New Trends in Mathematical Sciences
http://dx.doi.org/10.20852/ntmsci.2021.435

A secure communication scheme based on adaptive modified projective combination synchronization of fractional-order hyper-chaotic systems

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Received: 13 June 2021, Accepted: 16 June 2021
Published online: 12 July 2021.

Abstract: In this work, a novel technique called "adaptive modified projective combination synchronization (AMPCS)" for synchronizing non-identical fractional-order hyper-chaotic systems with unknown parameters has been introduced. The purpose of the suggested technique is to ensure synchronization between two non-identical master systems and one slave system by employing a diagonal matrix, Lyapunov stability theory, adaptive control, adaptive law of parameter, and some techniques of fractional calculus. An application of synchronization in secure communication has been performed. The important feature of the suggested (AMPCS) technique is to create high security in secure communication.

Keywords: Adaptive control, Hyper-chaotic systems, Combination synchronization, Secure communication.

1 Introduction

One of the most well-known strategies of chaos synchronization is the adaptive modified projective synchronization planned by Park in 2008 [1]. The advantages of this strategy are the various sorts of synchronization like projective synchronization, anti synchronization, and complete synchronization may be found. In 2011 Runzi et al. [2] proposed a new strategy for synchronizing two drive systems and one response system in which the trajectories of two drive systems and one response system become identical, it referred to as combination synchronization. Nowadays, there are several kinds for combination synchronization, projective combination synchronization [4], adaptive generalized combination synchronization [5], dual combination synchronization [6,7], combination-combination synchronization [8], dual-function projective synchronization [9]. This has actuated the researchers to develop a novel strategy up a diagonal matrix we synchronize both two different drive systems and one response system. The novelty and contribution of this research are to propose an easy technique to accomplish modified projective combination synchronization between almost all non-identical fractional-order chaotic and hyper-chaotic systems with unknown parameters by employing Lyapunov stability theory, diagonal matrix, adaptive control, the parameter update laws and some techniques of fractional calculus. Three points of interest that can make this study a great deal attractive, the primary we tend to apply the (AMPCS) technique for synchronizing non-identical fractional-order chaotic and hyper-chaotic systems with unknown or known parameters. The secondary, the proposed (AMPCS) technique will synchronize practically almost all non-identical fractional-order chaotic and hyper-chaotic systems. Then thirdly, this technique creates high security in secure communication and cryptography.

This paper is organized as follows:

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in section 2, we present some basic fractional calculus that we will utilize in our primary outcomes. In section 3, provides the (AMPCS) scheme. In section 4, we take three non-identical fractional-order hyper-chaotic systems with unknown parameters (Lorenz system, Chen system, and Lü system) to display the effectiveness and viability of the proposed (AMPCS) technique. Application of APMCS in secure communication is drawn in Section 5. Finally, section 6 points out concluding remarks.

2 Some basic about fractional calculus

We begin this section with the Gamma function playing a key role in the theory of fractional calculus. Then, we present the definition and some properties of the Caputo fractional derivative that we will use in our main results.
The Gamma function $\Gamma(z)$ is defined by the integral [10]:

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt. \tag{1}$$

The Caputo fractional derivative operator of order $q \in \mathbb{R}^+$ of function $x(t)$ is defined by [10]:

$${}^cD_t^q x(t) = \frac{1}{\Gamma(n-q)} \int_0^t (t-s)^{n-q-1} x^{(n)}(s) ds, \tag{2}$$

where $t \geq 0, n-1 < q < n \in \mathbb{Z}^+$ and $x^{(n)}(s)$ denotes the n-th derivative of x with respect to s.

Lemma 1. *[12] Let $x \in \mathbb{R}^n$ be a continuous and derivable function, then $\forall q \in (0.1)$*

$$\frac{1}{2}\,{}^cD_t^q [x^T(t)x(t)] \leq x^T(t)\,{}^cD_t^q x(t). \tag{3}$$

Lemma 2. *[10] The Caputo fractional derivative is a linear operator*

$${}^cD_t^q (x(t) + y(t)) = {}^cD_t^q x(t) + {}^cD_t^q y(t)$$

In order to investigate the stability of equilibrium points of fractional-order systems, we consider the following fractional-order system:

$${}^cD_t^q x(t) = F(x(t)) \tag{4}$$

where $0 < q < 1$ and $x \in \mathbb{R}^n$.

