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Semi-analytical solution for the mathematical modeling of yellow fever dynamics incorporating secondary host

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Abstract: In this paper we use Differential Transformation Method (DTM) to solve the mathematical modeling of yellow fever dynamics incorporating secondary host. The DTM numerical solution was compared with the MAPLE RungeKutta 4-th order. The variable and parameter values used for analytical solution were estimated from the data obtained from World Health Organization (WHO) and UNICEF. The results obtained are in good agreement with Runge-Kutta. The solution was also presented graphically and gives better understanding of the model. The graphical solution showed that vaccination rate and recovery rate play a vital role in eradicating the yellow fever in a community.

Keywords: Semi-analytical, mathematical modeling, yellow fever, dynamics, differential transformation method.

1 Introduction

Almost all epidemiological models are basically system of non-linear ordinary differential equations (ODEs). The work of mathematical biologist consists of model building, parameter estimation, sensitivity analysis of the model parameters and numerical simulation.

Yellow fever is caused by the yellow fever virus and is spread by the bite of the female mosquito. It only infects humans, other primates and several species of mosquito, [1].Yellow fever is one of the world infectious diseases. It was estimated that 200 000 cases and 30,000 deaths of yellow fever are reported per year globally, of which 90% are in Africa, [2].

The model equations are formulated using first order ordinary differential equation. Three populations were considered: human, vector (mosquito) and secondary host (monkey) populations.

The populations are sub- divided into compartments with assumptions of the nature and rate of transfer from one compartment to another. We consider the total population sizes denoted by $N_h(t)$, $N_v(t)$ and $N_m(t)$ for the humans, mosquitoes (Aedesaegypti) and monkeys respectively. The model equation is given below and the description of the variables and parameter is also given in table 1.

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$$\left.\begin{aligned}
\frac{dS_h}{dt} &= \Lambda_h - \frac{\alpha_1 S_h V_2}{N_h} - (\nu + \mu_h) S_h \\
\frac{dI_h}{dt} &= \frac{\alpha_1 S_h V_2}{N_h} - (\gamma_h + \mu_h + \delta_h) I_h \\
\frac{dR_h}{dt} &= \nu S_h + \gamma_h I_h - \mu_h R_h \\
\frac{dV_1}{dt} &= \Lambda_v - \frac{\alpha_2 V_1 I_h}{N_h} - \frac{\alpha_3 V_1 I_m}{N_m} - (\mu_v + \delta_v) V_1 \\
\frac{dV_2}{dt} &= \frac{\alpha_2 V_1 I_h}{N_h} + \frac{\alpha_3 V_1 I_m}{N_m} - (\mu_v + \delta_v) V_2 \\
\frac{dS_m}{dt} &= \Lambda_m - \frac{\alpha_4 S_m V_2}{N_m} - \mu_m S_m \\
\frac{dI_m}{dt} &= \frac{\alpha_4 S_m V_2}{N_m} - (\mu_m + \delta_m) I_m
\end{aligned}\right\} \tag{1}$$

The total populations are gives as

$$\left.\begin{aligned}
N_h &= S_h + I_h + R_h \\
N_v &= V_1 + V_2 \\
N_m &= S_m + I_m
\end{aligned}\right\} \tag{2}$$

Differential Transformation Method (DTM) is one of the methods used to solve linear and nonlinear differential equations. It was first proposed by Zhou, [3], for solving linear and nonlinear initial value problems in electrical circuit analysis. The DTM construct a semi-analytical numerical technique that uses Taylor series for the solution of differential equations in the form of a polynomial. DTM is a very effective and powerful tool for solving different kinds of differential equations. This technique has been used by different people to solve different kinds of problems such as; fractional differential equations, [4, 5], differential algebraic equations [6], nonlinear oscillatory system, [7], quadratic Riccati differential equation, [8], the numerical solution of Susceptible Infected Recovered (SIR) model, [9], the solution of prey and predator problem, [10], fourth-order parabolic partial differential equations, [11], Volterra integral equations, [12] and difference equations, [13]. The main advantage of this method is that it can be applied directly to linear and nonlinear Ordinary Differential Equations (ODEs) without linearization, discretization or perturbation.

In this paper we solved a system of seven first order ordinary differential equations (ODEs). We compared our numerical result with Rungr-Kutta and they are in agreement. We also represented the solution graphically.

2 Material and method

2.1 Differential transformation method (DTM)

An arbitrary function $f(t)$ can be expanded in Taylor series about a point $t = 0$ as

$$f(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \left[\frac{d^k f}{dt^k}\right]_{t=0} \tag{3}$$

The differential transformation of $f(t)$ is defined as

$$F(t) = \frac{1}{k!} \left[\frac{d^k f}{dt^k}\right]_{t=0} \tag{4}$$

Then the inverse differential transform is

$$f(t) = \sum_{k=0}^{\infty} t^k F(t) \tag{5}$$

In [14] if $y(t)$ and $g(t)$ are two uncorrelated functions with t where $Y(k)$ and $G(k)$ are the transformed functions corresponding to $y(t)$ and $g(t)$ then, the fundamental mathematical operations performed by differential transform can be proved easily and are listed as follows

Table 1: The fundamental mathematical operations by differential transformation method (DTM). Source: [14].

Original Function	Transformed Function
$y(t) = f(t) \pm g(t)$	$Y(k) = F(k) \pm G(k)$
$y(t) = af(t)$	$Y(k) = aF(k)$
$y(t) = \frac{df(t)}{dt}$	$Y(k) = (k+1)F(k+1)$
$y(t) = \frac{d^2f(t)}{dt^2}$	$Y(k) = (k+1)(k+2)F(k+2)$
$y(t) = \frac{d^mf(t)}{dt^m}$	$Y(k) = (k+1)(k+2)\ldots(k+m)F(k+m)$
$y(t) = 1$	$Y(k) = \delta(k)$
$y(t) = t$	$Y(k) = \delta(k-1)$
$y(t) = t^m$	$Y(k) = \delta(k-m) = \begin{cases} 1, & k = m \\ 0, & k \neq m \end{cases}$
$y(t) = f(t)g(t)$	$Y(k) = \sum_{m=0}^{k} G(m) f(k-m)$
$y(t) = e^{(\lambda t)}$	$Y(k) = \frac{\lambda^k}{k!}$
$y(t) = (1+t)^m$	$Y(k) = \frac{m(m-1)\ldots(m-k+1)}{k!}$

Table 2: Values for parameters used for analytical solutions.

