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On double framed B(T) soft Fuzzy ideals and doubt double framed soft Fuzzy algebras of BF-algebras

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Abstract: In this article, we aim to consider a new kind of double framed B(T)-soft fuzzy ideal (briefly, DFB(T)SF-deal), doubt double framed T-soft fuzzy-algebra (briefly, DDF-soft fuzzy-algebra) and doubt double framed T-soft fuzzy ideal (briefly, DDF-soft fuzzy-ideal) of BF-algebra over a $(U, [0, 1])$. The notions of DF soft fuzzy-deals, DDF-soft fuzzy-algebras and DDF-soft fuzzy-ideals are defined and their basic properties are investigated. With the support of examples, these notions are illustrated. Further, in DF-soft fuzzy-ideal the union product are studied and proved that their union product is also a DF-soft fuzzy-deal. Similarly, this property is also illustrated in DDF-soft fuzzy-algebra and DDF-soft fuzzy-ideal by using intersection-product. Furthermore, in this article the results which are proved for DDF-soft fuzzy-algebra of BF-algebra also satisfied for DDF-soft fuzzy-algebra of BCI/BCI-algebra except the concept of ideals of BF-algebra.

Keywords: Double framed soft fuzzy set, Double framed T(B)-soft fuzzy –algebra(ideal), doubt double framed T-soft fuzzy - algebra, doubt double framed T-soft fuzzy -ideal.

1 Introduction

In our daily life, the graph of uncertainty is very high in different fields like engineering, economics and in decision making problem. If we discussed about the classical mathematical tool which dealt with uncertainty is crisp set theory but with the passage of time uncertainty graph is increased. Then crisp theory [1] was not enough to handle this problem. At last fuzzy theory is introduced by L. A. Zadeh [2] in 1965 to handle vague concepts. This theory solved this problem and then it is used in many fields of our life like engineering, pharmacology, medical application and economics, among others. This is also affected by uncertainty because this concept based on membership function. We know that, sometime we feel hesitation to give membership value to any element which belong to fuzzy set. To overcome this problem, in 1999 Moldstov [3] introduced new concept which is known as soft set theory. He defined its applications in different areas like smoothness of function, measure integration, Riemannian integration, game theory and economics, among others.

After introduction of fuzzy theory and soft theory, now we discussed about the work of those researchers whose accepted this challenging theory and applied this concept in different algebraic structures like first time, Maji [4, 5] discussed this concept in decision making problem and combined the both concepts fuzzy set and soft set, and investigated their properties by giving examples. Cagman et al. [6] initiated to introduce this concept in group theory and discussed comparison between soft set, fuzzy set and rough sets and investigated its properties. Ali et.al [7] introduced some new operations on soft sets. These contradicted the some operations which are defined by Maji [5]. Jun and Ahn [8] initiated to introduce the concept of double framed soft and defined its notions. Further, they also discussed the product of double framed soft BCI/BCK-algebras. Furthermore, they studied the concept of double framed soft near ring. They also

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discussed major results related to this concept with the support of examples [9]. Naz [10] defined some operations on double framed double soft sets and introduced its notions. She also studied properties of BCI/BCK-algebras with the help of basic operations of double framed soft sets. Hadipour [11] applied the concept of double framed soft set in BF-algebras. Also the notations of double framed BF-algebras are investigated by examples. Shabir and Samreena [12], worked on double framed soft topological spaces and defined its notations. Bilal and Mahmood [13] initiated to introduce the concept of double framed T-soft fuzzy set and then concept is applied in BCI/BCI-algebra and investigated its properties to understand this concept. Further, they also discussed its union-product and defined its notions. Barbhuiya [14] introduced the doubt fuzzy ideal of BF-algebra and studied its properties like the major result in this article, Cartesian product of doubt fuzzy ideals also a doubt fuzzy ideal. For further information, we refer to reader papers [15-25] regarding soft and fuzzy algebraic structures.

In this paper, we applied the concept of double framed T-soft fuzzy set in BF-algebra to introduce DFSF-algebra, DFSF-deal, DDFSF-algebra and DDFSF-deal. Furthermore, we discussed union-product, intersection-product in DFSF-deal and DDFSF-deal, respectively.

2 Preliminaries

Let X be a non-empty set. Then $A = \{<\sigma, \mu_A(\sigma)> | \sigma \in X\}$ is called fuzzy set, where μ_A is a membership function which map each element of X in $[0,\ 1]$. Here we say that A is fuzzy subset of X. The set of all fuzzy subsets of a set X is denoted by $FP(X)$. Two fuzzy sets A and D are equal if and only if $\mu_A(\sigma) = \mu_D(\sigma)$ for all $\sigma \in X$. Where μ_A and μ_D are membership functions which map each element of X in $[0,\ 1]$.

A pair $(\mathscr{T}, E)$ is indicated to be soft set over if and only if $\mathscr{T}$ is a mapping from E to the all subset of U e.g

$$(\mathscr{T}, E) = \{(e,\ \mathscr{T}(e)) \mid e \in E \ and\ \mathscr{T}(e) \in P(U)\}$$

Where $P(U)$ is power set of U and $\mathscr{T}: E \to P(U)$. The function $\mathscr{T}$ is an approximation function of the soft set $(\mathscr{T}, E)$. It is easy to see that soft set is parameterized family of subsets of U.The set of all soft sets over U is denoted by $S(U)$.

A non-empty set X under binary operation "$\star$" with constant Θ is called a BF-algebra of type $(2,\ \Theta)$ is denoted by $(X,\ \star,\ \Theta)$ and defined as

$$for\ all\ \sigma,\ \eta \in X \begin{cases} \sigma \star \sigma = \Theta \\ \sigma \star \Theta = \sigma \\ \Theta \star (\sigma \star \eta) = \eta \star \sigma. \end{cases}$$

Definition 1. [8]. *A double framed pair $(\mathscr{T},\ \mathfrak{G})$ of X over U is said to double framed soft set, where $(\mathscr{T},\ \mathfrak{G})$ both are soft sets over the same universe U and denoted by $((\mathscr{T},\ \mathfrak{G}), X)$ over U.*

A $((\mathscr{T},\ \mathfrak{G}), X)$ over U is said to be double framed soft BCI/BCK-algebra if it satisfies $\mathscr{T}(\sigma \star \eta) \geq \mathscr{T}(\sigma) \cap \mathscr{T}(\eta)$ and $\mathfrak{G}(\sigma \star \eta) \subseteq \mathfrak{G}(\sigma) \cup \mathfrak{G}(\eta)$ for all $\sigma,\ \eta \in A$.

Definition 2. [13]. *A double framed pair $(\mathscr{T},\ \mu)$ of X over $(U,\ [0,\ 1])$ is said to DF-soft fuzzy set, where in $(\mathscr{T},\ \mu)$, $\mathscr{T}$ is a soft set over U, μ is fuzzy set over $[0,\ 1]$ and denoted by $((\mathscr{T},\ \mu), X)$ over $(U,\ [0,\ 1])$.*

Definition 3. [13]. *Let $((\mathscr{T}, \mu), A)$ and $((\mathfrak{G}, g), D)$ be two "DF-soft fuzzy set" over $(U,\ [0,\ 1])$ then $((\mathscr{T}, \mu), A)$ is said to be DF-soft fuzzy subset of $((\mathfrak{G}, g), D)$ if*

(1) $A \subseteq D$,

(2) $\mathscr{T}(a) \subseteq \mathfrak{G}(a)$ *and* $\mu(a) \geq g(a)$ *for all* $a \in A$.

We can write $((\mathscr{T},\mu),A)\widetilde{\widetilde{\sqsubset}}((\mathfrak{G},g),D)$. *In case* $((\mathscr{T},\mu),A)$ *is super set of* $((\mathfrak{G},g),D)$.

