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Inequalities involving Hadamard products of centrosymmetric matrices

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Abstract: Some new results on Kronecker and Hadamard products of centrosymmetric matrices and their equivalence forms are discussed, respectively. In addition, we derive an upper bound for the spectral radius of Hadamard product of two centrosymmetric matrices A and B with respect to p-norm of blocks of $A \circ B$, for $p \geq 2$.

Keywords: Spectral radius, Hadamard product, Kronecker product, p-norm, centrosymmetric matrix.

1 Introduction and Preliminaries

A centrosymmetric matrix being symmetric about its center has wide range of applications in antenna array, quantum physics, mechanical and electrical systems, pattern recognition, communication theory, speech analysis, digital filters and linear prediction. The structure of centrosymmetric matrices provides computationally efficient results in complex algorithms. Many applications such as pattern recognition feature selection, a uniform linear antenna array, vibration in structures and quantum mechanical oscillator benefit from this inherent structure of centrosymmetric matrices [2,3,4].

Let n be a positive integer. Then, $\mathbb{R}^{n\times n}$ denote the set of all real matrices throughout the paper. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two real $n \times n$ matrices. Then, we have $A \geq B$ $(> B)$ if $a_{ij} \geq b_{ij}$ $(> b_{ij})$ for all $1 \leq i \leq n$, $1 \leq j \leq n$. If $A \geq 0$ (> 0), we say A is nonnegative (positive) matrix, respectively. The spectral radius of A is denoted by $\rho(A)$. If A is a nonnegative matrix, the Perron-Frobenius theorem guarantees that $\rho(A) \in \sigma(A)$, where $\sigma(A)$ denotes the spectrum of A. The Hadamard product of $A, B \in \mathbb{R}^{n\times n}$ is denoted by $A \circ B = [a_{ij}b_{ij}] \in \mathbb{R}^{n\times n}$. The Kronecker product of two matrices $A = [a_{ij}] \in \mathbb{R}^{m\times n}$ and $B = [b_{kl}] \in \mathbb{R}^{p\times q}$ is the matrix $A \otimes B = [a_{ij}B]$ of order $mp \times nq$. The identity matrix in $\mathbb{R}^n$ is denoted by I_n. As usual, $A^* = (\overline{A})^T$ denotes the conjugate transpose of matrix A. A Hermitian matrix A is called positive semidefinite if $x^*Ax \geq 0$ for all $x \in \mathbb{C}^n$, and is called positive definite if $x^*Ax > 0$ for all nonzero $x \in \mathbb{C}^n$ [1,2,5,6,10]. The p-norm of an $m \times n$ matrix A is defined as

$$\|A\|_p = \left(tr\,(A^*A)^{p/2}\right)^{1/p}.$$

Definition 1. *$A = [a_{ij}]_{n\times n}$ is a centrosymmetric matrix, if $a_{ij} = a_{n-i+1,n-j+1}$ where $1 \leq i \leq n$, $1 \leq j \leq n$, or $J_nAJ_n = A$ where J_n is the flip matrix with ones on the secondary diagonal and zeroes elsewhere [5,8,9,11].*

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Lemma 1.*Let $A = [a_{ij}]_{nxn}$ be a centrosymmetric matrix in one of the following forms*

$$A = \begin{bmatrix} A_1 & J_m A_2 J_m \\ A_2 & J_m A_1 J_m \end{bmatrix} \quad or \quad A = \begin{bmatrix} A_1 & J_m a_2 & J_m A_2 J_m \\ a_1^T & \alpha & a_1^T J_m \\ A_2 & a_2 & J_m A_1 J_m \end{bmatrix}. \tag{1}$$

Then, for $B, C \in \mathbb{R}^{m\times m}$, $a_1, a_2 \in \mathbb{R}^{m\times 1}$, α is a scalar, we have either

$$Q_1^T A Q_1 = \begin{bmatrix} A_1 - J_m A_2 & 0 \\ 0 & A_1 + J_m A_2 \end{bmatrix} \tag{2}$$

or

$$Q_2^T A Q_2 = \begin{bmatrix} A_1 - J_m A_2 & 0 & 0 \\ 0 & \alpha & \sqrt{2} a_1^T \\ 0 & \sqrt{2} J_m a_2 & A_1 + J_m A_2 \end{bmatrix}, \tag{3}$$

where $Q_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} I_m & I_m \\ -J_m & J_m \end{bmatrix}$ *and* $Q_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} I_m & 0 & I_m \\ 0 & \sqrt{2} & 0 \\ -J_m & 0 & J_m \end{bmatrix}$.

