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Applications of the decomposition methods to some nonlinear partial differential equations

Hami Gündoğdu, Ömer Faruk Gözükızıl

Sakarya University, Department of Mathematics, Sakarya, Turkey

Abstract: In this paper, we consider the inhomogeneous Benjamin-Bona-Mahony-Peregrine-Burgers (BBMPB) equation with its initial condition. The decomposition methods such as Adomian and Laplace are applied to this equation. Then, the solution satisfying the given initial condition is gained by using each method. To demonstrate these decomposition methods are how effective and convenient for solving the nonlinear partial differential equations(NLPDEs), solutions of some NLPDEs subjected to initial conditions are attained by these method. In the application of the method, it is clearly noticed that there is no need to convert the nonlinear terms into the linear ones due to the Adomian polynomials. In addition to this simplicity, it is seen that each method gives the same solution with a fast convergence by taking the advantage of the noise terms.

Keywords: Adomian and Laplace Decomposition Methods, the noise term, Adomian polynomials, Benjamin Bona Mahony Peregrine Burgers equation.

1 Introduction

In almost all fields of science and engineering, most problems can be represented by linear or nonlinear partial differential equations (shortly, N/LPDEs). To proceed in these fields, solutions to this kind of problems should be obtained. Important information about the encountered problems can be achieved by getting solutions to these type of equations. So, it is of a very significance role in modern science and engineering. In this regard, numerous researches have been made to find ways to obtain the solutions of NLPDEs. Eventually, many analytical and numerical methods have been established and used to reach this goal. Some of the methods are; the perturbation method [1-5], the homotopy perturbation method [3-7], the delta perturbative method [8], the Modified Decomposition method [9-16], the Adomian Decomposition method [15-22], the Laplace Decomposition method [15, 16, 22-27].

Firstly, let us consider the Benjamin-Bona-Mahony-Burgers (BBMB) [28] given by

$$u_t - u_{xxt} - \alpha u_{xx} + \gamma u_x + f(u)_x = 0. \tag{1}$$

Here, $u(x,t)$ is speed of liquid flowing in horizontal x direction, α is a positive constant, γ is arbitrary real constant, and $f(u)$ is a smooth nonlinear function in C^2. When $f(u)_x = uu_x$ with $\alpha = 0$ and $\beta = 1$, equation (1) turns into the alternative regularized long-wase equation proposed by Peregrine [29] and Benjamin [30]. This equation characterizes a balance between nonlinear and dispersive effects, yet it does not take place in calculation of dissipation. In the perspective of physics, equation (1) with the dissipative term αu_{xx} is suggested when the good predictive power is wished. This kind of problem emerges in both the bore propagation and the water waves. Now, let us consider the Benjamin-Bona-Mahony

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* Corresponding author e-mail: hamigundogdu@sakarya.edu.tr

equations given by

$$u_t - \alpha u_{xx} + u_x + f(u)_x = 0. \tag{2}$$

As it is known that the dispersive effect of the equation (1) is same with the equation (2). Because of this fact, equation (1) is called the Benjamin-Boan-Mahony-Burgers (BBMB) equations. But, it is an interesting situation that it is not proposed by Benjamin or Mahanoy or Burgers, check [31]. These equations, (1) and (2), take a large place in the literature, see [32-34]. So, they have been studied in different points of view. Here, let us give some works related to these equations. In [32], new kinds of solitary wave solutions to the equation (1) by a variable coefficient balancing act method. In [33], equation (1) is considered with $f(u)_x = u^2$. In this work, the extended tanh method is applied to it and gained the solitary wave solutions. In [34], Kaya takes the equation (1) with $f(u)_x = u^p$, for any positive p. The solitary wave solutions are obtained by using the Adomian decomposition method. In this equation, if we take $\theta, \beta \neq 0$ and $f(u)_x = \theta u u_x + \beta u_{xxx}$ then the BBMB equation takes the form

$$u_t - u_{xxt} - \alpha u_{xx} + \gamma u_x + \theta u u_x + \beta u_{xxx} = 0 \tag{3}$$

which is suggested Peregrine, Benjamin, et al[30]. This equation is called Benjamin-Bona-Mahony-Peregrine-Burgers. In this study, we consider BBMPB equation of the form

$$u_t - u_{xxt} - \alpha u_{xx} + \gamma u_x + \theta u u_x + \beta u_{xxx} = x + \gamma t + \theta x t^2, \tag{4}$$

with initial condition $u(x,0) = 0$.