Definition 4. [13]. *Let* $((\mathscr{T},\mu),V)$ *and* $((\mathfrak{G},g),D)$ *are two "DF-soft fuzzy set" over* $(U,[0,1])$. *Then extended uni-int of* $((\mathscr{T},\mu),V)$ *and* $((\mathfrak{G},g),D)$ *is defined as a "DF-SS"* $\left(\left(\mathscr{T}\widetilde{\cup}\mathfrak{G},\ \mu\overline{\cap}g\right),V\cup D\right)$,*where* $\mathscr{T}\widetilde{\cup}\mathfrak{G}:(V\cup D)\longrightarrow P(U)$ *defined by*

$$p \longrightarrow \begin{cases} \mathscr{T}(p) \ if \ p \in V-D \\ \mathfrak{G}(p) \ if \ p \in D-V \\ \mathscr{T}(p)\cup\mathfrak{G}(p) \ if \ p \in V\cap D \end{cases}$$

and $\mu\overline{\cap}g:(V\cup D)\longrightarrow[0,1]$ *defined by*

$$p \longrightarrow \begin{cases} \mu(p) \ if \ p \in V-D \\ g(p) \ if \ p \in D-V \\ \mu(p)\wedge g(p) \ if \ p \in V\cap D. \end{cases}$$

It is denoted by $((\mathscr{T},\mu),V)\overline{\sqcup_{\mathscr{E}}}((\mathfrak{G},g),D)=\left(\left(\mathscr{T}\widetilde{\cup}\mathfrak{G},\ \mu\overline{\cap}g\right),V\cup D\right)$. *We shall call this uni-int "DF-soft fuzzy set" over* $(U,[0,1])$ *as union of "DF-soft fuzzy set" over* $(U,[0,1])$.

Definition 5. [13]. *Let* $((\mathscr{T},\mu),V)$ *and* $((\mathfrak{G},g),D)$ *are two "DFSFS" over* $(U,[0,1])$. *Then extended int-uni of* $((\mathscr{T},\mu),V)$ *and* $((\mathfrak{G},g),D)$ *is defined as a "DFSFS"* $\left(\left(\mathscr{T}\widetilde{\cap}\mathfrak{G},\ \mu\overline{\cup}g\right),V\cup D\right)$*over* $(U,[0,1])$, *where* $\mathscr{T}\widetilde{\cap}\mathfrak{G}:(V\cup D)\longrightarrow P(U)$ *defined by*

$$p \longrightarrow \begin{cases} \mathscr{T}(p) \ if \ p \in V-D \\ \mathfrak{G}(p) \ if \ p \in D-V \\ \mathscr{T}(p)\cap\mathfrak{G}(p) \ if \ p \in V\cap D \end{cases}$$

and $\mu\overline{\cup}g:(V\cup D)\longrightarrow[0,1]$ *defined by*

$$p \longrightarrow \begin{cases} \mu(p) \ if \ p \in V-D \\ g(p) \ if \ p \in D-V \\ \mu(p)\vee g(p) \ if \ p \in V\cap D \end{cases}$$

It is denoted by $((\mathscr{T},\mu),V)\overline{\sqcap_{\mathscr{E}}}((\mathfrak{G},g),D)=\left(\left(\mathscr{T}\widetilde{\cap}\mathfrak{G},\ \mu\overline{\cup}g\right),V\cup D\right)$. *We shall call this int-uni "DF-soft fuzzy set" over* $(U,[0,1])$ *as intersection of DF- soft fuzzy set over* $(U,[0,1])$.

Definition 6. [13]. *Let* $((\mathscr{T},\mu),A)$ *and* $((\mathscr{T},\mu),D)$ *be "DF-soft fuzzy set" over* $(U,[0,1])$. *Then their product is defined as*

$$\mathscr{T}_{A\vee D}: A\times D\longrightarrow P(U),$$

$$\mu_{A\wedge D}: A\times D\longrightarrow[0,1]$$

such that $(\sigma,\eta)\longrightarrow\begin{cases}\mathscr{T}(\sigma)\cup\mathscr{T}(\eta)\\ \mu(\sigma)\wedge\mu(\eta)\end{cases}$ *, and denoted by* $((\mathscr{T}_{\vee},\mu_{\wedge}),A\times D)$.

3 Related results

This section is divided in to two sub sections; we proposed the concept of DFSF-algebra, DFSF-deal in 1^{st} and in 2^{nd}, DFSF-algebra and DFSF-deal. Also, we investigated its properties with the support of examples.

For brevity we also call X a BF-algebra. A binary relation '$\leq$'on X can be defined by $\sigma\leq\eta$ if and only if $\sigma\star\eta=\Theta$

3.1 Double framed B(T)-soft Fuzzy-algebra and double framed B(T)-soft Fuzzy-ideals

In this subsection, we proposed the concept of DF-soft fuzzy-algebra and DF-soft fuzzy-ideal.

Definition 7. *A DF-soft fuzzy set* $((\mathscr{T},\mu),X)$ *over* $(U,\ [0,\ 1])$ *is said to be double framed B-soft fuzzy algebra of BF-algebra X if*

for all $\sigma,\eta \in X$ $\begin{cases} \mathscr{T}(\sigma\star\eta) \subseteq \mathscr{T}(\sigma)\cup\mathscr{T}(\eta) \\ \mu(\sigma\star\eta) \geq \mu(\sigma).\mu(\eta). \end{cases}$

Example 1. Let $U = \{\sigma_1,\ \sigma_2,\ \sigma_3,\ \sigma_4\}$ be an initial universe set and $X = \{p_\Theta,\ p_2,\ p_3,\ p_4\}$ be a set of parameters with following binary operation:

$\star$	p_Θ	p_2	p_3	p_4
p_Θ	p_Θ	p_2	p_3	p_4
p_2	p_2	p_Θ	p_2	p_2
p_3	p_3	p_2	p_Θ	p_2
p_4	p_4	p_2	p_2	p_Θ

Then $(X,\ \star,\ p_a)$ is a BF-algebra. Consider the DF-fuzzy soft set as follows:

$$\mathscr{T} : X \to P(U)$$

$$e \mapsto \begin{cases} \{\sigma_1,\ \sigma_2\} & \text{if } e \in \{p_\Theta,\ p_2\} \\ U & \text{if } e \in \{p_3,\ p_4\} \end{cases}$$

and $\mu : X \to [0,\ 1]$

$$e \mapsto \begin{cases} .5 & \text{if } e \in \{p_\Theta,\ p_2\} \\ .7 & \text{if } e \in \{p_3,\ p_4\} \end{cases}$$

It is routine to verify that $((\mathscr{T},\mu),X)$ over $(U,\ [0,\ 1])$ is a double framed B-soft fuzzy BF-algebra.

Definition 8. *A DF-soft fuzzy set* $((\mathscr{T},\mu),X)$ *over* $(U,\ [0,\ 1])$ *is said to be DFB-soft fuzzy sub algebra of BF-algebra X if for all* $\sigma,\eta \in X$

$$\begin{cases} \mathscr{T}(\sigma\star\eta) \subseteq \mathscr{T}(\sigma)\cup\mathscr{T}(\eta) \\ \mu(\sigma\star\eta) \geq \mu(\sigma).\mu(\eta). \end{cases}$$

Definition 9. *A DF-soft fuzzy set* $((\mathscr{T},\mu),X)$ *over* $(U,\ [0,\ 1])$ *is said to be double framed B-soft fuzzy ideal of X if for all* $\sigma,\eta \in X$

$$\begin{cases} \mathscr{T}(\Theta) \subseteq \mathscr{T}(\sigma) \text{ and } \mathscr{T}(\sigma) \subseteq \mathscr{T}(\sigma\star\eta)\cup\mathscr{T}(\eta) \\ \mu(\Theta) \geq \mu(\sigma)^2 \text{ and } \mu(\sigma) \geq \mu(\sigma\star\eta)\mu(\eta). \end{cases}$$

In Example 1, if we take

$$\mathscr{T} : X \to P(U)\, e \mapsto \begin{cases} \{\sigma_2\} & \text{if } e \in \{p_\Theta,\ p_2\} \\ \{\sigma_1,\ \sigma_2\} & \text{if } e \in \{p_3,\ p_4\} \end{cases}.$$

And $\mu : X \to [0,\ 1]\ e \mapsto \begin{cases} 0.4 & \text{if } e \in \{p_\Theta,\ p_2\} \\ 0.6 & \text{if } e \in \{p_3,\ p_4\} \end{cases}$. Then$((\mathscr{T},\mu),X)$ is a subalgebra. We note that $((\mathscr{T},\mu),X)$ over $(U,\ [0,\ 1])$ is also a DFB-soft fuzzy ideal.

