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A valid and advanced method for ranking the fuzzy numbers

Elmira Abbasi[1], *Rahim Saneifard*[2] *and Ercan Celik*[1]

[1]Ataturk University Faculty of Science, Department of Mathematics, Erzurum, Turkey
[2]Islamic Azad University Faculty of Science, Department of Mathematics, Oroumieh, Iran

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Abstract: With no doubt, ranking the fuzzy numbers are extremely effective and useful in different scientific fields such as Artificial Intelligence, Economics, Engineering and decision-making units and etc. The fuzzy quantities must be ranked before their engagement in the cycle of the applied functionalities. In this article, We offer a valid and advanced method for ranking the fuzzy numbers based on the Distance Measure Meter. In addition to the Distance Measure, we define a particular condition of the generalized fuzzy numbers. Having discussed some examples in this regard, we touch upon the advantages of this new method.

Keywords: Fuzzy numbers, ranking method, distance measure, generalized fuzzy numbers.

1 Introduction

Ranking fuzzy numbers is a topic and plays an very important role in linguistic decision making and some other fuzzy application systems, which has been studied by many researchers. Wang and Kerre [1] introduced reasonable properties for the ordering of fuzzy quantities. Some researchers introduced a distance and then compared the fuzzy numbers with it. Singh [2] proposed some distance measures based on the geometric distance model, the set-heoretic approach, and the matching functions. However, these existing distance measures for dual hesitant fuzzy sets do not satisfy such fundamental properties as triangle inequality which was stressed by Zhou and Wang [3] and Singh. However, there is no method which gives a satisfactory result to all situations. In [4], Tran and Duckstein introduced a notion of a distance based on interval numbers where the fuzzy numbers were transformed into interval numbers using α-cuts. Chakraborty and Chakraborty [5] introduced a fuzzy distance measure between two generalized fuzzy numbers as well as LR-type fuzzy numbers. Guha and Chakraborty [6] developed a fuzzy distance measure between two generalized fuzzy numbers and studied the metric properties of their distance measure. Distance measure between two fuzzy numbers is closely related to the concept of ranking fuzzy numbers. Therefore, it is possible to express ranking of fuzzy numbers and distance measure between fuzzy numbers with the aim of a functional relationship. In recent years, many methods have been presented for ranking fuzzy numbers. This article suggests a new method for ranking fuzzy numbers based on the concept of Euclidean distance between two generalized fuzzy numbers.

The article is organized as follows: Section 2. Shortly introduces the basic concepts and definitions of fuzzy numbers. Section 3. introduce the suggested new method. In Section 4, the suggested method has been explained with examples and compare the results with other authors, and describe the advantages and the disadvantages of the method.

* Corresponding author e-mail: elmira.math@yahoo.com

2 Definitions of fuzzy numbers

Fuzzy number has been introduced by Zadeh [19] in order to deal with imprecise numerical quantities in a practical way. The basic definitions of a fuzzy number are given in [1],[12,13,14] as follows:

Definition 1. *Let X be a universe set. A fuzzy set A of X is defined by a membership function $\mu_A(x) \to [0,1]$, where $\mu_A(x)$, $\forall x \in X$, indicates the degree of xin A.*

Definition 2. *A fuzzy subset A of universe set X is normal iff $sup_{x\in X}\ \mu_A(x) = 1$, where X is the universe set.*

Definition 3. *A fuzzy subset A of universe set X is convex iff*

$$\mu_A(\lambda x + (1-\lambda)y) \geq (\mu_A(x) \wedge \mu_A(y)),\ \forall x,y \in X,\ \forall \lambda \in [0,1].$$

In this article symbols $\wedge$ and $\vee$ denote the minimum and maximum operators, respectively.

Definition 4. *A fuzzy set Ais a fuzzy number iff A is normal and convex on X.*

Definition 5. *Assume that the fuzzy number $A \in F$ is represented by the following:*

$$A = \bigcup_{\alpha\in[0,1]} (\alpha, A_\alpha).$$

Here,

$$A_\alpha = \{x:\ \mu_A(x) \geq \alpha\},$$

is the α-level set of the fuzzy number A. This article considers normal and convex fuzzy numbers. Therefore, the α-level sets may be represented in the form of a segment,

$$\forall\alpha \in [0,1]:\ A_\alpha = [L_A(\alpha), R_A(\alpha)] \subset (-\infty,+\infty),$$