Theorem 1. *[11] If there exists a positive definite Lyapunov function $V(x)$ such that $D_t^q V(x) \leq 0$, for all $t \geq t_0$, then the trivial solution of system (4) is asymptotically stable.*

3 The scheme of (AMPCS)

Consider the two fractional-order drive systems with unknown parameters, respectively, as follows:

$${}^cD_t^q x = f(x) + F(x)\alpha \tag{5}$$

$${}^cD_t^q y = g(y) + G(y)\beta \tag{6}$$

and the one fractional-order response system with unknown parameters as:

$$ {}^{c}D_t^q z = h(z) + H(y)\delta + u \tag{7}$$

where $0 < q < 1$ are the fractional-orders, $f(x), g(y), h(z) \in \mathbb{R}^n$, are vector functions; $F(x) \in \mathbb{R}^{n\times m}, G(y) \in \mathbb{R}^{n\times p}$, $H(z) \in \mathbb{R}^{n\times r}$ are matrix functions; $\alpha \in \mathbb{R}^m, \beta \in \mathbb{R}^p, \delta \in \mathbb{R}^r$ are unknown parameter vectors. Our goal is to design (MPCS) between two drive systems (5,6) and one response system (7) by constructing an effective adaptive controller.

In this paper, the combination synchronization error between the two drive and response systems is defined by

$$e = (x+y) - \theta z,$$

where θ is diagonal matrix which called scaling factor matrix $\theta = diag(\theta_{11}, \theta_{22}, \ldots, \theta_{nn})$, $\theta_{ii} \neq 0$, $(i = 1\ldots n)$. Then

$$[c]l\ {}^{c}D_t^q e = ({}^{c}D_t^q x + {}^{c}D_t^q y) - \theta\, {}^{c}D_t^q z = (f(x) + F(x)\alpha + g(y) + G(y)\beta) - \theta h(z) - \theta H(z)\delta - \theta u. \tag{8}$$

Theorem 2. *The controller u is proposed as the following*

$$u = \theta^{-1} f(x) + \theta^{-1} F(x)\tilde{\alpha} + \theta^{-1} g(y) + \theta^{-1} G(y)\tilde{\beta} - h(z) - H(z)\delta + \theta^{-1} ke, \tag{9}$$

and adaptive law of parameter is taken as

$$\begin{cases} {}^{c}D_t^q \tilde{\alpha} = [F(x)]^T e + \varepsilon(\alpha - \tilde{\alpha}), \\ {}^{c}D_t^q \tilde{\beta} = [G(y)]^T e + \eta(\beta - \tilde{\beta}), \\ {}^{c}D_t^q \tilde{\delta} = [-H(z)]^T \theta e + \nu(\delta - \tilde{\delta}) \end{cases} \tag{10}$$

Then, the (MPCS) between two drive systems (5,6) and the one response system (7) can be achieved by using the controller (9) and parameter updating law (10).

Proof. According to the controller u, we get:

$${}^{c}D_t^q e = F(x)(\alpha - \tilde{\alpha}) + G(y)(\beta - \tilde{\beta}) - \theta H(z)(\delta - \tilde{\delta}) - ke.$$

The Lyapunov function is chosen as

$$V = \tfrac{1}{2} X^T X, \tag{11}$$

where $X = (e^T, (\alpha - \tilde{\alpha})^T, (\beta - \tilde{\beta})^T, (\delta - \tilde{\delta})^T)$. Next, taking the fractional-order derivative of the Lyapunov function

$${}^{c}D_t^q V = {}^{c}D_t^q (\tfrac{1}{2} X^T X).$$

Now, from lemma (1) we get:

