New Trends in Mathematical Sciences
http://dx.doi.org/10.20852/ntmsci.2019.345

Application of the improved $tan(\phi(\zeta)/2)$-expansion method for solving date-Jimbo-Kashiwara-Miwa equation

Berat Karaagac

Department of Mathematics Education, Adıyaman University, Adıyaman,02000, Turkey

Received: 12 March 2019, Accepted: 20 March 2019
Published online: 28 March 2019.

Abstract: This manuscript uses improved $tan(\phi(\zeta)/2)$-expansion method for obtaining novel exact solutions of Date-Jimbo-Kashiwara-Miwa equation. The method is based on using an auxiliary equation and traveling wave transformation. The implementation of the method with symbolic algorithms yield the various types of solutions including trigonometric, hyperbolic, exponential and rational function, successfully. Some graphical representations of the newly obtained solutions are presented. The results demonstrate that the mentioned method is efficient, active and can be used to obtain exact solutions of a large number of linear and nonlinear PDEs.

Keywords: Improved $tan(\phi(\zeta)/2)$-expansion method, exact solutions, Date-Jimbo-Kashiwara-Miwa equation.

1 Introduction

Most of the complicated phenomena in real life such as fluid mechanics, mass transfer, structure of a population,the propagation of waves, evolution of gases in fluid dynamics and so on are modelled by partial differential equations (PDEs). Thus, understanding PDEs allow making a much better prediction and much broader applications on nature and life. These advanced models prove to humanity why understanding solving partial differential equations are so important. Hence, investigation of many numerical and exact solution methods are illuminated to our scientific world.

In order to contribute to knowledge and literature, we have present an implementation of the improved $tan(\phi(\zeta)/2)$-expansion method for obtaining novel and more general exact solutions of the Date-Jimbo-Kashiwara-Miwa equation. The $tan(\phi(\zeta)/2)$-expansion method was presented and developed by Manafian and Lakestani [1]. The mentioned method has many advantage for providing novel exact solutions of the various partial differential equations (PDEs). One of them is obtaining more common and various solutions with some parameters. The second is the method solved PDEs a direct manner without a requirement of initial and boundary conditions. Additionally, it is very beneficial, easy to implementation.

Let us consider the Date-Jimbo-Kashiwara-Miwa equation (DJKM) as follows [2]

$$u_{xxxxt} + 4u_{xxy}u_x + 2u_{xxx}u_y + 6u_{xy}u_{xx} + u_{yyy} - 2u_{xxt} = 0 \tag{1}$$

where u is the real function of the dependent variables x, y and t. In the Literature, Yuan *et al.* [2] obtained bilinear and one-soliton Solutions of the equation via Hirota method and auxiliary variable. Adem *et al.* [3] obtained the complexiton solutions of DJKM equation using the extended transformed rational function algorithm. Sajid and Akram [4] applied $exp(-\Phi(\xi))$ method to the equation and investigated the exact solutions. In the literature, many authors have noted the

* Corresponding author e-mail: bkaraagac@adiyaman.edu.tr

importance of the method and present articles about the method, further knowledge see in references and there in [5,6,7,8,9].

This paper is organized as follows: in section 2, we present a brief information about the method. In section 3 is cover the application of the method to the DJKM equation. The newly obtained results and their graphical presentation are lie in the section 4. At the last conclusion is formed from outcomes of the present study.

2 The Improved $tan(\phi(\zeta)/2)$-expansion method (ITEM)

First of all, let us give basic structure of the considered method; firstly, we consider the given PDE respect to independent variables x and t given in the following general form

$$Q(z, z_x, z_t, z_{xx}, z_{xt}, z_{tt}, \ldots) = 0, \tag{2}$$

where $z(x,t)$ is an unknown function and Q is a polynomial involved highest order derivative and nonlinear terms. In order to obtain travelling wave solutions of Eq.(2), we combine the all independent variables into one variable with wave transform given as

$$z(x,t) = z(\zeta), \qquad \zeta = k(x - nt). \tag{3}$$

Thus, the transformation generates an ordinary differential equation given as

$$R(z, kz', -knz', k^2z'', -k^2nz'', k^2n^2z'', \ldots) = 0, \tag{4}$$

where prime denotes derivative respect to variable ζ. Any more, we are seeking for a solution for dependent variable u have a form