We have the following facts on the spectral radius of a matrix and the properties of Hadamard and Kronecker products [1, 2, 5, 6, 7, 8, 10].

Lemma 2. *Let $A, B \in \mathbb{R}^{n\times n}$. Then we have*

(1) $\rho(A) \le \|A\|$ *for any norm.*
(2) *If $|A| \le B$, then $\rho(A) \le \rho(|A|) \le \rho(B)$ where $|A| = \left[|a_{ij}|\right]_{n\times n}$ and $\rho(A)$ is the spectral radius of A.*

We use some properties of Hadamard and Kronecker products throughout. One we use is the following property of Kronecker product:

$$(A \otimes B)(C \otimes D) = AC \otimes BD \tag{4}$$

for $A_{p\times q}$, $B_{s\times t}$, $C_{q\times u}$ and $D_{t\times v}$. The other one is the relation between Kronecker and Hadamard products given in the following lemma.

Lemma 3. *Let A and B be $n\times n$ matrices. Then there exists an $n^2 \times n$ selection matrix $\mathbb{J}$ such that $\mathbb{J}^T \mathbb{J} = I$ and*

$$A \circ B = \mathbb{J}^T (A \otimes B) \mathbb{J} \tag{5}$$

where $\mathbb{J}^T$ is an $n\times n^2$ matrix $[E_{11}, E_{22}, ..., E_{nn}]$ and E_{ii} is the $n\times n$ matrix of zeros except for a one in the (i,i)th position.

2 Main results

In this section, we mainly discuss some results on Hadamard and Kronecker products of two centrosymmetric matrices. In [3], Chen, Wang, and Zhong mentioned the following result on centrosymmetric matrices. Here, we give a proof of the result.

Proposition 1. *Let $A = [a_{ij}]_{n\times n}$ and $B = [b_{ij}]_{n\times n}$ be two centrosymmetric matrices. Then $A \otimes B$ is a centrosymmetric matrix.*

Proof. Using the definition of Kronecker product for A and B , we have

$$A\otimes B=\begin{bmatrix} A_1\otimes B & (J_mA_2J_m)\otimes B \\ A_2\otimes B & (J_mA_1J_m)\otimes B \end{bmatrix} \text{ for } n=2m$$

and

$$A\otimes B=\begin{bmatrix} A_1\otimes B & J_ma_2\otimes B & (J_mA_2J_m)\otimes B \\ a_1^T\otimes B & \alpha\otimes B & a_1^TJ_m\otimes B \\ A_2\otimes B & a_2\otimes B & (J_mA_1J_m)\otimes B \end{bmatrix} for\ n=2m+1.$$

It is enough to show that

$$J_{mn}(A_1\otimes B)J_{mn}=(J_mA_1J_m)\otimes B \text{ and } J_n(a_1^T\otimes B)J_{mn}=(a_1^TJ_m)\otimes B.$$

Indeed since B is centrosymmetric and $J_{mn}=J_m\otimes J_n$ we have

$$J_{mn}(A_1\otimes B)J_{mn}=(J_m\otimes J_n)(A_1\otimes B)(J_m\otimes J_n)=(J_mA_1J_m)\otimes B$$

and

$$J_n(a_1^T\otimes B)J_{mn}=(1\otimes J_n)(a_1^T\otimes B)(J_m\otimes J_n)=a_1^TJ_m\otimes B.$$

Therefore, $A\otimes B$ is a centrosymmetric matrix.

It was given by Zhao, Li and Gong in [11] that Hadamard product of two centrosymmetric matrices is centrosymmetric. We give an alternative proof for this statement. For the proof, we need the following relation between flip and selection matrices.