Variables	Description	Values per year	Source
$S_h(0)$	Number of susceptible humans at time	177092484	E6
$I_h(0)$	Number of infectious humans at time	34200	E3
$R_h(0)$	Number of recovered/Immune human at time	29070	E4
$V_1(0)$	Number of non-carrier vectors at time	35000000	Assumed
$V_2(0)$	Number of carrier vectors at time	15000000	Assumed
$S_m(0)$	Number of susceptible secondary host at time	35000	Assumed
$I_m(0)$	Number of infectious secondary host at time	15000	Assumed
N_h	Total human population at time	177155754	E1
N_v	Total vector population at time	50000000	Assumed
N_m	Total secondary vector population at time	50000	E10
α_1	Effective virus Transmission rate from mosquito to humans	0.05	Assumed
α_2	Effective virus Transmission rate from humans to mosquito	0.48	[15] Chitnis
α_3	Effective virus Transmission rate from secondary host to mosquito	0.042	Assumed
α_4	Effective virus Transmission rate from mosquito to secondary host	0.001	Assumed
Λ_h	Recruitment number of human population	6865728	E2
Λ_v	Recruitment number of mosquito population	2000000	Assumed
Λ_m	Recruitment number of secondary vector population	5000	Assumed
δ_h	Disease-induced death rate of humans	0.15	E7
δ_v	Death rate of mosquito due to application of insecticide	0.001	Assumed
δ_m	Disease-induced death rate of secondary host	0.002	Assumed
μ_h	Natural death rate of human population	0.012	E8
μ_v	Natural death rate of mosquito population	0.02	Assumed
μ_m	Natural death rate of secondary host population	0.005	E11
γ_h	Recovery rate of human population due to drug administration	0.85	E5
v	vaccination rate for the human population	0.75	E9

2.2 Analytical solution of the model equations using differential transformation method (DTM)

In this section we are going to apply Differential Transformation Method to the Model equation and solve. Let the model equation be a function$q(t)$, $q(t)$ can be expanded in Taylor series about a point δ_v as

$$q(t)=\sum_{k=0}^{\infty}\frac{t^k}{k!}\left[\frac{d^kq}{dt^k}\right]_{t=0} \tag{6}$$

where,

$$q(t)=\{s_h(t),\ i_h(t),\ r_h(t),\ v_1(t),\ v_2(t),\ s_m(t),\ i_m(t)\} \tag{7}$$

The differential transformation of $q(t)$ is defined as

$$Q(t)=\frac{1}{k!}\left[\frac{d^kq}{dt^k}\right]_{t=0} \tag{8}$$

where,

$$Q(t)=\{S_h(t),\ I_h(t),\ R_h(t),\ V_1(t),\ V_2(t),\ S_m(t),\ I_m(t)\} \tag{9}$$

Then the inverse differential transform is

$$q(t)=\sum_{k=0}^{\infty}t^kQ(t) \tag{10}$$

Using the fundamental operations of differential transformation method in table 2.1, we obtain the following recurrence relation of equation as

$$S_h(k+1)=\frac{1}{k+1}\left[\Lambda_h-\frac{\alpha_1}{N_h}\sum_{m=0}^{k}S_h(m)V_2(k-m)-(v+\mu_h)S_h(k)\right] \tag{11}$$

$$I_h(k+1)=\frac{1}{k+1}\left[\frac{\alpha_1}{N_h}\sum_{m=0}^{k}S_h(m)V_2(k-m)-(\gamma_h+\mu_h+\delta_h)I_h(k)\right] \tag{12}$$

$$R_h(k+1)=\frac{1}{k+1}\left[vS_h(k)+\gamma_hI_h(k)-\mu_hR_h(k)\right] \tag{13}$$

$$V_1(k+1)=\frac{1}{k+1}\left[\Lambda_v-\frac{\alpha_2}{N_h}\sum_{m=0}^{k}V_1(m)I_h(k-m)-\frac{\alpha_3}{N_m}\sum_{m=0}^{k}V_1(m)I_m(k-m)-(\mu_v+\delta_v)V_1(k)\right] \tag{14}$$

$$V_2(k+1)=\frac{1}{k+1}\left[\frac{\alpha_2}{N_h}\sum_{m=0}^{k}V_1(m)I_h(k-m)-\frac{\alpha_3}{N_m}\sum_{m=0}^{k}V_1(m)I_m(k-m)-(\mu_v+\delta_v)V_2(k)\right] \tag{15}$$

$$S_m(k+1)=\frac{1}{k+1}\left[\Lambda_m-\frac{\alpha_4}{N_m}\sum_{m=0}^{k}S_m(m)V_2(k-m)-\mu_mS_m(k)\right] \tag{16}$$

$$I_m(k+1)=\frac{1}{k+1}\left[\frac{\alpha_4}{N_m}\sum_{m=0}^{k}S_m(m)V_2(k-m)-(\mu_m+\delta_m)I_m(k)\right] \tag{17}$$

with the initial conditions

$$\begin{aligned}&S_h(0)=170,638,700,\ I_h(0)=34,200,\ R_h(0)=6,482,854,\ V_1(0)=35,000,000,\\&V_2(0)=15,000,000,\ S_m(0)=35,000,\ I_m(0)=15,000\end{aligned} \tag{18}$$

The parameter values are

$$\begin{aligned}&N_h=177,155,754,\ N_v=50,000,000,\ N_m=50,000,\ \Lambda_h=6,865,728,\ \Lambda_v=2,000,000,\\&\Lambda_m=5,000,\ \alpha_1=0.005,\ \alpha_2=0.48,\ \alpha_3=0.042,\ \alpha_4=0.001,\ \delta_h=0.15,\ \delta_v=0.001,\\&\delta_m=0.002,\ \mu_h=0.012,\ \mu_v=0.02,\ \mu_m=0.005,\ v=0.75,\ \gamma_h=0.85\end{aligned}\tag{19}$$

we consider $k=0,\ 1,\ 2,\ 3.$

Cases A1 to B2 are the variation of different values of v and γ_h.

