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Determination of the curves of constant breadth according to Bishop Frame in Euclidean 3-space

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Abstract: In this paper, for the approximate solutions of the linear differential equation system with variable coefficients in normal form characterizing curves of constant breadth according to Bishop frame in Euclidean 3-space, it is used a collocation method based on Taylor polynomials and therefore the curves of constant breadth are determined. In addition, an error analysis based on residual function is given for the method. Also, to demonstrate the efficiency of the method, an example is given with the help of computer programmes written in Maple and Matlab.

Keywords: Curves of constant breadth; Bishop frame; Taylor polynomial and series; collocation points; system of differential equations.

1 Introduction

The curves of constant breadth were introduced by Euler in 1778 [1]. He investigated the constant breadth curves in the plane. Then, many mathematicians were interested in these special curves [2-11]. Mağden and Köse studied the curves of constant breadth in E^4-space in [12]. After then, the concepts related to space curve of constant breadth were extended to E^n-space in [13]. The differential equations characterizing curves of constant breadth were established and a criterion for these curves were given by Sezer in [14]. Moreover, Önder et al. gave the differential equations characterizing the timelike and spacelike curves of constant breadth in Minkowski 3-space in [15]. In addition Kocayiğit and Önder showed that in E_1^3 spacelike and timelike curves of constant breadth are related to helices, normal curves and spherical curves in some special cases [16].

In [17], the collocation method based on Taylor polynomials was given by Sezer et al. to find the approximate solutions of high-order systems of linear differential equations with variable coefficients. Also, in [25] Kocayiğit and Çetin investigated the curves of constant breadth according to Bishop frame in Euclidean 3-space and they gave differential equations and systems characterizing these curves.

In this study, we obtain the approximate solutions of the differential equation systems characterizing curves of constant breadth according to Bishop frame in Euclidean 3-space by using Taylor collocation method. Then we give an example and compare the results to demonstrate the efficiency of the method.

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2 Preliminaries

Now, we give some basic concepts on classical differential geometry of space curves. Let $\alpha(s)$ be a space curve, where s is an arc length parameter and let $\left\{\overrightarrow{T}(s), \overrightarrow{N}(s), \overrightarrow{B}(s)\right\}$ be Frenet frame of this curve. The elements of the frame $\overrightarrow{T}, \overrightarrow{N}, \overrightarrow{B}$ and are called the unit tangent vector, the unit principal normal vector and the unit binormal vector of the curve, respectively. Furthermore, $\kappa(s)$ and $\tau(s)$ are called curvature and torsion of the curve α, respectively. The Frenet formulae are also well known as

$$\begin{bmatrix} \overrightarrow{T}' \\ \overrightarrow{N}' \\ \overrightarrow{B}' \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} \overrightarrow{T} \\ \overrightarrow{N} \\ \overrightarrow{B} \end{bmatrix}$$

where $\left\langle \overrightarrow{T}, \overrightarrow{T} \right\rangle = \left\langle \overrightarrow{N}, \overrightarrow{N} \right\rangle = \left\langle \overrightarrow{B}, \overrightarrow{B} \right\rangle = 1$ and $\left\langle \overrightarrow{T}, \overrightarrow{N} \right\rangle = \left\langle \overrightarrow{N}, \overrightarrow{B} \right\rangle = \left\langle \overrightarrow{T}, \overrightarrow{B} \right\rangle = 0$.

The parallel transport frame is an alternative approach to defining a moving frame that is well-defined even when the curve has vanishing second derivative. We can parallel transport an orthonormal frame along a curve simply by parallel transporting each component of the frame [18].

Its mathematical properties derive from the observation that, while $\overrightarrow{T}(s)$ for a given curve model is unique, we may choose any convenient arbitrary basis $\left(\overrightarrow{N_1}(s), \overrightarrow{N_2}(s)\right)$ for the remainder of the frame, so long as it is in the normal plane perpendicular to $\overrightarrow{T}(s)$ at each point. If the derivatives of $\left(\overrightarrow{N_1}(s), \overrightarrow{N_2}(s)\right)$ depend only on $\overrightarrow{T}(s)$ and not each other, we can make $\overrightarrow{N_1}(s)$ and $\overrightarrow{N_2}(s)$ vary smoothly throughout the path regardless of the curvature. We may therefore choose the alternative frame equations

$$\begin{bmatrix} \overrightarrow{T}' \\ \overrightarrow{N_1}' \\ \overrightarrow{N_2}' \end{bmatrix} = \begin{bmatrix} 0 & k_1 & k_2 \\ -k_1 & 0 & 0 \\ -k_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} \overrightarrow{T} \\ \overrightarrow{N_1} \\ \overrightarrow{N_2} \end{bmatrix} \tag{1}$$

where $\left\langle \overrightarrow{T}, \overrightarrow{T} \right\rangle = \left\langle \overrightarrow{N_1}, \overrightarrow{N_1} \right\rangle = \left\langle \overrightarrow{N_2}, \overrightarrow{N_2} \right\rangle = 1$ and $\left\langle \overrightarrow{T}, \overrightarrow{N_1} \right\rangle = \left\langle \overrightarrow{N_1}, \overrightarrow{N_2} \right\rangle = \left\langle \overrightarrow{T}, \overrightarrow{N_2} \right\rangle = 0$ [19, 20]. One can show that [19]

$$\kappa(s) = \sqrt{k_1^2 + k_2^2}, \quad \theta(s) = \arctan\left(\frac{k_2}{k_1}\right), \quad \tau(s) = \frac{d\theta(s)}{ds}$$

$$k_1 = \kappa\cos(\theta), \quad k_2 = \kappa\sin(\theta)$$

and

$$\overrightarrow{T} = \overrightarrow{T}, \ \overrightarrow{N_1} = \overrightarrow{N}\cos(\theta) - \overrightarrow{B}\sin(\theta), \ \overrightarrow{N_2} = \overrightarrow{N}\sin(\theta) + \overrightarrow{B}\cos(\theta)$$

so that k_1 and k_2 effectively correspond to a Cartesian coordinate system for the polar coordinates κ, θ with $\theta = \int \tau(s)ds$. A fundamental ambiguity in the parallel transport frame compared to the Frenet frame thus arise from the arbitrary choice of an integration constant for θ_0, which disappears τ from due to the differentiation [20].

It is well-known that the curvature $\kappa(s)$ of the curve (C) is defined by

$$\lim_{\Delta s \to 0} \frac{\Delta\varphi}{\Delta s} = \frac{d\varphi}{ds} = \kappa(s)$$

where φ is the angle between the tangent $\overrightarrow{T}$ of the curve α and a given fixed direction at the point $\alpha(s)$.

3 Taylor Collocation Method for System of Linear Differential Equations with Variable Coefficients in Normal Form

A Taylor collocation method was given to find the approximate solutions of high-order systems of linear differential equations with variable coefficients in [17]. In this section we develop the Taylor collocation method for the systems of three linear differential equations with variable coefficients in the normal form

$$L[y_i(x)] = y_i'(x) - \sum_{j=1}^{3} p_{i,j}(x) y_j(x) = g_i(x) \quad (i = 1,2,3) \quad (0 \le a \le x \le b) \tag{2}$$

under the initial conditions

$$y_i(a) = c_i \tag{3}$$

where $y_i(x)$ $(i = 1,2,3)$ are unknown functions, $p_{i,j}(x)$ and $g_i(x)$ are the known continuous functions defined on interval $[a,b]$, and c_i $(i = 1,2,3)$ are the real constants. In this study, by developing the Taylor collocation method with the help of the residual error function used in [21-24], we obtain the approximate solutions of the system (2) expressed in the truncated Taylor series

$$y_{i,N,M}(x) = y_{i,N}(x) + e_{i,N,M}(x) \quad (i = 1,2,3)$$

where

$$y_i(x) \approx y_{i,N}(x) = \sum_{n=0}^{N} a_{i,n} x^n \tag{4}$$

is the Taylor polynomial solution and

$$e_{i,N,M}(x) = \sum_{n=0}^{M} a_{i,n}^{*} x^n \quad (M > N)$$

is the Taylor polynomial solution of the error problem obtained with the help of the residual error function. Here $a_{i,n}$ and $a_{i,n}^{*}$, $(n = 1,2,...,N)$ are the unknown Taylor coefficients.