Definition 10. *A DF-soft fuzzy set* $((\mathscr{T},\mu),X)$ *over* $(U,\ [0,\ 1])$ *is said to be DFT-soft fuzzy algebra of BF-algebra* X *if for all* $\sigma,\eta\in X$

$$\begin{cases} \mathscr{T}(\sigma\star\eta)\subseteq\mathscr{T}(\sigma)\cup\mathscr{T}(\eta) \\ \mu(\sigma\star\eta)\geq\mu(\sigma)\wedge\mu(\eta). \end{cases}$$

Example 2. Let $U=\{\sigma_1,\ \sigma_2,\ \sigma_3\}$ be an initial universe set and $X=\{p_\Theta,\ p_b,\ p_c\}$ be a set of parameters with following binary operation:

$\star$	p_Θ	p_b	p_c
p_Θ	p_Θ	p_b	p_c
p_b	p_b	p_Θ	p_Θ
p_c	p_c	p_Θ	p_Θ

Then $(X,\ \star,\ p_\Theta)$ is a BF-algebra. Consider the DF-fuzzy soft set as follows:

$$\mathscr{T}:X\to P(U)$$

$$e\mapsto\begin{cases} \{\sigma_1\} & \text{if } e{=}p_\Theta \\ \{\sigma_1,\ \sigma_2\} & \text{if } e\in\{p_b,\ p_c\} \end{cases}.$$

And $\mu:X\to[0,\ 1]$

$$e\mapsto\begin{cases} .5 & \text{if } e{=}p_\Theta \\ .4 & \text{if } e\in\{p_b,\ p_c\} \end{cases}$$

It is easy to see that $((\mathscr{T},\mu),X)$ over $(U,\ [0,\ 1])$ is a DFT-soft fuzzy BF-algebra. Note that, it is easy to see that all results which are proved in article [13] are satisfied if we replace X (BCI/BCK-algebra) as a BF-algebra than BCI/BCK algebra.

Definition 11. *A DF-soft fuzzy set* $((\mathscr{T},\mu),X)$ *over* $(U,\ [0,\ 1])$ *is said to be DFT-soft fuzzy sub algebra of BF-algebra* X *if for all* $\sigma,\eta\in X$

$$\begin{cases} \mathscr{T}(\sigma\star\eta)\subseteq\mathscr{T}(\sigma)\cup\mathscr{T}(\eta) \\ \mu(\sigma\star\eta)\geq\mu(\sigma)\wedge\mu(\eta). \end{cases}$$

Definition 12. *A DF-soft fuzzy set* $((\mathscr{T},\mu),X)$ *over* $(U,\ [0,\ 1])$ *is said to be DFT-soft fuzzy ideal of* X *if for all* $\sigma,\eta\in X$

$$\begin{cases} \mathscr{T}(\Theta)\subseteq\mathscr{T}(\sigma)\ \text{and}\ \mathscr{T}(\sigma)\subseteq\mathscr{T}(\sigma\star\eta)\cup\mathscr{T}(\eta) \\ \mu(\Theta)\geq\mu(\sigma)\ \text{and}\ \mu(\sigma)\geq\mu(\sigma\star\eta)\wedge\mu(\eta). \end{cases}$$

Proposition 1. *If* $((\mathscr{T},\mu),X)$ *over* $(U,\ [0,\ 1])$ *be a DFT-soft fuzzy ideal of* X *then following hold,*

(a) *if* $\sigma\leq\eta$ *then* $\begin{cases} \mathscr{T}(\sigma)\subseteq\mathscr{T}(\eta) \\ \mu(\sigma)\geq\mu(\eta), \end{cases}$

(b) *if* $\mathscr{T}(\sigma\star\eta)=\mathscr{T}(\Theta)$ *and* $\mu(\sigma\star\eta)=\mu(\Theta)$ *then* $\begin{cases} \mathscr{T}(\sigma)\subseteq\mathscr{T}(\eta) \\ \mu(\sigma)\geq\mu(\eta), \end{cases}$

(c) *if* $\sigma\star\eta\leq w$ *then* $\begin{cases} \mathscr{T}(\sigma)\subseteq\mathscr{T}(\eta)\cup\mathscr{T}(w) \\ \mu(\sigma)\geq\mu(\eta)\wedge\mathscr{T}(w), \end{cases}$

For all $\sigma,\eta,\ w\in X$.

Proof. (a) Let $\sigma \leq \eta$ then $\sigma \star \eta = \Theta$. Now by definition of $((\mathscr{T},\mu),X)$ such that $\mathscr{T}(\sigma) \supseteq \mathscr{T}(\sigma\star\eta) \cup \mathscr{T}(\eta)$ and $\mu(\sigma) \geq \mu(\sigma\star\eta) \wedge \mu(\eta)$ then $\mathscr{T}(\sigma) \subseteq \mathscr{T}(\Theta) \cup \mathscr{T}(\eta)$, $\mu(\sigma) \geq \mu(\Theta) \wedge \mu(\eta)$ implies that $\mathscr{T}(\sigma) \subseteq \mathscr{T}(\eta)$ and $\mu(\sigma) \geq \mu(\eta)$.

(b) Let $\mathscr{T}(\sigma\star\eta) = \mathscr{T}(\Theta)$ *and* $\mu(\sigma\star\eta) = \mu(\Theta)$. Then by definition of $(\mathscr{T},\mu),X)$, we have $\mathscr{T}(\sigma) \subseteq \mathscr{T}(\sigma\star\eta) \cup \mathscr{T}(\eta)$ and $\mu(\sigma) \geq \mu(\sigma\star\eta) \wedge \mu(\eta)$implies that $\mathscr{T}(\sigma) \subseteq \mathscr{T}(\Theta) \cup \mathscr{T}(\eta)$, $\mu(\sigma) \geq \mu(\Theta) \wedge \mu(\eta)$ implies that $\mathscr{T}(\sigma) \subseteq \mathscr{T}(\eta)$ and $\mu(\sigma) \geq \mu(\eta)$. (c) By definition of $((\mathscr{T},\mu),X)$, we have

$$\mathscr{T}(\sigma) \subseteq \mathscr{T}(\sigma\star\eta) \cup \mathscr{T}(\eta) \quad \textit{and} \quad \mu(\sigma) \geq \mu(\sigma\star\eta) \wedge \mu(\eta)\ldots \tag{1}$$

Let $\sigma\star\eta \leq w$ then $(\sigma\star\eta)\star w = \Theta$, $\mathscr{T}(\sigma\star\eta) \subseteq w$ and $\mu(\sigma\star\eta) \geq \mu(w)$. Now by following (1), we have

$\mathscr{T}(\sigma\star\eta) \subseteq \mathscr{T}((\sigma\star\eta)\star w) \cup \mathscr{T}(w)$ and $\mu(\sigma\star\eta) \geq \mu((\sigma\star\eta)\star w) \wedge \mu(w)$.

Implies $\mathscr{T}(\sigma\star\eta) \subseteq \mathscr{T}(\Theta) \cup \mathscr{T}(w)$ and $\mu(\sigma\star\eta) \geq \mu(\Theta) \wedge \mu(w)$

Implies $\mathscr{T}(\sigma\star\eta) \subseteq \mathscr{T}(w)$ and $\mu(\sigma\star\eta) \geq \mu(w)$

$$\text{Implies} \quad \mathscr{T}(\sigma\star\eta) \cup \mathscr{T}(\eta) \subseteq \mathscr{T}(\eta) \cup \mathscr{T}(w) \quad \textit{and} \quad \mu(\sigma\star\eta) \wedge \mu(\eta) \geq \mu(w) \wedge \mu(\eta). \tag{2}$$

From (1) and (2), we have $\mathscr{T}(\sigma) \subseteq \mathscr{T}(\eta) \cup \mathscr{T}(w)$ and $\mu(\sigma) \geq \mu(\eta) \wedge \mathscr{T}(w)$.