Case A1: High vaccination rate, $v=0.75$,

$$\left.\begin{aligned}&S_h(1)=-123233202.4,\ S_h(2)=50410489,\ S_h(3)=-10522906.5,\ S_h(4)=3722257.1\\&I_h(1)=37630.56,\quad I_h(2)=-44815.62,\ I_h(3)=22336.18,\quad I_h(4)=-6862.42\\&R_h(1)=127930301,\ R_h(2)=-46964039.7,\ R_h(3)=12777780.56,\ R_h(4)=-2006631.9\\&V_1(1)=820756.75,\quad V_1(2)=831582.47,\quad V_1(3)=664582.61,\quad V_1(4)=489515.59\\&V_2(1)=129243,\quad V_2(2)=158442.53,\quad V_2(3)=-4846.12,\quad V_2(4)=7020.79\\&S_m(1)=-5675,\quad S_m(2)=3320.20,\quad S_m(3)=1297.03,\quad S_m(4)=1154.30\\&I_m(1)=10395,\quad I_m(2)=-842.40,\quad I_m(3)=366.07,\quad I_m(4)=-93.44\end{aligned}\right\}\tag{20}$$

Then, the closed form of the solution where $k=0,\ 1,\ 2,\ 3$ can be written as

$$\left.\begin{aligned}&s_h(t)=\textstyle\sum_{k=0}^{\infty}S_h(k).t^k=170638700-123233202.4t+50410489t^2-10522906.5t^3+3722257.1t^4+\ldots\\&i_h(t)=\textstyle\sum_{k=0}^{\infty}I_h(k).t^k=34200+37630.56t-44815.62t^2+22336.18t^3-6862.42t^4+\ldots\\&r_h(t)=\textstyle\sum_{k=0}^{\infty}R_h(k).t^k=6482854+127930301t-46964039.7t^2+12777780.56t^3-2006631.9t^4+\ldots\\&v_1(t)=\textstyle\sum_{k=0}^{\infty}V_1(k).t^k=35000000+820756.75t+831582.47t^2+664582.61t^3+489515.59t^4+\ldots\\&v_2(t)=\textstyle\sum_{k=0}^{\infty}V_2(k).t^k=15000000+129243t+158442.53t^2-4846.12t^3+7020.79t^4+\ldots\\&s_m(t)=\textstyle\sum_{k=0}^{\infty}S_m(k).t^k=35000-5675t+3320.20t^2+1297.03t^3+1154.30t^4+\ldots\\&i_m(t)=\textstyle\sum_{k=0}^{\infty}I_m(k).t^k=15000+10395t-842.40t^2+366.07t^3-93.44t^4+\ldots\end{aligned}\right\}\tag{21}$$

Case A2: $v = 0.50$,

$$\left.\begin{array}{l}
S_h(1) = -80573527.36,\ S_h(2) = 24076431,\ S_h(3) = -1824022.32,\ S_h(4) = 1950173.5,\\
I_h(1) = 37630.56,\ I_h(2) = -35785.5,\ I_h(3) = 15625.66,\ I_h(4) = -4219.96,\\
R_h(1) = 85270626,\ R_h(2) = -20639012.6,\ R_h(3) = 4085155.40,\ R_h(4) = -236937.8,\\
V_1(1) = 820756.75,\ V_1(2) = 831582.47,\ V_1(3) = 664297.16,\ V_1(4) = 489695.70,\\
V_2(1) = 129243,\ V_2(2) = 158442.53,\ V_2(3) = -4560.67,\ V_2(4) = 6840.68,\\
S_m(1) = -5675,\ S_m(2) = 3320.20,\ S_m(3) = -1297.03,\ S_m(4) = 1154.25,\\
I_m(1) = 10395,\ I_m(2) = -842.40,\ I_m(3) = 366.07,\ I_m(4) = -93.49
\end{array}\right\} \quad (22)$$

Then, the closed form of the solution where $k = 0,\ 1,\ 2,\ 3$ can be written as

$$\left.\begin{array}{l}
s_h(t) = \sum_{k=0}^{\infty} S_h(k).t^k = 170638700 - 80573527.36t + 24076431t^2 - 1824022.32t^3 + 1950173.5t^4 + \ldots\\
i_h(t) = \sum_{k=0}^{\infty} I_h(k).t^k = 34200 + 37630.56t - 35785.5t^2 + 15625.66t^3 - 4219.96t^4 + \ldots\\
r_h(t) = \sum_{k=0}^{\infty} R_h(k).t^k = 6482854 + 85270626t - 20639012.6t^2 + 4085155.40t^3 - 236937.8t^4 + \ldots\\
v_1(t) = \sum_{k=0}^{\infty} V_1(k).t^k = 35000000 + 820756.75t + 831582.47t^2 + 664297.16t^3 + 489695.70t^4 + \ldots\\
v_2(t) = \sum_{k=0}^{\infty} V_2(k).t^k = 15000000 + 129243t + 158442.53t^2 - 4560.67t^3 + 6840.68t^4 + \ldots\\
s_m(t) = \sum_{k=0}^{\infty} S_m(k).t^k = 35000 - 5675t + 3320.20t^2 - 1297.03t^3 + 1154.25t^4 + \ldots\\
i_m(t) = \sum_{k=0}^{\infty} I_m(k).t^k = 15000 + 10395t - 842.40t^2 + 366.07t^3 - 93.49t^4 + \ldots
\end{array}\right\} \quad (23)$$

Case A3: $v = 0.25$,

$$\left.\begin{array}{l}
S_h(1) = -37913852.36,\ S_h(2) = 8407293,\ S_h(3) = 1552944.40,\ S_h(4) = 1614589.6\\
I_h(1) = 37630.56,\ I_h(2) = -26755.38,\ I_h(3) = 10420.16,\ I_h(4) = -2511.81\\
R_h(1) = 42610951,\ R_h(2) = -4978904.26,\ R_h(3) = 712942.67,\ R_h(4) = 97134.48\\
V_1(1) = 820756.75,\ V_1(2) = 831582.47,\ V_1(3) = 664011.72,\ V_1(4) = 489816.49\\
V_2(1) = 129243,\ V_2(2) = 158442.53,\ V_2(3) = -4275.22,\ V_2(4) = 6719.89,\\
S_m(1) = -5675,\ S_m(2) = 3320.20,\ S_m(3) = -1297.03,\ S_m(4) = 1154.20\\
I_m(1) = 10395,\ I_m(2) = -842.40,\ I_m(3) = 366.07,\ I_m(4) = -93.54
\end{array}\right\} \quad (24)$$

