_2(x))_k\right), \end{aligned} \tag{6}$$

where $\lambda_1(0), \lambda_2(0), \lambda_3(0)$ are written from the boundary conditions (4). From the above recursive relation for $k = 0, 1, 2, ...$, we have

$$\begin{aligned} &(\lambda_1(x))_{k+1} = \lambda_1(0) + L^{-1}\left(\frac{1}{\sqrt{1-x^2}}(\lambda_2(x))_k + 1\right), \\ &(\lambda_1(x))_0 = \lambda_1(0) + L^{-1}(1), \\ &k = 0,\ (\lambda_1(x))_1 = \int_0^x\left(\frac{1}{\sqrt{1-x^2}}(\lambda_2(x))_0\right)dx, \\ &k = 1,\ (\lambda_1(x))_2 = \int_0^x\left(\frac{1}{\sqrt{1-x^2}}(\lambda_2(x))_1\right)dx, \\ &k = 2,\ (\lambda_1(x))_3 = \int_0^x\left(\frac{1}{\sqrt{1-x^2}}(\lambda_2(x))_2\right)dx, \\ &\quad \ldots \end{aligned} \tag{7}$$

The approximate solution $\lambda_1(x)$ by ADM is given by

$$\begin{aligned} (\lambda_1(x))_{Approximate} = \sum_{k=0}^{5}(\lambda_1(x))_k =& (\lambda_1(x))_0 + (\lambda_1(x))_1 \\ &+ (\lambda_1(x))_2 + (\lambda_1(x))_3 + (\lambda_1(x))_4 + (\lambda_1(x))_5. \end{aligned}$$

$$
\begin{aligned}
&(\lambda_2(x))_{k+1} = \lambda_2(0) + L^{-1}\left(-\frac{1}{\sqrt{1-x^2}}(\lambda_1(x))_k - \frac{1}{\sqrt{1-x^2}}(\lambda_3(x))_k\right)\\
&(\lambda_2(x))_0 = \lambda_2(0),\\
&k=0,\ (\lambda_2(x))_1 = \int_0^x\left(-\frac{1}{\sqrt{1-x^2}}(\lambda_1(x))_0 - \frac{1}{\sqrt{1-x^2}}(\lambda_3(x))_0\right)dx,\\
&k=1,\ (\lambda_2(x))_2 = \int_0^x\left(-\frac{1}{\sqrt{1-x^2}}(\lambda_1(x))_1 - \frac{1}{\sqrt{1-x^2}}(\lambda_3(x))_1\right)dx,\\
&k=2,\ (\lambda_2(x))_3 = \int_0^x\left(-\frac{1}{\sqrt{1-x^2}}(\lambda_1(x))_2 - \frac{1}{\sqrt{1-x^2}}(\lambda_3(x))_2\right)dx,\\
&\quad\ldots
\end{aligned}
\tag{8}
$$

The approximate solution $\lambda_2(x)$ by ADM is given by

$$(\lambda_2(x))_{Approximate} = (\lambda_2(x))_0 + (\lambda_2(x))_1 + (\lambda_2(x))_2 + (\lambda_2(x))_3 + (\lambda_2(x))_4 + (\lambda_2(x))_5.$$

$$
\begin{aligned}
&(\lambda_3(x))_{k+1} = \lambda_3(0) + L^{-1}\left(-\frac{1}{\sqrt{1-x^2}}(\lambda_2(x))_k\right),\\
&(\lambda_3(x))_0 = \lambda_3(0),\\
&k=0,\ (\lambda_3(x))_1 = L^{-1}\left(-\frac{1}{\sqrt{1-x^2}}(\lambda_2(x))_0\right),\\
&k=1,\ (\lambda_3(x))_2 = L^{-1}\left(-\frac{1}{\sqrt{1-x^2}}(\lambda_2(x))_1\right),\\
&k=2,\ (\lambda_3(x))_3 = L^{-1}\left(-\frac{1}{\sqrt{1-x^2}}(\lambda_2(x))_2\right),\\
&\quad\ldots
\end{aligned}
\tag{9}
$$

The approximate solution $\lambda_3(x)$ by ADM is given by

$(\lambda_3(x))_{Approximate} = (\lambda_3(x))_0 + (\lambda_3(x))_1 + (\lambda_3(x))_2 + (\lambda_3(x))_3 + (\lambda_3(x))_4 + (\lambda_3(x))_5.$

Example 1 The system of differential equations of spherical curves are considered as

$$
\begin{cases}
\lambda'_1(x) = \frac{1}{\sqrt{1-x^2}}\lambda_2(x) + 1,\\
\lambda'_2(x) = -\frac{1}{\sqrt{1-x^2}}\lambda_1(x) - \frac{1}{\sqrt{1-x^2}}\lambda_3(x),\\
\lambda'_3(x) = \frac{-1}{\sqrt{1-x^2}}\lambda_2(x),
\end{cases}
\tag{10}
$$

