An algorithm for finding complementary nil dominating set in a fuzzy graph

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Abstract: In this paper, effective adjacency matrix of a fuzzy graph is introduced. An algorithm for finding a minimal complementary nil dominating set of fuzzy graph is studied with suitable examples.

Keywords: Fuzzy Graph, dominating set, complementary nil dominating set.

1 Introduction


In this paper, the notion of effective adjacency matrix is initiated and also introduced the algorithm for finding the minimal complementary nil dominating set of a fuzzy graph.

2 Preliminaries

Definition 1. [8] A fuzzy set $V$ is a mapping $\sigma$ from $V$ to $[0, 1]$. A fuzzy graph $G$ is a pair of functions $G = (\sigma, \mu)$ where $\sigma$ is a fuzzy subset of a non-empty set $V$ and $\mu$ is a symmetric fuzzy relation on $\sigma$. (i.e) $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$. The underlying crisp graph of $G = (\sigma, \mu)$ is denoted by $G^* = (V, E)$ where $E \subseteq V \times V$.

Definition 2. [8] The scalar cardinality of $S \subseteq V$ is defined by $\sum_{u \in S} \sigma(u)$. The order (denoted by $p$) and size (denoted by $q$) of a fuzzy graph $G$ are the scalar cardinality of $\sigma$ and $\mu$ respectively.

Definition 3. [10] An edge $e = (u, v)$ of a fuzzy graph is called an effective edge if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$.
Definition 4.\cite{10} Let $G = (\sigma, \mu)$ be a fuzzy graph and let $u, v \in V$. If $\mu(u, v) = \sigma(u) \land \sigma(v)$ then $u$ dominates $v$ (or $v$ is dominated by $u$) in $G$. A subset $D$ of $V$ is called a dominating set in $G$ if for every $v \notin D$ there exist $u \in D$ such that $u$ dominates $v$. The minimum scalar cardinality taken over all dominating set is called domination number and is denoted by the symbol $\gamma$. The maximum scalar cardinality of a minimal dominating set is called upper domination number and is denoted by the symbol $\Gamma$.

Definition 5.\cite{10} A set $S \subset V$ in a fuzzy graph $G = (\sigma, \mu)$ is said to be independent set if no two vertices of $S$ are adjacent.

Definition 6.\cite{5} Let $G = (\sigma, \mu)$ be a fuzzy graph on $V$. A set $S \subset V$ is said to be a complementary nil dominating set (or simply called cnd-set) of a fuzzy graph $G$ if $S$ is a dominating set and its complement $V - S$ is not a dominating set.

Definition 7.\cite{5} A cnd-set $S$ of a fuzzy graph $G = (\sigma, \mu)$ is called a minimal cnd-set if there is no cnd-set $S'$ such that $S' \subset S$.

Definition 8.\cite{5} A cnd-set $S$ of a fuzzy graph $G = (\sigma, \mu)$ is called a minimum cnd-set if there is no cnd-set $S'$ such that $|S'| < |S|$. The minimum scalar cardinality taken over all cnd-set is called a complementary nil domination number and is denoted by the symbol $\gamma_{cnd}$, the corresponding minimum cnd-set is denoted by $\gamma_{cnd}$-set.

3 Effective adjacency matrix

Definition 9. Let $G$ be a fuzzy graph on underlying graph $G^*$ with $n$ vertices. Then the effective adjacency matrix of $G$ is an $n \times n$ matrix whose entries are defined by

$$a_{ij} = \begin{cases} 1 & \text{if either } i = j \text{ or } (v_i, v_j) \text{ is an effective edge} \\ 0 & \text{otherwise} \end{cases}$$

and it is denoted by $EAM(A)$.

Example 1. Consider the fuzzy graph as given in Figure 1.

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Fig. 1: Illustration of the Algorithm
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The effective adjacency matrix $EAM(A) = \begin{pmatrix} (v_1) & (v_2) & (v_3) & (v_4) \\ (v_1) & 1 & 1 & 1 & 1 \\ (v_2) & 1 & 1 & 0 & 1 \\ (v_3) & 1 & 1 & 1 & 1 \\ (v_4) & 0 & 1 & 1 & 1 \end{pmatrix}$.
4 Algorithm for finding complementary nil dominating set in a fuzzy graph

Let us assume that $G = (V, E)$ be a fuzzy graph without isolated vertex but not complete.

**Step 1:**
Write the effective adjacency matrix $EAM(A)$ of a given fuzzy graph.

$EAM(A) = EAM_o(A)$

**Step 2:**
Calculate the number of 1’s occurred in each row of $EAM(A)$.

$EAM(A) = R(A)$. If $n$ rows has exactly two 1’s, then choose the vertex $v_i$ with minimum membership value among the vertices. (If more than one vertices has same membership values then choose a vertex arbitrarily, goto step 5. Otherwise goto step 4

**Step 4:**
In $R(A)$, Choose the row with maximum number of 1’s.(say $i^{th}$ row)
If more than one row has an equal number of 1’s, then choose a vertex with minimum membership value.(If more than one vertex has same membership value then choose arbitrarily one vertex).