Then, the closed form of the solution where $k=4$ can be written as

$$\left.\begin{array}{l} s_h(t)=\sum_{k=0}^{\infty}S_h(k).t^k=170638700-37913852.36t+8407293t^2+1552944.40t^3+1614589.6t^4+\ldots \\ i_h(t)=\sum_{k=0}^{\infty}I_h(k).t^k=34200+37630.56t-26755.38t^2+10420.16t^3-2511.81t^4+\ldots \\ r_h(t)=\sum_{k=0}^{\infty}R_h(k).t^k=6482854+42610951t-4978904.26t^2+712942.67t^3+97134.48t^4+\ldots \\ v_1(t)=\sum_{k=0}^{\infty}V_1(k).t^k=35000000+820756.75t+831582.47t^2+664011.72t^3+489816.49t^4+\ldots \\ v_2(t)=\sum_{k=0}^{\infty}V_2(k).t^k=15000000+129243t+158442.53t^2-4275.22t^3+6719.89t^4+\ldots \\ s_m(t)=\sum_{k=0}^{\infty}S_m(k).t^k=35000-5675t+3320.20t^2-1297.03t^3+1154.20t^4+\ldots \\ i_m(t)=\sum_{k=0}^{\infty}I_m(k).t^k=15000+10395t-842.40t^2+366.07t^3-93.54t^4+\ldots \end{array}\right\} \quad (25)$$

Case B1: $\gamma_h=0.65$

$$\left.\begin{array}{l} S_h(1)=-123233202.4, S_h(2)=50410489,\ S_h(3)=-10522907.02,\ S_h(4)=3722257.4 \\ I_h(1)=44470.56,\ I_h(2)=-43829.6,\ I_h(3)=19082.11,\ I_h(4)=-5085.28 \\ R_h(1)=127923461,\ R_h(2)=-46965538.7,\ R_h(3)=12780987.9,\ R_h(4)=-2008287.2 \\ V_1(1)=820756.75,\ V_1(2)=831258.15,\ V_1(3)=664550.01,\ V_1(4)=489611.70 \\ V_2(1)=129243,\ V_2(2)=158766.85,\ V_2(3)=-4813.52,\ V_2(4)=6924.69 \\ S_m(1)=-5675,\ S_m(2)=3320.20, S_m(3)=1296.96,\ S_m(4)=1154.31 \\ I_m(1)=10395,\ I_m(2)=-842.40,\ I_m(3)=366.14,\ I_m(4)=-93.43 \end{array}\right\} \quad (26)$$

Then, the closed form of the solution where $k=4$ can be written as

$$\left.\begin{array}{l} s_h(t)=\sum_{k=0}^{\infty}S_h(k).t^k=170638700-123233202.4t+50410489t^2-10522907.02t^3+3722257.4t^4+\ldots \\ i_h(t)=\sum_{k=0}^{\infty}I_h(k).t^k=34200+44470.56t-43829.6t^2+19082.11t^3-5085.28t^4+\ldots \\ r_h(t)=\sum_{k=0}^{\infty}R_h(k).t^k=6482854+127923461t-46965538.7t^2+12780987.9t^3-2008287.2t^4+\ldots \\ v_1(t)=\sum_{k=0}^{\infty}V_1(k).t^k=35000000+820756.75t+831258.15,t^2+664550.01t^3+489611.70t^4+\ldots \\ v_2(t)=\sum_{k=0}^{\infty}V_2(k).t^k=15000000+129243t+158766.85t^2-4813.52t^3+6924.69t^4+\ldots \\ s_m(t)=\sum_{k=0}^{\infty}S_m(k).t^k=35000-5675t+3320.20t^2+1296.96t^3+1154.31t^4+\ldots \\ i_m(t)=\sum_{k=0}^{\infty}I_m(k).t^k=15000+10395t-842.40t^2+366.14t^3-93.43t^4+\ldots \end{array}\right\} \quad (27)$$

Case B2: $\gamma_h = 0.35$

$$\left.\begin{array}{l} S_h(1) = -123233202.4,\ S_h(2) = 50410489,\ S_h(3) = -10522907.8,\ \ S_h(4) = 3722257.8 \\ I_h(1) = 54730.56,\ \ I_h(2) = -39785.58\ I_h(3) = 14009.76,\ \ I_h(4) = -3005.13 \\ R_h(1) = 127913201,\ \ R_h(2) = -46970352.2,\ \ R_h(3) = 12785861.92,\ \ R_h(4) = -2010176.9 \\ V_1(1) = 820756.75,\ V_1(2) = 830771.66,\ \ \ V_1(3) = 664420.04,\ \ V_1(4) = 489724.20 \\ V_2(1) = 129243,\ V_2(2) = 159253.34,\ \ V_2(3) = -4683.55, V_2(4) = 6812.18 \\ S_m(1) = -5675,\ \ S_m(2) = 3320.20,\ \ S_m(3) = 1296.84, S_m(4) = 1154.31 \\ I_m(1) = 10395,\ \ I_m(2) = -842.40,\ \ I_m(3) = 366.26,\ \ I_m(4) = -93.43 \end{array}\right\} \quad (28)$$

Then, the closed form of the solution where $k = 0,\ 1,\ 2,\ 3$ can be written as

$$\left.\begin{array}{l} s_h(t) = \sum_{k=0}^{\infty} S_h(k).t^k = 170638700 - 123233202.4t + 50410489t^2 - 10522907.8t^3 + 3722257.8t^4 + \ldots \\ i_h(t) = \sum_{k=0}^{\infty} I_h(k).t^k = 34200 + 54730.56t - 39785.58t^2 + 14009.76t^3 - 3005.13t^4 + \ldots \\ r_h(t) = \sum_{k=0}^{\infty} R_h(k).t^k = 6482854 + 127913201t - 46970352.2t^2 + 12785861.92t^3 - 2010176.9t^4 + \ldots \\ v_1(t) = \sum_{k=0}^{\infty} V_1(k).t^k = 35000000 + 820756.75t + 830771.66t^2 + 664420.04t^3 + 489724.20t^4 + \ldots \\ v_2(t) = \sum_{k=0}^{\infty} V_2(k).t^k = 15000000 + 129243t + 159253.34t^2 - 4683.55t^3 + 6812.18t^4 + \ldots \\ s_m(t) = \sum_{k=0}^{\infty} S_m(k).t^k = 35000 - 5675t + 3320.20t^2 + 1296.84t^3 + 1154.31t^4 + \ldots \\ i_m(t) = \sum_{k=0}^{\infty} I_m(k).t^k = 15000 + 10395t - 842.40t^2 + 366.26t^3 - 93.43t^4 + \ldots \end{array}\right\} \quad (29)$$

