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On integer additive set-filter graphs

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Abstract: Let $\mathbb{N}_0$ denote the set of all non-negative integers and $\mathscr{P}(\mathbb{N}_0)$ be its power set. An integer additive set-labeling (integer additive set-labeling) of a graph G is an injective function $f : V(G) \to \mathscr{P}(\mathbb{N}_0)$ such that the induced function $f^+ : E(G) \to \mathscr{P}(\mathbb{N}_0)$ is defined by $f^+(uv) = f(u) + f(v)$, where $f(u) + f(v)$ is the sumset of $f(u)$ and $f(v)$. In this paper, we introduce the notion of a particular type of integer additive set-indexers called integer additive set-filter labeling of given graphs and study their characteristics.

Keywords: Integer additive set-labeling, integer additive set-filter labeling, integer additive set-filter graphs.

1 Introduction

For all terms and definitions, not defined specifically in this paper, we refer to [3] and [8] and [19]. Unless mentioned otherwise, all graphs considered here are simple, finite and have no isolated vertices.
The *sumset* of two non-empty sets A and B, denoted by $A+B$, is defined as $A+B = \{a+b : a \in A, b \in B\}$. If either A or B is countably infinite, then their sumset $A+B$ is also countably infinite. Hence, all sets we mention here are finite. We denote the cardinality of a set A by $|A|$.
Using the concepts of sumsets, an integer additive set-labeling of a given graph G is defined as follows

Definition 1.[6] Let $\mathbb{N}_0$ denote the set of all non-negative integers and $\mathscr{P}(\mathbb{N}_0)$ be its power set. An *integer additive set-labeling* (integer additive set-labeling , in short) of a graph G is defined as an injective function $f : V(G) \to \mathscr{P}(\mathbb{N}_0)$ which induces a function $f^+ : E(G) \to \mathscr{P}(\mathbb{N}_0)$ such that $f^+(uv) = f(u) + f(v)$, $uv \in E(G)$. A graph which admits an integer additive set-labeling is called an *integer additive set-labeled graph*.

The notion of an integer additive set-indexers of graphs was introduced in [6].

Definition 2.*[6, 12] An integer additive set-labeling $f : V(G) \to \mathscr{P}(\mathbb{N}_0)$ of a graph G is said to be an* integer additive set-indexer *(integer additive set-labeling) if the induced function $f^+ : E(G) \to \mathscr{P}(\mathbb{N}_0)$ defined by $f^+(uv) = f(u) + f(v)$ is also injective. A graph which admits an integer additive set-labeling is called an* integer additive set-indexed graph *(integer additive set-labeling -graph).*

The existence of an integer additive set-labeling (or integer additive set-indexers) by a given graph was established in [12] and the admissibility of integer additive set-labeling (or integer additive set-indexers) by given graph operations and graph products was established in [18].

Theorem 1.[12] *Every graph G admits an integer additive set-labeling (or an integer additive set-indexer).*

The cardinality of the set-label of an element (a vertex or an edge) of a graph G is called the *set-indexing number* of that element. An element of G having a singleton set-label is called a *mono-indexed element* of G.
In this paper, we study the characteristic of graphs which admit a certain type of integer additive set-labeling, called integer additive set-filter labeling.

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2 Integer Additive Set-Filter Graphs

Note that all sets we consider in this paper are non-empty finite sets of non-negative integers. By the term a *ground set*, we mean a non-empty finite set of non-negative integers whose subsets are the set-labels of the elements of the given graph G. We denote the ground set used for labeling the elements of a graph G by X.
Motivated from the studies about topological integer additive set-labeled graphs, made in [14], we study a set-labeling of a given graph, in which the collection of all set-labels of the vertices of a given graph forms a filter of the ground set used for the labeling. Let us first recall the definition of the filter of a set.

