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The results of the functions obtained from class K by using algebraic operations

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Abstract: Functions selected from convex functions complex number of analytical and univalent conditions and unit D disk $|z| < 1$ was used. Whether the new function obtained by addition process is convex function is examined. Although $f(z) \in K$ and $g(z) \in K$ are convex function which is shown to be $f(z)+g(z) \notin K$, it is proved that the sums of two functions are not convex functions.

Keywords: Convex function, complex numbers, unit disk.

1 Introduction

It is known that $f(z)$ and $g(z)$ doesn't have value with single in $f(z)$ and $g(z)$ if provides a one-to-one match in domain D. In geometrical, the image shown in the complex plane can be visualized as a viable means a set of points. Function $f(z)$ and $g(z)$ is in D domain if the conditions $f(z_1) = f(z_2)$ and $g(z_1) = g(z_2)$, $z_1 \in D$ and $z_2 \in D$ so that $z_1 = z_2$ it is said to be univalent. Very simple and complex variable function theory would require some simple assumptions.

Definition 1. $D = \{z \in \mathbb{C} : |z| < 1\}$*in the unit disk , if analytic function* $f(z)$ *provides conditions* $f(0) = 0$, $f'(0) = 1$, *then*

$$f(z) = z+\sum_{n=2}^{\infty} a_n z^n$$

has a Taylor expansion. Such class of functions is indicated by class S. In the Class S,

$$k(z) = \frac{z}{(1-z^2)^2} = z+2z^2+3z^3+\ldots = \sum_{n=1}^{\infty} nz^n$$

the function shown in the form is called the Koebe function. This function converts disk D one-to-one $\mathbb{C}-(-\infty,-\frac{1}{4}]$ *on the region.* [4]

Definition 2. *Let B be an area in the complex plane. The one-to-one function* f *in B is called an univalent function and is selected* z_1 , z_2 *must provide the condition* $f(z_1) \neq f(z_2)$.

Definition 3. f, *complex variable and complex valued function is defined in a neighborhood of point* $z_0 \in \mathbb{C}$. *If*

$$\lim_{z\to z_0} \frac{f(z)-f(z_0)}{z-z_0}.$$

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This function can be distinguished from the so-called point z_0. If $f(z)$ is differentiated in the neighborhood of point z_0, its called analytic functions and it is show with A be differentiated in the point z_0. If $f(z)$ is differentiated in the point z_0 with the neighborhood, its called analytic functions and it is show with A [3].

Theorem 1. *$f : D \to \mathbb{C}$ be an analytic function. If $f(z)$ necessary and sufficient condition for being convex function $f'(0) = 1$ and*

$$\mathrm{Re}\left\{1+\frac{zf''(z)}{f'(z)}\right\} > 0. [3].$$

Theorem 2. *If $f(z)$ is the analytic of a suitable neighborhood of the point z_0, then*

$$f(z) = f(z_0)+\frac{f'(z_0)}{1!}(z-z_0)+\frac{f''(z_0)}{2!}(z-z_0)^2+\ldots$$

it is shown.

Main Theorem. $f(z) \in K$ and $g(z) \in K$ are convex function which is shown to be $f(z)+g(z) \notin K$, it is proved that the sums of two functions are not convex functions.

Proof. First of all we choose a function and $|z| < 1$, let's examine the convexity of this function. If we make a Taylor expansion for function $f(z) = \frac{z}{1-z}$.

$$f(z) = z+z^2+z^3+z^4+\ldots = z+\sum_{n=2}^{\infty} z^n.$$

$$f'(z) = \frac{1.(1-z)-z.(-1)}{(1-z)^2} = \frac{1}{(1-z)^2} = 1+2z+3z^2+4z^3+\ldots.$$

$$f''(z) = \frac{-1.(-1).2.(1-z)}{[(1-z)^2]^2} = \frac{2}{(1-z)^3} = 2+6z+12z^2+\ldots.$$

Then $f'(0) = 1$ and $f(0) = 0$ so function $f(z)$ is in class A and analytic. Now let's examine the univalent of this function. For $z_1 \neq z_2$, while, $f(z_1) \neq f(z_2)$ it is univalent. From here

$$\begin{aligned} z_1+\sum_{n=2}^{\infty} z_1^n-\left(z_2+\sum_{n=2}^{\infty} z_2^n\right) &= z_1-z_2+\sum_{n=2}^{\infty} z_1^n-\sum_{n=2}^{\infty} z_2^n \\ &= z_1-z_2+z_1^2-z_2^2+z_1^3-z_2^3+z_1^4-z_2^4+\ldots \\ &= (z_1-z_2)+(z_1-z_2)(z_1+z_2)+(z_1-z_2)\left(z_1^2+z_1z_2-z_2^2\right)+\ldots \\ &= (z_1-z_2)\left[1+(z_1+z_2)+\left(z_1^2+z_1z_2-z_2^2\right)+\ldots\right] \end{aligned}$$

for $z_1 \neq z_2$ we take

$$1+(z_1+z_2)+\left(z_1^2+z_1z_2-z_2^2\right)+\ldots \neq 0.$$

The function $f(z)$ is class A and because it is univalent, it provides properties of class S. Now, let's examine the convexity requirement. If $\mathrm{Re}\left\{1+\frac{zf''(z)}{f'(z)}\right\} > 0$. It would have a function that allows the processing function from the class K and convex.