Theorem 1. *Let $((\mathscr{T},\mu),X)$ and $((\mathfrak{G},g),X)$ over $(U,\ [0,\ 1])$ be two DFT-soft fuzzy ideals of X then prove that their intersection is also a DFT-soft fuzzy ideal of X.*

Proof. Let $((\mathscr{T},\mu),X)$ and $((\mathfrak{G},g),X)$ over $(U,\ [0,\ 1])$ be two DF-soft fuzzy ideals of X and $\sigma,\eta \in X$ then

$$\begin{cases} \mathscr{T}(\Theta) \subseteq \mathscr{T}(\sigma) \textit{ and } \mathscr{T}(\sigma) \subseteq \mathscr{T}(\sigma\star\eta) \cup \mathscr{T}(\eta) \\ \mu(\Theta) \geq \mu(\sigma) \textit{ and } \mu(\sigma) \geq \mu(\sigma\star\eta) \wedge \mu(\eta) \end{cases} \text{and} \begin{cases} \mathfrak{G}(\Theta) \subseteq \mathfrak{G}(\sigma) \textit{ and } \mathfrak{G}(\sigma) \subseteq \mathfrak{G}(\sigma\star\eta) \cup \mathfrak{G}(\eta) \\ g(\Theta) \geq g(\sigma) \textit{ and } g(\sigma) \geq g(\sigma\star\eta) \wedge g(\eta). \end{cases}$$

Now, union of $((\mathscr{T},\mu),X)$ and $((\mathfrak{G},g),X)$ is defined as $((\mathscr{T},\mu),X) \sqcup_{\mathscr{E}} ((\mathfrak{G},g),X) = ((H,\ h),\ X)$ where $H = \mathscr{T}\tilde{\cup}\mathfrak{G}$ and $h = \mu\bar{\cap}g$.

It is given that $\mathscr{T}(\Theta) \subseteq \mathscr{T}(\sigma)$, $\mu(\Theta) \geq \mu(\sigma)$ and $\mathfrak{G}(\Theta) \subseteq \mathfrak{G}(\sigma)$, $g(\Theta) \geq g(\sigma)$ then

$\mathscr{T}(\Theta) \cup \mathfrak{G}(\Theta) \subseteq \mathscr{T}(\sigma) \cup \mathfrak{G}(\sigma)$ and $\mu(\Theta) \wedge g(\Theta) \geq \mu(\sigma) \wedge g(\sigma)$,

$\Longrightarrow \mathscr{T}(\Theta) \cup \mathfrak{G}(\Theta) \subseteq \mathscr{T}(\sigma) \cup \mathfrak{G}(\Theta)$ and $\mu(\Theta) \wedge g(\Theta) \geq \mu(\sigma) \wedge g(\sigma)$

$\Longrightarrow \left(\mathscr{T}\tilde{\cup}\mathfrak{G}\right)(\Theta) \subseteq \left(\mathscr{T}\tilde{\cup}\mathfrak{G}\right)(\sigma)$ and $(\mu\bar{\cap}g)(\Theta) \geq (\mu\bar{\cap}g)(\sigma)$

Implies $H(\Theta) \subseteq H(\sigma)$ and $h(\Theta) \geq h(\sigma)$.

Further, $\mathscr{T}(\sigma) \subseteq \mathscr{T}(\sigma\star\eta) \cup \mathscr{T}(\eta)$, $\mu(\sigma) \geq \mu(\sigma\star\eta) \wedge \mu(\eta)$ and $\mathfrak{G}(\sigma) \subseteq \mathfrak{G}(\sigma\star\eta) \cup \mathfrak{G}(\eta)$, $g(\sigma) \geq g(\sigma\star\eta) \wedge g(\eta)$, we

$$\begin{aligned} &\mathscr{T}(\sigma) \cup \mathfrak{G}(\sigma) \subseteq (\mathscr{T}(\sigma\star\eta) \cup \mathscr{T}(\eta)) \cup (\mathfrak{G}(\sigma\star\eta) \cup \mathfrak{G}(\eta)) \quad \textit{and} \quad \mu(\sigma) \wedge g(\sigma) \geq (\mu(\sigma\star\eta) \wedge \mu(\eta)) \wedge (g(\sigma\star\eta) \wedge g(\eta)), \\ &\Longrightarrow \mathscr{T}(\sigma) \cup \mathfrak{G}(\sigma) \subseteq (\mathscr{T}(\sigma\star\eta) \cup \mathfrak{G}(\sigma\star\eta)) \cup (\mathfrak{G}(\eta) \cup \mathfrak{G}(\eta)) \quad \textit{and} \quad \mu(\sigma) \wedge g(\sigma) \geq (\mu(\sigma\star\eta) \wedge g(\sigma\star\eta)) \wedge (g(\eta) \wedge g(\eta)), \end{aligned}$$

$\Longrightarrow \left(\mathscr{T}\tilde{\cup}\mathfrak{G}\right)(\sigma) \subseteq \left(\mathscr{T}\tilde{\cup}\mathfrak{G}\right)(\sigma \star \eta) \cup \left(\mathscr{T}\tilde{\cup}\mathfrak{G}\right)(\eta)$ and $(\mu \bar{\sqcap} g)(\sigma) \geq (\mu \bar{\sqcap} g)(\sigma \star \eta) \wedge (\mu \bar{\sqcap} g)(\eta)$,

$\Longrightarrow H(\sigma) \subseteq H(\sigma \star \eta) \cup H(\eta)$ and $h(\sigma) \geq h(\sigma \star \eta) \wedge h(\eta)$.

Hence, result is proved.

Note that, intersection of $((\mathscr{T},\mu),X)$ and $((\mathfrak{G},g),X)$ over $(U,\ [0,\ 1])$ is also said to be extended int-uni of $((\mathscr{T},\mu),X)$ and $((\mathfrak{G},g),X)$ and known as extended int-uni doubt double T-soft fuzzy ideal of X.

Corollary 1. *Let $\left\{((\mathscr{T}_j,\mu_j),X) \text{ is the collection of ideals of } X : j = 1,\ 2,\ 3,\ldots\right\}$. Then their Intersection is also a DFT-soft fuzzy ideal of BF-algebra X.*

Theorem 2. *Let $((\mathscr{T}_1,\mu_1),X)$ and $((\mathscr{T}_2,\mu_2),X)$ be two DFT-soft fuzzy ideals of X over $(U,\ [0,\ 1])$ then prove that their union-product is also DFT-soft fuzzy ideal of $X \times X$.*

Proof. Let for any $(\sigma,\eta),\ (y,z) \in X \times X$, we have

$$(\mathscr{T}_1 \vee \mathscr{T}_2)(\Theta,\Theta) = \mathscr{T}_1(\Theta) \cup \mathscr{T}_2(\Theta) \subseteq \mathscr{T}_1(\sigma) \cup \mathscr{T}_2(\eta) = (\mathscr{T}_1 \vee \mathscr{T}_2)(\sigma,\eta)$$
$$\Longrightarrow (\mathscr{T}_1 \vee \mathscr{T}_2)(\Theta,\Theta) \subseteq (\mathscr{T}_1 \vee \mathscr{T}_2)(\sigma,\eta)\, (\mu_1 \wedge \mu_2)(\Theta,\Theta) = \mu_1(\Theta) \wedge \mu_2(\Theta) \geq \mu_1(\sigma) \wedge \mu_2(\eta)$$
$$\Longrightarrow (\mu_1 \wedge \mu_2)(\Theta,\Theta) \geq \mu_1 \wedge \mu_2(\sigma,\eta).$$