In order to find the solutions of the system (2) under the initial conditions (3), we can use the collocation points defined by

$$x_k = a + \frac{b-a}{N} k, \quad k = 1,2,...,N, \quad (0 \le a \le x \le b). \tag{5}$$

On the other hand, we can write the approximate solutions $y_{i,N}(x)$ given by Eq.(4) in the matrix form

$$y_{i,N}(x) = \mathbf{X}(x)\mathbf{A}_i, \quad (i = 1,2,3) \tag{6}$$

where

$$\mathbf{X}(x) = \begin{bmatrix} 1 \; x \; x^2 \; ... \; x^N \end{bmatrix}$$

and

$$\mathbf{A}_i = \begin{bmatrix} a_{i,0} \; a_{i,1} \; a_{i,2} \; ... \; a_{i,N} \end{bmatrix}^T.$$

From Eq.(6), the solutions $y_{i,N}(x)$ $(i = 1,2,3)$ can be expressed as

$$\mathbf{Y}(x) = \overline{\mathbf{X}}(x)\mathbf{A} \tag{7}$$

where

$$\mathbf{Y}(x)=\begin{bmatrix} y_{1,N}(x) \\ y_{2,N}(x) \\ y_{3,N}(x) \end{bmatrix},\quad \overline{\mathbf{X}}(x)=\begin{bmatrix} \mathbf{X}(x) & 0 & 0 \\ 0 & \mathbf{X}(x) & 0 \\ 0 & 0 & \mathbf{X}(x) \end{bmatrix},\quad \mathbf{A}=\begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \mathbf{A}_3 \end{bmatrix}.$$

Also, the relation between the matrix $\mathbf{X}(x)$ and its derivative $\mathbf{X}'(x)$ is

$$\mathbf{X}'(x)=\mathbf{X}(x)\mathbf{B} \tag{8}$$

where

$$\mathbf{B}=\begin{bmatrix} 0\,1\,0\,0 & \cdots & 0 \\ 0\,0\,2\,0 & \cdots & 0 \\ 0\,0\,0\,3 & \cdots & 0 \\ \vdots\,\vdots\,\vdots\,\vdots & \ddots & \vdots \\ 0\,0\,0\,0 & \cdots & N \\ 0\,0\,0\,0 & \cdots & 0 \end{bmatrix}.$$

By using the relations (6) and (8), we gain the following matrix relation

$$y'_{i,N}(x)=\mathbf{X}(x)\mathbf{B}\mathbf{A}_i \quad (i=1,2,3).$$

Hence, we can write the matrix relation as follows

$$\mathbf{Y}'(x)=\overline{\mathbf{X}}(x)\overline{\mathbf{B}}\mathbf{A} \tag{9}$$

where

$$\mathbf{Y}'(x)=\begin{bmatrix} y'_{1,N}(x) \\ y'_{2,N}(x) \\ y'_{3,N}(x) \end{bmatrix},\quad \overline{\mathbf{B}}(x)=\begin{bmatrix} \mathbf{B} & 0 & 0 \\ 0 & \mathbf{B} & 0 \\ 0 & 0 & \mathbf{B} \end{bmatrix}.$$

We can write the system (2) in the matrix form

$$\mathbf{Y}'(x)=\mathbf{P}(x)\mathbf{Y}(x)+\mathbf{G}(x) \tag{10}$$

where

$$\mathbf{P}(x)=\begin{bmatrix} p_{1,1}(x) & p_{1,2}(x) & p_{1,3}(x) \\ p_{2,1}(x) & p_{2,2}(x) & p_{2,3}(x) \\ p_{3,1}(x) & p_{3,2}(x) & p_{3,3}(x) \end{bmatrix},\quad \mathbf{G}(x)=\begin{bmatrix} g_1(x) \\ g_2(x) \\ g_3(x) \end{bmatrix}.$$

By using the collocation points given by (5) into Eq.(10), we obtain the system of matrix equations

$$\mathbf{Y}'(x_k)=\mathbf{P}(x_k)\mathbf{Y}(x_k)+\mathbf{G}(x_k) \quad (k=0,1,2,...,N).$$

Briefly, the fundamental matrix equation is

$$\mathbf{Y}'=\mathbf{P}\mathbf{Y}+\mathbf{G} \tag{11}$$

where

$$\mathbf{P}=\begin{bmatrix}\mathbf{P}(x_0) & 0 & \cdots & 0\\ 0 & \mathbf{P}(x_1) & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \mathbf{P}(x_N)\end{bmatrix},\quad \mathbf{Y}=\begin{bmatrix}\mathbf{Y}(x_0)\\ \mathbf{Y}(x_1)\\ \vdots\\ \mathbf{Y}(x_N)\end{bmatrix},$$

$$\mathbf{Y}'=\begin{bmatrix}\mathbf{Y}'(x_0)\\ \mathbf{Y}'(x_1)\\ \vdots\\ \mathbf{Y}'(x_N)\end{bmatrix},\ \mathbf{G}=\begin{bmatrix}\mathbf{G}(x_0)\\ \mathbf{G}(x_1)\\ \vdots\\ \mathbf{G}(x_N)\end{bmatrix}.$$

From the relations (7), (9) and the collocation points given by (5), we obtain

$$\mathbf{Y}(x_k)=\overline{\mathbf{X}}(x_k)\mathbf{A}\quad\text{and}\quad \mathbf{Y}'(x_k)=\overline{\mathbf{X}}(x_k)\overline{\mathbf{B}}\mathbf{A}\qquad (k=0,1,2,\ldots,N)$$

or briefly

$$\mathbf{Y}=\mathbf{X}\mathbf{A}\quad\text{and}\quad \mathbf{Y}'=\mathbf{X}\overline{\mathbf{B}}\mathbf{A} \tag{12}$$

where

$$\mathbf{X}=\begin{bmatrix}\overline{\mathbf{X}}(x_0)\\ \overline{\mathbf{X}}(x_1)\\ \vdots\\ \overline{\mathbf{X}}(x_N)\end{bmatrix},\quad \overline{\mathbf{X}}(x_k)=\begin{bmatrix}\mathbf{X}(x_k) & 0 & \cdots & 0\\ 0 & \mathbf{X}(x_k) & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \mathbf{X}(x_k)\end{bmatrix}.$$

By substituting the relations given by (12) into Eq.(11), we gain the fundamental matrix equation as

$$\left\{\mathbf{X}\overline{\mathbf{B}}-\mathbf{P}\mathbf{X}\right\}\mathbf{A}=\mathbf{G}. \tag{13}$$

In Eq.(13) the full dimensions of the matrices $\mathbf{P}$, $\mathbf{X}$, $\overline{\mathbf{B}}$, $\mathbf{A}$ and $\mathbf{G}$ are $3(N+1)\times 3(N+1)$, $3(N+1)\times 3(N+1)$, $3(N+1)\times 3(N+1)$, $3(N+1)\times 1$ and $3(N+1)\times 1$, respectively.

