Communication in Mathematical Modeling and Applications
http://ntmsci.com/cmma

Application of intuitionistic Fuzzy soft sets in decision making based on real life problems

Anjan Mukherjee, Sadhan Sarkar and Somen Debnath

Department Of Mathematics, Tripura University, Suryamaninagar, Agartala, Tripura

Received: 24 June 2016, Accepted: 3 November 2016
Published online: 18 December 2015.

Abstract: In 1999, Molodtsov introduced soft set theory which is a new mathematical tool for dealing with uncertainties and is free from the difficulties affecting the existing methods. Research works on soft set theory are progressing rapidly. Combining soft sets with fuzzy sets and intuitionistic fuzzy sets, Maji et al. , defined fuzzy soft sets and intuitionistic fuzzy soft sets which are rich potentials for solving decision making problems. It has been found that soft set, fuzzy set and rough set are closely related concepts. In this work, we define intuitionistic fuzzy soft set aggregative operator that allows constructing more efficient decision making method. Finally, we give an example which shows that the method can be successfully applied to many problems that contain uncertainties.

Keywords: Fuzzy sets, soft sets, Fuzzy soft sets, intuitionistic Fuzzy soft sets, intuitionistic Fuzzy aggregation.

1 Introduction

In the fuzzy set theory [**15**] there were no scope to think about the hesitation in the membership degree, which arise in various real life situations. To overcome these situations Atanassov [**1**] introduced theory of intuitionistic fuzzy set in 1986 as a generalization of fuzzy set.

Most of the problems in engineering, medical science, economics, environments etc have various uncertainties. Molodtsov[12] initiated the concept of soft set theory as a mathematical tool for dealing with uncertainties. Research works on soft set theory are progressing rapidly. Maji et al.[8] defined several operations on soft set theory. Combining soft sets with fuzzy sets and intuitionistic fuzzy sets, Feng et al.[7] and Maji et al.[9,10] defined fuzzy soft sets and intuitionistic fuzzy soft sets which are rich potentials for solving decision making problems.

Matrices play an important role in the broad area of science and engineering. The classical matrix theory cannot solve the problems involving various types of uncertainties. In [14] Yang et al, initiated a matrix representation of a fuzzy soft set and applied it in certain decision making problems. The concept of fuzzy soft matrix theory was studied by Borah et al. in [2]. In [5], Chetia et al. and in [13] Rajarajeswari et al. defined intuitionistic fuzzy soft matrix.

Again it is well known that the matrices are important tools to model/study different mathematical problems specially in linear algebra. Due to huge applications of imprecise data in the above mentioned areas, hence are motivated to study the different matrices containing these data. Soft set is also one of the interesting and popular subject, where different types of decision making problem can be solved. So attempt has been made to study the decision making problem by using intuitionistic fuzzy soft aggregation operator. Das and Kar [**6**] proposed an algorithmic approach for group decision making based on IF soft set. The authors [6] have used cardinality of IF soft set as a novel concept for assigning

* Corresponding author e-mail: anjan2002_m@yahoo.co.in

confident weight to the set of experts. Cagman and Enginogh[3,4] pioneered the concept of soft matrix to represent a soft set. Mao et al.[11] presented the concept of intuitionistic fuzzy soft matrix(IFSM) and applied it in group decision making problem.

In this paper,we define intuitionistic fuzzy soft set aggregative operator that allows constructing more efficient decision making method. Finally, we give an example which shows that the method can be successfully applied to many problems that contain uncertainties.

2 Preliminaries

In this section we briefly review some basic definitions related to fuzzy set, intuitionistic fuzzy sets, soft sets and their generalizations, which will be used in the rest of the paper.

Definition 1. *[15] Let X be a non-empty collection of objects denoted by x. Then a fuzzy set (FS for short) α in X is a set of ordered pairs having the form $\alpha = \{(x, \mu_\alpha(x)) : x \in X\}$.*

Where the function $\mu_\alpha : X \to [0,1]$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in α.The interval M = $[0,1]$is called membership space.