3 Result and discussion

3.1 Numerical solution

We only consider case A1 for the numerical solution

Table 3: Numerical solution of susceptible humans.

t	DTM	RUNGE-KUTTA
0	170638700.0000	170638700.0000
0.1	158808961.7435	158773440.8068
0.2	147924295.8280	147779184.3165
0.3	137921564.8145	137591991.8407
0.4	128737631.2640	128152618.1621
0.5	120309357.7375	119406165.8516
0.6	112573606.7960	111301769.6829
0.7	105467241.0005	103792298.6814
0.8	98927122.9120	96834078.4782
0.9	92890115.0915	90386644.1674
1	87293080.1000	84412501.1746

Table 4: Numerical solution of infected humans.

t	DTM	RUNGE-KUTTA
0	34200.0000	34200.0000
0.1	37537.1800	37536.0791
0.2	40112.0646	40097.7604
0.3	42058.6711	41993.5847
0.4	43511.0163	43318.9995
0.5	44603.1175	44157.8393
0.6	45468.9917	44583.6265
0.7	46242.6559	44660.7545
0.8	47058.1274	44445.5439
0.9	48049.4230	43987.1459
1	49350.5600	43328.3858

Table 5: Numerical solution of recovered/immune humans.

t	DTM	RUNGE-KUTTA
0	6482854.0000	6482854.0000
0.1	18819021.5396	18817939.9880
0.2	30292574.9685	30282186.1550
0.3	40980180.9701	40939434.1593
0.4	50958506.2278	50848843.7416
0.5	60304217.4250	60065237.1135
0.6	69093981.2450	68639413.4068
0.7	77404464.3711	76618445.5887
0.8	85312333.4867	84045957.1847
0.9	92894255.2752	90962368.6286
1	100226896.4200	97405135.6545

Table 6: Numerical solution of non-carrier vector.

t	DTM	RUNGE-KUTTA
0	35000000.0000	35000000.0000
0.1	35091056.0823	35080407.1303
0.2	35202731.3097	35157538.6644
0.3	35339013.1778	35231484.6159
0.4	35503889.1822	35302332.1707
0.5	35701346.8188	35370165.8500
0.6	35935373.5830	35435067.6589
0.7	36209956.9705	35497117.2237
0.8	36529084.4771	35556391.9193
0.9	36896743.5984	35612966.9801
1	37316921.8300	35666915.6074

Table 7: Numerical solution of carrier vector.

t	DTM	RUNGE-KUTTA
0	15000000.0000	15000000.0000
0.1	15014503.8792	15014493.1895
0.2	15032147.5322	15032062.8936
0.3	15052901.8825	15052619.5164
0.4	15076737.8531	15076076.2887
0.5	15103626.3675	15102349.1053
0.6	15133538.3489	15131356.3760
0.7	15166444.7205	15163018.8884
0.8	15202316.4058	15197259.6814
0.9	15241124.3278	15234003.9327
1	15282839.4100	15273178.8525

Table 8: Numerical solution of susceptible monkeys.

t	DTM	RUNGE-KUTTA
0	35000.0000	35000.0000
0.1	34466.9990	34440.5880
0.2	34008.1842	33896.9068
0.3	33631.3378	33368.3130
0.4	33344.2419	32854.1937
0.5	33154.6788	32353.9653
0.6	33070.4305	31867.0716
0.7	33099.2793	31392.9824
0.8	33249.0074	30931.1922
0.9	33527.3969	30481.2188
1	33942.2300	30042.6022

Table 9: Numerical solution of infected monkeys.

t	DTM	RUNGE-KUTTA
0	15000.0000	15000.0000
0.1	16031.4421	16024.5061
0.2	17048.2326	17019.7262
0.3	18052.5679	17986.6737
0.4	19046.6445	18926.3118
0.5	20032.6588	19839.5569
0.6	21012.8071	20727.2815
0.7	21989.2860	21590.3166
0.8	22964.2918	22429.4546
0.9	23940.0210	23245.4515
1	24918.6700	24039.0291

3.2 Graphical Representation of Solutions of the Model Equations

The graphical representations are from the analytical solutions of the model equations. They are plotted using MAPLE software.

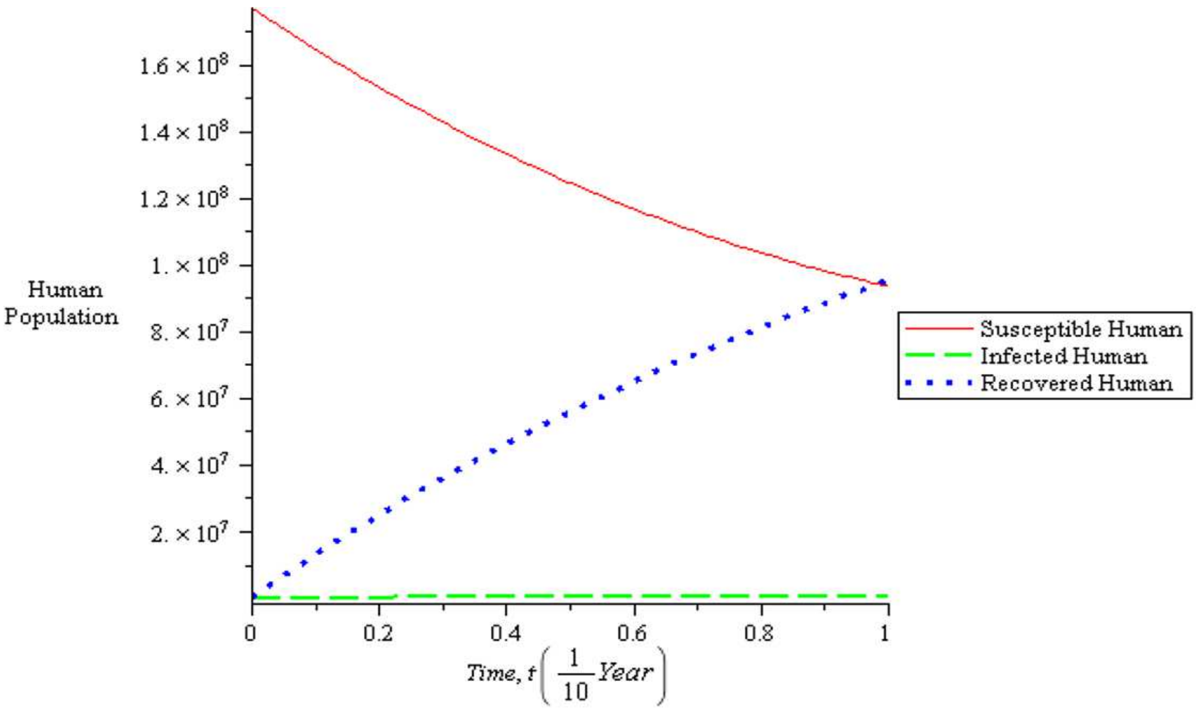

Fig. 1: The effect of high vaccination rate on the humane populations.