Since $(\sigma,\eta),\ (y,z) \in X \times X$, we have $(\mathscr{T}_1 \vee \mathscr{T}_2)(\sigma,\eta) = \mathscr{T}_1(\sigma) \cup \mathscr{T}_2(\eta)$. As $((\mathscr{T}_1,\mu_1),X)$ and $((\mathscr{T}_2,\mu_2),X)$ both are ideals so we have

$$\begin{aligned}
&(\mathscr{T}_1 \vee \mathscr{T}_2)(\sigma,\eta) = \mathscr{T}_1(\sigma) \cup \mathscr{T}_2(\eta)\\
&\subseteq (\mathscr{T}_1(\sigma \star y) \cup \mathscr{T}_1(y)) \cup (\mathscr{T}_2(\eta \star z) \cup \mathscr{T}_2(z))\\
&= (\mathscr{T}_1(\sigma \star y) \cup \mathscr{T}_2(\eta \star z)) \cup (\mathscr{T}_1(y) \cup \mathscr{T}_2(z))\\
&= ((\mathscr{T}_1 \vee \mathscr{T}_2)(\sigma \star y,\ \eta \star z)) \cup ((\mathscr{T}_1 \vee \mathscr{T}_2)(y,z))\\
&= ((\mathscr{T}_1 \vee \mathscr{T}_2)((\sigma,\eta) \star\ (y,z))) \cup ((\mathscr{T}_1 \vee \mathscr{T}_2)(y,z))\\
&\Longrightarrow (\mathscr{T}_1 \vee \mathscr{T}_2)(\sigma,\eta) \subseteq ((\mathscr{T}_1 \vee \mathscr{T}_2)((\sigma,\eta) \star\ (y,z))) \cup ((\mathscr{T}_1 \vee \mathscr{T}_2)(y,z))\\
&(\mu_1 \wedge \mu_2)(\sigma,\eta) = \mu_1(\sigma) \wedge \mu_2(\eta)\\
&\geq (\mu_1(\sigma \star y) \wedge \mu_1(y)) \wedge (\mu_2(\eta \star z) \wedge \mu_2(z))\\
&= (\mu_1(\sigma \star y) \wedge \mu_2(\eta \star z)) \wedge (\mu_1(y) \wedge \mu_2(z))\\
&= ((\mu_1 \vee \mu_2)(\sigma \star y,\ \eta \star z)) \wedge ((\mu_1 \vee \mu_2)(y,z))\\
&= ((\mu_1 \wedge \mu_2)((\sigma,\eta) \star\ (y,z))) \wedge ((\mu_1 \wedge \mu_2)(y,z))\\
&\Longrightarrow (\mu_1 \wedge \mu_2)(\sigma,\eta) \geq ((\mu_1 \wedge \mu_2)((\sigma,\eta) \star\ (y,z))) \wedge ((\mu_1 \wedge \mu_2)(y,z)).
\end{aligned}$$

Hence, $((\mathscr{T}_\vee,\ \mu_\wedge),A \times D)$ is a "DFT-soft fuzzy ideal" over $(U,\ [0,\ 1])$.

3.2 Doubt double framed soft Fuzzy-algebras of BF or BCI/BCK-algebras

In this subsection, we proposed the concept of doubt DF-soft fuzzy-algebra of BF or BCI/BCK-algebra and properties are investigated with the support of examples.

Note that in this section, we only proved results for BF-algebras. The results and definitions are also hold for doubt DF-soft fuzzy algebras of BCI/BCK-algebra which are prove for BF-algebra if we replace X as a BCI/BCK-algebra.

Definition 13. *A DF-soft fuzzy set* $((\mathscr{T},\mu),X)$ *over* $(U,\ [0,\ 1])$ *is said to be doubt DF-soft fuzzy set if behaviour of both sets is changed for example, a DF-soft fuzzy set* $((\mathscr{T},\mu),X)$ *over* $(U,\ [0,\ 1])$ *is said to be doubt DF-soft fuzzy algebra (DDF-soft fuzzy algebra) of BF-algebra* X *if for all* $\sigma,\eta \in X$

$$\begin{cases} \mathscr{T}(\sigma\star\eta) \supseteq \mathscr{T}(\sigma)\cap\mathscr{T}(\eta) \\ \mu(\sigma\star\eta) \leq \mu(\sigma)\vee\mu(\eta). \end{cases}$$

Lemma 1. *Every "DDF-soft fuzzy algebra"* $((\mathscr{T},\mu),\delta)$ *over* $(U,\ [0,\ 1])$ *satisfies the following conditions*

$$\mathscr{T}(\Theta) \supseteq \mathscr{T}(\sigma) \ \textit{and}\ \mu(\Theta) \leq \mu(\sigma)\text{for all } \sigma\in\delta.$$

Proof. By definition of $((\mathscr{T},\mu),\delta)$, we have $\mathscr{T}(\sigma\star\sigma) \supseteq \mathscr{T}(\sigma)\cap\mathscr{T}(\sigma)$ for all $\sigma\in\delta$ implies $\mathscr{T}(\sigma\star\sigma) = \mathscr{T}(\Theta) \supseteq \mathscr{T}(\sigma)$ because $\sigma\star\sigma=\Theta$.

Now $\mu(\sigma\star\sigma) \leq \mu(\sigma)\vee\ \mu(\sigma)$ for all $\sigma\in\delta$. Since $\sigma\star\sigma=\Theta$ so $\mu(\sigma\star\sigma)=\mu(\Theta)\leq\mu(\sigma)$.

Hence, proof is complete.

Theorem 3. *For a "DDF-soft fuzzy algebra"* $((\mathscr{T},\mu),\delta)$ *over* $(U,\ [0,\ 1])$*, the following are equivalent;*

$$(1)\mathscr{T}(\Theta) = \mathscr{T}(\sigma)\ \textit{and}\ \mu(\Theta)=\mu(\sigma)\text{for all } \sigma\in\delta,$$

$$\mathscr{T}(\sigma\star\eta) \supseteq \mathscr{T}(\eta)\ \textit{and}\ \mu(\sigma\star\eta)\leq\mu(\eta)\text{for all } \sigma,\ \eta\in\delta. \tag{3}$$

Proof. $(1)\Rightarrow(2)$, We suppose that (1) is valid. Now by definition of $((\mathscr{T},\mu),\delta)$, we have (for all $\sigma\in\delta$) $\mathscr{T}(\sigma\star\eta)\supseteq\mathscr{T}(\sigma)\cap\mathscr{T}(\eta)=\mathscr{T}(\Theta)\cap\mathscr{T}(\eta)=\mathscr{T}(\eta)$ because $\mathscr{T}(\Theta)=\mathscr{T}(\sigma)$ for all $\sigma\in\delta$, now (for all $\sigma\in\delta$)$\mu(\sigma\star\eta)\leq\mu(\sigma)\vee\mu(\eta)=\mu(\Theta)\vee\mu(\eta)=\mu(\eta)$ because $\mu(\Theta)=\mu(\sigma)$ for all $\sigma\in\delta$. $(2)\Rightarrow(1)$, Assume that (2) is valid. Then for $\eta=\Theta$ such that $\mathscr{T}(\sigma\star\Theta)\supseteq\mathscr{T}(\Theta)$ and $\mu(\sigma\star\Theta)\leq\mu(\Theta)$ implies $\mathscr{T}(\sigma)\supseteq\mathscr{T}(\Theta)$ and $\mu(\sigma)\leq\mu(\Theta)$ because $\sigma\star\Theta=\sigma$, then by 3.8. Lemma we have, $\mathscr{T}(\Theta)\supseteq\mathscr{T}(\sigma)\ \textit{and}\ \mu(\Theta)\leq\mu(\sigma)$ for all $\sigma\in\delta$, so $\mathscr{T}(\sigma)=\mathscr{T}(\Theta)$ and $\mu(\sigma)=\mu(\Theta)$.

Theorem 4. *The extended int-uni of "DDF-soft fuzzy set" over* $(U,\ [0,\ 1])$ *of two doubt double-framed soft fuzzy algebras* $((\mathscr{T},\ \mu),X)$ *and* $((\mathfrak{G},\ g),X)$ *over* $(U,\ [0,\ 1])$ *is "DDF-soft fuzzy algebra" over* $(U,\ [0,\ 1])$.