Definition 2. *[1] Let X be a non empty set. Then an intuitionistic fuzzy set (IFS for short) A is a set having the form $A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}$ where the functions $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ represents the degree ofmembership and the degree of non-membership respectively of each element $x \in X$ and $0 \le \mu_A(x)+\gamma_A(x) \le 1$ for each $x \in X$.*

Definition 3. *[8, 12] Let U be an initial universe and E be a set of parameters. Let P(U) denotes the power set of U and $A \subseteq E$. Then the pair (F,A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.*

Definition 4. *[9] Let U be an initial universe and E be a set of parameters. Let I^U be the set of all fuzzy subsets of U and $A \subseteq E$. Then the pair (F, A) is called a **fuzzy soft set** over U, where F is a mapping given by $F: A \to I^U$.*

For any $\varepsilon \in A$, $F(\varepsilon)$ is a fuzzy subset of U. Let us denote the membership degree that object x holds parameter ε by $\mu_{F(\varepsilon)}(x)$, where $x \in U$ and $\varepsilon \in A$. Then $F(\varepsilon)$ can be written as a fuzzy set such that $F(\varepsilon) = \{(x, \mu_{F(\varepsilon)}(x)) : x \in U\}$.

Definition 5. *[10] Let U be an initial universe and E be a set of parameters. Let IF^U be the set of all intuitionistic fuzzy subsets of U and $A \subseteq E$. Then the pair (F, A) is called an **intuitionistic fuzzy soft set** over U, where F is a mapping given by F. $A \to IF^U$.*

For any $\varepsilon \in A$, $F(\varepsilon)$ is an intuitionistic fuzzy subset of U. Let us denote $\mu_{F(\varepsilon)}(x)$ and $\gamma_{F(\varepsilon)}(x)$ by the membership degree & non-membership degree respectively that object x holds parameter ε, where $x \in U$ and $\varepsilon \in A$. Then $F(\varepsilon)$ can be written as an intuitionistic fuzzy set such that $F(\varepsilon) = \{(x, \mu_{F(\varepsilon)}(x),\ \gamma_{F(\varepsilon)}(x)) : x \in U\}$.

3 Intuitionistic Fuzzy soft aggregation operator

In this section we define an intuitionistic fuzzy soft (IF Soft)- aggregation operator that produces an aggregate IF set from an IF soft set and its cardinal set. The approximate functions of an IF Soft –set are IF set. An IF Soft-set aggregation operator on the IF sets is an operation by which several approximate functions of an IF Soft –set are combined to produce a single IF set which is the aggregate IF set of the IF Soft-set.

We restate an intuitionistic fuzzy soft set in the following way- **An intuitionistic fuzzy soft set(IF soft set)** Γ_A over U is a set defined by the functions $\mu_{\lambda_A} : E \to IF(U)$and $\upsilon_{\eta_A} : E \to IF(U)$,

$$\Gamma_A = \left\{ \frac{(\mu_{\lambda_A}(x), \upsilon_{\eta_A}(x))}{x} : x \in E, \lambda_A, \eta_A \in IFS(U) \right\}$$

$IFS(U)$ is the set of all IF soft sets over U. λ_A is called intuitionistic fuzzy approximate function of the IF soft set Γ_A and the value $\lambda_A(x)$ is a set called x-element of the IF soft set for all $x \in E$. η_A is called non intuitionistic fuzzy approximate function of the IF soft set Γ_A and the value $\eta_A(x)$ is a set called non x-element of the IF soft set for all $x \in E$.