$$z(\zeta) = \sum_{k=0}^{N} A_k [p + \tan(\phi(\zeta)/2)]^k + \sum_{k=1}^{N} B_k [p + \tan(\phi(\zeta)/2)]^{-k}, \tag{5}$$

where A_k, B_k $(k = 0, 1, 2, \ldots, N)$ and p are constants to be determined and $\phi(\zeta)$ is the solution of the following differential equation

$$\phi'(\zeta) = a\sin(\phi(\zeta)) + b\cos(\phi(\zeta)) + c \tag{6}$$

and positive integer N is the degree of $z(\zeta)$. The degree of the function can be determined using a formula which is consisting the highest order derivative and nonlinear term determine given as follow

$$D\left[\frac{d^q z(\zeta)}{d\zeta^q}\right] = N + q$$
$$D\left[(z(\zeta))^r \left(\frac{d^q z(\zeta)}{d\zeta^q}\right)^s\right] = Nr + s(q + N). \tag{7}$$

Substituting (5) into (4) yields a polynomial in $\tan(\phi(\zeta))$ and $\cot(\phi(\zeta))$. Collecting the coefficients of $(\tan(\phi(\zeta)))^k$ and $(\cot(\phi(\zeta)))^k$ and setting the to zero yields a system of nonlinear algebraic equations including parameters $A_k, B_k, a, b, c, p, m, k$ and n. One can solve the system with Matlab, Mapple or mathematica, thus the desired solution is reached using obtained values of the parameters.

Manafian[10] considered the following nineteen families special solutions of the special equation given in (6).

Family 1 If $a^2+b^2-c^2<0$, and $b\neq c$ then
$$\phi(\zeta)=2\tan^{-1}\left(\frac{a}{b-c}-\frac{\sqrt{-a^2-b^2+c^2}\tan\left(\frac{1}{2}(C+\zeta)\sqrt{-a^2-b^2+c^2}\right)}{b-c}\right)$$
Family 2 If $a^2+b^2-c^2>0$, and $b\neq c$ then
$$\phi(\zeta)=2\tan^{-1}\left(\frac{\sqrt{a^2+b^2-c^2}\tan\left(\frac{1}{2}(C+\zeta)\sqrt{a^2+b^2-c^2}\right)}{b-c}+\frac{a}{b-c}\right)$$
Family 3 If $a^2+b^2-c^2>0, b\neq 0$ and $c=0$ then
$$\phi(\zeta)=2\tan^{-1}\left(\frac{\sqrt{a^2+b^2}\tanh\left(\frac{1}{2}\sqrt{a^2+b^2}(C+\zeta)\right)}{b}+\frac{a}{b}\right)$$
Family 4 If $a^2+b^2-c^2<0, c\neq 0$ and $b=0$ then
$$\phi(\zeta)=2\tan^{-1}\left(\frac{\sqrt{c^2-a^2}\tanh\left(\frac{1}{2}\sqrt{c^2-a^2}(C+\zeta)\right)}{c}-\frac{a}{c}\right)$$
Family 5 If $a^2+b^2-c^2>0, b\neq c$ and $a=0$ then
$$\phi(\zeta)=2\tan^{-1}\left(\sqrt{\frac{b+c}{b-c}}\tanh\left(\frac{1}{2}\sqrt{b^2-c^2}(C+\zeta)\right)\right)$$
Family 6 If $a=c=0$ then $\phi(\zeta)=\tan^{-1}\left(\frac{e^{2b(C+\xi)}-1}{e^{2b(C+\zeta)}+1}\right)$

Family 7 If $b=c=0$ then $\phi(\zeta)=\tan^{-1}\left(\frac{2e^{a(C+\zeta)}+1}{e^{2a(C+\zeta)}+1}\right)$

Family 8 If $a^2+b^2=c^2$ then $\phi(\zeta)=-2\tan^{-1}\left(\frac{(b+c)(a(C+\zeta)+2)}{a^2(C+\zeta)}\right)$

Family 9 If $a=b=c=ka$ then $\phi(\zeta)=2\tan^{-1}\left(e^{ak(cc+\zeta)}-1\right)$