Proof. Let $\sigma,\eta\in X$ then we have

$$\begin{aligned}\left(\mathscr{T}\tilde{\cap}\mathfrak{G}\right)(\sigma\star\eta) &= \mathscr{T}(\sigma\star\eta)\cap\mathfrak{G}(\sigma\star\eta)\supseteq(\mathscr{T}(\sigma)\cap\mathscr{T}(\eta))\cap(\mathfrak{G}(\sigma)\cap\mathfrak{G}(\eta))\\ &= (\mathscr{T}(\sigma)\cap\mathfrak{G}(\sigma))\cap(\mathscr{T}(\eta)\cap\mathfrak{G}(\eta)) = \left(\mathscr{T}\tilde{\cap}\mathfrak{G}\right)(\sigma)\cap\left(\mathscr{T}\tilde{\cap}\mathfrak{G}\right)(\eta),\\ (\mu\overline{\cup}g)(\sigma\star\eta) &= \mu(\sigma\star\eta)\vee g(\sigma\star\eta)\leq(\mu(\sigma)\vee\mu(\eta))\vee(g(\sigma)\wedge g(\eta))\\ &= (\mu(\sigma)\vee g(\sigma))\vee(\mu(\eta)\vee g(\eta)) = (\mu\overline{\cup}g)(\sigma)\vee(\mu\overline{\cup}g)(\eta).\end{aligned}$$

Hence, $V_{(\mathscr{T},\mu)}\overline{\sqcap_{\mathscr{E}}}D_{(\mathfrak{G},\ g)}$ is a "DDF-soft fuzzy algebra" over $(U,\ [0,\ 1])$.

Theorem 5. *The extended uni-int of two doubt double-framed soft fuzzy algebras* $((\mathscr{T},\ \mu),X)$ *and* $((\mathfrak{G},\ g),X)$ *over* $(U,\ [0,\ 1])$ *is "DDF-soft fuzzy algebra" over* $(U,\ [0,\ 1])$ *if* $((\mathscr{T},\mu),X)\ \widetilde{\subset}((\mathfrak{G},g),X)$.

Proof. Let $\sigma,\eta \in X$ then we have

$$\begin{aligned}\left(\mathscr{T}\widetilde{\cup}\mathfrak{G}\right)(\sigma\star\eta) &= \mathscr{T}(\sigma\star\eta)\cup\mathfrak{G}(\sigma\star\eta) = \mathfrak{G}(\sigma\star\eta)\supseteq\mathfrak{G}(\sigma)\cap\mathfrak{G}(\eta)\\ &= (\mathscr{T}(\sigma)\cap\mathfrak{G}(\sigma))\cap(\mathscr{T}(\eta)\cap\mathfrak{G}(\eta)) = \left(\mathscr{T}\widetilde{\cup}\mathfrak{G}\right)(\sigma)\cup\left(\mathscr{T}\widetilde{\cup}\mathfrak{G}\right)(\eta),\\ (\mu\overline{\cap}g)(\sigma\star\eta) &= \mu(\sigma\star\eta)\wedge g(\sigma\star\eta) = g(\sigma\star\eta)\leq g(\sigma)\vee g(\eta)\\ &= (\mu(\sigma)\vee g(\sigma))\wedge(\mu(\eta)\vee g(\eta)) = (\mu\overline{\cap}g)(\sigma)\wedge(\mu\overline{\cap}g)(\eta).\end{aligned}$$

Hence, $V_{(\mathscr{T},\ \mu)}\overline{\sqcup}_{\mathscr{E}}D_{(\mathfrak{G},\ g)}$ is an extended uni-int "DDF-soft fuzzy algebra" over $(U,\ [0,\ 1])$.

Definition 14. *If* $((\mathscr{T}_1,\mu_1),C)$ *and* $((\mathscr{T}_2,\mu_2),D)$ *be two doubt DF-soft fuzzy ideal of* X *over* $(U,\ [0,\ 1])$ *the their intersection product is defined as* $\begin{cases}\mathscr{T}_1\wedge\mathscr{T}_2 : C\times D\longrightarrow P(U)\\ \mu_1\vee\mu_2 : C\times D\longrightarrow[0,\ 1]\end{cases}$ *, such that* $(c,\ d)\longrightarrow\begin{cases}\mathscr{T}(c)\cap\mathscr{T}(d)\\ \mu(c)\vee\mu(d)\end{cases}$ *And denoted by* $((\mathscr{T}_\wedge,\ \mu_\vee),C\times D)$.

Theorem 6. *Let* $((\mathscr{T},\ \mu),C)$ *and* $((\mathscr{T},\ \mu),D)$ *be doubt DF-soft fuzzy algebras over* $(U,\ [0,\ 1])$*. Then prove that* $((\mathscr{T}_\wedge,\ \mu_\vee),C\times D)$ *is also a "DDF-soft fuzzy algebra" over* $(U,\ [0,\ 1])$.

Proof. We know that $(C\times D,\ \diamond,\ (\Theta,\Theta)$ is also a BF-algebra. Then we only prove that $((\mathscr{T}_\wedge,\ \mu_\vee),C\times D)$ is a "DDF-soft fuzzy algebra" over $(U,\ [0,\ 1])$. Let $(x,y),\ (\sigma,\eta)\in C\times D$, we have

$$\begin{aligned}\mathscr{T}_{C\wedge D}((x,y)\diamond(\sigma,\ \eta)) &= \mathscr{T}_{C\wedge D}(x\star\sigma,\ y\star\eta)\\ &= \mathscr{T}(x\star\sigma)\cap\mathscr{T}(y\star\eta)\supseteq(\mathscr{T}(x)\cap\mathscr{T}(\sigma))\cap(\mathscr{T}(y)\cap\mathscr{T}(\eta))\\ &= (\mathscr{T}(x)\cap\mathscr{T}(y))\cap(\mathscr{T}(\sigma)\cap\mathscr{T}(\eta))\\ &= \mathscr{T}_{C\wedge D}(x,\sigma)\cap\mathscr{T}_{C\wedge D}(y,\eta).\\ \mu_{C\vee D}((x,y)\diamond(\sigma,\ \eta)) &= \mu_{C\vee D}(x\star\sigma,\ y\star\eta) = \mu(x\star\sigma)\vee\mu(y\star\eta)\\ &\leq(\mu(x)\vee\mu(\sigma))\vee(\mu(y)\vee\mu(\eta))\\ &= (\mu(x)\vee\mu(y))\vee(\mu(\sigma)\vee\mu(\eta))\\ &= \mu(x\star y)\vee\mu(\sigma\star\eta)\\ &= \mu_{C\vee D}(x,y)\vee\mu_{C\vee D}(\sigma,\eta).\end{aligned}$$

Hence, $((\mathscr{T}_\wedge,\ \mu_\vee),C\times D)$ is a "DDF-soft fuzzy algebra" over $(U,\ [0,\ 1])$.

3.2.1 Doubt double framed soft Fuzzy ideals of BF-algebras

In this subsection, we proposed the concept of doubt DF-soft fuzzy ideals of BF-algebra.