Definition 6. *Let $\Gamma_A \in IFS(U)$. Assume that $U = \{u_1, u_2, u_3, ..., u_m\}$ and $E = \{x_1, x_2, x_3, ..., x_n\}$ and $A \subseteq E$, then the IF soft set Γ_A is as follows.*

Γ_A	x_1	x_2	...	x_n
u_1	$(\mu_{\lambda_A}(x_1)(u_1), \upsilon_{\eta_A}(x_1)(u_1))$	$(\mu_{\lambda_A}(x_2)(u_1), \upsilon_{\eta_A}(x_2)(u_1))$	...	$(\mu_{\lambda_A}(x_n)(u_1), \upsilon_{\eta_A}(x_n)(u_1))$
u_2	$(\mu_{\lambda_A}(x_1)(u_2), \upsilon_{\eta_A}(x_1)(u_2))$	$(\mu_{\lambda_A}(x_2)(u_2), \upsilon_{\eta_A}(x_2)(u_2))$	...	$(\mu_{\lambda_A}(x_n)(u_2), \upsilon_{\eta_A}(x_n)(u_2))$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
u_m	$(\mu_{\lambda_A}(x_1)(u_m), \upsilon_{\eta_A}(x_1)(u_m))$	$(\mu_{\lambda_A}(x_2)(u_m), \upsilon_{\eta_A}(x_2)(u_m))$	...	$(\mu_{\lambda_A}(x_2)(u_m), \upsilon_{\eta_A}(x_2)(u_m))$

Here

$$\Gamma_A = \left\{ \frac{(\mu_{\lambda_A}(x), \upsilon_{\eta_A}(x))}{x} : x \in E, \lambda_A(x), \eta_A(x) \in IF(U) \right\}.$$

$\mu_{\lambda_A}(x)$is the membership function of $\lambda_A$and $\upsilon_{\eta_A}(x)$ is the non-membership function of η_A. If $a_{ij} = \mu_{\lambda_A}(x_j)(u_i)$and $b_{ij} = \upsilon_{\eta_A}(x_j)(u_i)$, i = 1,2,3, . . . , m and $j = 1,2,3,\ldots,n$ then the IF soft set Γ_A is given by the matrix

$$[a_{ij}, b_{ij}]_{m\times n} = \begin{bmatrix} (a_{11}, b_{11}) & \ldots & (a_{1n}, b_{1n}) \\ (a_{21}, b_{21}) & \ldots & (a_{2n}, b_{2n}) \\ & \ldots & \\ (a_{m1}, b_{m1}) & \ldots & (a_{mn}, b_{mn}) \end{bmatrix}$$

is called IF soft matrix of the IF soft set Γ_A over U.

Definition 7. *Let $\Gamma_A \in IFS(U)$ then the cardinal set of Γ_A is denoted by $c\Gamma_A$, where $c\Gamma_A = \left\{ \frac{(\mu_{c\lambda_A}(x), \upsilon_{c\eta_A}(x))}{x} : x \in E \right\}$ is an IF set over E. $\mu_{c\lambda_A} : E \to [0,1]$ and $\mu_{c\lambda_A}(x) = \frac{|\lambda_A(x)|}{|U|}$, $\upsilon_{c\eta_A} : E \to [0,1]$ and $\upsilon_{c\eta_A}(x) = \frac{|\eta_A(x)|}{|U|}$, where $|U|$ is the cardinality of the universe U, $|\lambda_A(x)|$ and $|(\eta_A x)|$ are the scalar cardinality of the IF sets $\lambda_A(x)$and $\eta_A(x)$ respectively.*

Note that the set of all cardinal sets of the IF soft sets over U will be denoted by $cIFS(U)$.