The fundamental matrix equation (13) corresponding to Eq.(2) can be written in the form

$$\mathbf{W}\mathbf{A}=\mathbf{G}\quad\text{or}\quad [\mathbf{W};\mathbf{G}]. \tag{14}$$

This is a linear system of $3(N+1)$ algebraic equations in $3(N+1)$ the unknown Taylor coefficients such that

$$\mathbf{W}=\mathbf{X}\overline{\mathbf{B}}-\mathbf{P}\mathbf{X}=[w_{p,q}]\qquad p,q=1,2,\ldots,3(N+1).$$

By using the conditions given by (5) and the relations (7), the matrix form for the conditions is obtained as

$$\overline{\mathbf{X}}(a)\mathbf{A}=\mathbf{C} \tag{15}$$

where

$$\mathbf{C}=\begin{bmatrix}c_1 & c_2 & c_3\end{bmatrix}^T.$$

Hence, the fundamental matrix form for conditions is

$$\mathbf{U}\mathbf{A}=\mathbf{C}\quad\text{or}\quad [\mathbf{U};\mathbf{C}] \tag{16}$$

such that

$$\mathbf{U} = \overline{\mathbf{X}}(a).$$

Consequently, we obtain the Taylor polynomial solution of the system (2) under the initial conditions (3) by replacing the row matrices (16) by last rows of the matrix (14). Then, we obtain the new augmented matrix

$$\widetilde{\mathbf{W}}\mathbf{A} = \widetilde{\mathbf{G}} \quad \text{or} \quad \left[\widetilde{\mathbf{W}};\widetilde{\mathbf{G}}\right]. \tag{17}$$

If $rank\,\widetilde{\mathbf{W}} = rank\left[\widetilde{\mathbf{W}};\widetilde{\mathbf{G}}\right] = 3(N+1)$, then we can write

$$\mathbf{A} = \left(\widetilde{\mathbf{W}}\right)^{-1}\widetilde{\mathbf{G}}. \tag{18}$$

By solving this linear system, the unknown Taylor coefficients matrix $\mathbf{A}$ is determined and $a_{i,0}, a_{i,1}, ..., a_{i,N}\ \ (i = 1,2,3)$ are substituted in Eq.(4). Thus, we find the Taylor polynomial solutions

$$y_{i,N}(x) = \sum_{n=0}^{N} a_{i,n}x^n \quad (i = 1,2,3).$$

4 Residual Correction and Error Estimation

In this section, we give an error estimation for the Taylor polynomial solutions (4) with the residual error function [21-24]. Also, we develop the Taylor polynomial solutions (4) via the residual error function. Firstly, we can define the residual function of the Taylor collocation method as

$$R_{i,N}(x) = L\left[y_{i,N}(x)\right] - g_i(x) \quad (i = 1,2,3). \tag{19}$$

Here $y_{i,N}(x)$, represent the Taylor polynomial solutions given by (4) of the problem (2)-(3), and satisfies the problem

$$\begin{cases} y'_{i,N}(x) - \sum\limits_{j=1}^{3} p_{i,j}(x)y_{j,N}(x) = g_i(x) + R_{i,N}(x), & (i = 1,2,3) \\ y_{i,N}(a) = c_i, \quad (i = 1,2,3). \end{cases}$$

Also, the error function $e_{i,N}(x)$ can be defined as

$$e_{i,N}(x) = y_i(x) - y_{i,N}(x) \tag{20}$$

where $y_i(x)$ are the exact solutions of the problem (2) and (3). From Eqs.(2), (3), (19) and (20), we obtain the system of error differential equations

$$L\left[e_{i,N}(x)\right] = L\left[y_i(x)\right] - L\left[y_{i,N}(x)\right] = -R_{i,N}(x)$$

with the homogeneous initial conditions

$$e_{i,N}(a) = 0 \qquad (i = 1,2,3)$$

or clearly, the error problem can be expressed as

$$\begin{cases} e'_{i,N}(x) - \sum\limits_{j=1}^{3} p_{i,j}(x)e_{j,N}(x) = -R_{i,N}(x), & (i = 1,2,3) \\ e_{i,N}(a) = 0, \quad (i = 1,2,3). \end{cases} \tag{21}$$

Here, the nonhomogeneous initial conditions

$$y_i(a) = c_i \quad \text{and} \quad y_{i,N}(a) = c_i$$

are reduced to homogeneous initial conditions

$$e_{i,N}(a) = 0.$$

The error problem (21) can be solved by using the procedure given in Section 3. Thus, we obtain the approximation

$$e_{i,N,M}(x) = \sum_{n=0}^{M} a_{i,n}^{*} x^n \qquad (M > N, \quad i = 1,2,3)$$

to $e_{i,N}(x)$. Consequently, the corrected Taylor polynomial solution $y_{i,N,M}(x) = y_{i,N}(x) + e_{i,N,M}(x)$ is obtained by means of the polynomials $y_{i,N}(x)$ and $e_{i,N,M}(x)$. Also, we construct the error function $e_{i,N}(x) = y_i(x) - y_{i,N}(x)$, the estimated error function $e_{i,N,M}(x)$ and the corrected error function $E_{i,N,M}(x) = e_{i,N}(x) - e_{i,N,M}(x) = y_i(x) - y_{i,N,M}(x)$.

5 Illustration

In this section, we give an example which is related to space curve pair of constant breadth according to Bishop frame in Euclidean 3-space. The computations connected with the example are calculated by using a computer programme which is called Maple and the figures are drawn in Matlab. In tables and figures, we calculate the values of the Taylor polynomial solution $y_{i,N}(x)$, the corrected Taylor polynomial solution $y_{i,N,M}(x) = y_{i,N}(x) + e_{i,N,M}(x)$, the actual absolute error function $|e_{i,N}(x)| = |y_i(x) - y_{i,N}(x)|$,the estimated absolute error function $|e_{i,N,M}(x)|$.

Definition 1. A pair of space curves (C) and (C^*) in E^3 for which the tangents at the corresponding points $\alpha(s)$ and $\alpha^*(s^*)$ are parallel and in opposite directions, and the distance between these points is always constant are called space curve pair of constant breadth [11].

Let (C) and (C^*) be a pair of unit-speed curves in Euclidean 3-space with non-zero Bishop curvatures and let those curves have parallel tangents in opposite directions at the corresponding points $\alpha(s)$ and $\alpha^*(s^*)$, respectively. Hence, the position vector of the curve (C^*) at the point $\alpha^*(s^*)$ can be written as

$$\overrightarrow{\alpha^*}(s^*) = \overrightarrow{\alpha}(s) + \lambda_1(s)\overrightarrow{T}(s) + \lambda_2(s)\overrightarrow{N_1}(s) + \lambda_3(s)\overrightarrow{N_2}(s) \tag{22}$$

where $\lambda_i(s)$ $(i = 1,2,3)$ are differentiable functions of s which is arc length of (C). Denote by $\left\{\overrightarrow{T}, \overrightarrow{N_1}, \overrightarrow{N_2}\right\}$, k_1 and k_2 the moving Bishop frame, Bishop curvatures along the curve (C), respectively. And denote by $\left\{\overrightarrow{T^*}, \overrightarrow{N_1^*}, \overrightarrow{N_2^*}\right\}$, k_1^* and k_2^* the moving Bishop frame, Bishop curvatures along the curve (C^*), respectively.