**Step 5:**
Put $v_i$ in $D$.

**Step 6:**
Delete the row and column corresponding to $v_i$ in $R(A)$.

**Step 7:**
Calculate $D_1 = V - D$.
Delete the row corresponding to $v_i$ in $EAM(A)$.

**Step 8:**
Calculate the column total of $EAM(A)$.If any one of the total is zero, then $D_1 = V - D$ is not a dominating set. Goto step 9
Otherwise $D_1 = V - D$ is a dominating set, then goto step 4

**Step 9:** [Check D is a dominating set or not]
Delete all the rows corresponding to the vertices of $D_1$ in $EAM_o(A)$
Calculate the column total of $EAM_o(A)$.
If all the column totals are non-zero, then goto step 10. Otherwise goto step 4.

**Step 10:**
$D$ is a dominating set and also it is a minimal complementary nil dominating set.

**Example 2.** Consider the fuzzy graph as given in Figure 2.

*Step: 1*

![Fig. 2: Illustration of the Algorithm](image-url)
The effective adjacency matrix $EAM(A)$:

$$
EAM(A) = \begin{pmatrix}
(v_1) & (v_2) & (v_3) & (v_4) & (v_5) \\
(v_1) & 1 & 1 & 0 & 1 & 0 \\
(v_2) & 1 & 1 & 1 & 0 & 0 \\
(v_3) & 0 & 1 & 1 & 1 & 0 \\
(v_4) & 1 & 1 & 1 & 1 & 1 \\
(v_5) & 0 & 0 & 0 & 1 & 1
\end{pmatrix}
$$

Step: 2
The number of 1’s occurred in row 1, row 2, row 3, row 4 and row 5 are 3, 4, 3, 4 and 2 respectively.

Step: 3
$EAM(A) = R(A)$ The row 5 has exactly two 1’s. Choose $v_5$.

Step: 5
Put $v_5$ in $D$.

Step: 6
Delete a row and a column corresponding to $v_5$ in $R(A)$.

Step: 7
Calculate $D_1 = V - D = \{v_1, v_2, v_3, v_4\}$.

Step: 8
The column totals of $EAM(A)$ are 3, 4, 3, 4 and 1. Hence all the column totals are non-zero. Goto step 4.

Step: 4
$R(A) = \begin{pmatrix}
(v_1) & (v_2) & (v_3) & (v_4) \\
(v_1) & 1 & 1 & 0 & 1 \\
(v_2) & 1 & 1 & 1 & 1 \\
(v_3) & 0 & 1 & 1 & 1 \\
(v_4) & 1 & 1 & 1 & 1
\end{pmatrix}$

The number of 1’s occurred in row 1, row 2, row 3 and row 4 are 3, 4, 3 and 4 respectively. The rows 2 and 4 has equal of no of 1’s. Then minimum weight of $v_2$ and $v_4$ is $v_4$. Choose $v_4$.

Step: 5
Put $v_4$ in $D$.

Step: 6
Delete the row and column corresponding to $v_4$ in $R(A)$. 

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\[ R(A) = \begin{pmatrix} (v_1) & (v_2) & (v_3) \\ (v_1) & 1 & 1 & 0 \\ (v_2) & 1 & 1 & 1 \\ (v_3) & 0 & 1 & 1 \end{pmatrix} \]

**Step: 7**
Calculate \( D_1 = V - D = \{v_1, v_2, v_3\} \).
Delete the row corresponding to \( v_4 \) in \( EAM(A) \).

**Step: 8**
\[ EAM(A) = \begin{pmatrix} (v_1) & (v_2) & (v_3) & (v_4) & (v_5) \\ (v_1) & 1 & 1 & 0 & 1 & 0 \\ (v_2) & 1 & 1 & 1 & 1 & 0 \\ (v_3) & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \]

The column totals of \( EAM(A) \) are 2, 3, 2, 3 and 0. Hence one of the column total is zero. Goto step 9.

**Step: 9** Check \( D \) is a dominating set or not
Delete all the rows corresponding to the vertices of \( D_1 \) in \( EAM_o(A) \).
Calculate the column total of \( EAM_o(A) \).
\[ EAM_o(A) = \begin{pmatrix} (v_1) & (v_2) & (v_3) & (v_4) & (v_5) \\ (v_4) & 1 & 1 & 1 & 1 & 1 \\ (v_5) & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \]

All the column totals are non-zero. goto step 10.

**Step: 10**
\( D = \{v_4, v_5\} \) is a dominating set and also it is a minimal complementary nil dominating set.

**Example 3.** Consider a fuzzy graph \( G = (\sigma, \mu) \) given in FIGURE 3, where 
\( \sigma = \{u_1/0.3, u_2/0.4, u_3/0.6, u_4/0.2, u_5/0.8, u_6/0.7\} \) and \( \mu = \{(u_1, u_2)/0.3, (u_2, u_3)/0.4, (u_3, u_4)/0.2, (u_4, u_5)/0.2, (u_5, u_6)/0.7, (u_6, u_1)/0.3, (u_3, u_6)/0.4\} \),
\( S = \{u_1, u_2, u_3, u_4\} \)- is \( \gamma_{cd}-\)set.

![Fig. 3: Fuzzy Graph](image-url)
Competing interests

The authors declare that they have no competing interests.

Authors’ contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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