Definition 8. *Let $\Gamma_A \in IFS(U)$ and $c\Gamma_A \in cIFS(U)$. Assume that $E = \{x_1, x_2, x_3, \ldots, x_n\}$ and $A \subseteq E$ then $c\Gamma_A$ can be represented by the tabular form,*

E	x_1	x_2	$\cdots$	x_n
$c\Gamma_A$	$(\mu_{c\lambda_A}(x_1), \upsilon_{c\eta_A}(x_1))$	$(\mu_{c\lambda_A}(x_2), \upsilon_{c\eta_A}(x_n))$	$\cdots$	$(\mu_{c\lambda_A}(x_n), \upsilon_{c\eta_A}(x_n))$

If $(a_{1j}, b_{1j}) = \mu_{c\Gamma_A}(x_j)$ for $j = 1,2,3,\ldots,n$ then $c\Gamma_A$ is represented by a matrix given by

$$[a_{1j}, b_{ij}]_{1\times n} = [(a_{11}, b_{11})(a_{12}, b_{12})\ldots(a_{1n}, b_{1n})]$$

Which is called the cardinal matrix of the cardinal set $c\Gamma_A$ over E.

Definition 9. *Let $\Gamma_A \in IFS(U)$ and $c\Gamma_A \in cIFS(U)$. Then the IF soft aggregation operator denoted by IFS_{agg} is defined by $IFS_{agg} : cIFS(U) \times IFS(U) \to IF(U)$. So $IFS_{agg}(c\Gamma_A, \Gamma_A) = \Gamma_A{}^*$, where*

$$\Gamma_A{}^* = \left\{ \frac{\mu_{\Gamma^*(A)}(u)}{u} : u \in U \right\} = \left\{ \frac{\left(\mu_{\lambda^*{}_A}(u), \upsilon_{\eta^*{}_A}(u)\right)}{u} : u \in U \right\}$$

is a IF set over U. $\Gamma_A{}^*$is called aggregate IF set of the IF soft set Γ_A. Then the membership and non-membership function of $\Gamma_A{}^*$ is defined as

$$\mu_{\lambda_A{}^*} : U \to [0,1]$$

$$\mu_{\lambda_A{}^*}(u) = \frac{1}{|E|} \sum_{x \in E} \left(\mu_{c\lambda_A}(x), \mu_{\lambda_A}(x)\right)(u)$$

$\upsilon_{\eta_A{}^*} : U \to [0,1]$ *and*

$$\upsilon_{\eta_A{}^*}(u) = \frac{1}{|E|} \sum_{x \in E} \left(\upsilon_{c\eta_A}(x), \upsilon_{\eta_A}(x)\right)(u)$$

where $|E|$is called cardinality of E.

Definition 10. *Let $\Gamma_A \in IFS(U)$and $\Gamma_A{}^*$ be its aggregate IF set. Assume that $U = \{u_1, u_2, ..., u_m\}$then $\Gamma_A{}^*$ can be presented as*

Γ_A	$\mu_{\Gamma_A^*}$
u_1	$\mu_{\Gamma_A^*}(u_1) = \left(\mu_{\lambda^*{}_A}(u_1), \upsilon_{\eta^*{}_A}(u_1)\right)$
u_2	$\mu_{\Gamma_A^*}(u_2) = \left(\mu_{\lambda^*{}_A}(u_2), \upsilon_{\eta^*{}_A}(u_2)\right)$
$\vdots$	$\vdots$
u_m	$\mu_{\Gamma_A^*}(u_m) = \left(\mu_{\lambda^*{}_A}(u_m), \upsilon_{\eta^*{}_A}(u_m)\right)$

If $[a_{i1,bi1}] = \mu_{\Gamma_A^*}(u_i)$ for $I = 1, 2, 3, \ldots, m$ then $\Gamma_A{}^*$ is uniquely characterized by the matrix

$$[a_{i1,bi1}]_{m\times 1} = \begin{bmatrix} (a_{11}, b_{11}) \\ (a_{21}, b_{21}) \\ \vdots \\ (a_{m1}, b_{m1}) \end{bmatrix}$$

which is called the aggregate matrix of $\Gamma_A{}^*$ over U.