Theorem 1. The general differential equation systems characterizing space curve pair of constant breadth according to Bishop frame in E^3 are as follows [25].

$$\begin{cases} \frac{d\lambda_1}{ds} = k_1\lambda_2 + k_2\lambda_3 \\ \frac{d\lambda_2}{ds} = -k_1\lambda_1 \\ \frac{d\lambda_3}{ds} = -k_2\lambda_1 \end{cases} \tag{23}$$

where k_1 and k_2 are Bishop curvatures and

$$\begin{cases} \frac{d\lambda_1}{d\varphi} = \mu_1\lambda_2 + \mu_2\lambda_3 \\ \frac{d\lambda_2}{d\varphi} = -\mu_1\lambda_1 \\ \frac{d\lambda_3}{d\varphi} = -\mu_2\lambda_1 \end{cases} \tag{24}$$

where $\mu_1 = \rho k_1 = \frac{k_1}{\kappa} = \cos(\theta)$, $\mu_2 = \rho k_2 = \frac{k_2}{\kappa} = \sin(\theta)$, $\left(\theta = \int \tau ds\right)$.

We can express $\overrightarrow{T}, \overrightarrow{N_1}$ and $\overrightarrow{N_2}$ in terms of $\alpha', \alpha'', \alpha'''$ and k_1, k_2 via the Bishop formulae as follows.

$$\overrightarrow{T} = \alpha' \tag{25}$$

$$N_1 = \frac{1}{\mu}\left(-k_2\alpha''' + k_2'\alpha'' - k_2(k_1^2 + k_2^2)\alpha'\right) \tag{26}$$

$$N_2 = \frac{1}{\mu}\left(k_1\alpha''' - k_1'\alpha'' + k_1(k_1^2 + k_2^2)\alpha'\right) \tag{27}$$

where

$$\mu = k_1^2\left(\frac{k_2}{k_1}\right)'.$$

Substituting (25), (26) and (27) in (22), we obtain

$$\begin{aligned} \overrightarrow{\alpha^*}(s^*) = {} & \left(\frac{k_1\lambda_3 - k_2\lambda_2}{\mu}\right)\overrightarrow{\alpha}''' + \left(\frac{k_2'\lambda_2 - k_1'\lambda_3}{\mu}\right)\overrightarrow{\alpha}'' \\ & + \left[\frac{(k_1^2 + k_2^2)(k_1\lambda_3 - k_2\lambda_2)}{\mu} + \lambda_1\right]\overrightarrow{\alpha}' + \overrightarrow{\alpha}. \end{aligned} \tag{28}$$

Example 1. We consider the curve $\alpha : [0, 2\pi] \to E^3$ given by

$$\alpha(s) = \left(3\cos\left(\frac{s}{5}\right), 3\sin\left(\frac{s}{5}\right), \frac{4s}{5}\right).$$

For the curve α, the Frenet vectors, curvature and torsion are obtained as follows

$$\overrightarrow{T}(s) = \left(-\frac{3}{5}\sin\left(\frac{s}{5}\right), \frac{3}{5}\cos\left(\frac{s}{5}\right), \frac{4}{5}\right), \quad \overrightarrow{N}(s) = \left(-\cos\left(\frac{s}{5}\right), -\sin\left(\frac{s}{5}\right), 0\right)$$

$$\overrightarrow{B}(s) = \left(\frac{4}{5}\sin\left(\frac{s}{5}\right), -\frac{4}{5}\cos\left(\frac{s}{5}\right), \frac{3}{5}\right), \quad \kappa = \frac{3}{25} \quad \text{and} \quad \tau = \frac{4}{25}$$

respectively. Also, we can calculate the Bishop elements of the curve α as follows

$$\theta = \frac{4s}{25}$$

$$k_1 = \frac{3}{25}\cos\left(\frac{4s}{25}\right), \quad k_2 = \frac{3}{25}\sin\left(\frac{4s}{25}\right) \tag{29}$$

and

$$N_1=\left(\frac{-4}{5}\sin\left(\frac{s}{5}\right)\sin\left(\frac{4s}{25}\right)-\cos\left(\frac{s}{5}\right)\cos\left(\frac{4s}{25}\right),\frac{4}{5}\cos\left(\frac{s}{5}\right)\sin\left(\frac{4s}{25}\right)-\sin\left(\frac{s}{5}\right)\cos\left(\frac{4s}{25}\right),\frac{-3}{5}\sin\left(\frac{4s}{25}\right)\right)$$

$$N_2=\left(\frac{4}{5}\sin\left(\frac{s}{5}\right)\cos\left(\frac{4s}{25}\right)-\cos\left(\frac{s}{5}\right)\sin\left(\frac{4s}{25}\right),\frac{-4}{5}\cos\left(\frac{s}{5}\right)\cos\left(\frac{4s}{25}\right)-\sin\left(\frac{s}{5}\right)\sin\left(\frac{4s}{25}\right),\frac{3}{5}\cos\left(\frac{4s}{25}\right)\right)$$

Substituting (29) in (23), we obtain

$$\begin{cases}\lambda_1'(s)=\frac{3}{25}\cos\left(\frac{4s}{25}\right)\lambda_2(s)+\frac{3}{25}\sin\left(\frac{4s}{25}\right)\lambda_3(s).\\ \lambda_2'(s)=\frac{-3}{25}\cos\left(\frac{4s}{25}\right)\lambda_1(s).\\ \lambda_3'(s)=\frac{-3}{25}\sin\left(\frac{4s}{25}\right)\lambda_1(s).\end{cases}\tag{30}$$

We can find approximate solutions of the problem (30) by using Taylor collocation method above mentioned. We suppose that the initial conditions for $\lambda_1(s)$, $\lambda_2(s)$ and $\lambda_3(s)$ as follows

$$\lambda_1(0)=2,\ \ \lambda_2(0)=1,\ \ \ \lambda_3(0)=3.$$

The approximate solutions $\lambda_{1,3}(s)$, $\lambda_{2,3}(s)$ and $\lambda_{3,3}(s)$ by the truncated Taylor series for $N=3$ are given by

$$\lambda_{i,3}(s)=\sum_{n=0}^{3}a_{i,n}s^n,\quad (i=1,2,3).$$

The set of the collocation points for $a=0$, $b=2\pi$ and $N=3$ is calculated as

$$\left\{s_0=0,\ \ s_1=\frac{2\pi}{3},\ \ s_2=\frac{4\pi}{3},\ \ s_3=2\pi\right\}.$$

We can write the fundamental matrix equation of the problem (26) from Eq.(13) as

$$\left\{\mathbf{S}\overline{\mathbf{B}}-\mathbf{PS}\right\}\mathbf{A}=\mathbf{G}.$$

By using the technique in Section 3, the approximate solutions of the problem (30) for $N=3$ are obtained as

$$\lambda_{1,3}(s)=2+0.1199999999999999994s+(0.148701113607149546e-1)s^2-(0.108758588173734316e-2)s^3.$$
$$\lambda_{2,3}(s)=1-0.2399999999999999990s-(0.873633174412091763e-2)s^2+(0.115528896583677382e-2)s^3.$$
$$\lambda_{3,3}(s)=3-(0.195847736542368278e-1)s^2-(0.683135379920168538e-3)s^3.$$