Definition 3.*[9, 11] Given a set X, a partial ordering $\subseteq$ can be defined on the power set $\mathscr{P}(X)$ by subset inclusion, turning $(\mathscr{P}(X),\subseteq)$ into a lattice. A* filter *on X, denoted by $\mathscr{F}$, is a non-empty subset of the power set $\mathscr{P}(X)$ of X which has the following properties*

(i) $X \in \mathscr{F}$.
(ii) $A,B \in \mathscr{F} \implies A\cap B \in \mathscr{F}$. *($\mathscr{F}$ is closed under finite intersection).*
(iii) $\emptyset \notin \mathscr{F}$. *($\mathscr{F}$ is a proper filter).*
(iv) $A \in \mathscr{F}, A \subset B, \implies B \in \mathscr{F}$ *where B is a non-empty subset of X.*

In view of Definition 3, we define the notion of an integer additive set-filter labeling of a given graph as follows.

Definition 4. *Let X be a finite set of non-negative integers. Then, an integer additive set-labeling $f : V(G) \to \mathscr{P}(X)$ is said to be an* integer additive set-filter labeling *of G if $\mathscr{F} = f(V)$ is a proper filter on X. A graph G which admits an integer additive set-filter labeling is called an* integer additive set-filter graph.

Note that the null set can not be the set-label of any element of the graph G, with respect to an integer additive set-labeling defined on it.
Does every given graph admit an integer additive set-filter labeling? If not so, what is the condition required for a graph to admit an integer additive set-filter labeling? As answers to both questions, we establish a necessary and sufficient condition for an integer additive set-labeling f of a given graph G to be an integer additive set-filter labeling of G as follows

Theorem 2. *An integer additive set-labeling f defined on a given graph G with respect to a non-empty ground set X is an integer additive set-filter labeling of G if and only if the following conditions hold.*

(i) $0 \in X$.
(ii) every subset of X containing 0 is the set-label of some vertex in G.
(iii) 0 is an element of the set-label of every vertex in G.

Proof. Let f be an integer additive set-filter labeling defined on a given graph G, with respect to a non-empty set X. Then, $\mathscr{F} = f(V)$ is a filter on X. Therefore, $X \in \mathscr{F}$. Since, for any non-zero element $a \in X$, the sets X and $X+\{a\}$ are of same cardinality, but indeed $X \subsetneq X+\{a\}$. Hence, $\{0\}$ must also be an element of $\mathscr{F}$. Hence, we notice that 0 is an element of X. Then, by condition (iv) of Definition 3, every subset of X containing 0 must belong to $\mathscr{F}$. For any two subsets X_i and X_j of X, $0 \in X_i$, $0 \in X_j \implies 0 \in X_i \cap X_j$ and hence $X_i \cap X_j$ also belongs to $\mathscr{F}$. If possible, let a set-label X_i of a vertex v_i of G does not contain 0. Then, $\{0\}\cap X_i = \emptyset$, which can not be the set-label of any vertex of G, contradicting the fact that $\mathscr{F}$ is a filter on X. Hence, no subset of X which does not contain 0, belongs to $\mathscr{F}$.
Conversely, assume that the set-label of every vertex of G contains 0 and every subset of X containing 0 is the set-label of some vertex of G. Since $0 \in X$, $X \in \mathscr{F}$. If X_i and X_j are the set-labels of two vertices in G, then both X_i and X_j contain the element 0 and hence $X_i \cap X_j$ also contains 0. Therefore, by the assumption, $X_i \cap X_j$ is also the set-label of some vertex in G. That is, $X_i, X_j \in \mathscr{F} \implies X_i \cap X_j \in \mathscr{F}$. As the set-label X_i of any vertex v_i of G contains 0, then every super set X_j of X_i also contains the element 0. Therefore, by the hypothesis, X_j is also the set-label of some vertex of G. That is, $X_i \in \mathscr{F}$, $X_i \subset X_j \implies X_j \in \mathscr{F}$. Therefore, $\mathscr{F}$ is a filter on X. Hence, f is an integer additive set-filter labeling on G.