Theorem 1. *Let $\Gamma_A \in IFS(U)$ and $A \subseteq E$. If M_{Γ_A}, $M_{c\Gamma_A}$, $M_{\Gamma_A^*}$ are representative matrices of Γ_A, $c\Gamma_A$and Γ_A^* respectively then $|E| \times M_{\Gamma_A^*} = M_{\Gamma_A} \times M_{c\Gamma_A}^T$, where $M_{c\Gamma_A}^T$ is the transpose of $M_{c\Gamma_A}$ and $|E|$ is the cardinality of E.*

Proof. It is sufficient to consider$|E| \times [(a_{i1}, b_{i1})] = [(a_{ij}, b_{ij})]_{m\times n} \times \left[(a_{1j}, b_{1j})\right]_{1\times n}$.

4 Algorithm

We have an aggregate IF set, now it is necessary to choose the best alternative form of this set. We can make a decision by the following algorithm.

(1) Construct an IF soft set Γ_A over U.
(2) Find the cardinal set $c\Gamma_A$ of Γ_A i.e. $c\mu_{\lambda_A}$ and $c\upsilon_{\eta_A}$.
(3) Find the aggregate IF set Γ_A^* of Γ_A.
(4) Find the best alternative form of the set that has the largest membership grade by max $\mu_{\lambda_A^*}(u)$ and the smallest non membership grade by min $\upsilon_{\eta_A^*}(u)$.

4.1 Case study

Suppose a farm or institution wants to fill a position of faculty. There are eight candidates who form the set of alternatives U = $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$. So $|U| = 8$. The expert committee consider a set parameters $E = \{x_1, x_2, x_3, x_4, x_5\}$, $|E| = 5$. For $i = 1,2,3,4,5$ the parameters x_i stands for "experience", "computer knowledge", "young age", "good speaking" and "friendly" respectively.

After a discussion and interview each candidate is evaluated from the goals and constraint point of view according to a chosen subset $A = \{x_2, x_3, x_4\}$ of E. Finally the expert committee applies the following steps.

(1) The committee construct an IF soft set Γ_A over U.

$$\Gamma_A = \left\{ \left(x_2, \left(\frac{(0.3,0.5)}{u_2}, \frac{(0.5,0.4)}{u_3}, \frac{(0.1,0.7)}{u_4}, \frac{(0.8,0.2)}{u_5}, \frac{(0.7,0.2)}{u_7} \right) \right), \left(x_3, \left(\frac{(0.4,0.6)}{u_1}, \frac{(0.4,0.5)}{u_2}, \frac{(0.9,0.1)}{u_3}, \frac{(0.3,0.6)}{u_4} \right) \right), \left(x_4, \left(\frac{(0.2,0.6)}{u_1}, \frac{(0.5,0.4)}{u_2}, \frac{(0.1,0.4)}{u_5}, \frac{(0.7,0.3)}{u_7}, \frac{(0.1,0.6)}{u_8} \right) \right) \right\}$$

(2) Compute cardinality

$$c\Gamma_A = \left\{ \frac{(0.3,0.25)}{x_2}, \frac{(0.25,0.225)}{x_3}, \frac{(0.2,0.2875)}{x_4} \right\}$$

(3) The aggregate IF set is obtained by using theorem 1.

$$M_{\Gamma_A^*} = \left\{ \frac{1}{5} \begin{bmatrix} 0 & 0 & 0.4 & 0.2 & 0 \\ 0 & 0.3 & 0.4 & 0.5 & 0 \\ 0 & 0.5 & 0.9 & 0 & 0 \\ 0 & 0.1 & 0.3 & 0 & 0 \\ 0 & 0.8 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0.7 & 0 \\ 0 & 0 & 0 & 0.1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0.3 \\ 0.25 \\ 0.2 \\ 0 \end{bmatrix} \right\}, \left\{ \frac{1}{5} \begin{bmatrix} 0 & 0 & 0.6 & 0.6 & 0 \\ 0 & 0.5 & 0.5 & 0.4 & 0 \\ 0 & 0.4 & 0.1 & 0 & 0 \\ 0 & 0.7 & 0.6 & 0 & 0 \\ 0 & 0.2 & 0 & 0.4 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0.2 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0.6 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0.25 \\ 0.225 \\ 0.2875 \\ 0 \end{bmatrix} \right\} = \left[\begin{pmatrix} 0.028 \\ 0.058 \\ 0.075 \\ 0.021 \\ 0.052 \\ 0.000 \\ 0.070 \\ 0.041 \end{pmatrix}, \begin{pmatrix} 0.0615 \\ 0.0705 \\ 0.0245 \\ 0.0620 \\ 0.0330 \\ 0.9625 \\ 0.0272 \\ 0.0345 \end{pmatrix} \right]$$