The numerical solution of different compartment of the model have been shown in table 3.1 to table 3.7. The DTM solution is in agreement with the Runge-Kutta in Maple software.

Figure 3.1, 3.2 and 3.3 are the effect of the high, moderate and low vaccination rate on the human population

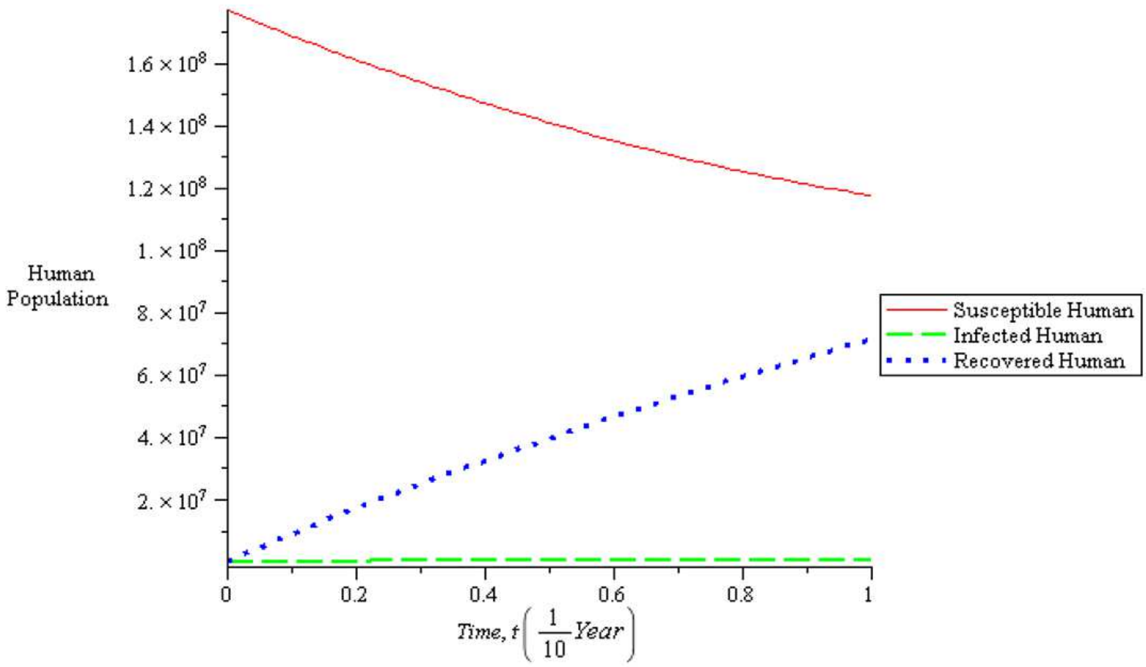

Fig. 2: The effect of moderate vaccination rate on the human populations.

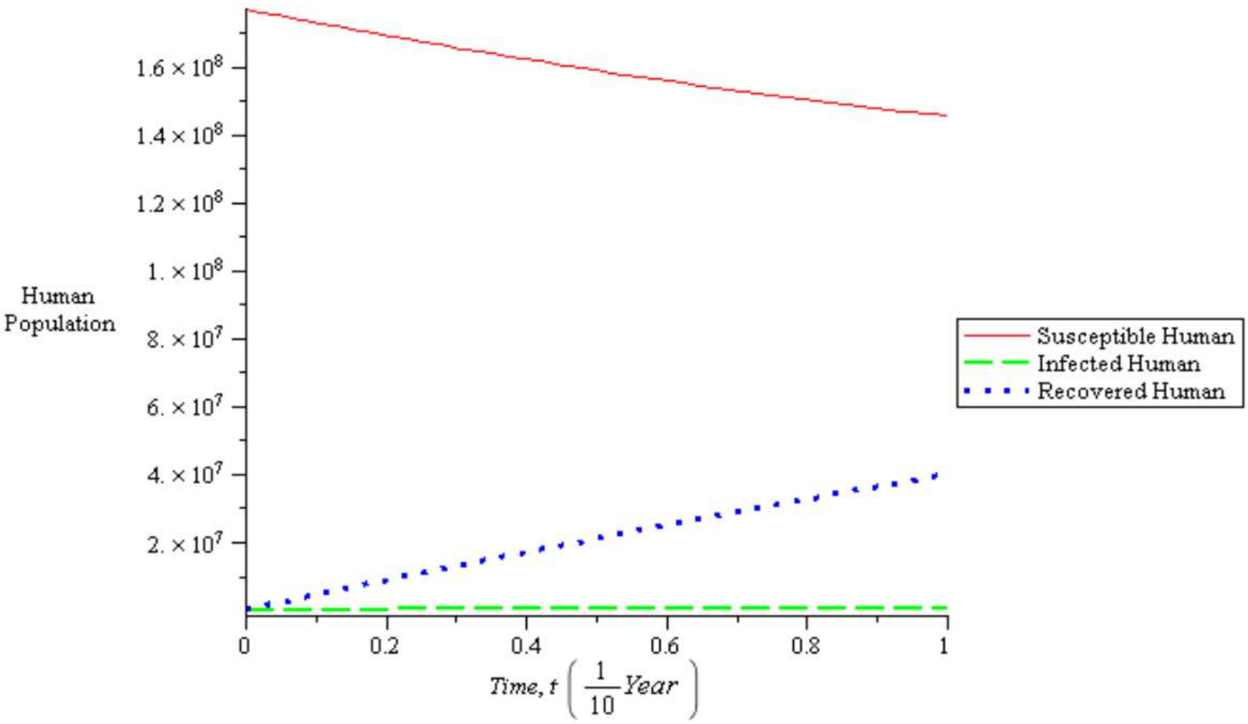

Fig. 3: The effect of low vaccination rate on the human population.