In order to calculate the corrected Taylor polynomial solutions, let us consider the error problem

$$\begin{cases}e_{1,3}'(s)-\frac{3}{25}\cos\left(\frac{4s}{25}\right)e_{2,3}(s)-\frac{3}{25}\sin\left(\frac{4s}{25}\right)e_{3,3}(s)=-R_{1,3}(s).\\ e_{2,3}'(s)+\frac{3}{25}\cos\left(\frac{4s}{25}\right)e_{1,3}(s)=-R_{2,3}(s).\\ e_{3,3}'(s)+\frac{3}{25}\sin\left(\frac{4s}{25}\right)e_{1,3}(s)=-R_{3,3}(s).\end{cases}\tag{31}$$

such that $e_{1,3}(0)=0$, $e_{2,3}(0)=0$, $e_{3,3}(0)=0$ and the residual functions are

$$\begin{cases}R_{1,3}(s)=\lambda_{1,3}'(s)-\frac{3}{25}\cos\left(\frac{4s}{25}\right)\lambda_{2,3}(s)-\frac{3}{25}\sin\left(\frac{4s}{25}\right)\lambda_{3,3}(s).\\ R_{2,3}(s)=\lambda_{2,3}'(s)+\frac{3}{25}\cos\left(\frac{4s}{25}\right)\lambda_{1,3}(s).\\ R_{3,3}(s)=\lambda_{3,3}'(s)+\frac{3}{25}\sin\left(\frac{4s}{25}\right)\lambda_{1,3}(s).\end{cases}$$

By solving the error problem (31) for $M=4$, the estimated Taylor error function approximations $e_{1,3,4}(s)$, $e_{2,3,4}(s)$ and $e_{3,3,4}(s)$ to $e_{1,3}(s)$, $e_{2,3}(s)$ and $e_{3,3}(s)$ are obtained as

$$e_{1,3,4}(s) = -(0.341067270660595030e-3)s^2 + (0.190358881883564312e-3)s^3 - (0.241570139950251427e-4)s^4.$$
$$e_{2,3,4}(s) = (0.151791194512943678e-2)s^2 - (0.715789891362811458e-3)s^3 + (0.844881912401404312e-4)s^4.$$
$$e_{3,3,4}(s) = (0.784722476727751960e-3)s^2 - (0.392449258278592588e-3)s^3 + (0.484628169544237858e-4)s^4.$$

Hence, we can calculate the corrected Taylor polynomial solutions for $M=4$ as

$$\lambda_{1,3,4}(s) = 2 + 0.119999999999999994s + (0.1452904409e-1)s^2 - (0.8972270001e-3)s^3 - (0.241570139950251427e-4)s^4.$$
$$\lambda_{2,3,4}(s) = 1 - 0.239999999999999990s - (0.7218419799e-2)s^2 + (0.4394990746e-3)s^3 + (0.844881912401404312e-4)s^4.$$
$$\lambda_{3,3,4}(s) = 3 - (0.1880005117e-1)s^2 - (0.1075584638e-2)s^3 + (0.484628169544237858e-4)s^4.$$

Similarly, we can calculate corrected Taylor polynomial solutions of the problem for different values of M as follows. For $M=5$, the approximate solutions are

$$\lambda_{1,3,5}(s) = 2 + 0.119999999999999994s + (0.1439737414e-1)s^2 - (0.7947228696e-3)s^3 - (0.508657095719278348e-4)s^4 + (0.226278147175747614e-5)s^5.$$
$$\lambda_{2,3,5}(s) = 1 - 0.239999999999999990s - (0.7121186842e-2)s^2 + (0.3615042855e-3)s^3 + (0.105565171620638706e-3)s^4 - (0.184964326733999626e-5)s^5.$$
$$\lambda_{3,3,5}(s) = 3 - (0.1920195851e-1)s^2 - (0.7626538882e-3)s^3 - (0.329828771379642208e-4)s^4 + (0.690853227461905316e-5)s^5.$$

For $M=6$, the approximate solutions are

$$\lambda_{1,3,6}(s) = 2 + 0.119999999999999994s - (0.489933391146528524e-4)s^4 + (0.279612930756887982e-7)s^6 + (0.1439876390e-1)s^2 + (0.187592327211527806e-5)s^5 - (0.7982653977e-3)s^3.$$
$$\lambda_{2,3,6}(s) = 1 - 0.239999999999999990s - (0.438073448317362830e-6)s^6 + (0.4488424909e-3)s^3 + (0.691386308539103172e-4)s^4 + (0.476573407873323844e-5)s^5 - (0.7200265850e-2)s^2.$$
$$\lambda_{3,3,6}(s) = 3 - (0.1921223552e-1)s^2 + (0.796884680628572582e-5)s^5 - (0.385065566486739328e-4)s^4 - (0.7502724205e-3)s^3 - (0.735345232520348584e-7)s^6.$$

For $M=7$, the approximate solutions are

$$\lambda_{1,3,7}(s) = 2 + 0.119999999999999994s + (0.680723473252911355e-7)s^6 - (0.7999978004e-3)s^3 - (0.479825459755605068e-4)s^4 + (0.158713217431771746e-5)s^5 + (0.1439997195e-1)s^2 - (0.216393272754547384e-8)s^7.$$
$$\lambda_{2,3,7}(s) = 1 - 0.239999999999999990s - (0.494476969184058441e-6)s^6 + (0.4508911392e-3)s^3 + (0.678646405673082616e-4)s^4 + (0.515131126235351736e-5)s^5 - (0.7201590929e-2)s^2 + (0.318876134473695714e-8)s^7.$$
$$\lambda_{3,3,7}(s) = 3 - (0.1919998595e-1)s^2 + (0.495050218888017330e-5)s^5 - (0.280426817302835939e-4)s^4 - (0.7680733225e-3)s^3 + (0.350111688119908828e-6)s^6 - (0.231051655239871117e-7)s^7.$$

For $M = 8$, the approximate solutions are

$$\lambda_{1,3,8}(s) = 2 + 0.119999999999999994s + (0.659440054032486062e-7)s^6 - (0.8000151877e-3)s^3$$
$$-(0.479862996416925864e-4)s^4 + (0.159324423040038568e-5)s^5 + (0.1440000746e-1)s^2$$
$$-(0.184727975432647118e-8)s^7 - (0.175647261556320578e-10)s^8.$$
$$\lambda_{2,3,8}(s) = 1 - 0.239999999999999990s - (0.296452061743570304e-6)s^6 + (0.4479985321e-3)s^3$$
$$+(0.700883710680355620e-4)s^4 + (0.425489541190857338e-5)s^5 - (0.7200001520e-2)s^2$$
$$-(0.194890934014232106e-7)s^7 + (0.105207625088382470e-8)s^8.$$
$$\lambda_{3,3,8}(s) = 3 - (0.1919982286e-1)s^2 + (0.484261911331022518e-5)s^5 - (0.277865487879685406e-4)s^4$$
$$-(0.7683896860e-3)s^3 + (0.374860308800117502e-6)s^6 - (0.260342571361117485e-7)s^7$$
$$+(0.139893638198862298e-9)s^8.$$

Table 1. Comparison of the approximate solutions, $\lambda_{1,N,M}(s)$ for $N = 3$ and $M = 4,5,6,7,8$.