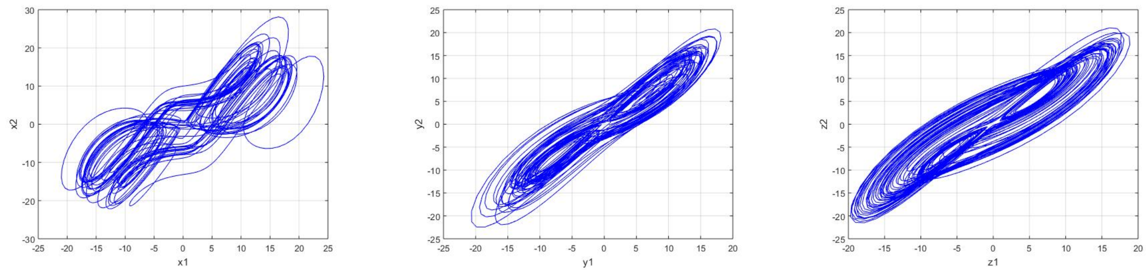
$${}^{c}D_t^q V = {}^{c}D_t^q (\tfrac{1}{2} X^T X) \leq X^T\, {}^{c}D_t^q X$$

from which we get

$$\begin{aligned} {}^{c}D_t^q V &= e^T\, {}^{c}D_t^q e - (\alpha - \tilde{\alpha})^T\, {}^{c}D_t^q \tilde{\alpha} - (\beta - \tilde{\beta})^T\, {}^{c}D_t^q \tilde{\beta} - (\delta - \tilde{\delta})^T\, {}^{c}D_t^q \tilde{\delta} \\ &= -e^T ke - (\alpha - \tilde{\alpha})^T \varepsilon(\alpha - \tilde{\alpha}) - (\beta - \tilde{\beta})^T \eta(\beta - \tilde{\beta}) - (\delta - \tilde{\delta})^T \nu(\delta - \tilde{\delta}). \end{aligned} \tag{12}$$

Let $\lambda = \min(k_i, \varepsilon_j, \eta_\kappa, \delta_\iota.) 1 \leq i \leq n, 1 \leq j \leq m, 1 \leq \kappa \leq p, 1 \leq \iota \leq r$. Then

$$\begin{aligned} {}^{c}D_t^q V &= -\lambda(e^T e + (\alpha - \tilde{\alpha})^T (\alpha - \tilde{\alpha}) + (\beta - \tilde{\beta})^T)(\beta - \tilde{\beta}) + (\delta - \tilde{\delta})^T (\delta - \tilde{\delta})) \\ &\leq -\lambda \parallel X \parallel^2 \end{aligned} \tag{13}$$



(1) (2) (3)

Fig. 1: (1): Strange attractors of FO hyper-chaotic Lorenz system. (2): Strange attractors of FO hyper-chaotic Chen system. (3): Strange attractors of FO hyper-chaotic Lü system.

Then the error system is globally asymptotically stable . Therefore, the two drive systems (5,6) and one response system (7) synchronized in the sense of (MPCS).

4 Example

In this section, we apply (AMPCS) for synchronizing three different fractional-order hyper-chaotic systems. We take the fractional-order Lorenz system [16] and the fractional-order Chen system [17] as the two drive systems and the fractional-order Lü system [18] as the one response system. The first drive system is presented by:

$$\begin{cases} {}^cD_t^q x_1 = \alpha_1(x_2 - x_1) + x_4 \\ {}^cD_t^q x_2 = \alpha_2 x_1 - x_1 x_3 - x_2 \\ {}^cD_t^q x_3 = x_1 x_2 - \alpha_3 x_3 \\ {}^cD_t^q x_4 = -x_2 x_3 + \alpha_4 x_4 \end{cases} \tag{14}$$

Where $(x_1, x_2, x_3, x_4)^T$ is the state vector of the system. When the parameters are selected as $\alpha_1 = 10, \alpha_2 = 28, \alpha_3 = 8/3, \alpha_4 = -1$ and $q = 0.98$ and the initial value is $(x_1(0), x_2(0), x_3(0), x_4(0))^T = (1.5, 3, -1, 3)$, system exhibits chaotic behaviors as shown in Fig.1.1. The second drive system is presented by:

$$\begin{cases} {}^cD_t^q y_1 = \beta_1(y_2 - y_1) + y_4 \\ {}^cD_t^q y_2 = \beta_2 y_1 - y_1 y_3 + \beta_3 y_2 \\ {}^cD_t^q y_3 = y_1 y_2 - \beta_4 y_3 \\ {}^cD_t^q y_4 = y_2 y_3 + \beta_5 y_4 \end{cases} \tag{15}$$