So

$$\Gamma_A^* = \left\{ \frac{(0.028,0.0615)}{u_1}, \frac{(0.058,0.0705)}{u_2}, \frac{(0.075,0.0245)}{u_3}, \frac{(0.021,0.062)}{u_4}, \frac{(0.052,0.033)}{u_5}, \frac{(0,0.9625)}{u_6}, \frac{(0.070,0.0272)}{u_7}, \frac{(0.040,0.0345)}{u_8} \right\}$$

(4) The candidate u_3 has the largest membership grade i.e. 0.075 and smallest non membership grade i.e. 0.0245. Hence candidate u_3 may be selected for the job.

5 Conclusion

In this work we define intuitionistic fuzzy soft aggregation operator for intuitionistic fuzzy soft set and construct an algorithm using intuitionistic fuzzy soft aggregation operator for the decision making problem. Finally we apply the algorithm to solve a group decision making problem. Here we get better result by using intuitionistic fuzzy soft matrix than fuzzy soft matrix (as there are largest membership grade and smallest non membership grade)

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

References

[1] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20(1986) 87-96.
[2] M. J. Borah, T. J. Neog and D. K. Sut, Fuzzy soft matrix theory and its decision making, International Journal of Modern Engineering Research, 2(2012) 121-127.
[3] N. Cagman and S. Enginogh, Soft matrix theory and its decision making, Computers and Mathematics with Applications 59(10) (2010) 3308-3314.
[4] N. Cagman and S. Enginogh, Soft set theory and uni-int Decision making, European journal of operational research 207(2)(2010) 848-855.
[5] B. Chetia and P. K. Das, Some results of intuitionistic fuzzy soft matrix theory, Advances in Applied Science Research, 3(1) (2012) 412-423.
[6] S. Das and S. Kar, Group decision making in medical system: an intuitionistic fuzzy soft set approach, Applied soft computing 24(2014) 196-211.
[7] F. Feng, C. X. Li, B. Davvaz, M. I. Ali, Soft sets combined with fuzzy sets and rough sets: a tentative approach, Soft Computing 14(2010) 899-911.
[8] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Computers and Mathematics with Applications 45 (4-5) (2003) 555-562.
[9] P. K. Maji, R. Biswas, A. R. Roy, Fuzzy soft sets, The Journal Fuzzy Mathematics 9(2001)589-602.
[10] P. K. Maji, R. Biswas, A. R. Roy, Intuitionistic fuzzy soft sets, The Journal Fuzzy Mathematics 9(3)(2001) 677-692.
[11] J. Mao, D. Yao, C. Wang, Group decision making method based on intuitionistic fuzzy soft matrices, Applied Mathematical Modeling 37(2013) 6425-6436.
[12] D. Molodtsov, Soft set theory-first results, Computers and Mathematics with Applications, 37(4-5) (1999) 19-31.
[13] P. Rajarajeswari and P. Dhanalakshmi, Intuitionistic fuzzy soft matrix theory and its application in decision making, International Journal of Engineering Research and Technology, 2(2013), 1100-1111.
[14] Y. Yang, J. Chenli, Fuzzy soft matrices and their applications part I, Lecture notes in Computer Science, 7002(2011), 618-627.
[15] L. A. Zadeh, Fuzzy sets, Information and Control 8(1965) 338-353.