Family 10 If $a=-b=c=ka$ then $\phi(\zeta)=2\tan^{-1}\left(\frac{e^{ak(cc+\xi)}}{e^{ak(cc+\zeta)}-1}\right)$

Family 11 If $c=a$ then $\phi(\zeta)=-2\tan^{-1}\left(\frac{(a+b)e^{b(cc+\zeta)}-1}{(a-b)e^{b(cc+\zeta)}-1}\right)$

Family 12 If $a=c$ then $\phi(\zeta)=2\tan^{-1}\left(\frac{(b+c)e^{b(cc+\zeta)}+1}{(b-c)e^{b(cc+\zeta)}+1}\right)$

Family 13 If $c=-a$ then $\phi(\zeta)=2\tan^{-1}\left(\frac{-a+e^{b(cc+\zeta)}+b}{-a+e^{b(cc+\zeta)}-b}\right)$

Family 14 If $b=-c$ then $\phi(\zeta)=-2\tan^{-1}\left(\frac{ae^{a(cc+\zeta)}}{ce^{a(cc+\zeta)}-1}\right)$

Family 15 If $b=0$ and $a=c$ then $\phi(\zeta)=-2\tan^{-1}\left(\frac{c(cc+\zeta)+2}{c(cc+\zeta)}\right)$

Family 16 If $a=0$ and $b=c$ then $\phi(\zeta)=2\tan^{-1}(c(C+\zeta))$

Family 17 If $a=0$ and $b=-c$ then $\phi(\zeta)=-2\tan^{-1}\left(\frac{1}{c(C+\zeta)}\right)$

Family 18 If $a=b=0$ then $\phi(\zeta)=c(C+\zeta)$

Family 19 If $b=c$ then $\phi(\zeta)=2\tan^{-1}\left(\frac{e^{a(C+\zeta)}-c}{a}\right)$

where C is an integration constant.

3 Application of the Method to (2+1) dimensional Date-Jimbo-Kashiwara–Miwa equation

In this section we are going to apply improved $tan(\phi(\zeta)/2)$-expansion method (ITEM) to the (2+1) dimensional Date-Jimbo-Kashiwara-Miwaequation given in Eq.(1). We start with reducing procedure and implementing travelling wave transformation $u(x,y,t)=\eta(\zeta)$, $\zeta=k(x+my-nt)$. Integrating once, we get;

$$k^2m\eta^{(4)}+6km\eta''\eta'+\left(m^3+2n\right)\eta''=0 \tag{8}$$

where the integrant constants are zero. When we set $u=\eta'$ and according to the balance rule between order of u''' and uu' we get that $N=2$ [4].

$$k^2mu^{(3)}+6kmuu'+\left(m^3+2n\right)u'=0. \tag{9}$$

According to ITEM, we obtain a travelling wave solution of (9) in the form

$$u(\zeta)=A_0+A_1[p+\tan(\phi(\zeta)/2)]+A_2[p+\tan(\phi(\zeta)/2)]^2+B_1[p+\tan(\phi(\zeta)/2)]^{-1}+B_2[p+\tan(\phi(\zeta)/2)]^{-2}. \tag{10}$$

When we substitute (10) into (8) and collecting all terms with the same powers of $\tan(\phi(\zeta))$ and $\cot(\phi(\zeta))$ and setting to zero, we get an algebraic equation system. Solving the system of equations yields to get the following coefficients.

Set 1:

$$A_0 = \frac{m\left(k^2\left(-a^2+6cp(a-b)+2b^2-3p^2(b-c)^2-2c^2\right)-m^2\right)-2n}{6km}$$
$$A_1 = kpb^2 - abk - ack + 2bckp + c^2kp$$
$$A_2 = -\frac{b^2k}{2} + bck - \frac{c^2k}{2}, \quad B_1 = 0, \quad B_2 = O$$

Set 2:

$$A_0 = \frac{2a^2k^2m+2b^2k^2m-2c^2k^2m-m^3-2n}{6(km)}$$
$$A_1 = 0, \quad B_1 = 0, \quad B_2 = -\frac{k\left(a^4+2a^2b^2-2a^2c^2+b^4-2b^2c^2+c^4\right)}{6\left(b^2-2bc+c^2\right)}$$
$$A_2 = -\frac{b^2k}{2} + bck - \frac{c^2k}{2}.$$