Proposition 2. *Let J_m and $\mathbb{J}_{m^2\times m}$ be the flip and the selection matrices, respectively. Then*

(1) $\mathbb{J}^T_{m\times m^2}J_{m^2}=J_m\mathbb{J}^T_{m\times m^2}$
(2) $J_{m^2}\mathbb{J}_{m^2\times m}=\mathbb{J}_{m^2\times m}J_m$.

Proof. If J_m is an $m\times m$ flip matrix and $\mathbb{J}^T_{m\times m^2}$ is an $m\times m^2$ selection matrix,

$$\begin{aligned}\mathbb{J}^T_{m\times m^2}J_{m^2} &= [E_{11},E_{22},...,E_{mm}]J_{m^2} \\ &= [E_{mm}J_m,E_{m-1,m-1}J_m,\ldots,E_{22}J_m,E_{11}J_m] \\ &= [J_mE_{11},J_mE_{22},\ldots,J_mE_{m-1,m-1},J_mE_{mm}] \\ &= J_m[E_{11},E_{22},\ldots,E_{mm}] \\ &= J_m\mathbb{J}^T_{m\times m^2}.\end{aligned}$$

Thus, the proof of part (1) is completed. The second part clearly follows from (1) since $J_m^T=J_m$.

We define a centrosymmetric matrix B in one of the following two forms

$$B=\begin{bmatrix} B_1 & J_mB_2J_m \\ B_2 & J_mB_1J_m \end{bmatrix} \quad or \quad B=\begin{bmatrix} B_1 & J_mb_2 & J_mB_2J_m \\ b_1^T & \beta & b_1^TJ_m \\ B_2 & b_2 & J_mB_1J_m \end{bmatrix} \tag{6}$$

for $n=2m$ or $n=2m+1$, respectively.

Theorem 1. *Let $A=[a_{ij}]_{nxn}$ and $B=[b_{ij}]_{nxn}$ be two centrosymmetric matrices as in (1) and (6), then $A\circ B$ is a centrosymmetric matrix.*

*Proof.*Using the definition of Hadamard product of A and B for $n=2m$ and $n=2m+1$, we have

$$A\circ B=\begin{bmatrix} A_1\circ B_1 & (J_mA_2J_m)\circ(J_mB_2J_m) \\ A_2\circ B_2 & (J_mA_1J_m)\circ(J_mB_1J_m)\end{bmatrix}$$

and

$$A\circ B=\begin{bmatrix} A_1\circ B_1 & J_ma_2\circ J_mb_2 & (J_mA_2J_m)\circ(J_mB_2J_m) \\ a_1^T\circ b_1^T & \alpha\beta & a_1^TJ_m\circ b_1^TJ_m \\ A_2\circ B_2 & a_2\circ b_2 & (J_mA_1J_m)\circ(J_mB_1J_m)\end{bmatrix}.$$

To prove $A\circ B$ is a centrosymmetric matrix, we need to show that

$$(J_mA_1J_m)\circ(J_mB_1J_m)=J_m(A_1\circ B_1)J_m \text{ and } J_1(a_1^T\circ b_1^T)J_m=a_1^TJ_m\circ b_1^TJ_m.$$

Note that the second equation clearly follows. For the first one, by Proposition 2 and Lemma 3, we have

$$\begin{aligned}(J_mA_1J_m)\circ(J_mB_1J_m) &= \mathbb{J}_{m\times m^2}^T[(J_mA_1J_m)\otimes(J_mB_1J_m)]\mathbb{J}_{m^2\times m}\\ &= \mathbb{J}_{m\times m^2}^T[(J_m\otimes J_m)(A_1\otimes B_1)(J_m\otimes J_m)\mathbb{J}_{m^2\times m}\\ &= \mathbb{J}_{m\times m^2}^T(J_{m^2}(A_1\otimes B_1)J_{m^2})\mathbb{J}_{m^2\times m}\\ &= \mathbb{J}_{m\times m^2}^TJ_{m^2}(A_1\otimes B_1)J_{m^2}\mathbb{J}_{m^2\times m}\\ &= J_m\mathbb{J}_{m\times m^2}^T(A_1\otimes B_1)\mathbb{J}_{m^2\times m}J_m\\ &= J_m\left(\mathbb{J}_{m\times m^2}^T(A_1\otimes B_1)\mathbb{J}_{m^2\times m}\right)J_m\\ &= J_m(A_1\circ B_1)J_m.\end{aligned}$$