$\lambda_1(0) = 0,\ \lambda_2(0) = -1,\ \lambda_3(0) = 0.$

The exact solution of this problem is

$\lambda_1(\mathrm{x}) = 0,\ \lambda_2(\mathrm{x}) = \sqrt{1-x^2},\ \lambda_3(\mathrm{x}) = -x.$

By applying the ADM with five iterations, according to Equations (5)-(10), we obtain for $k = 0,1,2,3,4,5,$

$$
\begin{aligned}
&(\lambda_1(x))_{k+1} = \int_0^x\left(\frac{1}{\sqrt{1-x^2}}(\lambda_2(x))_k\right)dx,\\
&(\lambda_1(x))_0 = \int_0^x dx = x,
\end{aligned}
$$

$$
\begin{aligned}
&(\lambda_2(x))_{k+1} = \int_0^x\left(-\frac{1}{\sqrt{1-x^2}}(\lambda_1(x))_k - \frac{1}{\sqrt{1-x^2}}(\lambda_3(x))_k\right)dx,\\
&(\lambda_2(x))_0 = -1,
\end{aligned}
$$

$$
\begin{aligned}
&(\lambda_3(x))_{k+1} = \int_0^x\left(-\frac{1}{\sqrt{1-x^2}}(\lambda_2(x))_k\right)dx,\\
&(\lambda_3(x))_0 = 0.
\end{aligned}
$$

Utilizing the above recurrence relations for $k = 0,1,2,...,$ we find solutions of the system of differential equations of spherical curves in 3-dimensional euclidean space as

$(\lambda_1(x))_0 = x,$
$(\lambda_1(x))_1 = -\mathrm{arcsin(x)},$
$(\lambda_1(x))_2 = \mathrm{x} - \mathrm{arcsin(x)},$
$(\lambda_1(x))_3 = 0.3333333333arcsin(x)^3,$
$(\lambda_1(x))_4 = 0.3333333333arcsin(x)^3 + 2x - 2arcsin(x),$
$\lambda_1(x) = \sum_{k=0}^{5} (\lambda_1(x))_k = 4x - 4arcsin(x) + 6666666666arcsin(x)^3 - (03333333332e-1)\,arcsin(x)^5,$
$(\lambda_2(x))_0 = -1,$
$(\lambda_2(x))_1 = -1 + \frac{1}{\sqrt{1-x^2}} - \frac{x^2}{\sqrt{1-x^2}},$
$(\lambda_2(x))_2 = arcsin(x)^2,$
$(\lambda_2(x))_3 = -2 + arcsin(x)^2 + 2\sqrt{1-x^2},$
$(\lambda_2(x))_4 = -0.1666666666arcsin(x)^4,$
$\lambda_2(x) = \sum_{k=0}^{5} (\lambda_2(x))_k = -8 + \frac{1}{\sqrt{1-x^2}} - \frac{x^2}{\sqrt{1-x^2}} + 4arcsin(x)^2 + 6\sqrt{1-x^2} - 0.3333333332arcsin(x)^4,$
$(\lambda_3(x))_0 = 0,$
$(\lambda_3(x))_1 = -\mathrm{arcsin(x)},$
$(\lambda_3(x))_2 = \mathrm{x} - \mathrm{arcsin(x)},$
$(\lambda_3(x))_3 = 0.3333333333arcsin(x)^3,$
$(\lambda_3(x))_4 = 0.3333333333arcsin(x)^3 + 2x - 2arcsin(x),$
$\lambda_3(x) = \sum_{k=0}^{5} (\lambda_3(x))_k = -4arcsin(x) + 3x + 0.6666666666arcsin(x)^3 - 0.3333333332arcsin(x)^5.$

After these calculations with five iterations, we obtain the following results of both the exact solution and approximate solution for different values of x.

Table 1: Comparison of exact solution and approximate solution for $\lambda_1(x)$.

x	$\lambda_1(x)_{Exact}$	$\lambda_1(x)_{Approximate}$	$Error$
0	0.00000000	0.00000000	0.00000000
$\frac{3\pi}{40}$	0.00000000	-0.00000000	0.00000000
$\frac{3\pi}{20}$	0.00000000	0.00000541	0.00000541
$\frac{9\pi}{40}$	0.00000000	-0.00014460	0.00014460
$\frac{3\pi}{10}$	0.00000000	-0.00330969	0.00330969

Table 2: Comparison of exact solution and approximate solution for $\lambda_2(x)$.

x	$\lambda_2(x)_{Exact}$	$\lambda_2(x)_{Approximate}$	$Error$
0	-1.00000000	-1.00000000	0.00000000
$\frac{3\pi}{40}$	-0.97184539	- 0.97184740	0.00000201
$\frac{3\pi}{20}$	-0.88200561	-0.88216005	0.00015444
$\frac{9\pi}{40}$	-0.70735512	-0.70992764	0.00257252
$\frac{3\pi}{10}$	-0.33426875	-0.37171644	0.03744769

The results obtained with the ADM are almost the same as the results found with the exact solution. It is clear in tables and figures that these results not only give rapidly convergent results but also accurately compute the solutions.