Here, $L\ :[0,1] \to (-\infty,+\infty)$ is a monotonically non-decreasing left continuous function, and $R\ :[0,1] \to (-\infty,+\infty)$ is a monotonically non-increasing right-continuous function. The functions $L(.)$and $R(.)$ express the left and right sides of a fuzzy number, respectively. In other words,

$$L(\alpha) = \mu_{\uparrow}^{-1}(\alpha)\quad,\quad R(\alpha) = \mu_{\downarrow}^{-1}(\alpha),$$

where $L(\alpha) = \mu_{\uparrow}^{-1}(\alpha)$ and $R(\alpha) = \mu_{\downarrow}^{-1}(\alpha)$ denote quasi-inverse functions of the increasing and decreasing parts of the membership functions $\mu(t)$respectively. As a result, the decomposition representation of the fuzzy number A, called the $L-R$ representation, has the following form:

$$A = \bigcup_{\alpha\in[0,1]} (\alpha, [L_A(\alpha), R_A(\alpha)]).$$

Definition 6. *The following values constitute the weighted average and weighted width, respectively, of the fuzzy number A:*

$$I(A) = \int_0^1 (cL_A(\alpha) + (1-c)R_A(\alpha))\, p(\alpha) d\alpha,$$

and

$$D(A) = \int_0^1 (R_A(\alpha) - L_A(\alpha))\, p(\alpha) d\alpha.$$

Here $0 \leq c \leq 1$ *denotes an "optimism/pessimism" coefficient in conducting operations on fuzzy numbers. The function* $p : [0,1] \to [0,+\infty)$ *denotes the distribution density of the importance of the degrees of fuzziness, where* $\int_0^1 p(\alpha)d\alpha = 1$. *In particular cases, it may be assumed that*

$$p(\alpha) = (k+1)\alpha^k, \quad k = 0,1,2,\cdots.$$

Definition 7. *For arbitrary fuzzy numbers A and B, the quantity*

$$d_p(A,B) = \sqrt{[I(A) - I(B)]^2 + [D(A) - D(B)]^2},$$

is called the parametric distance between the fuzzy numbers A and B.

Definition 8. *The membership function of Generalized Trapzoidal Fuzzy Number A=(a,b,c,d;ω) where a≤ b ≤ c ≤ d, 0< ω <1 is defined as:*

$$\mu_A(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{d-x}{d-c}, & c \leq x \leq d \\ 0, & x > d \end{cases}$$

If $\omega = 1 \to$ *A is normal trapzoidal fuzzy number.*
If $a = b and c = d \to$ *A is crisp interval.*
If $b = c \to$ *A is a generalized triangular fuzzy number.*
If $a = b = c = d$ *and* $\omega = 1 \to A$ *is a real number.*

3 New fuzzy distance measure for generalized fuzzy numbers base on parametric interval

3.1 Construction of the fuzzy parametric interval distance measure

Suppose $\tilde{A}_1 = (a_1, b_1, c_1, d_1, \alpha_1)$ *and* $\tilde{A}_2 = (a_2, b_2, c_2, d_2, \alpha_2)$ be two generalized fuzzy numbers. The interval distance between calculate as follows.

Abbreviation of this interval distance measure considered by : ER(.), that is taken from the first letters of the names of the authors.

$$ER(\tilde{A}_1, \tilde{A}_2) = \int_0^1 \left(\gamma \sqrt{\left(\tilde{A}_{1\omega_1}^{\ -} - \tilde{A}_{2\omega_2}^{-}\right)^2 + (\omega_1 - \omega_2)^2} + (\gamma - 1)\sqrt{\left(\tilde{A}_{1\omega_1}^{\ +} - \tilde{A}_{2\omega_2}^{+}\right)^2 + (\omega_1 - \omega_2)^2} \right) d\alpha$$

where parametric interval from [20] for each fuzzy number is :

Furthermore in this article:

$$\left\{ \begin{array}{l} \omega_1 = \gamma\alpha_1 \\ \omega_2 = \gamma\alpha_2 \end{array} \right\} \to \left\{ \begin{array}{l} \omega_1 = 0 \leftrightarrow \gamma = 0 \\ \omega_1 = \alpha_1 \leftrightarrow \gamma = 1 \\ \omega_2 = 0 \leftrightarrow \gamma = 0 \\ \omega_2 = \alpha_2 \leftrightarrow \gamma = 1 \end{array} \right\} \quad \text{and} \quad \gamma + (1-\gamma) = 1.$$