$$\mathrm{Re}\left\{1+\frac{z\left(2+6z+12z^2+\ldots\right)}{1+2z+3z^2+4z^3+\ldots}\right\} = \frac{1+2z+3z^2+4z^3+\ldots+2z+6z^2+12z^3+\ldots}{1+2z+3z^2+4z^3+\ldots}.$$

If we do a simple division operation

$$1+\frac{2z+2z^2+2z^3+...}{1+2z+3z^2+4z^3+...}>0.$$

So we obtained $f(z)\in K$. Now we choose a different function $g(z)$ and $|z|<1$, let's examine the convexity of this function

$$g(z)=\frac{z(-z-1)}{z-1}.$$

If we make a Taylor expansion for function

$$g(z)=z+2z^2+2z^3+2z^4+...=z+\sum_{n=2}^{\infty}2z^n.$$

Here in $g(z)$ function $g'(z)=\frac{(2z+1)(1-z)-(z^2+z)(-1)}{(1-z)^2}=\frac{-z^2+2z+1}{(1-z)^2},=1+4z+6z^2+8z^3+...,g''(z)=\frac{4}{(1-z)^3}=4+12z+24z^2+40z^3...$. Then $g'(0)=1$ and $g(0)=0$ so function $g(z)$ is in class A and it is analytic. Now we need to show $g(z)$ is univalent in unit disk D. For $z_1\neq z_2$, while, $g(z_1)\neq g(z_2)$ it is univalent. From here if

$$z_1\neq z_2,$$

so $g(z_1)-g(z_2)\neq 0$ and $g(z)$ is univalent. Then

$$\begin{aligned} z_1+\sum_{n=2}^{\infty}2z_1^n-\left(z_2+\sum_{n=2}^{\infty}2z_2^n\right) &= z_1-z_2+\sum_{n=2}^{\infty}2z_1^n-\sum_{n=2}^{\infty}2z_2^n \\ &= z_1-z_2+2z_1^2-2z_2^2+2z_1^3-2z_2^3+2z_1^4-2z_2^4+... \\ &= (z_1-z_2)\left[1+2(z_1+z_2)+2\left(z_1^2+z_1z_2-z_2^2\right)+...\right] \end{aligned}$$

for $z_1-z_2\neq 0$. We obtained $1+2(z_1+z_2)+2\left(z_1^2+z_1z_2-z_2^2\right)+...\neq 0$. The function $g(z)$ is class A and because it is univalent, it provides properties of class S. Now, let's examine the convexity requirement.

$$\mathrm{Re}\left\{1+\frac{zg''(z)}{g'(z)}\right\}>0$$

would have a function that allows the processing function from the class K and convex.

$$\mathrm{Re}\left\{1+\frac{z\left(4+12z+24z^2+40z^3...\right)}{1+4z+6z^2+8z^3+...}\right\}=\frac{1+4z+6z^2+8z^3+...+4z+12z^2+24z^3+40z^4...}{1+4z+6z^2+8z^3+...}.$$

If we do a simple division operation

$$1+\frac{4z-4z^2}{1+4z+6z^2+8z^3+...}>0.$$

We obtained that $g(z)\in K$.

Now we need to prove that $f(z)\in K$ and $g(z)\in K$ but $f(z)+g(z)\notin K$ is not in class K and it is not a convex function. It is enough to show that the sum of the given functions is not in the class K and it is not convex function. So

$$f(z)+g(z)=\frac{z}{1-z}+\frac{(-z-1)z}{z-1}=\frac{2z+z^2}{1-z}.$$

If $f(z)+g(z)=\frac{2z+z^2}{1-z}=h(z)$ from taylar expansion we take $h(z)=\frac{2z+z^2}{1-z}=2z+3z^2+3z^3+3z^4+3z^5+\ldots$. If we take the derivative

$$h^{'}(z)=\frac{-z^2+2z+2}{(1-z)^2}=2z+3z^2+3z^3+3z^4+3z^5+\ldots.$$

$h(0)=0$ but $h^{'}(0)=2\neq 1$. Therefore, it does not provide convexity conditions.We proved $h(z)\notin A$ and $h(z)\notin K$.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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