respectively. It was shown that as the vaccination rate increases the susceptible human population decreases and the

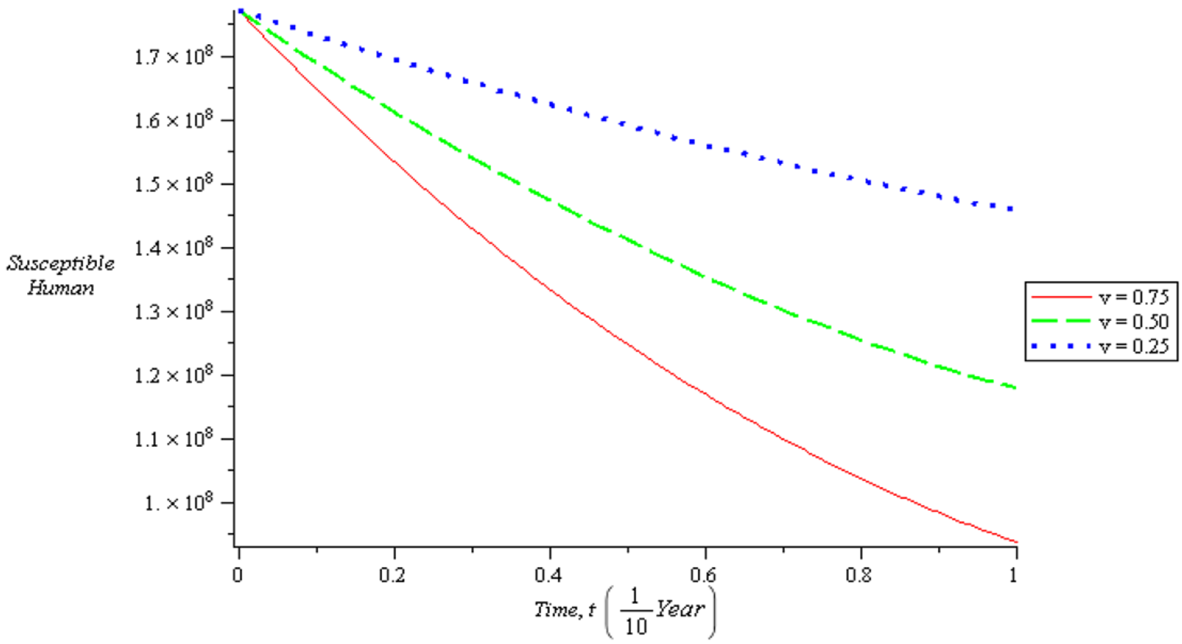

Fig. 4: The effect of different vaccination rate on susceptible humans.

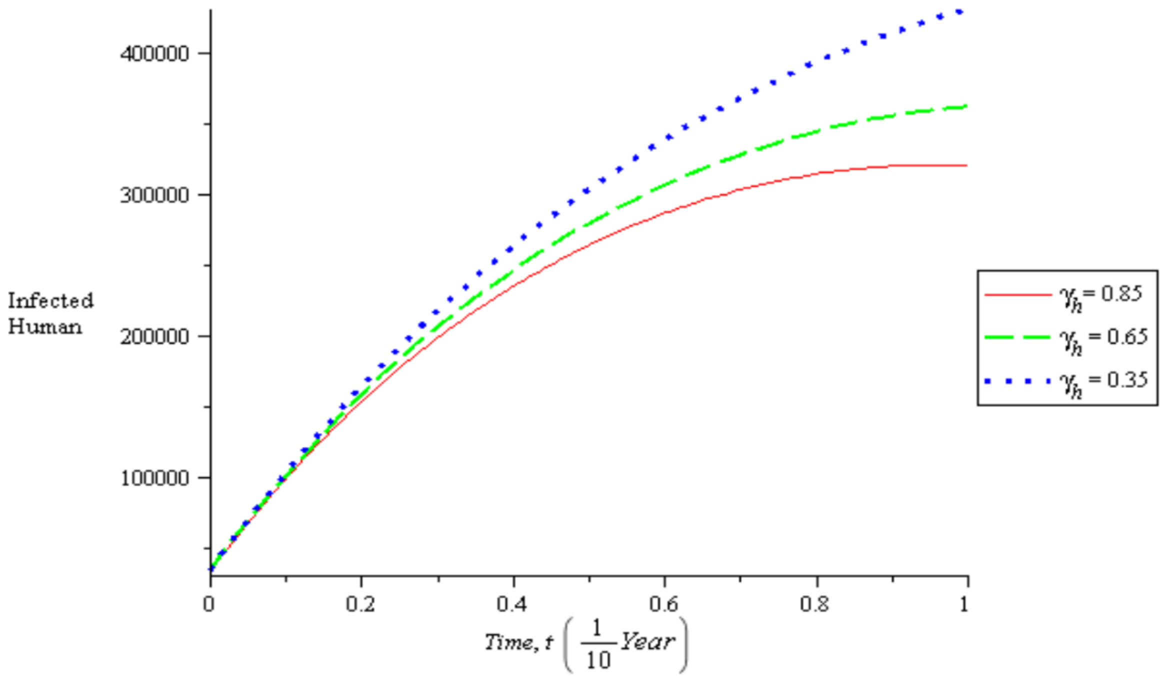

Fig. 5: The effect of different recovery rate on infected humans.

recovered human population increases. This is due to the fact that as susceptible humans are vaccinated they move to recovered class. It was also shown that with high vaccination rate, the recovered population will grow more than the