s_i	$\lambda_{1,3,4}(s_i)$	$\lambda_{1,3,5}(s_i)$	$\lambda_{1,3,6}(s_i)$	$\lambda_{1,3,7}(s_i)$	$\lambda_{1,3,8}(s_i)$
0	2	2	2	2	2
$\frac{\pi}{4}$	2.10276615	2.10272510	2.10272485	2.10272506	2.10272508
$\frac{2\pi}{4}$	2.22072001	2.22065145	2.22064927	2.22064948	2.22064951
$\frac{3\pi}{4}$	2.35092258	2.35087357	2.35086934	2.35086956	2.35086960
$\frac{4\pi}{4}$	2.49021425	2.49018378	2.49017854	2.49017886	2.49017892
$\frac{5\pi}{4}$	2.63521483	2.63515334	2.63514678	2.63514712	2.63514719
$\frac{6\pi}{4}$	2.78232348	2.78221358	2.78220426	2.78220473	2.78220482
$\frac{7\pi}{4}$	2.92771879	2.92773705	2.92773000	2.92773067	2.92773076
$\frac{8\pi}{4}$	3.06735875	3.06811867	3.06814505	3.06814178	3.06814167

Table 2. Comparison of the approximate solutions, $\lambda_{2,N,M}(s)$ for $N = 3$ and $M = 4,5,6,7,8$.

s_i	$\lambda_{2,3,4}(s_i)$	$\lambda_{2,3,5}(s_i)$	$\lambda_{2,3,6}(s_i)$	$\lambda_{2,3,7}(s_i)$	$\lambda_{2,3,8}(s_i)$
0	1	1	1	1	1
$\frac{\pi}{4}$	0.80729683	0.80732649	0.80730804	0.80730784	0.80730803
$\frac{2\pi}{4}$	0.60741592	0.60746417	0.60744247	0.60744230	0.60744245
$\frac{3\pi}{4}$	0.40279215	0.40282701	0.40281320	0.40281301	0.40281317
$\frac{4\pi}{4}$	0.19663196	0.19666035	0.19664290	0.19664265	0.19664280
$\frac{5\pi}{4}$	−0.00708665	−0.00702544	−0.00704710	−0.00704729	−0.00704714
$\frac{6\pi}{4}$	−0.20361413	−0.20352127	−0.20352510	−0.20352548	−0.20352531
$\frac{7\pi}{4}$	−0.38742938	−0,38748569	−0.38751117	−0.38751146	−0.38751147
$\frac{8\pi}{4}$	−0.55223974	−0,55301120	−0.55341351	−0.55340509	−0.55339973

Table 3. Comparison of the approximate solutions, $\lambda_{3,N,M}(s)$ for $N = 3$ and $M = 4,5,6,7,8$.

s_i	$\lambda_{3,3,4}(s_i)$	$\lambda_{3,3,5}(s_i)$	$\lambda_{3,3,6}(s_i)$	$\lambda_{3,3,7}(s_i)$	$\lambda_{3,3,8}(s_i)$
0	3	3	3	3	3
$\frac{\pi}{4}$	2.98790053	2.98777530	2.98777315	2.98777526	2.98777528
$\frac{2\pi}{4}$	2.94973904	2.94953045	2.94952849	2.94953038	2.94953039
$\frac{3\pi}{4}$	2.88305284	2.88290643	2.88290551	2.88290716	2.88290717
$\frac{4\pi}{4}$	2.78582178	2.78573852	2.78573672	2.78573879	2.78573880
$\frac{5\pi}{4}$	2.65646830	2.65630404	2.65630230	2.65630376	2.65630377
$\frac{6\pi}{4}$	2.49385741	2.49357012	2.49357240	2.49357451	2.49357452
$\frac{7\pi}{4}$	2.29729668	2.29744144	2.29743700	2.29743938	2.29743936
$\frac{8\pi}{4}$	2.06653623	2.06900799	2.06892336	2.06887642	2.06887730

Table 1-3 display that the approximate solutions are almost identical. We can write the distance function $d_{N,M}$ from (22) as

$$d_{N,M} = \sqrt{\lambda_{1,N,M}^2 + \lambda_{2,N,M}^2 + \lambda_{3,N,M}^2} = k, \quad k \in \mathbb{R}.$$

Now, let us calculate the values of $d_{N,M}$ for $N = 3$ and $M = 4,5,6,7,8$. Hence,

$$\begin{aligned} d_{3,4} = {} & (0.2e-10)[(0.138367955738275e19)s^4 + (0.719757435500000e19)s^2 - (0.552901398520000e19)s^3 \\ & -(0.958165040494845e17)s^5 - (0.397211986269680e16)s^6 + 33404728162078s^7 \\ & +25176151028361s^8 + (0.350000000000000e23)]^{\frac{1}{2}}. \end{aligned}$$

$$\begin{aligned} d_{3,5} = {} & (0.4e-12)[(0.846073850000000e21)s^2 - (0.989167242750000e21)s^3 + (0.451409944795980e21)s^4 \\ & -(0.926227946415095e209s^5 + (0.670821758671850e19)s^6 + (0.210416759380431e18)s^7 \\ & -(0.407698907964350e16)s^8 - (0.672773956672850e16)s^9 + 351682364967490s^{10} \\ & +(0.875000000000000e26)]^{\frac{1}{2}}. \end{aligned}$$

$$\begin{aligned} d_{3,6} = {} & (0.2e-13)[-(0.197223050000000e24)s^2 + (0.287049530000000e24)s^3 - (0.168713254720665e24)s^4 \\ & +(0.482011956982625e23)s^5 - (0.642042431060345e22)s^6 + (0.802038008421965e17)s^{10} \\ & -(0.131063683600192e17)s^{11} + 495243765352585s^{12} + (0.235612475686618e21)s^7 \\ & +(0.198200774605652e20)s^8 - (0.116519926251896e19)s^9 + (0.350000000000000e29)]^{\frac{1}{2}}. \end{aligned}$$

$$\begin{aligned} d_{3,7} = {} & (0.1e-14)[-(0.320975800000000e25)s^2 + (0.610805572000000e25)s^3 - (0.502264751168900e25)s^4 \\ & +(0.222316100225410e25)s^5 - (0.556993377301340e24)s^6 + (0.752406710477370e23)s^7 \\ & -(0.421612125203490e22)s^8 - (0.444368881479050e20)s^9 + (0.210952808421099e19)s^{10} \\ & +(0.524459315793662e18)s^{11} + (0.168938876626434e18)s^{12} - (0.196269230626158e17)s^{13} \\ & +548699477653873s^{14} + (0.140000000000000e32)]^{\frac{1}{2}}. \end{aligned}$$

$$d_{3,8} = (0.1e-15)[(0.108964000000000e27)s^2 - (0.239928260000000e27)s^3 + (0.230274497692750e27)s^4$$
$$-(0.120638545022320e27)s^5 + (0.366926990526920e26)s^6 - (0.635088985330760e25)s^7$$
$$+(0.536652347489610e24)s^8 + (0.140000000000000e34)$$
$$-(0.548645006355880e22)s^9 - (0.201483174705506e22)s^{10}$$
$$+(0.965946771587340e20)s^{11} - (0.497417074083528e19)s^{12}$$
$$+(0.204499269053910e18)s^{13} + (0.539803969393183e17)s^{14}$$
$$-(0.482271846089033e16)s^{15} + 112674318728718s^{16}]^{\frac{1}{2}}.$$

Table 4. Numerical results of distance functions, $d_{N,M}$ for $N = 3$ and $M = 4,5,6,7,8$.

s_i	$d_{3,4}(s_i)$	$d_{3,5}(s_i)$	$d_{3,6}(s_i)$	$d_{3,7}(s_i)$	$d_{3,8}(s_i)$
0	3.741657387	3.741657387	3.741657387	3.741657387	3.741657386
$\frac{\pi}{4}$	3.741778086	3.741661414	3.741655578	3.741657335	3.741657400
$\frac{2\pi}{4}$	3.741859416	3.741662122	3.741655757	3.741657345	3.741657397
$\frac{3\pi}{4}$	3.741800656	3.741660812	3.741655960	3.741657342	3.741657399
$\frac{4\pi}{4}$	3.741742125	3.741661353	3.741655605	3.741657343	3.741657398
$\frac{5\pi}{4}$	3.741821912	3.741661886	3.741656066	3.741657343	3.741657399
$\frac{6\pi}{4}$	3.741938992	3.741660763	3.741655561	3.741657341	3.741657396
$\frac{7\pi}{4}$	3.741551934	3.741660935	3.741655332	3.741657346	3.741657407
$\frac{8\pi}{4}$	3.739549499	3.741653059	3.741687379	3.741657497	3.741657103

Now, let us draw the graphics of the distance functions $d_{N,M}$ for $N = 3$ and $M = 4,5,6,7,8$.