The purpose of this work is to show that the Laplace and Adomian decomposition methods are usefull in solving NLPDEs subject to the initial conditions. Therefore, we apply these decomposition methods to the inhomogeneous BBMPB equation and some NLPDEs with their initial conditions. The applications of these methods point out that these methods are completely suitable for getting the solutions of NLPDEs with initial conditions.

This study is arranged as follows. In section 2, descriptions of the methods are given in detail. In section 3, ADM and LDM are applied to BBMPB equation given in (4) and some other NLPDEs, and the solutions which satisfy the given initial conditions. As a result of this work, some conclusions are given in section 4.

2 Outline of the methods

2.1 The Laplace decomposition method

This method is given as in the following manner. First, let us consider the nonlinear partial differential equation in operator form

$$Du(x,t) + Ru(x,t) + Nu(x,t) = h(x,t) \tag{5}$$

with initial conditions $u(x,0) = f(x)$ and $u_t(x,0) = g(x)$. Here, D is a second order partial differential operator w.r.t t, R is a remaining linear operator, N represents a general nonlinear differantial operator, $h(x,t)$ is a source term.

Firstly, the laplace transform is applied to both sides of (5). Then, we have

$$L[Du(x,t)] + L[Ru(x,t)] + L[Nu(x,t)] = L[h(x,t)]. \tag{6}$$

By the differentiation property of the Laplace transform, we get

$$L[u(x,t)] = \frac{f(x)}{s} + \frac{g(x)}{s^2} + \frac{1}{s^2}L[h(x,t)] - \frac{1}{s^2}[L[Ru(x,t) + L[Nu(x,t)]]] \tag{7}$$

where s is Laplace domain function. In this method, $u(x,t)$ is defined as

$$u(x,t) = \sum_{n=0}^{\infty} u_n. \tag{8}$$

The nonlinear terms, N, given by infinite series

$$N(u(x,t)) = \sum_{n=0}^{\infty} A_n(u) \tag{9}$$

where A_n represents the Adomian polynomials. Adomian polynomials are defined as

$$A_n = \frac{1}{n!}\frac{d^n}{d\alpha^n}\left[N(\sum_{n=0}^{\infty}\alpha^n u_n\right], \quad n = 0, 1, 2, ... \tag{10}$$

By using this definition, the first few terms can be given as follows:

$$A_0 = N(u_0), \tag{11}$$

$$A_1 = u_1 N'(u_0), \tag{12}$$

$$A_2 = u_2 N'(u_0) + \frac{1}{2!}u_1^2 N''(u_0), \tag{13}$$

and other terms can be derived in a similar way. Now, substituting (8) and (9) into equation (7) gives us

$$L\left(\sum_{n=0}^{\infty} u_n\right) = \frac{f(x)}{s} + \frac{g(x)}{s^2} + \frac{1}{s^2}L[h(x,t)] - \frac{1}{s^2}L[Ru_n(x,t)] - \frac{1}{s^2}L\left(\sum_{n=0}^{\infty} A_n(u)\right). \tag{14}$$

Linearity of laplace transform converts it into the following form:

$$\sum_{n=0}^{\infty} L(u_n) = \frac{f(x)}{s} + \frac{g(x)}{s^2} + \frac{1}{s^2}L[h(x,t)] - \frac{1}{s^2}L[Ru_n(x,t)] - \frac{1}{s^2}\sum_{n=0}^{\infty}[L(A_n(u))]. \tag{15}$$

Comparing both sides of (15) yields the following equalities:

$$L(u_0) = \frac{f(x)}{s} + \frac{g(x)}{s^2} + \frac{1}{s^2}L[h(x,t)], \tag{16}$$

$$L(u_1) = -\frac{1}{s^2}L[Ru_0(x,t)] - \frac{1}{s^2}\sum_{n=0}^{\infty}[L(A_0(u))], \tag{17}$$

$$L(u_2) = -\frac{1}{s^2}L[Ru_1(x,t)] - \frac{1}{s^2}\sum_{n=0}^{\infty}[L(A_1(u))], \tag{18}$$

and so on. In general, the recursive relation is given by

$$L(u_{n+1}) = -\frac{1}{s^2}L[Ru_n(x,t)] - \frac{1}{s^2}\sum_{n=0}^{\infty}[L(A_n(u))], n \geq 0. \tag{19}$$

To get $u_0, u_1,..., u_{n+1}$, we take the inverse laplace transform of this equation. Then, we obtain

$$u_0 = f(x) + tg(x) + H(x,t), \tag{20}$$

$$u_{n+1} = -L^{-1}[\frac{1}{s^2}L[Ru_n(x,t)] - \frac{1}{s^2}\sum_{n=0}^{\infty}[L(A_n(u))]] \tag{21}$$

where $H(x,t) = L^{-1}[\frac{1}{s^2}L[h(x,t)]]$.