4 Results and Graphical Representations

The procedure that given in Section 3 ends with substituting reached values into solution families given in previous Section 2. Therefore, this section presents newly obtained solutions of (2+1) dimensional Date-Jimbo-Kashiwara-Miwa equation and their three-dimensional plots. Using coefficients and variables given in Set 1 and families 1,2,3,5,10 and 18 , the novel exact solutions can be written as

$$\begin{aligned} u_1(x,y,t) =& \frac{k\sqrt{-a^2-b^2+c^2}(a(b-c)+cp(c-2b))\tan\left(\frac{1}{2}\sqrt{-a^2-b^2+c^2}(\mathrm{C}+k(my-nt+x))\right)}{b-c} \\ &+\frac{1}{2}k\left(a^2+b^2-c^2\right)\tan^2\left(\frac{1}{2}\sqrt{-a^2-b^2+c^2}(\mathrm{C}+k(my-nt+x))\right) \\ &-\frac{2k^2m\left(2a^2-b^2+c^2\right)+m^3+2n}{6km}+\frac{ckp(a(3b-2c)+b(c-b))}{b-c}+ckp^2(2b-c) \end{aligned} \quad (11)$$

$$\begin{aligned} u_2(x,y,t) =& \frac{b^2kp\left(\sqrt{a^2+b^2-c^2}\tan\left(\frac{1}{2}\sqrt{a^2+b^2-c^2}(\mathrm{C}+k(my-nt+x))\right)+a+p(b-c)\right)^2}{b-c} \\ &-\frac{1}{2}k\left(\sqrt{a^2+b^2-c^2}\tan\left(\frac{1}{2}\sqrt{a^2+b^2-c^2}(\mathrm{C}+k(my-nt+x))\right)+a+p(b-c)\right) \\ &+\frac{m\left(k^2\left(-a^2+6cp(a-b)+2b^2-3p^2(b-c)^2-2c^2\right)-m^2\right)-2n}{6km} \end{aligned} \quad (12)$$

$$\begin{aligned} u_3(x,y,t) =& \frac{m\left(k^2\left(-a^2+6cp(a-b)+2b^2-3p^2(b-c)^2-2c^2\right)-m^2\right)-2n}{6km} \\ &-\frac{k(b-c)^2\left(\sqrt{a^2+b^2}\tanh\left(\frac{1}{2}\sqrt{a^2+b^2}(\mathrm{C}+k(my-nt+x))\right)+a+bp\right)^2}{2b^2} \\ &+bkp\left(\sqrt{a^2+b^2}\tanh\left(\frac{1}{2}\sqrt{a^2+b^2}(\mathrm{C}+k(my-nt+x))\right)+a+bp\right) \end{aligned} \quad (13)$$

$$\begin{aligned} u_5(x,y,t) =& \frac{m\left(k^2\left(-\left(a^2-6acp+6cp(-2bp+b+cp)-2(b-c)(b+c)\right)\right)-m^2\right)}{6km} \\ &\frac{+3k^2m\tanh\left(\frac{1}{2}\sqrt{(b-c)(b+c)}(\mathrm{cc}+k(my-nt+x))\right)}{6km} \\ &\frac{\left(\left(c^2-b^2\right)\tanh\left(\frac{1}{2}\sqrt{(b-c)(b+c)}(\mathrm{C}+k(my-nt+x))\right)+2cp(2b-c)\sqrt{\frac{b+c}{b-c}}\right)-2n}{6km} \end{aligned} \quad (14)$$

$$u_{10}(x,y,t) = \frac{1}{6}\left(-a^2k - \frac{6k\left(cp(c-2b)+(b-c)^2\right)}{e^{ak(\mathrm{C}+k(my-nt+x))}-1}\right) - \frac{3k(b-c)^2}{6\left(e^{ak(\mathrm{C}+k(my-nt+x))}-1\right)^2} + ackp + \frac{m\left(k^2\left(-\left(b^2-6bc\left(2p^2+p+1\right)+c^2(6p(p+1)+5)\right)\right)-m^2\right)-2n}{6km} \quad (15)$$

$$u_{18}(x,y,t) = \frac{m\left(k^2\left(-\left(a^2-6acp+6cp(-2bp+b+cp)-2(b-c)(b+c)\right)\right)-m^2\right)}{6km} \quad (16)$$
$$+ \frac{k^2 m \tan\left(\frac{1}{2}c(\mathrm{C}+k(my-nt+x))\right)}{2km} \times \left(2cp(2b-c)-(b-c)^2\tan\left(\frac{1}{2}c(\mathrm{cc}+k(my-nt+x))\right)\right) - 2n$$