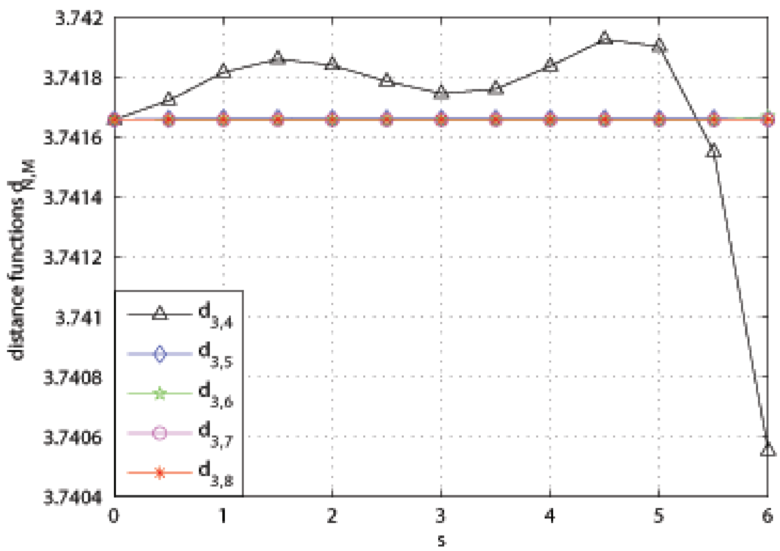


Fig. 1. Comparison of distance functions

It is seen from Table 4 and Figure 1 that accuracy of the solution of system (30) increase when the value of M is increased. In addition, $d_{N,M}$ is closing a constant value as the value of M is selected big. This value is breadth of the curve pair of constant breadth. Hence, we can say that the present method is very effective.

Furthermore, let us calculate and compare the estimated absolute error functions $|e_{i,N,M}(s)|$ for and $N=3$ and $M=4,5,6,7,8$ $(i=1,2,3)$.

$$|e_{1,3,4}(s)| = |(0.34106727e-3)s^2 - (0.1903588819e-3)s^3 + (0.241570139950251427e-4)s^4|.$$

$$\begin{aligned}|e_{1,3,5}(s)| = &|-(0.47273722e-3)s^2 + (0.2928630124e-3)s^3 - (0.508657095719278348e-4)s^4\\ &+(0.226278147175747614e-5)s^5|.\end{aligned}$$

$$\begin{aligned}|e_{1,3,6}(s)| = &|-(0.47134746e-3)s^2 + (0.2893204843e-3)s^3 - (0.489933391146528524e-4)s^4\\ &+(0.187592327211527806e-5)s^5 + (0.279612930756887982e-7)s^6|.\end{aligned}$$

$$\begin{aligned}|e_{1,3,7}(s)| = &|(0.47013941e-3)s^2 - (0.2875880816e-3)s^3 + (0.479825459755605068e-4)s^4\\ &-(0.158713217431771746e-5)s^5 - (0.680723473252911355e-7)s^6 + (0.216393272754547384e-8)s^7|.\end{aligned}$$

$$\begin{aligned}|e_{1,3,8}(s)| = &|(0.47010390e-3)s^2 - (0.2875706943e-3)s^3 + (0.479862996416925864e-4)s^4\\ &-(0.159324423040038568e-5)s^5 - (0.659440054032486062e-7)s^6 + (0.184727975432647118e-8)s^7\\ &+(0.175647261556320578e-10)s^8|.\end{aligned}$$

Table 5. Comparison of the estimated absolute error functions, $|e_{1,N,M}(s)|$ for $N=3$ and $M=4,5,6,7,8$.

s_i	$\lvert e_{1,3,4}(s_i)\rvert$	$\lvert e_{1,3,5}(s_i)\rvert$	$\lvert e_{1,3,6}(s_i)\rvert$	$\lvert e_{1,3,7}(s_i)\rvert$	$\lvert e_{1,3,8}(s_i)\rvert$
0	0	0	0	0	0
$\frac{\pi}{4}$	$1.2736e-4$	$1.6840e-4$	$1.6866e-4$	$1.6844e-4$	$1.6843e-4$
$\frac{2\pi}{4}$	$2.5083e-4$	$3.1939e-4$	$3.2157e-4$	$3.2136e-4$	$3.2133e-4$
$\frac{3\pi}{4}$	$1.4796e-4$	$1.9700e-4$	$2.0122e-4$	$2.0101e-4$	$2.0096e-4$
$\frac{4\pi}{4}$	$1.8301e-4$	$1.5254e-4$	$1.4730e-4$	$1.4761e-4$	$1.4767e-4$
$\frac{5\pi}{4}$	$5.2338e-4$	$4.6190e-4$	$4.5534e-4$	$4.5567e-4$	$4.5574e-4$
$\frac{6\pi}{4}$	$4.3375e-4$	$3.2385e-4$	$3.1454e-4$	$3.1501e-4$	$3.1509e-4$
$\frac{7\pi}{4}$	$7.4586e-4$	$7.2761e-4$	$7.3466e-4$	$7.3399e-4$	$7.3390e-4$
$\frac{8\pi}{4}$	$3.8960e-3$	$3.1361e-3$	$3.1097e-3$	$3.0031e-3$	$3.1131e-3$

$$|e_{2,3,4}(s)| = |(0.1517911945e-2)s^2 - (0.7157898914e-3)s^3 + (0.844881912401404312e-4)s^4|.$$

$$\begin{aligned}|e_{2,3,5}(s)| = &|-(0.1615144902e-2)s^2 + (0.7937846805e-3)s^3 - (0.105565171620638706e-3)s^4\\ &+(0.184964326733999626e-5)s^5|.\end{aligned}$$

$$\begin{aligned}|e_{2,3,6}(s)| = &|-(0.1536065894e-2)s^2 + (0.7064464751e-3)s^3 - (0.691386308539103172e-4)s^4\\ &-(0.476573407873323844e-5)s^5 + (0.438073448317362830e-6)s^6|\end{aligned}$$

$$|e_{2,3,7}(s)| = |(0.1534740815e-2)s^2 - (0.7043978268e-3)s^3 + (0.678646405673082616e-4)s^4 + (0.515131126235351736e-5)s^5 - (0.494476969184058441e-6)s^6 + (0.318876134473695714e-8)s^7|.$$

$$|e_{2,3,8}(s)| = |(0.1536330224e-2)s^2 - (0.7072904339e-3)s^3 + (0.700883710680355620e-4)s^4 + (0.425489541190857338e-5)s^5 - (0.296452061743570304e-6)s^6 - (0.194890934014232106e-7)s^7 + (0.105207625088382470e-8)s^8.$$