We note that by Lemma 1, $A\circ B$ has one of the following two forms

$$Q_1^T(A\circ B)Q_1=\begin{bmatrix} P-J_mS & 0 \\ 0 & P+J_mS\end{bmatrix} \; for\ n=2m \quad or \tag{7}$$

$$Q_2^T(A\circ B)Q_2=\begin{bmatrix} P-J_mS & 0 \\ 0 & K\end{bmatrix} \; for\ n=2m+1, \tag{8}$$

where $P=A_1\circ B_1$, $S=A_2\circ B_2$, and $K=\begin{bmatrix} \alpha\beta & \sqrt{2}(a_1^T\circ b_1^T) \\ \sqrt{2}J_m(a_2\circ b_2) & P+J_mS\end{bmatrix}$.

Li, Zhao, Dai and Su in [8] had proved that for a nonnegative centrosymmetric matrix it is possible to find the spectral radius of a matrix in terms of its blocks. In fact, if A and B are centrosymmetric matrices, then we have

$$\begin{aligned}&\rho(A\circ B)=\rho\{A_1\circ B_1+J_m(A_2\circ B_2)\} \quad \text{for } n=2m \quad \text{and}\\ &\rho(A\circ B)=\max\{\rho(K),\rho(A_1\circ B_1-J_m(A_2\circ B_2))\} \quad \text{for } n=2m+1.\end{aligned} \tag{9}$$

Now we give an upper bound for the spectral radius of Hadamard product of two nonnegative centrosymmetric matrices.

Proposition 3. *Let $A=[a_{ij}]_{n\times n}$ and $B=[b_{ij}]_{n\times n}$ be two nonnegative centrosymmetric matrices, then*

$$\rho(A\circ B)\leq \rho(A_1^TB_1)+m^{1/p}\rho(A_2^TB_2) \text{ for } n=2m \quad \text{and}$$
$$\rho(A\circ B)\leq \max\{\|K\|_p,\|A_1\circ B_1-J_m(A_2\circ B_2)\|_p\} \text{ for } n=2m+1.$$

Proof. For $n=2m$, by equation (9) and part (1) in Lemma 2, we have

$$\begin{aligned}\rho(A\circ B) &= \rho((A_1\circ B_1)+J_m(A_2\circ B_2))\\ &\leq \|A_1\circ B_1+J_m(A_2\circ B_2)\|_p\\ &\leq \|A_1\circ B_1\|_p+\|J_m(A_2\circ B_2)\|_p\\ &\leq \|A_1\circ B_1\|_p+\|J_m\|_p\|A_2\circ B_2\|_p.\end{aligned}$$

For any nonnegative matrices, we have $\|A\circ B\|\leq\rho(A^TB)$ by [6], then

$$\rho(A\circ B)\leq\rho(A_1^TB_1)+\|J_m\|_p\,\rho(A_2^TB_2).$$

Since $\|J_m\|_p=m^{1/p}$, we get the result. The second inequality follows directly.

Lemma 4. *Let $X,Y,Z,W\in\mathbb{C}^{n\times n}$. Then, for $2\leq p<\infty$,*

$$\left\|\begin{bmatrix}X & Y\\ W & Z\end{bmatrix}\right\|_p \leq 2^{1/p}\left(\frac{p-1}{2}Tr(\alpha^2)+\frac{2-p}{4}\left(Tr(\alpha)\right)^2\right)^{1/2},$$

where

$$\alpha=\begin{bmatrix}\|X\|_p & \left(\frac{1}{2}\left(\|Y\|_p^p+\|W\|_p^p\right)\right)^{1/p}\\ \left(\frac{1}{2}\left(\|Y\|_p^p+\|W\|_p^p\right)\right)^{1/2} & \|Z\|_p\end{bmatrix}$$

as in [7].

Now we give an upper bound for the spectral radius of Hadamard product of two centrosymmetric matrices in terms of p-norms.