Graphical simulations of obtained solutions associated with (2+1) dimensional Date-Jimbo-Kashiwara-Miwa equation for set 1 are presented in figures 2. we used Mathematica software for plotting graphics of $u(x,y,t)$ when $-10 \le x,y \le 10$. Also, we chose $a = 2, b = 3, c = 4,2,1$ as suitable for the method conditions and other parameters which are indicated in the captions.

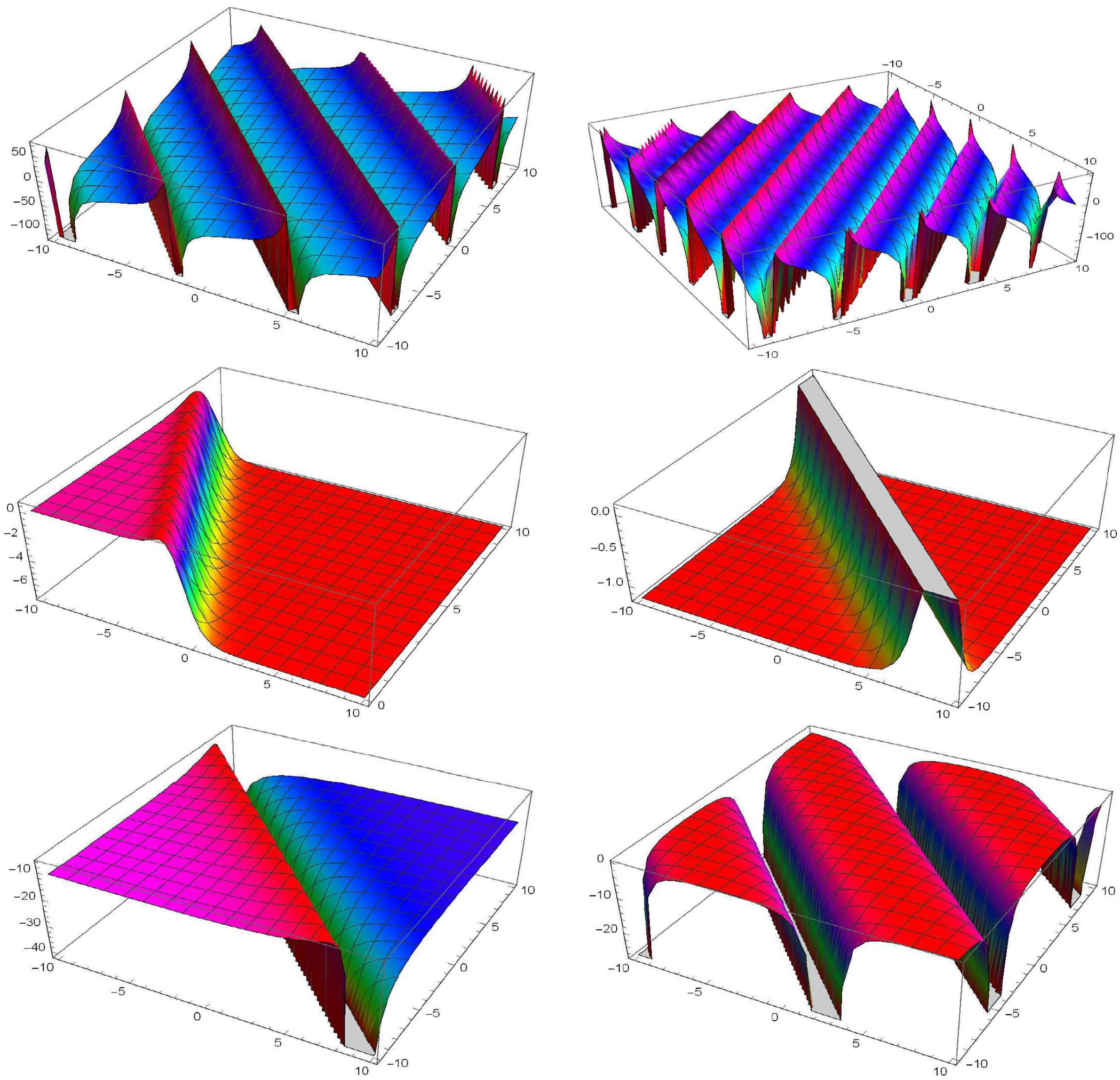

Fig. 1: Eq. (11)-(16)plotted by $3D$ for $m = 1, n = 0.3, k = 0.5$ and $C = 1$,respectively.

The newly obtained solutions and their graphical presentations are presented for Set 2 and families1,2,3,5,6 and 10 the following

$$u_1(x,y,t)=\frac{k\left(a^2+b^2-c^2\right)}{\cos\left(\sqrt{-a^2-b^2+c^2}(\mathrm{C}+k(my-nt+x))\right)+1}+\frac{1}{6}k\left(a^2+b^2-c^2\right)\csc^2\left(\frac{1}{2}\sqrt{-a^2-b^2+c^2}(\mathrm{C}+k(my-nt+x))\right)-\frac{2k^2m\left(a^2+b^2-c^2\right)+m^3+2n}{6km} \tag{17}$$

$$u_2(x,y,t)=-\frac{k^2m\left(a^2+b^2-c^2\right)\tan^2\left(\frac{1}{2}\sqrt{a^2+b^2-c^2}(\mathrm{C}+k(my-nt+x))\right)}{6km}-\frac{\left(\cot^4\left(\frac{1}{2}\sqrt{a^2+b^2-c^2}(\mathrm{C}+k(my-nt+x))\right)+3\right)-2k^2m\left(a^2+b^2-c^2\right)+m^3+2n}{6km} \tag{18}$$