Where $(y_1, y_2, y_3, y_4)^T$ is the state vector of the system. This system exhibits chaotic behaviors when $\beta_1 = 35$, $\beta_2 = 7$, $\beta_3 = 12$, $\beta_4 = 3$, $\beta_5 = 0.3$, $q = 0.98$ and the initial value is $y(0) = (2, 3, 5)$. Attractors of the fractional-order Lü system are shown in Fig.1.2. The response system is

$$\begin{cases} {}^cD_t^q z_1 = \delta_1(z_2 - z_1) + z_4 + u_1 \\ {}^cD_t^q z_2 = -z_1 z_3 + \delta_2 z_2 + u_2 \\ {}^cD_t^q z_3 = z_1 z_2 - \delta_3 z_3 + u_3 \\ {}^cD_t^q z_4 = z_1 z_3 + \delta_4 z_4 + u_4 \end{cases} \tag{16}$$

where $(z_1,z_2,z_3,z_4)^T$ is the state vector of the system $u=(u_1,u_2,u_3,u_4)^T$ is the controller.. When $\delta_1=36$, $\delta_2=20$, $\delta_3=3$, $\delta_4=-1$, $q=0.98$, $(u_1,u_2,u_3,u_4)=(0,0,0)$ and the initial value is $x(0)=(0.2,0,2)$, system exhibits chaotic behaviors as shown in Fig.1.3. Compare systems (14,15) and (16) with systems (5,6) and (7), we get

$$f(x)=\begin{pmatrix} x_4\\ -x_2-x_1x_3\\ x_1x_2\\ -x_2x_3\end{pmatrix},\ F(x)=\begin{pmatrix} x_2-x_1 & 0 & 0 & 0\\ 0 & x_1 & 0 & 0\\ 0 & 0 & -x_3 & 0\\ 0 & 0 & 0 & x_4\end{pmatrix}$$

$\alpha=(\alpha_1,\alpha_2,\alpha_3,\alpha_4)^T$ is the unknown parameter vector of the system (14).

$$g(y)=\begin{pmatrix} y_4\\ -y_1y_3\\ y_1y_2\\ y_2y_3\end{pmatrix},\ G(y)=\begin{pmatrix} y_2-y_1 & 0 & 0 & 0 & 0\\ 0 & y_1 & y_2 & 0 & 0\\ 0 & 0 & 0 & -y_3 & 0\\ 0 & -y_3 & 0 & 0 & y_4\end{pmatrix}$$

$\beta=(\beta_1,\beta_2,\beta_3,\beta_4,\beta_5)^T$ is the unknown parameter vector of the system (15).

$$h(z)=\begin{pmatrix} z_4\\ -z_1z_3\\ z_1z_2\\ z_1z_3\end{pmatrix},\ H(z)=\begin{pmatrix} z_2-z_1 & 0 & 0 & 0\\ 0 & z_2 & 0 & 0\\ 0 & 0 & -z_3 & 0\\ 0 & 0 & 0 & z_4\end{pmatrix}$$

$\delta=(\delta_1,\delta_2,\delta_3,\delta_4)^T$ is the unknown parameter vector of the system (16). According to theorem (2), the controller is taken as:

$$\begin{aligned}
u =&\theta^{-1}f(x)+\theta^{-1}F(x)\tilde{\alpha}+\theta^{-1}g(y)+\theta^{-1}G(y)\tilde{\beta}-h(z)-H(z)\delta+\theta^{-1}ke\\
=&\begin{bmatrix} \frac{1}{\theta_{11}} & 0 & 0 & 0\\ 0 & \frac{1}{\theta_{22}} & 0 & 0\\ 0 & 0 & \frac{1}{\theta_{33}} & 0\\ 0 & 0 & 0 & \frac{1}{\theta_{44}}\end{bmatrix}\begin{bmatrix} x_4\\ -x_2-x_1x_3\\ x_1x_2\\ -x_2x_3\end{bmatrix}+\begin{bmatrix} \frac{1}{\theta_{11}} & 0 & 0 & 0\\ 0 & \frac{1}{\theta_{22}} & 0 & 0\\ 0 & 0 & \frac{1}{\theta_{33}} & 0\\ 0 & 0 & 0 & \frac{1}{\theta_{44}}\end{bmatrix}\\
&\begin{bmatrix} x_2-x_1 & 0 & 0 & 0\\ 0 & x_1 & 0 & 0\\ 0 & 0 & -x_3 & 0\\ 0 & 0 & 0 & x_4\end{bmatrix}\begin{bmatrix}\tilde{\alpha}_1\\ \tilde{\alpha}_2\\ \tilde{\alpha}_3\\ \tilde{\alpha}_4\end{bmatrix}+\begin{bmatrix} \frac{1}{\theta_{11}} & 0 & 0 & 0\\ 0 & \frac{1}{\theta_{22}} & 0 & 0\\ 0 & 0 & \frac{1}{\theta_{33}} & 0\\ 0 & 0 & 0 & \frac{1}{\theta_{44}}\end{bmatrix}\begin{bmatrix} y_4\\ -y_1y_3\\ y_1y_2\\ y_2y_3\end{bmatrix}\\
&+\begin{bmatrix} \frac{1}{\theta_{11}} & 0 & 0 & 0\\ 0 & \frac{1}{\theta_{22}} & 0 & 0\\ 0 & 0 & \frac{1}{\theta_{33}} & 0\\ 0 & 0 & 0 & \frac{1}{\theta_{44}}\end{bmatrix}\begin{bmatrix} y_2-y_1 & 0 & 0 & 0 & 0\\ 0 & y_1 & y_2 & 0 & 0\\ 0 & 0 & 0 & -y_3 & 0\\ 0 & -y_3 & 0 & 0 & y_4\end{bmatrix}\begin{bmatrix}\tilde{\beta}_1\\ \tilde{\beta}_2\\ \tilde{\beta}_3\\ \tilde{\beta}_4\\ \tilde{\beta}_5\end{bmatrix}-\begin{bmatrix} z_4\\ -z_1z_3\\ z_1z_2\\ z_1z_3\end{bmatrix}\\
&-\begin{bmatrix} z_2-z_1 & 0 & 0 & 0\\ 0 & z_2 & 0 & 0\\ 0 & 0 & -z_3 & 0\\ 0 & 0 & 0 & z_4\end{bmatrix}\begin{bmatrix}\tilde{\delta}_1\\ \tilde{\delta}_2\\ \tilde{\delta}_3\\ \tilde{\delta}_4\end{bmatrix}+\begin{bmatrix} \frac{1}{\theta_{11}} & 0 & 0 & 0\\ 0 & \frac{1}{\theta_{22}} & 0 & 0\\ 0 & 0 & \frac{1}{\theta_{33}} & 0\\ 0 & 0 & 0 & \frac{1}{\theta_{44}}\end{bmatrix}\begin{bmatrix} k_1e_1\\ k_2e_2\\ k_3e_3\\ k_4e_4\end{bmatrix}
\end{aligned}$$

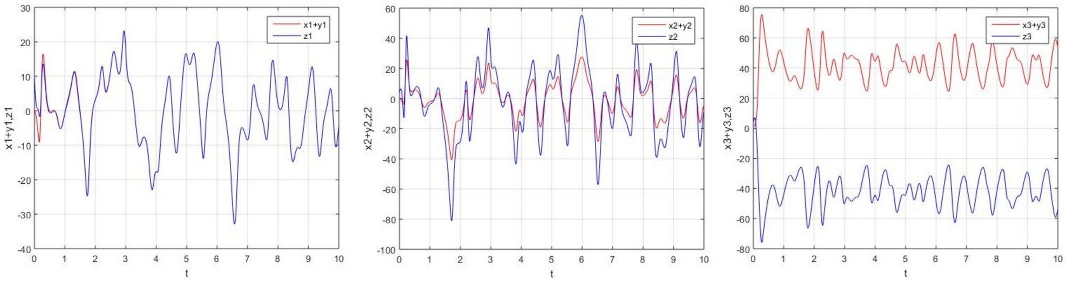


Fig. 2: State trajectories of FO Lorenz, FO Chen and FO Lü hyper-chaotic systems for $\theta_{11} = 1$, $\theta_{22} = 0.5$ and $\theta_{33} = -1$