2.2 The Adomian decomposition method

Let us consider the nonlinear partial differential equation (5). It can be written as

$$L_t u(x,t) + Ru(x,t) + Nu(x,t) = h(x,t) \tag{22}$$

where $L_t = \frac{\partial}{\partial t^2}$. Firstly, let us rewrite the equation (22) as

$$L_t u(x,t) = h(x,t) - Ru(x,t) - Nu(x,t). \tag{23}$$

Then, applying L_t^{-1} to both sides of this equation and using the initial conditions gives us

$$u(x,t) = f(x) + tg(x) + L_t^{-1}h(x,t) - L_t^{-1}Ru(x,t) - L_t^{-1}Nu(x,t) \tag{24}$$

where $L_t^{-1} = \int_0^t \int_0^t (.) \, dt \, dt$. For the linear terms, we use the decomposition series given by

$$u(x,t) = \sum_{n=0}^{\infty} u_n. \tag{25}$$

For the nonlinear terms, we use the following infinite series of Adomian polynomials given by

$$N(u(x,t)) = \sum_{n=0}^{\infty} A_n(u) \tag{26}$$

where A_n are Adomian polynomials. We obtain the recursive relation:

$$u_0(x,t) = f(x) + tg(x) + L_t^{-1}h(x,t), \tag{27}$$

$$u_{k+1}(x,t) = -L_t^{-1}(Ru(x,t)) - L_t^{-1}(A_k). \tag{28}$$

This relation provides us with the components of the solution $u(x,t)$. The first few terms of solution are given as

$$u_0(x,t) = f(x) + tg(x) + L_t^{-1}h(x,t) \tag{29}$$

$$u_1(x,t) = -L_t^{-1} R u_0(x,t) - L_t^{-1}(A_0) \tag{30}$$

$$u_2(x,t) = -L_t^{-1} R u_1(x,t) - L_t^{-1}(A_1). \tag{31}$$

In both methods, the solution $u(x,t)$ is given by

$$u(x,t) = \sum_{n=0}^{\infty} u_n = u_0 + u_1 + u_2 + ...u_n + ... \tag{32}$$

Putting these components in $u(x,t) = \sum_{n=0}^{\infty} u_n$ and using the noise terms phenomena gives us the solution rapidly.

3 Application of the methods

The methods are applied to the BBMPB equation and some other inhomogenous NLPDEs with initial conditions.

3.1 Solving BBMPB equation by LDM

In this section, LDM is applied to BBMPB equation given by (4). Taking the laplace transform of both sides, we get

$$L[u_t] = L[u_{xxt}] + \alpha L[u_{xx}] - \gamma L[u_x] - \beta L[u_{xxx}] + L[x + \gamma t - \theta x t^2] - L[N(u)], \tag{33}$$

where $N(u) = uu_x$. Using the properties of laplace transform gives

$$L[u] = \frac{1}{s}(L[u_{xxt}] + \alpha L[u_{xx}] - \gamma L[u_x] - \beta L[u_{xxx}] + L[x + \gamma t - \theta x t^2] - L[N(u)]). \tag{34}$$

So, we have the following recursive relation:

$$L[u_0] = \frac{1}{s} L[x + t\gamma + x t^2 \theta), \tag{35}$$

$$L[u_{k+1}] = \frac{1}{s} L[u_{k_{xxt}}] + \alpha \frac{1}{s} L[u_{k_{xx}}] - \gamma \frac{1}{s} L[u_{k_x}] - \beta \frac{1}{s} L[u_{k_{xxx}}] - \theta \frac{1}{s} L[A_k]. \tag{36}$$

Suppose that the inverse of laplace transform exists. Then taking the inverse of laplace transform, we get

$$u_0 = xt + \frac{\gamma}{2} t^2 + \frac{\theta}{3} x t^3, \tag{37}$$

$$u_1 = -\frac{\gamma}{2} t^2 - \frac{7\gamma\theta}{24} t^4 - \frac{\gamma\theta^2}{18} t^6 - \frac{\theta}{3} x t^3 - \frac{4\theta^2}{15} x t^5 - \frac{4\theta^3}{63} x t^2, \tag{38}$$

and so on for the other components. By canceling the noise terms, $\frac{\gamma}{2} t^2$ and $\frac{\theta}{3} x t^3$, from the component u_0 gives us the solution:

$$u(x,t) = xt. \tag{39}$$

3.2 Solving BBMPB equation by ADM

Firstly, let us write the equation (4) as

$$L_t(u) = u_{xxt} + u_{xx}\alpha - u_x\gamma - uu_x\theta - u_{xxx}\beta + x + t\gamma + xt^2\theta, u(x,0) = 0, \tag{40}$$

where L_t is a first order derivative w.r.t t. Then, we get

$$u = L_t^{-1}(u_{xxt} + \alpha u_{xx} - \gamma u_x - \beta u_{xxx}) + L_t^{-1}(x + \gamma t + \theta xt^2) - L_t^{-1}N(u), \tag{41}$$

where $L_t^{-1} = \int_0^t (.)\,dt$. Using the decomposition series for the linear terms and Adomian polynomial for the nonlinear terms gives the following relation

$$u_0 = L_t^{-1}(x + \gamma t + \theta xt^2), \tag{42}$$

$$u_1 = L_t^{-1}(u_{0_{xxt}} + \alpha u_{0_{xx}} - \gamma u_{0_x} - \beta u_{0_{xxx}}) - L_t^{-1}(A_0) \tag{43}$$

and so on for the other components. Here, $A_0(u) = u_0 u_{0_x}$. Then, we obtain

$$u_0 = xt + \frac{\gamma}{2}t^2 + \frac{\theta}{3}xt^3, \tag{44}$$

$$u_1 = -\frac{\gamma}{2}t^2 - \frac{7\gamma\theta}{24}t^4 - \frac{\gamma\theta^2}{18}t^6 - \frac{\theta}{3}xt^3 - \frac{4\theta^2}{15}xt^5 - \frac{4\theta^3}{63}xt^2. \tag{45}$$

So, the noise terms are $\frac{\gamma}{2}t^2$ and $\frac{\theta}{3}xt^3$. Canceling these terms from the component u_0 gives the solution

$$u_0 = xt. \tag{46}$$

3.3 Examples

In this part, the LDM and ADM are applied to some inhomogeneous NLPDEs with their initial values.

Example 1. Consider the following equation

$$u_{tt} - u_{xx} + u_{xxt} - 2u_{xtt} - u_x u_t = sin(t) - cos(t) \tag{47}$$

with the initial conditions $u(x,0) = x + 1$ and $u_t(x,0) = 0$.

(a) Let us apply the LDM to this equation. Then, we have

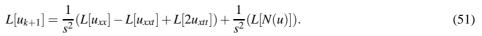$$L[u_{tt}] = L[u_{xx}] - L[u_{xxt}] + L[2u_{xtt}] + L[N(u)] + L[sin(t) - cos(t)] \tag{48}$$

where $N(u) = u_x u_t$. The differentiation property of the laplace transform and initial conditions give

$$L[u] = \frac{x+1}{s} + \frac{0}{s^2} + \frac{1}{s^2}L[sin(t) - cos(t)] + \frac{1}{s^2}(L[u_{xx}] - L[u_{xxt}] + L[2u_{xtt}]) + \frac{1}{s^2}(L[N(u)]). \tag{49}$$

Then, we get the following recursive relation:

$$L[u_0] = \frac{x+1}{s} + \frac{1}{s^2}L[sin(t) - cos(t)], \tag{50}$$

$$L[u_{k+1}] = \frac{1}{s^2}\left(L[u_{xx}] - L[u_{xxt}] + L[2u_{xtt}]\right) + \frac{1}{s^2}\left(L[N(u)]\right). \tag{51}$$

Assume that the inverse of laplace transform exists. Then taking the inverse of laplace transform, we obtain

$$u_0 = x + t + cos(t) - sin(t), \tag{52}$$

$$u_1 = -1 - t + \frac{t^2}{2} + cos(t) + sin(t). \tag{53}$$

Here, we have the noise terms, t and $sin(t)$. Removing the noise terms from u_0 provides us with the solution

$$u(x,t) = x + cos(t). \tag{54}$$

(b) The ADM is applied to the equation (47). It can be rewritten as

$$L_{tt}(u) = u_{xx} - u_{xxt} + 2u_{xtt} + u_x u_t + sin(t) - cos(t) \tag{55}$$

where L_{tt} is a second order derivative w.r.t t. Applying the inverse of L_{tt} gives

$$u = L_{tt}^{-1}(u_{xx} - u_{xxt} + 2u_{xtt}) + L_{tt}^{-1}(u_x u_t) + L_{tt}^{-1}(sin(t) - cos(t)) \tag{56}$$

where $L_t t^{-1} = \int_0^t \int_0^t (.)\, dt\, dt$. Using the decomposition series for the linear terms and Adomian polynomial for the nonlinear terms gives the following relation