$$u_3(x,y,t)=\left(-\frac{b^2k}{2}+bck-\frac{c^2k}{2}\right)\left(\frac{\sqrt{a^2+b^2}\tanh\left(\frac{1}{2}\sqrt{a^2+b^2}(\mathrm{C}+k(my-nt+x))\right)}{b}-\frac{a}{b-c}+\frac{a}{b}\right)^2+\frac{2a^2k^2m+2b^2k^2m-2c^2k^2m-m^3-2n}{6km}-\frac{k\left(a^4+2a^2b^2-2a^2c^2+b^4-2b^2c^2+c^4\right)}{6\left(b^2-2bc+c^2\right)\left(\frac{\sqrt{a^2+b^2}\tanh\left(\frac{1}{2}\sqrt{a^2+b^2}(\mathrm{C}+k(my-nt+x))\right)}{b}-\frac{a}{b-c}+\frac{a}{b}\right)^2} \tag{19}$$

$$u_5(x,y,t)=\frac{2a^2k^2m+2b^2k^2m-2c^2k^2m-m^3-2n}{6km}-\frac{k\left(a^4+2a^2b^2-2a^2c^2+b^4-2b^2c^2+c^4\right)}{6\left(b^2-2bc+c^2\right)\left(\sqrt{\frac{b+c}{b-c}}\tanh\left(\frac{1}{2}\sqrt{b^2-c^2}(\mathrm{cc}+k(my-nt+x))\right)-\frac{a}{b-c}\right)^2}+\left(-\frac{b^2k}{2}+bck-\frac{c^2k}{2}\right)\left(\sqrt{\frac{b+c}{b-c}}\tanh\left(\frac{1}{2}\sqrt{b^2-c^2}(\mathrm{cc}+k(my-nt+x))\right)-\frac{a}{b-c}\right)^2 \tag{20}$$

$$u_6(x,y,t)=\frac{2a^2k^2m+2b^2k^2m-2c^2k^2m-m^3-2n}{6km}-\frac{k\left(a^4+2a^2b^2-2a^2c^2+b^4-2b^2c^2+c^4\right)}{6\left(b^2-2bc+c^2\right)\left(\tan\left(\frac{1}{2}\tan^{-1}\left(\frac{e^{2b(\mathrm{cc}+k(my-nt+x))}-1}{e^{2b(\mathrm{cc}+k(my-nt+x))}+1}\right)\right)-\frac{a}{b-c}\right)^2}+\left(-\frac{b^2k}{2}+bck-\frac{c^2k}{2}\right)\left(\tan\left(\frac{1}{2}\tan^{-1}\left(\frac{e^{2b(\mathrm{cc}+k(my-nt+x))}-1}{e^{2b(\mathrm{cc}+k(my-nt+x))}+1}\right)\right)-\frac{a}{b-c}\right)^2 \tag{21}$$