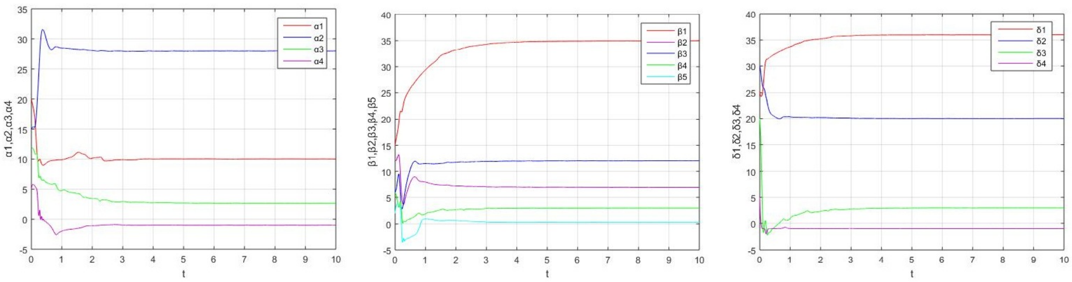


Fig. 3: State trajectories of adaptive parameters (17) and (18).

and, the updating laws to identify the unknown system parameters can be designed as

$$\begin{cases} {}^cD_t^q\tilde{\alpha}_1 = (x_2 - x_1)e_1 + \varepsilon_1(\alpha_1 - \tilde{\alpha}_1) \\ {}^cD_t^q\tilde{\alpha}_2 = x_1e_2 + \varepsilon_2(\alpha_2 - \tilde{\alpha}_2) \\ {}^cD_t^q\tilde{\alpha}_3 = -x_3e_3 + \varepsilon_3(\alpha_3 - \tilde{\alpha}_3) \\ {}^cD_t^q\tilde{\alpha}_4 = x_4e_4 + \varepsilon_2(\alpha_4 - \tilde{\alpha}_4) \end{cases}, \quad \begin{cases} {}^cD_t^q\tilde{\beta}_1 = (y_2 - y_1)e_1 + \eta_1(\beta_1 - \tilde{\beta}_1) \\ {}^cD_t^q\tilde{\beta}_2 = y_1e_2 + \eta_2(\beta_2 - \tilde{\beta}_2) \\ {}^cD_t^q\tilde{\beta}_3 = y_2e_2 + \eta_3(\beta_3 - \tilde{\beta}_3) \\ {}^cD_t^q\tilde{\beta}_4 = -y_3e_3 + \eta_4(\beta_4 - \tilde{\beta}_4) \\ {}^cD_t^q\tilde{\beta}_5 = y_4e_4 + \eta_5(\beta_5 - \tilde{\beta}_5) \end{cases} \tag{17}$$

and

$$\begin{cases} {}^cD_t^q\tilde{\delta}_1 = -\theta_{11}(z_2 - z_1)e_1 + \nu_1(\delta_1 - \tilde{\delta}_1) \\ {}^cD_t^q\tilde{\delta}_2 = -\theta_{22}z_2e_2 + \nu_2(\delta_2 - \tilde{\delta}_2) \\ {}^cD_t^q\tilde{\delta}_3 = \theta_{33}z_3e_3 + \nu_3(\delta_3 - \tilde{\delta}_3) \\ {}^cD_t^q\tilde{\delta}_4 = -\theta_{44}z_4e_3 + \nu_4(\delta_4 - \tilde{\delta}_4) \end{cases} \tag{18}$$

For this numerical simulation, the Adams-Bashforth-Moulton method has been used. Also, a time step size of 0.01 was employed. The initial values of the two drive systems and the response system are $x(0) = (1.5, 3, -1, 3), y(0) = (-1, -3, 2, 5)$ and $z(0) = (-10, -14, 12, 10)$, respectively, $q = 0.98$ and the initial values of the uncertain parameters are arbitrarily taken as $\tilde{\alpha}(0) = (20, 35, -2, -5)$, $\tilde{\beta}(0) = (40, 15, 20, 6, 2)$, $\delta(0) = (40, 35, 5, 2)$ and the initial states of the error system are $e(0) = (-9.5, -2, 5, 13)$; $\varepsilon_i = 1, (i = 1 \dots 4), \eta_i = 1, (i = 1 \dots 5)$ and $\delta_i = 1 (i = 1 \dots 4)$, the control gain is given by $k_i = 1, (i = 1 \dots 4)$, we choose arbitrarily the scaling matrix as $\theta_{11} = 1, \theta_{22} = 0.5, \theta_{33} = -1, \theta_{33} = -0.5$. The numerical simulation is shown in Fig.2.
The state trajectories for the behavior of the error system is illustrated in Fig.4, which shows the error vectors converge to zero in infinite time with the control law. Fig.3 illustrates the estimated values of the unknown parameter tend to $\alpha_1 = 10, \alpha_2 = 28, \alpha_3 = 8/3, \alpha_3 = -1$, $\beta_1 = 35, \beta_2 = 7, \beta_3 = 12, \beta_4 = 3, \beta_5 = 0.3$, and $\delta_1 = 36, \delta_2 = 20, \delta_3 = 3, \delta_4 = -1$ as time goes to infinity, respectively.