From the above theorem we notice that all graphs do not possess IASFLs. Hence, a characterisation of the graphs that admit IASFLs arouses much interest. In view of Theorem 2, we now proceed to find the characteristics and properties of the graphs which admit IASFLs.
The following results is are immediate consequences of Theorem 2

Corollary 1. *If a graph G admits an IASFL, then G has $2^{|X|-1}$ vertices.*

Proof. Note that $0 \in X$ and let $|X| = n$. The number of r-element subsets of X with a common element 0 is $\binom{|X|-1}{r-1}$. Therefore, the number of subsets of X containing the element 0 is $\sum\limits_{i=0}^{n-1} \binom{n-1}{i} = 2^{n-1}$. This completes the proof.

Corollary 2. *If a given graph G admits an integer additive set-filter labeling f, then only one vertex of G can have a singleton set-label.*

Proof. Let G be an IASF-graph. Then, by Theorem 2, $\{0\}$ is a set-label of some vertex in G. Let a be a non-zero element in X. If $\{a\}$ is the set-label of some vertex of G, then the set $\{0\} \cap \{a\} = \emptyset$ must belong to $\mathscr{F} = f(V)$, which is a contradiction to Condition (iii) of Definition 3. Therefore, only one vertex of G can have a singleton set-label. (That is, the only possible singleton set-label in $\mathscr{F}$ is $\{0\}$).

Next, we establish the relation between the collection of the set-labels of vertices and the collection of the set-labels of the edges of an IASF-graph G in the following result.

Proposition 1. *If f is an integer additive set-filter labeling of a graph G, then $f^+(E(G)) \subseteq f(V(G))$.*

Proof. If u and v are any two adjacent vertices of the integer additive set-filter graph G, then $f(u)$ and $f(v)$ contains 0 and hence, being the sumset of $f(u)$ and $f(v)$, the set-label $f^+(uv)$ also contains the element 0. Since every subset of X containing 0 is the set-label of some vertex in G, the set label of the edge uv will also be a set-label of some vertex in G. Therefore, $f^+(E) \subseteq f(V)$.

The following theorem is a consequence of Theorem 2.

Theorem 3. *If a given graph G admits an integer additive set-filter labeling f, then every element of the collection $\mathscr{F} = f(V(G))$ belongs to some finite chain of sets in $\mathscr{F}$ of the form $\{0\} = f(v_1) \subset f(v_2) \subset f(v_3) \subset \ldots\ldots \subset f(v_r) = X$.*

Proof. Let f be an integer additive set-filter labeling defined on a graph G and $\mathscr{F}$ be the collection of all set-labels of the vertices in G. Then, by Theorem 2, both $\{0\}$ and X are in $\mathscr{F}$. Since every set-label in $\mathscr{F}$ contains 0, $\{0\}$ is the subset of all set-labels in $\mathscr{F}$. Since $\mathscr{F} \subseteq \mathscr{P}(X)$, X is the maximal set in $\mathscr{F}$ containing all sets in $\mathscr{F}$. Since $\mathscr{F}$ is a filter on X, if a subset X_i of X belongs to $\mathscr{F}$ implies every subset of X containing X_i is also in $\mathscr{F}$. Therefore, there exists some finite sequence $\{0\} \subset \ldots\ldots X_i \subset X_j \ldots\ldots X$ of subsets of X in $\mathscr{F}$. Therefore, every set-label in $\mathscr{F}$ is contained in some finite chain of subsets of X whose least element is $\{0\}$ and the maximal element is X.

We have already identified the number of vertices required for a graph to admit an integer additive set-labeling with respect to a given ground set X. In this context, it is interesting to examine certain structural properties of a graph that admit an integer additive set-filter labeling. Hence, we have

Theorem 4. *If a graph G admits an integer additive set-filter labeling, with respect to a non-empty ground set X, then G must have at least $2^{|X|-2}$ pendant vertices that are incident on a single vertex of G.*

Proof. Let f be an integer additive set-filter labeling defined on a given graph G. Then by Theorem 2, every subset of X containing the element 0 must belong to $\mathscr{F}$. Let x_l be the maximal element of X. Then, for any non-zero element x in X, $x + x_l \notin X$. Therefore, if X_l is a subset of X containing x_l, then the vertex having X_l as its set-label can not be adjacent to any vertex of G other than the one that has the set-label $\{0\}$. Hence, all the subsets of X containing x_l, including X itself, can be adjacent only to the vertex having the set-label $\{0\}$. Note that the number of subsets of X containing 0 and x_l is $2^{|X|-2}$. Therefore, the minimum number of pendant vertices in G is $2^{|X|-2}$.