$$u_{10}(x,y,t)=\frac{2a^2k^2m+2b^2k^2m-2c^2k^2m-m^3-2n}{6km}-\frac{k\left(a^4+2a^2b^2-2a^2c^2+b^4-2b^2c^2+c^4\right)}{6\left(b^2-2bc+c^2\right)\left(\frac{e^{ak(\mathrm{cc}+k(my-nt+x))}}{e^{ak(\mathrm{cc}+k(my-nt+x))}-1}-\frac{a}{b-c}\right)^2}+\left(-\frac{b^2k}{2}+bck-\frac{c^2k}{2}\right)\left(\frac{e^{ak(\mathrm{cc}+k(my-nt+x))}}{e^{ak(\mathrm{cc}+k(my-nt+x))}-1}-\frac{a}{b-c}\right)^2 \tag{22}$$

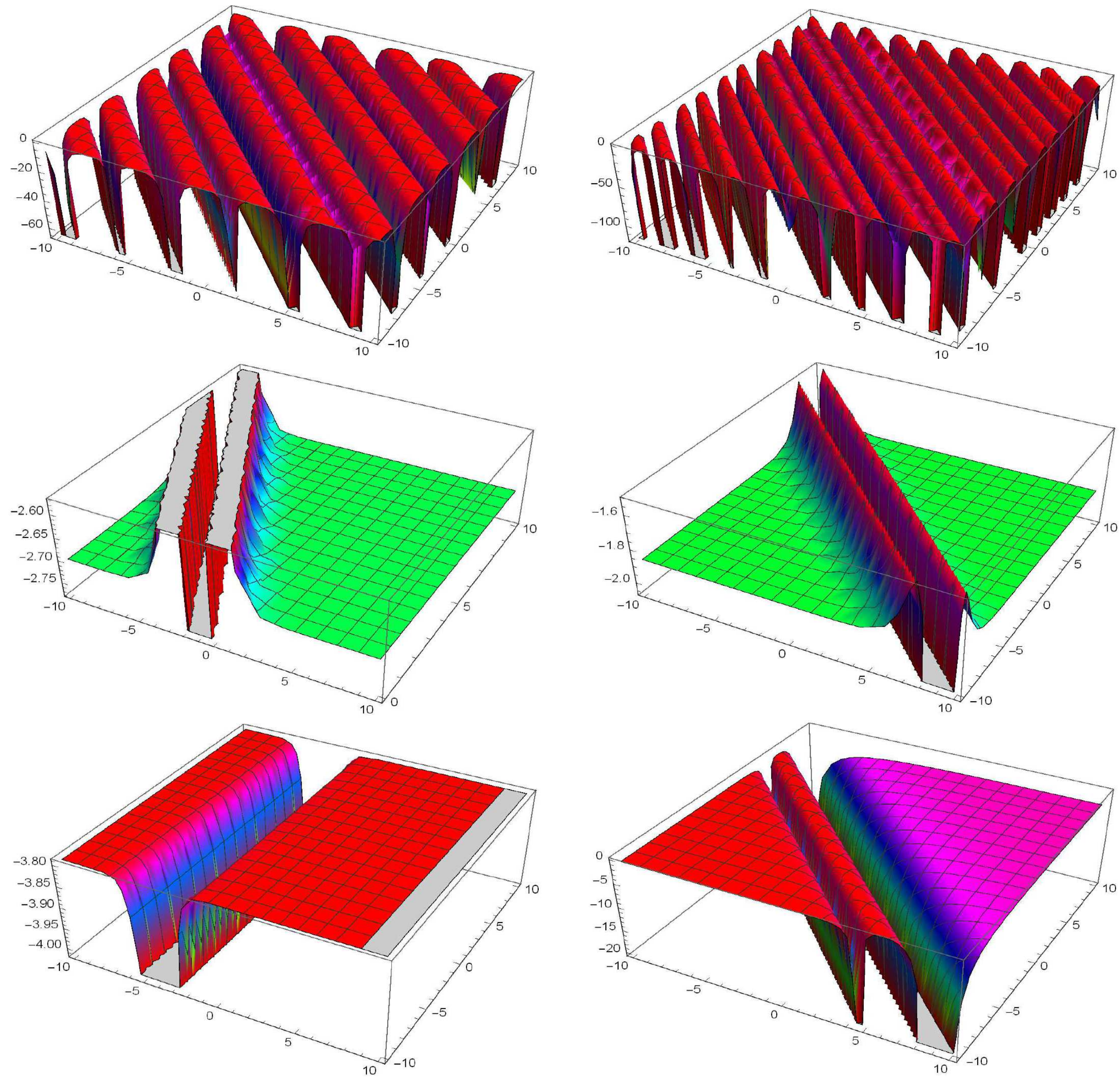

Fig. 2: Eq. (17)-(22)plotted by $3D$ for $a = 2, b = 3, C = 1, m = 1, n = 0.3, 0.5$ and $c = 2$, respectively.

5 Conclusion

In summary, the (2+1) dimensional Date-Jimbo-Kashiwara-Miwa equation is studied in this manuscript. Rich and more general form of solutions including trigonometric,hyperbolic,exponential and rational types of the DJKM equation are constructed. The Maple and Mathematica softwares are used of computation and graphical representation. The newly obtained results show that the proposed method is efficient and straightforward for finding novel solutions of linear or nonlinear various partial differential equations appearing in applied nonlinear sciences.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

References

[1] Manafian, J., Lakestani, M., New improvement of the expansion methods for solving the generalized Fitzhugh- Nagumo equation with time-dependent coefficients, Int. J. Eng. Math, (**2015**), p 1-35, (2015).

[2] Yuan, Y. Q., Tian, B., Sun, W. R., Chai, J., Liu, L., Wronskian and Grammian solutions for a (2+1)-dimensional Date-Jimbo-Kashiwara-Miwa equation, Computers & Mathematics with Applications, (**74**)4, p 873-879, (2017).

