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A Remark on "On $\bar{G}-J$ anti-invariant submanifolds of almost complex contact metric manifolds

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Abstract: In this paper, we present some mathematical mistakes of the paper titled "*On $\bar{G}-J$ anti-invariant submanifolds of almost complex contact metric manifolds, New Trends in Mathematical Sciences, NTMSCI 4, No. 3, 277-289 (2016).*"

Keywords: Normal complex contact manifold, submanifolds.

1 Introduction

Complex contact manifolds have been studied since, 1959 the time Kobayashi's first paper [17] was published in this area. Afterwards several authors have been studied on these manifolds [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,20,21,22]. Along with that there are several open problems in this area. One of them is submanifold theory and this theory involves both complex and contact manifolds.

In the article *"On $\bar{G}-J$ anti-invariant submanifolds of almost complex contact metric manifolds, New Trends in Mathematical Sciences, NTMSCI 4, No. 3, 277-289 (2016) "* Cumali Yildirim and Feyza Esra Erdogan aimed to construct a complex almost contact structure on a submanifold of a complex almost contact metric manifold. Also they intended to study anti-invariant submanifold of a complex almost contact metric manifold. But they did not give any result about anti-invariant submanifolds. On the other hand their construction about complex almost contact structure on a submanifold of a complex almost contact metric manifold is not possible.

In this remark, we give important mathematical mistakes and inaccuracy of their results.

2 Comments on abstract and introduction

In abstract the sentence " studied anti-invariant submanifolds of almost complex contact metric manifolds." was written two times. Unfortunately, they didn't give any results on anti-invariant submanifold in the paper [25]. In introduction the authors claim that there is no any work on submanifold theory, but Imada [12] constructed complex almost contact metric structures on complex hypersurfaces in hyperkähler manifolds and give some fundamental equations. Also Mihai [19] gave some definitions for submanifolds.

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3 Mistakes and errors on the section-3: *G-J anti- invariant submanifolds of almost complex contact metric manifolds*

The definition of ambient manifold's structure is not clear. Is this structure "almost complex contact" , "complex almost contact" or "normal complex contact"?

On the other hand as we know $\bar{G}$ is $(1,1)-$tensor on $\bar{M}$ but what is the G? There is no any explanation about this. Similarly the notation $\bar{J}$ is not use for ambient manifold and also any information about complex structure on submanifold is not given. Let $W \in \Gamma(TM)$ then from

$$JT_mM \subseteq T_mM^{\perp} \ \ and \ \ GT_mM \subseteq T_mM^{\perp} \tag{1}$$

we have $\bar{J}W \in \Gamma(TM^{\perp})$ and $\bar{G}W \in \Gamma(TM^{\perp})$. Since the definition of "G" is not given we could not write $\bar{G}\bar{J}W = -JGW$.Assume that $\bar{G}\bar{J}W = -JGW$ is true. Then by applying $\bar{J}$ we get $\bar{G}W = GW$. Other question in here is the new structure vector fields, 1-forms and (1,1)-tensor fields are globally or locally defined? They gave the following equation for $W \in \Gamma(TM)$

$$\bar{G}^2JW = JG^2W = -JW + \bar{g}(JW,\bar{X})\bar{X} + \bar{g}(JW,\bar{Y})\bar{Y}$$

and if the structure vector fields normal to M they obtained $G^2 = -I$. But in this case we have $G^2W = -W + \bar{x}(W)\bar{X} + \bar{y}(W)\bar{Y}$. From now on, authors tried to obtain structure vector field for new contact structure from ambient manifold. They choose $\bar{X} = -JX$ and $\bar{Y} = -JY$ for unit vector field X,Y on M. Then we get

$$J\bar{X} = J(-JX) = -J^2X = X.$$

From definition we know $J\bar{X} \in \Gamma(TM^{\perp})$ and also by assumption $X \in \Gamma(TM)$. Therefore two different elements of different spaces are equal and so must be $X = 0$. Similarly we get $Y = 0$. In the page 284 line 12 authors prove the main theorem as following:

Theorem 1. *Let M be a $G-J$ anti invariant submanifold of an almost complex contact manifold $\bar{M}$, such that $\bar{X}$ and $\bar{Y}$ are normal to M. Then M has almost complex contact structure $(M,X,Y,x,y,H = GJ,g)$.*

Since we get $X = 0$, M has not almost complex contact structure $(M,X,Y,x,y,H = GJ,g)$. Unfortunately, the main theorem is not true. Since the other results in paper are originated from this theorem they also are not true.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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