Table 3: Comparison of exact solution and approximate solution for $\lambda_3(x)$.

x	$\lambda_3(x)_{Exact}$	$\lambda_3(x)_{Approximate}$	$Error$
0	0.00000000	-1.00000000	0.00000000
$\frac{3\pi}{40}$	-0.23561944	- 0.23561948	0.00000004
$\frac{3\pi}{20}$	-0.47123889	-0.47124431	0.00000542
$\frac{9\pi}{40}$	-0.70685834	-0.70700295	0.00014461
$\frac{3\pi}{10}$	-0.94247779	-0.94578749	0.00330970

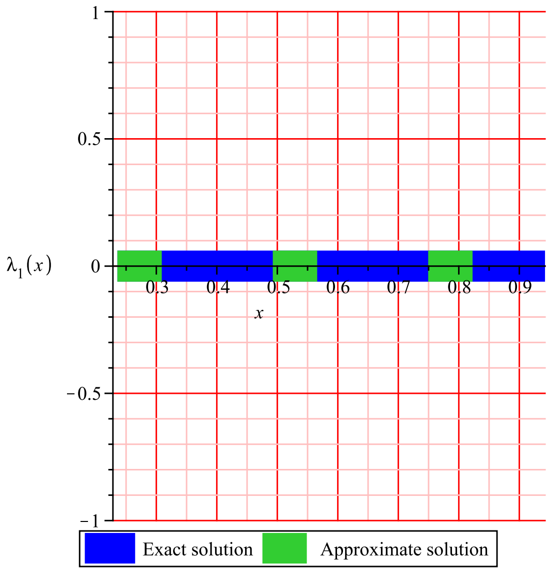

Fig. 1: Curves of exact solution and approximate solution for $\lambda_1(x)$.

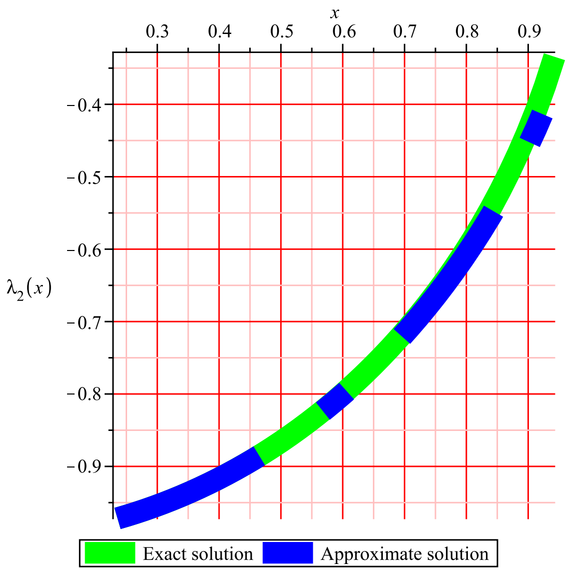

Fig. 2: Curves of exact solution and approximate solution for $\lambda_2(x)$.

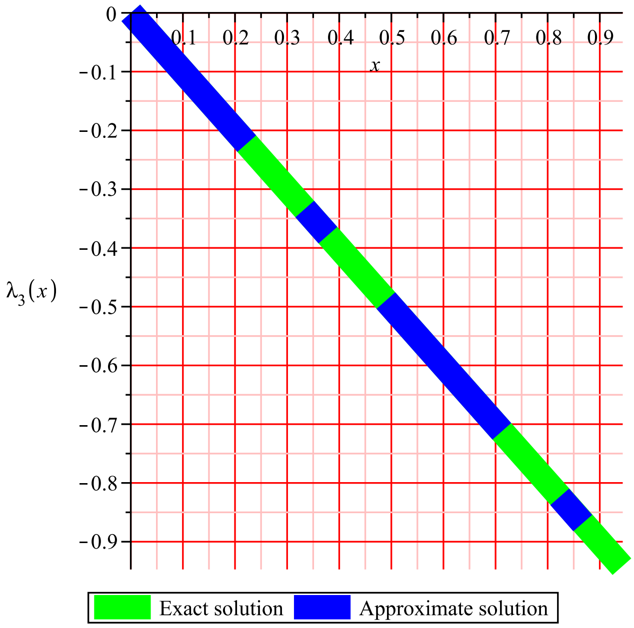

Fig. 3: Curves of exact solution and approximate solution for $\lambda_3(x)$.

3 Conclusion

We reached high approximate solutions that are very close to exact solutions with five iterations. All the figures and tables show that successive approximation methods such as ADM are an accurate, reliable and simple method for solving the system of differential equations of spherical curves in 3-dimensional euclidean space. ADM can also be easily applied to differential equations of spherical curves in high dimensional Euclidean space.

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