Thus, we have :

$$
\begin{aligned}
ER(\tilde{A}_1,\tilde{A}_2) &= \int_0^1 \gamma\sqrt{(\tilde{A}_{1\omega_1}{}^{-} - \tilde{A}^{-}_{2\omega_2})^2 + (\omega_1-\omega_2)^2}d\alpha + \int_0^1 (\gamma-1)\sqrt{(\tilde{A}_{1\omega_1}{}^{+} - \tilde{A}^{+}_{2\omega_2})^2 + (\omega_1-\omega_2)^2}\ d\alpha \\
&= \gamma\int_0^1 \sqrt{(\tilde{A}_{\gamma\alpha_1}{}^{-} - \tilde{A}^{-}_{\gamma\alpha_2})^2 + (\gamma\alpha_1-\gamma\alpha_2)^2}d\alpha + (\gamma-1)\int_0^1 \sqrt{(\tilde{A}_{\gamma\alpha_1}{}^{+} - \tilde{A}^{+}_{\gamma\alpha_2})^2 + (\gamma\alpha_1-\gamma\alpha_2)^2}\ d\alpha \\
&= \gamma + (1-\gamma)\int_0^1 (\sqrt{(\tilde{A}_{\gamma\alpha_1}{}^{-} - \tilde{A}^{-}_{\gamma\alpha_2})^2 + (\gamma\alpha_1-\gamma\alpha_2)^2} + \sqrt{(\tilde{A}_{\gamma\alpha_1}{}^{+} - \tilde{A}^{+}_{\gamma\alpha_2})^2 + (\gamma\alpha_1-\gamma\alpha_2)^2}\ )d\alpha \\
&= \int_0^1 (\sqrt{(\tilde{A}_{\gamma\alpha_1}{}^{-} - \tilde{A}^{-}_{\gamma\alpha_2})^2 + (\gamma\alpha_1-\gamma\alpha_2)^2} + \sqrt{(\tilde{A}_{\gamma\alpha_1}{}^{+} - \tilde{A}^{+}_{\gamma\alpha_2})^2 + (\gamma\alpha_1-\gamma\alpha_2)^2}\ )d\alpha.
\end{aligned}
$$

3.2 Metric properties

Here, we show that the new distance measure satisfies the following properties of a distance metric:

(I) $d(\tilde{A},\tilde{B}) \geq 0$, for any two generalized fuzzy numbers $\tilde{A},\tilde{B}$.
(II) $d(\tilde{A},\tilde{B}) = d(\tilde{B},\tilde{A})$, for any two generalized fuzzy numbers $\tilde{A},\tilde{B}$.
(III) $d(\tilde{A},\tilde{B}) + d(\tilde{B},\tilde{C}) \geq d(\tilde{A},\tilde{C})$, for any three generalized fuzzy numbers $\tilde{A},\tilde{B},\tilde{C}$.

The accuracy of the first and second properties is quite clear. Here we proved the third property. Suppose three generalized fuzzy numbers $\tilde{A},\tilde{B},\tilde{C}$, where $\alpha_1 \leq \alpha_2 \leq \alpha_3$, also consider $\omega_1,\omega_2,\omega_3$ to be the cuts corresponding $\tilde{A},\tilde{B}$, $\tilde{C}$, respectively.

For each γ,for the three points for the left side $(A^{-}_{\omega_1}\ ,\ \omega_1)$, $(B^{-}_{\omega_2}\ ,\omega_2)$ and ($C^{-}_{\omega_3}, \omega_3)$, we have:

$$
\sqrt{(A_{\omega_1}{}^{-} - B_{\omega_2}{}^{-})^2 + (\omega_1-\omega_2)^2} + \sqrt{(B_{\omega_2}{}^{-} - C_{\omega_3}{}^{-})^2 + (\omega_2-\omega_3)^2} \ \geq\ \sqrt{(A_{\omega_1}{}^{-} - C_{\omega_3}{}^{-})^2 + (\omega_1-\omega_3)^2}
$$

Therefore :

$$
\int_0^1 (\sqrt{(A_{\omega_1}{}^{-} - B_{\omega_2}{}^{-})^2 + (\omega_1-\omega_2)^2} + \sqrt{(B_{\omega_2}{}^{-} - C_{\omega_3}{}^{-})^2 + (\omega_2-\omega_3)^2})\ d\alpha \geq
$$

$$
\int_0^1 (\sqrt{(A_{\omega_1}{}^{-} - C_{\omega_3}{}^{-})^2 + (\omega_1-\omega_3)^2}\ )d\alpha
$$