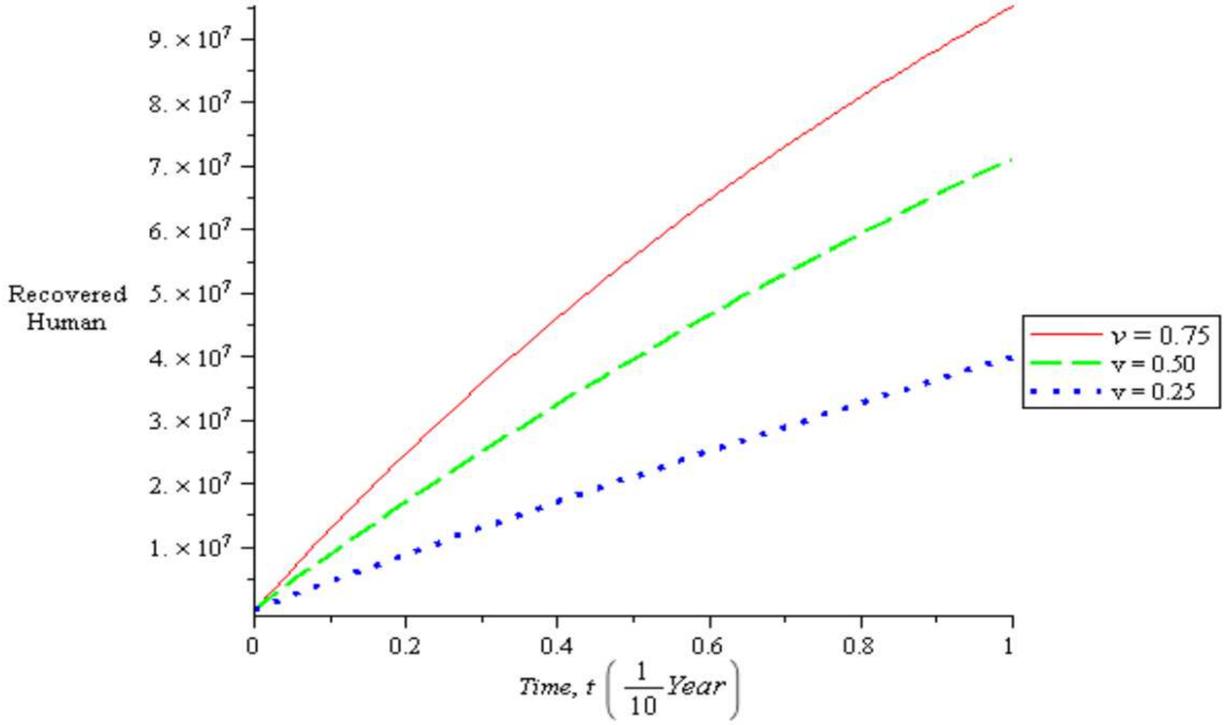

Fig. 6: The effect of different vaccination rate on recovered humans.

susceptible. So also, as the vaccination rate decreases the susceptible population decreases a little and the recovered population increases a little also. Figure 3.4 is the effect of different vaccination rate on susceptible human population. The higher the vaccination rate the lower the susceptible population. The highest percentage almost decreased to zero. This shows that as the susceptible population is vaccinated they are moving to recovered population. Figure 3.5, shows the effect of different recovery rates on infected human population. Infected human population increase with low recovery rate and decreases with high recovery rate. The infected population increases but with treatment and natural healing, it begin to decreases. Figure 3.6, shows the effect of different vaccination rate on recovered human population. The recovered population increased with high vaccination rate and decreased with low vaccination rate. The vaccinated susceptible individuals moved to recovered class.

4 Conclusion

The numerical solution of DTM was validated with Runge-Kutta in Maple. It was discovered from the graphical solutions that, the vaccination of susceptible human population will reduce the outbreak of yellow fever in a community.

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Appendix

Estimation of Variables and Parameter Values

It is difficult to get a reliable data, we estimated the parameter values based on the available data from the World Health Organization (WHO), UNICEF and reliable related literature. The estimates are clearly explained in the following sub-sections.

E1: The Total Human Population of Nigeria, N_h
According to the WHO (2015), Nigeria total human population is at 177,155,754.

$$N_h = 177,155,754$$

E2: Recruitment Number of human in Nigeria, Λ_h
The number of surviving infants in Nigeria in 2014 is 6,865,728. Therefore,

$$\Lambda_h = 6,865,728$$

E3: Infected Humans in Nigeria, I_h
The WHO estimate that, there are 200, 000 cases of yellow fever worldwide each year, resulting in 30,000 deaths of which 90% are in Africa.
90% of 200,000 = 180, 000 cases in Africa
90% of 30,000 = 27, 000 deaths in Africa
According to WHO in 2014 Africa total population is 951,820,000, and Nigeria total population is 19% of Africa total population. Therefore,
19% of 180, 000 = 34,200 cases in Nigeria
19% of 27,000 = 5,130 death in Nigeria

$$I_h = 34,200$$

E4: Recovered/Immune Human population in Nigeria, R_h
Recovered/Immune Human population, R_h= recovered + immune
From E3 the number of cases is 34,200 and number of death is 5130.
Recovered= 34, 200 -5,130 = 29,070the number of surviving infants in 2014 is 6,865,728 and the percentage of vaccinated is 94%. Therefore,
Vaccinated = 94% of 6,865,728 = 6,453,784.
Hence, Recovered/Immune Human population, R_h= 29,070+6,453,784= 6,482,854.

E5: Recovery Rate of Human, γ_h
From E3 and E4

$$\gamma_h = \frac{\text{Recovered/Immune}}{\text{Number of cases}}$$

$$\gamma_h = \frac{29,070}{34,200} = 0.85$$

E6: Susceptible Human population in Nigeria, S_h
Recall $N_h = S_h + I_h + R_h$ therefore,

$$S_h = N_h - (I_h + R_h)$$

$$S_h = 177,155,754 - (34,200 - 6,482,854) = 170,638,700$$

E7: Disease Induce death rate of Human, δ_h
From E3 the number of cases of yellow fever is 34,200 and the number of death from yellow fever is 5,130

$$\delta_h = \frac{\text{Number of Death from yellow fever}}{\text{Number of cases}}$$

$$\delta_h = \frac{5,130}{34,200} = 0.15$$

E8: Natural Death Rate of Human, μ_h
According to WHO, the death rate of Nigeria is 12.01 deaths per 1,000. Therefore,

$$\mu_h = \frac{12.01}{1000} = 0.012$$

E9: Vaccination rate of Human, v
The average percentage of vaccinated infants from 2005 to 2014 is 65.2%. Therefore, we estimate the vaccination rate as

75%, i.e.

$$v = 0.75$$

E10: Total Number of Monkeys N_m

In [16] about 8,000 Drill monkey are found in Cross River State of Nigeria. However 50,000 monkeys are estimated for Nigeria.

Hence, the number of recruitment of monkeys is given by;

$$N_m = 50,000$$

E11: Natural Death Rate of Monkeyμ_m

In [17] the lifespan of monkeys in the forest is 15-30years. Hence,

$$\mu_m = \frac{5}{1000} = 0.005.$$