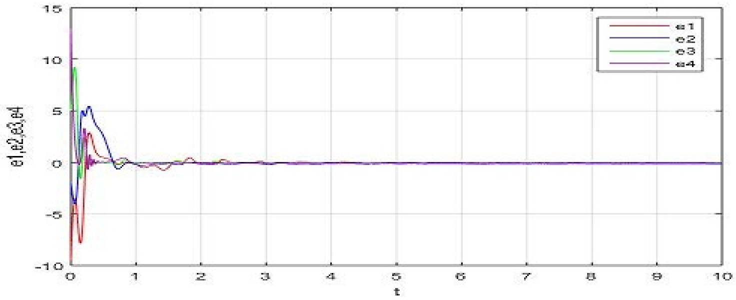
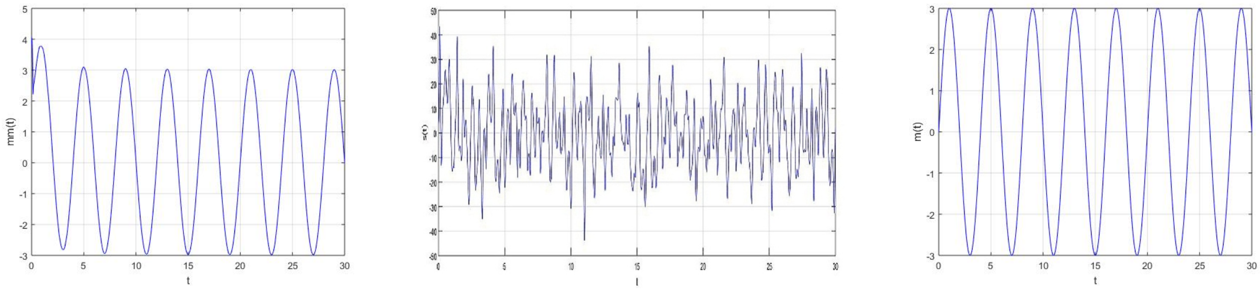


Fig. 4: State trajectories of the synchronization error system.



(1) (2) (3)

Fig. 5: (1) the original message signal. (2) the encrypted signal. (3) decrypted signal.

5 Application to secure communication

In this section, we apply the proposed (AMPCS) to secure communication. An original message signal $m(t) = 3 * \sin(\pi t/2)$ is mixed with master signals $x_1 + y_1$ and encrypted as $s(t)$. $s(t)$ is attached with slave signal. The decrypted message signal is given by $mm(t) = s(t) - z_1$. The original message signal is recovered by performing the desired adaptive modified projective combination synchronization on applying controllers at the receiving end. Fig.5 (1), (2), (3) gives the original message signal, the encrypted signal and decrypted signal respectively.

6 Conclusions

(AMPCS) of fractional-order hyperchaotic systems with uncertain parameters are studied in the present article. By utilizing a diagonal matrix, Lyapunov stability theory, adaptive control and some techniques of fractional calculus. This technique applies particularly to almost all fractional-order chaotic and hyper-chaotic systems. Furthermore, various kinds of synchronization could be found by this technique such as projective combination synchronization, anti combination synchronization and complete combination synchronization. The suggested scheme of (AMPCS) will be helpful for a specialist during their practical applications in the field of secure communication and cryptography. In the future direction, we can investigate another control for improving the modified projective combination synchronization technique.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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