As well as , for the right side, we have:

$$
\int_0^1 (\sqrt{(A_{\omega_1}{}^{+} - B_{\omega_2}{}^{+})^2 + (\omega_1-\omega_2)^2} + \sqrt{(B_{\omega_2}{}^{+} - C_{\omega_3}{}^{+})^2 + (\omega_2-\omega_3)^2})\ d\alpha \geq
$$

$$
\int_0^1 (\sqrt{(A_{\omega_1}{}^{+} - C_{\omega_3}{}^{+})^2 + (\omega_1-\omega_3)^2})d\alpha.
$$

Consequently;

$$
\begin{aligned}
&\int_0^1 (\sqrt{(A_{\omega_1}{}^{-} - B_{\omega_2}{}^{-})^2 + (\omega_1-\omega_2)^2} + \sqrt{(A_{\omega_1}{}^{+} - B_{\omega_2}{}^{+})^2 + (\omega_1-\omega_2)^2})\ d\alpha \\
&+ \int_0^1 (\sqrt{(B_{\omega_2}{}^{-} - C_{\omega_3}{}^{-})^2 + (\omega_2-\omega_3)^2} + \sqrt{(B_{\omega_2}{}^{+} - C_{\omega_3}{}^{+})^2 + (\omega_2-\omega_3)^2})\ d\alpha \\
&\geq \int_0^1 (\sqrt{(A_{\omega_1}{}^{-} - C_{\omega_3}{}^{-})^2 + (\omega_1-\omega_3)^2} + \sqrt{(A_{\omega_1}{}^{+} - C_{\omega_3}{}^{+})^2 + (\omega_1-\omega_3)^2}\ )d\alpha,
\end{aligned}
$$

$$d\left(\tilde{A},\tilde{B}\right)+d\left(\tilde{B},\tilde{C}\right)\geq d\left(\tilde{A},\tilde{C}\right).$$

4 Ranking generalized fuzzy numbers

In this section we require to consider a fuzzy origin number. Therefore, this singleton fuzzy number has membership values of zero for all x values, except at the x value 0 where it is 1. Another words membership function value for $x=0$is1, and for $x\neq 0$ is 0. (In this article we showed origin number with O.)

$$\mu_O(x)=\left\{\begin{array}{ll}0, & x<0\\ 1, & x=0\\ 0, & x>0\end{array}\right\}$$

and consequently

$$\tilde{O}^{-}=\int_0^1 L_O\left(\alpha\right).p\left(\alpha\right)d\alpha$$

$$\tilde{O}^{+}=\int_0^1 R_O\left(\alpha\right).p\left(\alpha\right)d\alpha$$

and, $\tilde{O}^{-}=\tilde{O}^{+}=0$. Therefore

$$ER\left(\tilde{A},\tilde{O}\right)=\int_0^1\sqrt{\left(\tilde{A}^{-}-\tilde{O}^{-}\right)^2}\,.\,d\alpha+\int_0^1\sqrt{\left(\tilde{A}^{+}-\tilde{O}^{+}\right)^2}\,.d\alpha$$

$$ER\left(\tilde{A},\tilde{O}\right)=\int_0^1\left|\tilde{A}^{-}\right|\,.\,d\alpha\;+\;\int_0^1\left|\tilde{A}^{+}\right|\,.d\alpha.$$

Remark. For any $A\in E$, it is clear that $ER\left(\tilde{A},\tilde{O}\right)=ER\left(-\tilde{A},\tilde{O}\right)$.

Definition 9.*For A and B$\in E$, we define the ranking of A and B by ER, i.e.*

(1) $ER\left(\tilde{A},\tilde{O}\right)\;>\;ER\left(\tilde{B},\tilde{O}\right)\rightarrow A>B$
(2) $ER\left(\tilde{A},\tilde{O}\right)\;<\;ER\left(\tilde{B},\tilde{O}\right)\rightarrow A<B$
(3) $ER\left(\tilde{A},\tilde{O}\right)\;=\;ER\left(\tilde{B},\tilde{O}\right)\rightarrow A=B.$

4.1 Numerical examples

In this part of this article we compare the proposed method with others [7], [8], [10], [16], [18].

4.1.1 Example

Suppose three fuzzy numbers $A=(6,1,1)$, $B=(6,0.1,1)$, and $C=(6,0,1)$. These data are used in [10].