Theorem 2. *Let A and B be two centrosymmetric matrices as in (7) or(8), and let $2\leq p<\infty$. Then for $n=2m$ and $n=2m+1$, respectively, we have*

$$\rho(A\circ B)\leq 2^{1/p}(p-1)\left(1+m^2\right)\left(\|P\|_p^2+\|S\|_p^2\right)^{1/2}$$

$$\rho(A\circ B)\leq\|P-J_mS\|_p+2^{1/p}\left(\frac{p-1}{2}\left(\alpha^2\beta^2+2M^2+\|P+J_mS\|_p^2\right)+\frac{2-p}{4}\left(\alpha\beta+\|P+J_mS\|_p\right)^2\right)^{1/2}$$

where $P=A_1\circ B_1$, $S=A_2\circ B_2$, and

$$M=\left\{\frac{1}{2}\left(\left\|\sqrt{2}J_m(a_2\circ b_2)\right\|_p^p+\left\|\sqrt{2}(a_1^T\circ b_1^T)\right\|_p^p\right)\right\}^{1/p}.$$

Proof. If $n = 2m$, we define a symmetric matrix such that

$$\alpha = \begin{bmatrix} \|A_1 \circ B_1\|_p & \left(\frac{1}{2}\left(\|J_m(A_2 \circ B_2)J_m\|_p^p + \|A_2 \circ B_2\|_p^p\right)\right)^{1/p} \\ \left(\frac{1}{2}\left(\|J_m(A_2 \circ B_2)J_m\|_p^p + \|A_2 \circ B_2\|_p^p\right)\right)^{1/p} & \|J_m(A_1 \circ B_1)J_m\|_p \end{bmatrix}$$

$$= \begin{bmatrix} \|P\|_p & \left(\frac{1}{2}\left(\|J_mSJ_m\|_p^p + \|S\|_p^p\right)\right)^{1/p} \\ \left(\frac{1}{2}\left(\|J_mSJ_m\|_p^p + \|S\|_p^p\right)\right)^{1/p} & \|J_mPJ_m\|_p \end{bmatrix}.$$

Note that since $\|J_m\|_p = \left(Tr(J_m^*J_m)^{p/2}\right)^{1/p} = (Tr(I_m))^{1/p} = m^{1/p}$, we have

$$\begin{aligned} \left(\frac{1}{2}\left(\|J_mSJ_m\|_p^p + \|S\|_p^p\right)\right)^{2/p} &\leq 2^{-2/p}\left(\|J_m\|_p^p\|S\|_p^p\|J_m\|_p^p + \|S\|_p^p\right)^{2/p} \\ &= 2^{-2/p}\left((m^2+1)\|S\|_p^p\right)^{2/p} \\ &= 2^{-2/p}(m^2+1)^{2/p}\|S\|_p^2. \end{aligned} \tag{10}$$

Then we get

$$\begin{aligned} (Tr(\alpha))^2 &= (\|P\|_p + \|J_mPJ_m\|_p)^2 \\ &\leq (\|P\|_p + \|J_m\|_p\|P\|_p\|J_m\|_p)^2 \\ &\leq \left(\|P\|_p + m^{2/p}\|P\|_p\right)^2 = (1+m^{2/p})^2\|P\|_p^2 \end{aligned}$$

and by equation (10), we obtain

$$\begin{aligned} Tr(\alpha)^2 &= \|P\|_p^2 + \|J_mPJ_m\|_p^2 + 2\left(\frac{1}{2}\left(\|J_mSJ_m\|_p^p + \|S\|_p^p\right)\right)^{2/p} \\ &\leq \|P\|_p^2 + m^{4/p}\|P\|_p^2 + 2^{1-2/p}(m^2+1)^{2/p}\|S\|_p^2 \\ &= (m^{4/p}+1)\|P\|_p^2 + 2^{1-2/p}(m^2+1)^{2/p}\|S\|_p^2. \end{aligned}$$