Figure 1 elucidates an integer additive set-filter graph with $2^{|X|-2}$ pendant vertices incident on a single vertex, where $X = \{0,1,2,3,4\}$.

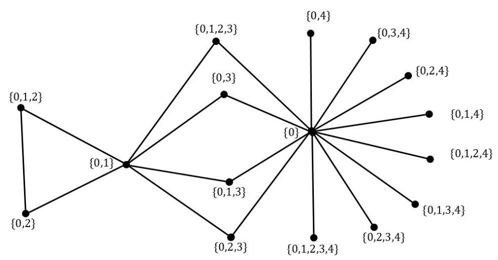

Fig. 1: An example to an integer additive set-filter graph

In view of the discussions we have made so far, we notice the following.

1. The existence of an integer additive set-filter labeling is not a hereditary property. That is, an integer additive set-filter labeling of a graph need not induce an integer additive set-filter labeling for all of its subgraphs.
2. For $n \geq 3$, no paths P_n admits an integer additive set-filter labeling.
3. No cycles admit IASFLs and as a result neither Eulerian graphs nor Hamiltonian graphs admit IASFLs.
4. Neither complete graphs nor complete bipartite graphs admit IASFLs. For $r > 2$, complete r-partite graphs also do not admit IASFLs.
5. Graphs having odd number of vertices never admits an integer additive set-filter labeling.

Another important property of IASFLs is that the existence of an integer additive set-filter labeling is a *monotone* property. That is, removing any non-leaf edge of an integer additive set-filter labeling graph preserves the integer additive set-filter labeling of that graph.

3 Relation Between Different integer additive set-labeling s

In this section, let us verify the relation between an integer additive set-filter labeling of a graph G with certain other types of integer additive set-labeling s of G. First recall the definition of an exquisite integer additive set-labeling of a given graph G.

Definition 5.[14] *An* exquisite integer additive set-labeling *(Einteger additive set-labeling , in short) is defined as an integer additive set-labeling $f : V(G) \to \mathscr{P}(\mathbb{N}_0)$ with the induced function $f^+ : E(G) \to \mathscr{P}(\mathbb{N}_0)$ defined by $f^+(uv) = f(u) + f(v)$, $uv \in E(G)$, holds the condition $f(u), f(v) \subseteq f^+(uv)$ for all adjacent vertices $u, v \in V(G)$.*

The following theorem is a necessary and sufficient condition for an integer additive set-labeling of a graph G to be an Einteger additive set-labeling of G.

Theorem 5.[14] *Let f be an integer additive set-labeling of a given graph G. Then, f is an Einteger additive set-labeling of G if and only if 0 is an element in the set-label of every vertex in G.*

Invoking Theorem 5, we establish the following relation between an integer additive set-filter labeling and an exquisite integer additive set-labeling of a given graph G.

Proposition 2. *Every integer additive set-filter labeling of a graph G is also an exquisite integer additive set-labeling of G.*

Proof. Let f be an integer additive set-filter labeling of a given graph G. Then, by Theorem 2, the set-label of every vertex of G contains 0. Then by theorem 5, f is also an exquisite integer additive set-labeling of G.

It is to be noted that, for an exquisite integer additive set-labeling f of a graph G, $f(V)$ need not contain all the subsets of the ground set X containing 0. Therefore, every exquisite integer additive set-labeling of a graph G need not be an integer additive set-filter labeling of G. Figure 2 depicts a topological integer additive set-labeling of a graph G with respect to the ground set $X = \{0,1,2,3,4\}$, which is not an integer additive set-filter labeling of G.