$$u_0 = x + 1 + L_t t^{-1}(sin(t) - cos(t)), \tag{57}$$

$$u_1 = L_t t^{-1}(u_{xx} - u_{xxt} + 2u_{xtt}) + L_t t^{-1}(A_0(u)) \tag{58}$$

where $A_0(u) = N(u) = u_{0_x} u_{0_t}$. Then, we get

$$u_0 = x + t - sin(t) + cos(t), \tag{59}$$

$$u_1 = -1 - t + \frac{t^2}{2} + cos(t) + sin(t). \tag{60}$$

Clearly, the noise terms are t and $sin(t)$. If we remove these terms from u_0, we gain the solution as

$$u(x,t) = x + cos(t). \tag{61}$$

Example 2. Consider the following equation

$$u_t - u_{xx} + u_{xxtt} + \frac{1}{2}u_{xx}u_{tt} = 2xt - 2xt^2 \tag{62}$$

with the initial condition $u(x,0) = -x^2$.

(a) Let us apply the LDM to the equation (62). The laplace transform of both sides is taken.Then, we have

$$L[u_t] = L[u_{xx}] - L[u_{xxtt}] - L[N(u)] + L[2xt - 2xt^2] \qquad (63)$$

where $N(u) = \frac{1}{2}u_{xx}u_{tt}$. By using the differentiation property of the laplace transform and initial condition, we get

$$L[u] = \frac{-x^2}{s} + + \frac{1}{s}L[2xt - 2xt^2] + \frac{1}{s}(L[u_{xx}] - L[u_{xxtt}] - \frac{1}{s}(L[N(u)]). \qquad (64)$$

Then, this equality gives the following recursive relation:

$$L[u_0] = \frac{-x^2}{s} + \frac{1}{s}L[2xt - 2xt^2], \qquad (65)$$

$$L[u_{k+1}] = \frac{1}{s}(L[u_{xx}] - L[u_{xxtt}] - \frac{1}{s}(L[N(u)]). \qquad (66)$$

Assume that the inverse of laplace transform exists. If we take the inverse of laplace transform, then we get

$$u_0 = -x^2 + xt^2 - 2xt + 2t, \qquad (67)$$

$$u_1 = -2t + 2xt. \qquad (68)$$

Comparing u_0 and u_1 gives the noise terms as $2t$ and $-2xt$. Removing the noise terms from u_0 provides us with the solution

$$u(x,t) = xt^2 - x^2. \qquad (69)$$

(b) The ADM is applied to the equation (62). It can be rewritten as

$$L_t(u) = u_{xx} - u_{xxtt} - \frac{1}{2}u_{xx}u_{tt} + 2xt - 2xt^2 \qquad (70)$$

where L_t is a first order derivative w.r.t t. By applying the inverse of L_t to both sides, we get

$$u = L_t^{-1}(u_{xx} - u_{xxtt}) + L_t^{-1}(\frac{1}{2}u_{xx}u_{tt}) + L_t^{-1}(2xt - 2xt^2) \qquad (71)$$

where $L_t^{-1} = \int_0^t dt$. If we use the decomposition series for the linear terms and Adomian polynomial for the nonlinear terms, then we have the following relation:

$$u_0 = -x^2 + L_t^{-1}(2xt - 2xt^2), \qquad (72)$$

$$u_1 = L_t^{-1}(u_{xx} - u_{xxtt}) + L_t^{-1}(N(u)) \qquad (73)$$

where $A_0(u) = N(u) = \frac{1}{2}u_{xx}u_{tt}$. Then, we obtain

$$u_0 = 2t - 2xt + xt^2 - x^2, \qquad (74)$$

$$u_1 = -2t - 2xt. \tag{75}$$

It is obvious that the noise terms are $2t$ and $-2xt$. If we remove these terms from u_0, we gain the solution as

$$u(x,t) = xt^2 - x^2. \tag{76}$$

4 Conclusion

In this study, we consider the BBMPB equation with its initial condition. To obtain the solution of this equation, ADM, and LDM are applied. Then, it is seen that the same exact solution is gained by both methods. Then, these methods are applied to some NLPDEs with initial conditions and the solutions are gained. In these applications, it is shown that these decomposition methods are powerful in solving nonlinear ones with initial and boundary conditions in addition to their effectiveness and usefulness in solving linear partial differential equations. Compared to other methods for solving nonlinear partial differential equations with initial conditions, there is no need for linearization of nonlinear terms thanks to the Adomian polynomials. Moreover, the solution can easily and rapidly be attained with the help of the noise terms.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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