Definition 15. *A double framed pair* $(\mathscr{T},\mu)$ *of* X *over* $(U,\ [0,\ 1])$ *is said to be doubt DF-soft fuzzy sub algebra of BF-algebra* X *if for all* $\sigma,\eta\in X$

$$\begin{cases}\mathscr{T}(\sigma\star\eta)\supseteq\mathscr{T}(\sigma)\cap\mathscr{T}(\eta)\\ \mu(\sigma\star\eta)\leq\mu(\sigma)\vee\mu(\eta).\end{cases}$$

Definition 16. *A DF-soft fuzzy set* $((\mathscr{T},\mu),X)$ *over* $(U,\ [0,\ 1])$ *is said to be doubt DF-soft fuzzy ideal of* X *if for all* $\sigma,\eta\in X$

$$\begin{cases}\mathscr{T}(\Theta)\supseteq\mathscr{T}(\sigma) & \text{and} \quad \mathscr{T}(\sigma)\supseteq\mathscr{T}(\sigma\star\eta)\cap\mathscr{T}(\eta)\\ \mu(\Theta)\leq\mu(\sigma) & \text{and} \quad \mu(\sigma)\leq\mu(\sigma\star\eta)\vee\mu(\eta).\end{cases}$$

Proposition 2. *If $((\mathscr{T},\mu),X)$ over $(U,\ [0,\ 1])$ be a doubt DF-soft fuzzy ideal of X then following hold,*

(a) *if* $\sigma \leq \eta$ *then* $\begin{cases} \mathscr{T}(\sigma) \supseteq \mathscr{T}(\eta) \\ \mu(\sigma) \leq \mu(\eta), \end{cases}$

(b) *if* $\mathscr{T}(\sigma \star \eta) = \mathscr{T}(\Theta)$ *and* $\mu(\sigma \star \eta) = \mu(\Theta)$ *then* $\begin{cases} \mathscr{T}(\sigma) \supseteq \mathscr{T}(\eta) \\ \mu(\sigma) \leq \mu(\eta), \end{cases}$

(c) *if* $\sigma \star \eta \leq w$ *then* $\begin{cases} \mathscr{T}(\sigma) \supseteq \mathscr{T}(\eta) \cap \mathscr{T}(w) \\ \mu(\sigma) \leq \mu(\eta) \vee \mathscr{T}(w), \end{cases}$

For all $\sigma, \eta,\ w \in X$.

Proof. **(a)** Let $\sigma \leq \eta$ then $\sigma \star \eta = \Theta$. Now by definition of $((\mathscr{T},\mu),X)$ such that $\mathscr{T}(\sigma) \supseteq \mathscr{T}(\sigma \star \eta) \cap \mathscr{T}(\eta)$ and $\mu(\sigma) \leq \mu(\sigma \star \eta) \vee \mu(\eta)$ then $\mathscr{T}(\sigma) \supseteq \mathscr{T}(\Theta) \cap \mathscr{T}(\eta)$, $\mu(\sigma) \leq \mu(\Theta) \vee \mu(\eta)$ implies that $\mathscr{T}(\sigma) \supseteq \mathscr{T}(\eta)$ and $\mu(\sigma) \leq \mu(\eta)$.

(b) Let $\mathscr{T}(\sigma \star \eta) = \mathscr{T}(\Theta)($ *and* $\mu(\sigma \star \eta) = (\Theta)$. Then by definition of $(\mathscr{T},\mu),X)$, we have $\mathscr{T}(\sigma) \supseteq \mathscr{T}(\sigma \star \eta) \cap \mathscr{T}(\eta)$ and $\mu(\sigma) \leq \mu(\sigma \star \eta) \vee \mu(\eta)$implies that $\mathscr{T}(\sigma) \supseteq \mathscr{T}(\Theta) \cap \mathscr{T}(\eta)$, $\mu(\sigma) \leq \mu(\Theta) \vee \mu(\eta)$ implies that $\mathscr{T}(\sigma) \supseteq \mathscr{T}(\eta)$ and $\mu(\sigma) \leq \mu(\eta)$.

(c) By definition of $((\mathscr{T},\mu),X)$, we have

$$\mathscr{T}(\sigma) \supseteq \mathscr{T}(\sigma \star \eta) \cap \mathscr{T}(\eta) \quad \textit{and} \quad \mu(\sigma) \leq \mu(\sigma \star \eta) \vee \mu(\eta). \tag{4}$$

Let $\sigma \star \eta \leq w$ then $(\sigma \star \eta) \star w = \Theta$, $\mathscr{T}(\sigma \star \eta) \supseteq w$ and $\mu(\sigma \star \eta) \leq \mu(w)$. Now by following (4), we have
$\mathscr{T}(\sigma \star \eta) \supseteq \mathscr{T}((\sigma \star \eta) \star w) \cap \mathscr{T}(w) \quad \textit{and} \quad \mu(\sigma \star \eta) \leq \mu((\sigma \star \eta) \star w) \vee \mu(w)$

Implies $\mathscr{T}(\sigma \star \eta) \supseteq \mathscr{T}(\Theta) \cap \mathscr{T}(w) \quad \textit{and} \quad \mu(\sigma \star \eta) \leq \mu(\Theta) \vee \mu(w)$

Implies $\mathscr{T}(\sigma \star \eta) \supseteq \mathscr{T}(w) \quad \textit{and} \quad \mu(\sigma \star \eta) \leq \mu(w)$.

$$\textit{Implies} \quad \mathscr{T}(\sigma \star \eta) \cap \mathscr{T}(\eta) \supseteq \mathscr{T}(\eta) \cap \mathscr{T}(w) \quad \textit{and} \quad \mu(\sigma \star \eta) \vee \mu(\eta) \leq \mu(w) \vee \mu(\eta). \tag{5}$$

From (4) and (5), we have $\mathscr{T}(\sigma) \supseteq \mathscr{T}(\eta) \cap \mathscr{T}(w)$ and $\mu(\sigma) \leq \mu(\eta) \vee \mathscr{T}(w)$.

Theorem 7. *Let $((\mathscr{T},\mu),X)$ and $((\mathfrak{G},g),X)$ over $(U,\ [0,\ 1])$ be two doubt DF-soft fuzzy ideal of X then prove that their intersection is also a doubt DF-soft fuzzy ideal of X.*

Proof. Let $((\mathscr{T},\mu),X)$ and $((\mathfrak{G},g),X)$ over $(U,\ [0,\ 1])$ be two doubt DF-soft fuzzy ideal of X and $\sigma, \eta \in X$ then

$$\begin{cases} \mathscr{T}(\Theta) \supseteq \mathscr{T}(\sigma) \quad \textit{and} \quad \mathscr{T}(\sigma) \supseteq \mathscr{T}(\sigma \star \eta) \cap \mathscr{T}(\eta) \\ \mu(\Theta) \leq \mu(\sigma) \quad \textit{and} \quad \mu(\sigma) \leq \mu(\sigma \star \eta) \vee \mu(\eta). \end{cases} \quad \textit{and} \quad \begin{cases} \mathfrak{G}(\Theta) \supseteq \mathfrak{G}(\sigma) \quad \textit{and} \quad \mathfrak{G}(\sigma) \supseteq \mathfrak{G}(\sigma \star \eta) \cap \mathfrak{G}(\eta) \\ g(\Theta) \leq g(\sigma) \quad \textit{and} \quad g(\sigma) \leq g(\sigma \star \eta) \vee g(\eta). \end{cases}$$

Now, intersection of $((\mathscr{T},\mu),X)$ and $((\mathfrak{G},g),X)$ is defined as $((\mathscr{T},\mu),X) \sqcup_{\mathscr{E}} ((\mathfrak{G},g),X) = ((H,\ h),\ X)$ where $H = \mathscr{T} \tilde{\cap} \mathfrak{G}$ and $h = \mu \overline{\cup} g$.

It is given that $\mathscr{T}(\Theta) \supseteq \mathscr{T}(\sigma)$, $\mu(\Theta) \leq \mu(\sigma)$ and $\mathfrak{G}(\Theta) \supseteq \mathfrak{G}(\sigma)$, $g(\Theta) \leq g(\sigma)$ then $\mathscr{T}(\Theta) \cap \mathfrak{G}(\Theta) \supseteq \mathscr{T}(\sigma) \cap \mathfrak{G}(\sigma)$ and $\mu(\Theta) \vee g(\Theta) \leq \mu(\sigma) \vee g(\sigma) \Longrightarrow (\mathscr{T} \tilde{\cap} \mathfrak{G})(\Theta) \supseteq (\mathscr{T} \tilde{\cap} \mathfrak{G})(\sigma)$ and $(\mu \overline{\cup} g)(\Theta) \leq (\mu \overline{\cup} g)(\sigma)$.