By utilization parametric Interval that it is base of new fuzzy distance measure, the values of parametric interval for each number are:

$$(\tilde{A}^{-}=2.83,$$

$\tilde{A}^{+}=3.16)$, $(\tilde{B}^{-}=2.98,\tilde{B}^{+}=3.16)$ and $(\tilde{C}^{-}=3,\;\tilde{C}^{+}=3.16)$. Thus the ranking of fuzzy numbers according to the new method is $A<B\;<C$. in table 1 we compare the results of another ranking method with Abbasbandi [9,10], [7] and [8].

Table 1: Comparative Results of Example 4.1.1.

Fuzzy numbers→ **Methods↓**	$A=(6,1,1)$	$B=(6,0.1,1)$	$C=(6,0,1)$	**Results**
ER measure (New approach)	5.99	6.14	6.16	$A<B<C$
Sign Distance with $p=1$	6.12	12.45	12.50	$A<B<C$
Sign Distance with $p=2$	8.52	8.82	8.85	$A<B<C$
Chu-Tsao	3.000	3.126	3.085	$A<C<B$
Chen Distance	6.021	6.349	6.351	$A<B<C$

4.1.2 Example

Consider the three fuzzy numbers $A=(1,2,5)$, $B=(0,3,4)$and $C=(2,2.5,3)$ as shown in Figure 1. By using this new approach method, we have :

$$(\tilde{A}^{-}=0.83,$$

$\tilde{A}^{+}=1.5$), (therefore the ranking order is $A<C<B$. See the compare of the results in Table 2.

Table 2: Comparative Results of Example 4.1.2.

Fuzzy numbers→ **Methods↓**	$A=(1,2,5)$	$B=(0,3,4)$	$C=(2,2.5,3)$	**Results**
ER measure (New approach)	2.33	2.66	2.49	$A<C<B$
Sign Distance with $p=1$	3	3	3	$A\sim B\sim C$
Sign Distance with $p=2$	2.16	2.70	2.70	$A<B\sim C$
Chen Distance	2.50	2.50	2.50	$A\sim B\sim C$
Chu-Tsao	0.74	0.74	0.75	$A\sim B<C$

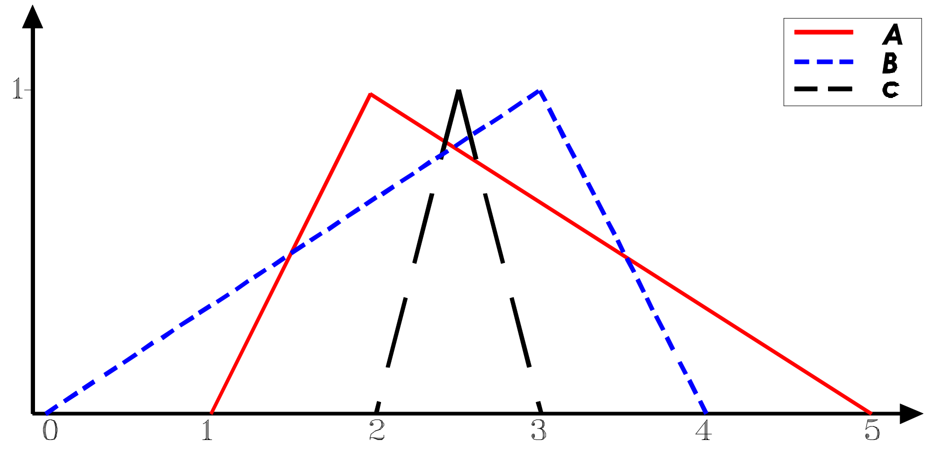

Fig. 1

As you seen the new advanced ranking method has multiple precision and indicates explicit result for ranking fuzzy numbers. All the above examples show the results of this effort to be more efficient and consistent than the other ranking methods, and overcomes the shortcomings of the previous methods.

5 Conclusions

Ranking Fuzzy numbers through distance method is one of the most pervasive methods in ranking fuzzy numbers, but because of its indiscriminativeness, it does not lead to desired and highly appropriate consequences. This article is tasked to render a new method based on the distance measure and also interval value of fuzzy numbers to rank generalized fuzzy numbers. Having utlizied the proposed interval distance measure, we defined particular conditions such as origine number of generalized fuzzy numbers. This is used to rank generalized fuzzy numbers with more optimal and desired results in hand. In doing so, some numerical instances were also illustrated in order to compare the proposed method with other efforts in this regard. At the end, it is elaborated that the method this article is tasked to offer is rather useful and optimal.

Compliance with ethical standards

The author declares that he has no conflict of interest. This article does not contain any studies with human participants performed by the author.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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