Table 6. Comparison of the estimated absolute error functions, $|e_{2,N,M}(s)|$ for $N = 3$ and $M = 4,5,6,7,8$.

s_i	$\lvert e_{2,3,4}(s_i)\rvert$	$\lvert e_{2,3,5}(s_i)\rvert$	$\lvert e_{2,3,6}(s_i)\rvert$	$\lvert e_{2,3,7}(s_i)\rvert$	$\lvert e_{2,3,8}(s_i)\rvert$
0	0	0	0	0	0
$\frac{\pi}{4}$	$6.2169e-4$	$6.5135e-4$	$6.3290e-4$	$6.3269e-4$	$6.3289e-4$
$\frac{2\pi}{4}$	$1.4854e-3$	$1.5337e-3$	$1.5120e-3$	$1.5118e-3$	$1.5120e-3$
$\frac{3\pi}{4}$	$1.6678e-3$	$1.7027e-3$	$1.6889e-3$	$1.6887e-3$	$1.6889e-3$
$\frac{4\pi}{4}$	$1.0171e-3$	$1.0455e-3$	$1.0281e-3$	$1.0278e-3$	$1.0280e-3$
$\frac{5\pi}{4}$	$1.5307e-4$	$2.1427e-4$	$1.9261e-4$	$1.9242e-4$	$1.9257e-4$
$\frac{6\pi}{4}$	$4.6696e-4$	$5.5981e-4$	$5.5599e-4$	$5.5561e-4$	$5.5577e-4$
$\frac{7\pi}{4}$	$4.1217e-3$	$4.0653e-3$	$4.0399e-3$	$4.0396e-3$	$4.0396e-3$
$\frac{8\pi}{4}$	$1.4052e-2$	$1.3280e-2$	$1.2878e-2$	$1.2886e-2$	$1.2892e-2$

$$|e_{3,3,4}(s)| = |(0.78472248e-3)s^2 - (0.3924492581e-3)s^3 + (0.484628169544237858e-4)s^4|.$$

$$\left|e_{3,3,5}(s)\right| = |(0.38281514e-3)s^2 - (0.795185083e-4)s^3 - (0.329828771379642208e-4)s^4 + (0.690853227461905316e-5)s^5|.$$

$$\left|e_{3,3,6}(s)\right| = |-(0.37253813e-3)s^2 + (0.671370406e-4)s^3 + (0.385065566486739328e-4)s^4 - (0.796884680628572582e-5)s^5 + (0.735345232520348584e-7)s^6|.$$

$$|e_{3,3,7}(s)| = |-(0.38478770e-3)s^2 + (0.849379426e-4)s^3 + (0.280426817302835939e-4)s^4 - (0.495050218888017330e-5)s^5 - (0.350111688119908828e-6)s^6 + (0.231051655239871117e-7s^7|.$$

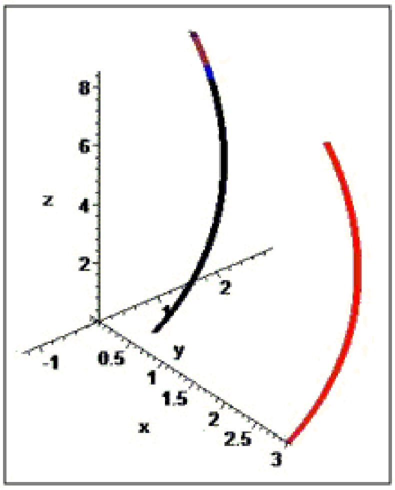
$$|e_{3,3,8}(s)| = |(0.38495079e-3)s^2 - (0.852543061e-4)s^3 - (0.277865487879685406e-4)s^4 + (0.484261911331022518e-5)s^5 + (0.374860308800117502e-6)s^6 - (0.260342571361117485e-7)s^7 + (0.139893638198862298e-9)s^8|$$

Table 7. Comparison of the estimated absolute error functions, $|e_{3,N,M}(s)|$ for $N = 3$ and $M = 4,5,6,7,8$.

s_i	$\lvert e_{3,3,4}(s_i)\rvert$	$\lvert e_{3,3,5}(s_i)\rvert$	$\lvert e_{3,3,6}(s_i)\rvert$	$\lvert e_{3,3,7}(s_i)\rvert$	$\lvert e_{3,3,8}(s_i)\rvert$
0	0	0	0	0	0
$\frac{\pi}{4}$	$3.1237e-4$	$1.8713e-4$	$1.8499e-4$	$1.8709e-4$	$1.8711e-4$
$\frac{2\pi}{4}$	$7.1022e-4$	$5.0163e-4$	$4.9966e-4$	$5.0155e-4$	$5.0157e-4$
$\frac{3\pi}{4}$	$7.1663e-4$	$5.7023e-4$	$5.6931e-4$	$5.7095e-4$	$5.7096e-4$
$\frac{4\pi}{4}$	$2.9723e-4$	$2.1398e-4$	$2.1217e-4$	$2.1424e-4$	$2.1425e-4$
$\frac{5\pi}{4}$	$1.3979e-4$	$3.0405e-4$	$3.0579e-4$	$3.0432e-4$	$3.0432e-4$
$\frac{6\pi}{4}$	$2.5635e-4$	$3.0942e-5$	$2.8663e-5$	$2.6548e-5$	$2.6544e-5$
$\frac{7\pi}{4}$	$2.7790e-3$	$2.9237e-3$	$2.9193e-3$	$1.1504e-2$	$2.9217e-3$
$\frac{8\pi}{4}$	$9.1640e-3$	$1.1636e-2$	$1.1551e-2$	$1.1150e-2$	$1.1505e-2$

In Tables 5-6-7, we have considered the estimated absolute error functions. It is seen from these tables that the results are almost identical and approximate solutions are very close to absolute solutions of system. In addition, we say that the Taylor collocation method is very effective for solving differential equations with variable coefficients. Because, It is very difficult to find the analytical solutions of these differential equations systems.

Now, let us draw the graphics of curves of constant breadth.



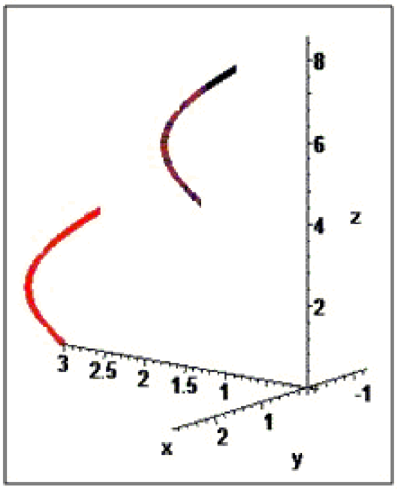


Fig. 2. (Red) graphic of α, (Black) graphic of α^* for $N = 3, M = 4$. (Brown) graphic of α^* for $N = 3, M = 6$ and (Blue) graphic of α^* for $N = 3, M = 8$.

6 Conclusions

In this study, we have developed a Taylor collocation method for system of three linear differential equations in normal form with the help of the residual error function. Then, we have given the system of linear differential equations characterizing curves of constant breadth according to Bishop frame in Euclidean 3-space E^3 and then we have obtained approximate solutions of system of differential equations characterizing curves of constant breadth by using Taylor collocation method. We have given an example to show efficiency of this method.

In Figure 1, we have obtained the graphics of the distance function. In Figure 2, the graphics of curves of constant breadth are drawn. Also, we have studied the residual error analysis. It is seen from these comparisons that the approximate solutions are very close to absolute solutions when the values of N and M are selected big. Also, Taylor collocation method used for approximate solutions is very effective.

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