Implies $H(\Theta) \supseteq H(\sigma)$ and $h(\Theta) \leq h(\sigma)$. Further, $\mathscr{T}(\sigma) \supseteq \mathscr{T}(\sigma \star \eta) \cap \mathscr{T}(\eta)$, $\mu(\sigma) \leq \mu(\sigma \star \eta) \vee \mu(\eta)$ and $\mathfrak{G}(\sigma) \supseteq \mathfrak{G}(\sigma \star \eta) \cap \mathfrak{G}(\eta)$, $g(\sigma) \leq g(\sigma \star \eta) \vee g(\eta)$, we $\mathscr{T}(\sigma) \cap \mathfrak{G}(\sigma) \supseteq (\mathscr{T}(\sigma \star \eta) \cap \mathscr{T}(\eta)) \cap (\mathfrak{G}(\sigma \star \eta) \cap \mathfrak{G}(\eta))$ and
$\mu(\sigma) \vee g(\sigma) \leq (\mu(\sigma \star \eta) \vee \mu(\eta)) \vee (g(\sigma \star \eta) \vee g(\eta))$, $\Longrightarrow \mathscr{T}(\sigma) \cap \mathfrak{G}(\sigma) \supseteq (\mathscr{T}(\sigma \star \eta) \cap \mathfrak{G}(\sigma \star \eta)) \cap (\mathfrak{G}(\eta) \cap \mathfrak{G}(\eta))$ and $\mu(\sigma) \vee g(\sigma) \leq (\mu(\sigma \star \eta) \vee g(\sigma \star \eta)) \vee (g(\eta) \vee g(\eta))$, $\Longrightarrow (\mathscr{T} \tilde{\cap} \mathfrak{G})(\sigma) \supseteq (\mathscr{T} \tilde{\cap} \mathfrak{G})(\sigma \star \eta) \cap (\mathscr{T} \tilde{\cap} \mathfrak{G})(\eta)$ and $(\mu \tilde{\cup} g)(\sigma) \leq (\mu \tilde{\cup} g)(\sigma \star \eta) \vee (\mu \tilde{\cup} g)(\eta)$, $\Longrightarrow H(\sigma) \supseteq H(\sigma \star \eta) \cap H(\eta)$ and $h(\sigma) \leq h(\sigma \star \eta) \vee h(\eta)$.
Hence, result is proved.

Note that, intersection of $((\mathscr{T}, \mu), X)$ and $((\mathfrak{G}, g), X)$ over $(U, [0, 1])$ is also said to be extended int-uni of $((\mathscr{T}, \mu), X)$ and $((\mathfrak{G}, g), X)$, known as extended int-uni doubt double T-soft fuzzy ideal of X.

Theorem 8. *Let $\{((\mathscr{T}_j, \mu_j), X)$ is the collection of ideals of $X : j = 1, 2, 3, \ldots\}$. Then their Intersection is also a doubt DF-soft fuzzy ideal of BF-algebra X.*

Definition 17. *If $((\mathscr{T}_1, \mu_1), X)$ and $((\mathscr{T}_2, \mu_2), X)$ be two doubt DF-soft fuzzy ideal of X over $(U, [0, 1])$ the their intersection product is defined as*

$$\begin{cases} \mathscr{T}_1 \wedge \mathscr{T}_2 : X \times X \longrightarrow P(U) \\ \mu_1 \vee \mu_2 : X \times X \longrightarrow [0, 1] \end{cases}, \quad such that \quad (\sigma, \eta) \longrightarrow \begin{cases} \mathscr{T}(\sigma) \cap \mathscr{T}(\eta) \\ \mu(\sigma) \vee \mu(\eta) \end{cases}$$

and denoted by $((\mathscr{T}_{\wedge}, \mu_{\vee}), X?X)$.

Theorem 9. *Let $((\mathscr{T}_1, \mu_1), X)$ and $((\mathscr{T}_2, \mu_2), X)$ be two doubt DF-soft fuzzy ideal of X over $(U, [0, 1])$ then prove that their intersection product is also doubt DF- soft fuzzy ideal of $X \times X$.*

Proof. Let for any (σ, η), $(y, z) \in X \times X$, we have

$$\begin{aligned} (\mathscr{T}_1 \wedge \mathscr{T}_2)(\Theta, \Theta) &= \mathscr{T}_1(\Theta) \cap \mathscr{T}_2(\Theta) \supseteq \mathscr{T}(\sigma) \cap \mathscr{T}(\eta) = (\mathscr{T}_1 \wedge \mathscr{T}_2)(\sigma, \eta) \\ &\Longrightarrow (\mathscr{T}_1 \wedge \mathscr{T}_2)(\Theta, \Theta) \supseteq (\mathscr{T}_1 \wedge \mathscr{T}_2)(\sigma, \eta)\, (\mu_1 \vee \mu_2)(\Theta, \Theta) = \mu_1(\Theta) \vee \mu_2(\Theta), \leq \mu_1(\sigma) \vee \mu_2(\eta) \\ &\Longrightarrow (\mu_1 \vee \mu_2)(\Theta, \Theta) \leq \mu_1 \vee \mu_2(\sigma, \eta). \end{aligned}$$

Since (σ, η), $(y, z) \in X \times X$, we have $(\mathscr{T}_1 \wedge \mathscr{T}_2)(\sigma, \eta) = \mathscr{T}_1(\sigma) \cap \mathscr{T}_2(\eta)$. Since, $((\mathscr{T}_1, \mu_1), X)$ and $((\mathscr{T}_2, \mu_2), X)$ both are ideals so we have

$$\begin{aligned} &(\mathscr{T}_1 \wedge \mathscr{T}_2)(\sigma, \eta) = \mathscr{T}_1(\sigma) \cap \mathscr{T}_2(\eta) \supseteq (\mathscr{T}_1(\sigma \star y) \cap \mathscr{T}_1(y)) \cap (\mathscr{T}_2(\eta \star z) \cap \mathscr{T}_2(z)) \\ &= (\mathscr{T}_1(\sigma \star y) \cap \mathscr{T}_2(\eta \star z)) \cap (\mathscr{T}_1(y) \cap \mathscr{T}_2(z)) = ((\mathscr{T}_1 \wedge \mathscr{T}_2)(\sigma \star y, \eta \star z)) \cap ((\mathscr{T}_1 \vee \mathscr{T}_2)(y, z)) \\ &= ((\mathscr{T}_1 \vee \mathscr{T}_2)((\sigma, \eta) \star (y, z))) \cap ((\mathscr{T}_1 \vee \mathscr{T}_2)(y, z)) \\ &\Longrightarrow (\mathscr{T}_1 \vee \mathscr{T}_2)(\sigma, \eta) \supseteq ((\mathscr{T}_1 \vee \mathscr{T}_2)((\sigma, \eta) \star (y, z))) \cap ((\mathscr{T}_1 \vee \mathscr{T}_2)(y, z))\, (\mu_1 \vee \mu_2)(\sigma, \eta) = \mu_1(\sigma) \vee \mu_2(\eta) \\ &\leq (\mu_1(\sigma \star y) \vee \mu_1(y)) \vee (\mu_2(\eta \star z) \vee \mu_2(z)) = (\mu_1(\sigma \star y) \vee \mu_2(\eta \star z)) \vee (\mu_1(y) \vee \mu_2(z)) \\ &= ((\mu_1 \vee \mu_2)(\sigma \star y, \eta \star z)) \vee ((\mu_1 \vee \mu_2)(y, z)) = ((\mu_1 \vee \mu_2)((\sigma, \eta) \star (y, z))) \vee ((\mu_1 \vee \mu_2)(y, z)) \\ &\Longrightarrow (\mu_1 \vee \mu_2)(\sigma, \eta) \leq ((\mu_1 \vee \mu_2)((\sigma, \eta) \star (y, z))) \vee ((\mu_1 \vee \mu_2)(y, z)). \end{aligned}$$

Hence, $(((\mathscr{T}_1 \wedge \mathscr{T}_2), (\mu_1 \vee \mu_2)), X \times X)$ is a "DDF-soft fuzzy algebra" over $(U, [0, 1])$.

4 Conclusion

We discussed the DF-soft fuzzy BF-algebra, doubt DF-soft fuzzy BF-algebra and their ideals. By using union product and intersection product, the above mentioned concepts are investigated. We can easily see that DF-soft algebra is not also a double framed B-soft algebra by 1.Example. Similar case for DFSB-soft fuzzy ideal and DFT-soft fuzzy ideal.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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