Thus, since $\|A \circ B\|_p = \|Q_1(A \circ B)Q_1\|_p$ and $Q_1(A \circ B)Q_1$ is as in (7), by Lemma 4, for $p \geq 2$ we have

$$\begin{aligned} \rho(A \circ B) &\leq \|A \circ B\|_p \\ &\leq 2^{1/p}\left(\frac{p-1}{2}(m^{4/p}+1)\|P\|_p^2 + (p-1)2^{-2/p}(m^2+1)^{2/p}\|S\|_p^2 + \frac{2-p}{4}(1+m^{2/p})^2\|P\|_p^2\right)^{1/2} \\ &= 2^{1/p}\left(\frac{p}{4}(m^{4/p}+1)\|P\|_p^2 + m^{2/p}\left(1-\frac{p}{2}\right)\|P\|_p^2 + (p-1)2^{-2/p}(m^2+1)^{2/p}\|S\|_p^2\right)^{1/2} \\ &= 2^{1/p}\left(\frac{p}{4}\left(m^{4/p} - \frac{2(p-2)}{p}m^{2/p}+1\right)\|P\|_p^2 + (p-1)2^{-2/p}(m^2+1)^{2/p}\|S\|_p^2\right)^{1/2} \\ &\leq 2^{1/p}\left((p-1)\left(m^{2/p}+1\right)^2\|P\|_p^2 + (p-1)2^{-2/p}(m^2+1)^{2/p}\|S\|_p^2\right)^{1/2} \\ &\leq 2^{1/p}\left((p-1)\left(m^2+1\right)^2\|P\|_p^2 + (p-1)2^{-2/p}(m^2+1)^2\|S\|_p^2\right)^{1/2} \\ &\leq 2^{1/p}(p-1)\left(m^2+1\right)\left(\|P\|_p^2 + \|S\|_p^2\right)^{1/2}. \end{aligned}$$

Inequalities follow from the fact that $m^{4/p}-\frac{2(p-2)}{p}m^{2/p}+1\leq\left(m^{2/p}+1\right)^2$ and $2^{-2/p}<1$ for $p\geq 2$.

If $n=2m+1$, note that

$$\rho(A\circ B)\leq\|A\circ B\|_p=\|Q_2^T(A\circ B)Q_2\|_p=\|A_1\circ B_1-J_m(A_2\circ B_2)\|_p+\|K\|_p \tag{11}$$

where $K=\begin{bmatrix}\alpha\beta & \sqrt{2}(a_1^T\circ b_1^T)\\ \sqrt{2}J_m(a_2\circ b_2) & (A_1\circ B_1)+J_m(A_2\circ B_2)\end{bmatrix}$. Then define a symmetric matrix $\tilde{K}$ such that

$$\begin{aligned}\tilde{K}&=\begin{bmatrix}\alpha\beta & \left(\frac{1}{2}\left(\left\|\sqrt{2}J_m(a_2\circ b_2)\right\|_p^p+\left\|\sqrt{2}(a_1^T\circ b_1^T)\right\|_p^p\right)\right)^{1/p}\\ \left(\frac{1}{2}\left(\left\|\sqrt{2}J_m(a_2\circ b_2)\right\|_p^p+\left\|\sqrt{2}(a_1^T\circ b_1^T)\right\|_p^p\right)\right)^{1/p} & \|(A_1\circ B_1)+J_m(A_2\circ B_2)\|_p\end{bmatrix}\\ &=\begin{bmatrix}\alpha\beta & M\\ M & \|P+J_mS\|_p\end{bmatrix}.\end{aligned}$$

Then $Tr(\tilde{K})=\alpha\beta+\|P+J_mS\|_p$ and $Tr(\tilde{K}^2)=\alpha^2\beta^2+2M^2+\|P+J_mS\|_p^2$. By Lemma 4, we have

$$\|K\|_p\leq 2^{1/p}\left[\frac{1}{2}\left(\alpha^2\beta^2+2M^2+\|P+J_mS\|_p^2\right)+\frac{2-p}{4}\left(\alpha\beta+\|P+J_mS\|_p\right)^2\right]^{1/2} \tag{12}$$

Thus, combining equations (11) and (12), we obtain the result.

3 Conclusion

The algebraic relationships between the Kronecker and Hadamard product of centrosymmetric matrices have been examined in this work. Specifically, the upper bound for the spectral radius of Hadamard product of two centrosymmetric matrices have been obtained analytically.

Furthermore, the alternative proof of the statement which has two different dimensions of the Hadamard product of two centrosymmetric matrices have been pointed out. Finally, the upper bound for the spectral radius of Hadamard product of two centrosymmetric matrices in terms of p-norms of submatrices were given.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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