[3] Adem, A. R., Yildirim, Y., Yaşar, E., Complexiton solutions and soliton solutions: (2+1)-dimensional Date-Jimbo-Kashiwara-Miwa equation, Pramana (**92**)3, p 1-12,(2019).

[4] Sajid, N., Akram, G., The Application of the $exp(-\Phi\xi)$-Expansion Method for Finding the Exact Solutions of Two Integrable Equations,Hindawi Mathematical Problems in Engineering, (**2018**), p 1-10, (2018).

[5] Hosseini, K., Manafian, J., Samadani, F., Foroutan, M., Mirzazadeh, M. Zhou, Q. ,Resonant optical solitons with perturbation terms and fractional temporal evolution using improved $tan(\Phi(\eta)/2)$-expansion method and exp function approach, Optik, (**158**), p 933-939, (2018).

[6] Kadkhoda, N., Feckan, M., Application of $tan(\Phi(\xi)/2)$-expansion method to burgers and foam drainage equations, Mathematica Slovaca (**68**)5, p 1057-1064, (2018).

[7] Liu, H. Z., Zhang, T,A note on the improved $tan(\phi(\xi)/2)$-expansion method, Optik, (**131**), p 273-278, (2017).

[8] Khan, U., Irshad, A.,Ahmed, N., Mohyud-Din, S. T., Improved tan $\frac{\phi(\xi)}{2}$-expansion method for (2+1)-dimensional KP-BBM wave equation, Optical and Quantum Electronics, (**50**)3, p 135, (2018).

[9] Hammouch, Z., Mekkaoui, T., Agarwal, P., Optical solitons for the Calogero-Bogoyavlenskii-Schiff equation in (2+ 1) dimensions with time-fractional conformable derivative, The European Physical Journal Plus (**133**)248, p 1-6,(2018).

[10] J. Manafian, Optical soliton solutions for Schrödinger type nonlinear evolution equations by the $tan(\phi(\xi)/2)$-expansion method, Optik, (**127**), p 4222-4245, (2016).