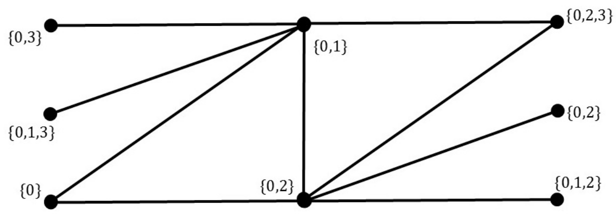

Fig. 2: An example to a strong integer additive set-labeling of G which is not an integer additive set-filter labeling of G.

Let us now consider the notions of integer additive set-graceful graphs and integer additive set-sequential graphs, which are defined as follows

Definition 6.*[16,17] Let $f : V(G) \to \mathscr{P}(X) - \{\emptyset\}$ be an integer additive set-labeling defined on a graph G. Then, f is called an* integer additive set-graceful labeling *(IASGL) of G if $f^+(E(G)) = \mathscr{P}(X) - \{\emptyset, \{0\}\}$ and f is called an* integer additive set-sequential labeling *(IASSL) of G if $f(V(G)) \cup f^+(E(G)) = \mathscr{P}(X) - \{\emptyset, \{0\}\}$.*

The following result checks whether an integer additive set-filter labeling of a given graph G can be an IASGL of the graph G.

Proposition 3. *No integer additive set-filter labeling defined on a given graph G is an IASGL of G.*

Proof. Let f be an integer additive set-filter labeling defined on G. By Proposition 1, $f(E(G)) \subseteq f(V(G))$. Hence, set-labels of all edges of G also contain the element 0. That is, any subset X_r of X that does not contain 0 will not be in $f(E(G))$. Therefore, $f(E(G)) \neq \mathscr{P}(X) - \{\{0\}, \emptyset\}$. Hence, f is not an IASGL of G.

The following results can also be proved in a similar manner.

Proposition 4. *No integer additive set-filter labeling defined on a given graph G is an integer additive set-sequential labeling of G.*

Proof. We have already proved that the set-labels of all elements of an integer additive set-filter graph G contain the element 0. Therefore, the set $f(V) \cup f^+(E)$ contains only those subsets of X which contain 0. That is, $f(V(G)) \cup f^+(E(G)) \neq \mathscr{P}(X) - \{\emptyset, \{0\}\}$. Hence, f is not an integer additive set-sequential labeling of G.

Another important integer additive set-labeling known to us, is a topological integer additive set-labeling , which is defined in [15] as follows

Definition 7.*[15] An integer additive set-labeling $f : V(G) \to \mathscr{P}(X) - \{\emptyset\}$ is called a* topological integer additive set-labeling *(Tinteger additive set-labeling) of G if $f(V(G)) \cup \{\emptyset\}$ is a topology of X.*

Can an integer additive set-filter labeling of a given graph G be a topological integer additive set-labeling of G? A relation between an integer additive set-filter labeling and an Tinteger additive set-labeling of a graph G is established in the following result.

Proposition 5. *Every integer additive set-filter labeling of a graph G is also a topological integer additive set-labeling of G.*

Proof. Let f be an integer additive set-filter labeling of a given graph G, with respect to a non-empty set X. Then $\mathscr{F} = f(V(G))$ is a filter on X. Let $\mathscr{T} = \mathscr{F} \cup \{\emptyset\}$. To show that f is a Tinteger additive set-labeling of G, we need to show that $\mathscr{T}$ is a topology on X. Since $X \in \mathscr{F}$, we have $\emptyset, X \in \mathscr{T}$. Since X is a finite set and $\mathscr{F}$ contains all subsets of X consisting of 0, the union of any number of elements of $\mathscr{T}$ is a set containing the element 0 and hence belongs to $\mathscr{T}$. Similarly, the intersection of any two sets in $\mathscr{F}$ contains at least one element 0 and hence the intersection of any number of elements in $\mathscr{T}$ is also in $\mathscr{T}$. Therefore, $\mathscr{T}$ is a topology on X. This completes the proof.

If an integer additive set-labeling f of a graph G is an integer additive set-filter labeling of G, then $f(V(G))$ contains only those subsets of X consisting of the element 0 and hence not all topological integer additive set-labeling s of G, with respect to X, can be the IASFLs of G. Figure 3 depicts a topological integer additive set-labeling of a graph G which is not an integer additive set-filter labeling of G with respect to the ground set $X = \{0, 1, 2, 3\}$.

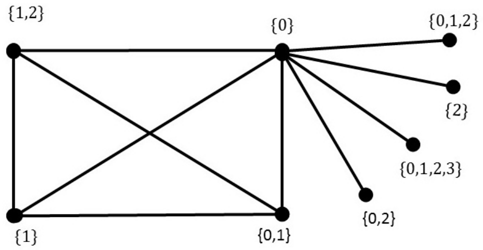

Fig. 3: An example to a topological integer additive set-labeling of G which is not an integer additive set-filter labeling of G.

Another important type integer additive set-labeling of an given graph G is a weak integer additive set-labeling of G, which is defined as follows

Definition 8.[7] *A* weak integer additive set-labeling *(Winteger additive set-labeling) of a graph G is an integer additive set-labeling f such that $|f^+(uv)| = \max(|f(u)|,|f(v)|)$ for all $u,v \in V(G)$.*

The following is a necessary and sufficient condition for an integer additive set-labeling to be a weak integer additive set-labeling of a given graph G.

Lemma 1.[7] *Let f be an integer additive set-labeling defined on a given graph G. Then, f is a Winteger additive set-labeling of G if and only if at least one end vertex of every edge of G is mono-indexed.*

An interesting question in this context is whether an integer additive set-filter labeling of a given graph G can be a weak integer additive set-labeling . The following result provides an answer to this question.

Proposition 6. *An integer additive set-filter labeling of a graph G is a weak integer additive set-labeling of G if and only if G is a star.*

Proof. Let $G = K_{1,n}$, where $n = 2^{|X|-1} - 1$, X being the ground set that is used for set-labeling and let f be an integer additive set-filter labeling defined on G. Then, label the vertex v at the centre of G by the set $\{0\}$ and other vertices by other subsets of X containing 0. Therefore, every edge of G has one mono-indexed end vertex. Hence by Lemma 1, f is a weak integer additive set-labeling of G. Conversely, assume that an integer additive set-filter labeling f of G is a weak integer additive set-labeling of G. Then, by Lemma 1, every edge of G must have at least one mono-indexed end vertex. But by Corollary 2, the only singleton set-label in $\mathscr{F}$ is $\{0\}$. Therefore, the vertex,say v, having the set-label $\{0\}$ must be adjacent to all other vertices of G and the graph $G - v$ is a trivial graph. Therefore, G is a star.

Next, recall the definition of a strong integer additive set-labeling of a given graph G.

Definition 9.[13] *A* strong integer additive set-labeling *(Sinteger additive set-labeling) of G is an integer additive set-labeling such that if $|f^+(uv)| = |f(u)|\,|f(v)|$ for all $u,v \in V(G)$.*

The *difference set* of a set A is the set of all positive differences between the elements of A. The difference set of a set A is denoted by D_A. Then, the following result is a necessary and sufficient condition for an integer additive set-labeling (or integer additive set-labeling) to be a Sinteger additive set-labeling (or Sinteger additive set-labeling) of a given graph G.

Lemma 2.[13] *Let f be an integer additive set-labeling defined on a given graph G. Then, f is a Sinteger additive set-labeling of G if and only if the difference sets of any two adjacent vertices of G are disjoint.*

Can a given integer additive set-filter labeling f of a given graph G be a strong integer additive set-labeling of G? We know that f is a strong integer additive set-labeling of G if the difference sets of the set-labels of any two adjacent vertices of G are disjoint. Using this result, we wish to verify whether there is any relation between an integer additive set-filter labeling and a strong integer additive set-labeling of G. Invoking Lemma 2 and Theorem 2, we propose the following result

Proposition 7.*If an integer additive set-filter labeling f of a graph G is a strong integer additive set-labeling of G, then $f(u) \cap f(v) = \{0\}$, where u and v of G are two adjacent vertices of G.*

Proof. Assume that f is an integer additive set-filter labeling defined on a graph G. Let u and v be two adjacent vertices of G. Now, assume that f is a strong integer additive set-labeling . Then, by Lemma 2, $D_{f(v_i)} \cap D_{f(v_i)} = \emptyset$. If $f(u)$ and $f(v)$ have a common non-zero element, say a, then both $D_{f(v_i)}$ and $D_{f(v_i)}$ also contain the element a, contradicting the fact that f is a strong integer additive set-labeling . Therefore, the set-labels of any two adjacent vertices have only one common element 0.

It can be noted that the conditions $f(u) \cap f(v) = \{0\}$ and $D_{f(v_i)} \cap D_{f(v_i)} = \emptyset$, even together, do not produce the idea that every subset of X containing 0 is the set-label of some vertex of G. Therefore, every strong integer additive set-labeling of G need not be an integer additive set-filter labeling of G. Figure 4 depicts a strong integer additive set-labeling of a graph G, with respect to the ground set $\{0,1,2,3\}$, which is not an integer additive set-filter labeling of G.

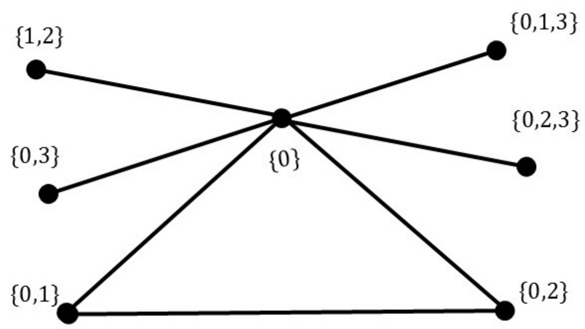

Fig. 4: An example to a strong integer additive set-labeling of G which is not an integer additive set-filter labeling of G.

Another type integer additive set-labeling which remains to be considered in this occasion is an arithmetic integer additive set-labeling of a graph G. An arithmetic integer additive set-labeling of a graph G is an integer additive set-labeling f, with respect to which, the set-labels of all elements of G are AP-sets. (An AP-set is a set whose elements are in an arithmetic progression). Since an AP-sets must have at least three elements, an integer additive set-filter labeling of a graph G can not be an Arithmetic integer additive set-labeling .

4 Conclusion

In this paper, we have introduced a new type of integer additive set-labeling and called an integer additive set-filter labeling and have discussed certain characteristics and structural properties of graphs which admit this type of integer additive set-labeling . We have also discussed the relations, if any, with the other known types of integer additive set-labeling s. There are several other problems in this area are still open. The following are some of the problems we have identified in this area which need further investigation.

Problem 1. Determine a necessary and sufficient condition for an integer additive set-filter labeling of a given graph G to be an integer additive set-filter indexer of G.

Problem 2. Characterise the graphs which admit integer additive set-filter indexers.

Problem 3. Check the admissibility of integer additive set-filter labeling by different operations and products of integer additive set-filter graphs.

Problem 4. Check the admissibility of integer additive set-filter labeling by the complement of integer additive set-filter graphs.

Problem 5. Check the admissibility of integer additive set-filter labeling by different certain graph classes.

Problem 6. Check the admissibility of an induced integer additive set-filter labeling by certain associated graphs such as line graphs, total graphs, subdivisions, homeomorphic graphs etc. of given integer additive set-filter graphs.

An integer additive set-labeling (or integer additive set-labeling) is said to be *k-uniform* if $|f^+(e)| = k$ for all $e \in E(G)$. That is, a connected graph G is said to have a k-uniform integer additive set-labeling (or integer additive set-labeling) if all of its edges have the same set-indexing number k.

Problem 7. Determine the conditions required for an integer additive set-filter labeling of a given graph to be a uniform integer additive set-filter labeling.

Studies on certain other types of integer additive set-labeling of graphs, both uniform and non-uniform, seem to be much promising. The integer additive set-labelings under which the vertices of a given graph are labeled by different standard sequences of non negative integers, are also worth studying. All these facts highlight a wide scope for further studies in this area.

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