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Soft sets combined with interval valued intuitionistic fuzzy sets of type-2 and rough sets

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Abstract: Fuzzy set theory, rough set theory and soft set theory are all mathematical tools dealing with uncertainties. The concept of type-2 fuzzy sets was introduced by Zadeh in 1975 which was extended to interval valued intuitionistic fuzzy sets of type-2 by the authors.This paper is devoted to the discussions of the combinations of interval valued intuitionistic sets of type-2, soft sets and rough sets.Three different types of new hybrid models, namely-interval valued intuitionistic fuzzy soft sets of type-2, soft rough interval valued intuitionistic fuzzy sets of type-2 and soft interval valued intuitionistic fuzzy rough sets of type-2 are proposed and their properties are derived.

Keywords: Soft set, rough set, soft rough set, soft rough fuzzy set, soft fuzzy rough set, interval valued intuitionistic fuzzy set of type-2,interval valued intuitionistic fuzzy soft set of type-2, soft roughinterval valued intuitionistic fuzzy set of type-2,soft interval valued intuitionistic fuzzy rough set of type-2.

1 Introduction

The soft set theory, initiated by Molodtsov in 1999, is a completely generic mathematical tool for modeling vague concepts.In soft set theory there is no limited condition to the description of objects; so researchers can choose the form of parameters they need, which greatly simplifies the decision making process and make the process more efficient in the absence of partial information.Although many mathematical tools are available for modeling uncertainties such as probability theory, fuzzy set theory, rough set theory, interval valued mathematics etc,but there are inherent difficulties associated with each of these techniques. Moreover all these techniques lack in the parameterization of the tools and hence they could not be applied successfully in tackling problems especially in areas like economic, environmental and social problems domains. Soft set theory is standing in a unique way in the sense that it is free from the above difficulties and it has a unique scope for many applications in a multi-dimentional way.

Soft set theory has a rich potential for application in many directions, some of which are reported by Molodtsov [18] in his work. He successfully applied soft set theory in areas such as the smoothness of functions, game theory, operations research, Riemann integration and so on. Later on Maji et al. [14] presented some new definitions on soft sets such as subset, union, intersection and complements of soft sets and discussed in detail the application of soft set in decision making problem. Based on the analysis of several operations on soft sets introduced in [14], Ali et al. [2] presented some new algebraic operations for soft sets and proved that certain De Morgan's law holds in soft set theory with respect to

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these new definitions. Chen [5]presented a new definition of soft set parameterization reduction and compared this definition with the related concept of knowledge reduction in the rough set theory. Kong et al. [13] introduced the definition of normal parameter reduction into soft sets and then presented a heuristic algorithm to compute normal parameter reduction of soft sets. By amalgamating the soft sets and algebra, Aktas and Cagman [1] introduced the basic properties of soft sets, compared soft sets to the related concepts of fuzzy sets [27] and rough sets[20], pointed out that every fuzzy set and every rough set may be considered as a soft set. Jun[12] applied soft sets to the theory of BCK/BCI-algebra and introduced the concept of soft BCK/BCI-algebras. Feng et al.[7]defined soft semi rings and several related notions to establish a connection between the soft sets and semi rings. Sun et al.[22] presented the definition of soft modules and constructed some basic properties using modules and Molodtsov's definition of soft sets.Maji et al. [15]presented the concept of the fuzzy soft set which is based on a combination of the fuzzy set and soft set models.Roy and Maji[21] presented a fuzzy soft set theoretic approach towards a decision making problem. Yang et al.[25]defined the operations on fuzzy soft sets, which are based on three fuzzy logic operations: negation, triangular norm and triangular co-norm. Xiao et al.[23] proposed a combined forecasting approach based on fuzzy soft set theory. Yang et al.[24] introduced the concept of interval valued fuzzy soft set and a decision making problem was analyzed by the interval valued fuzzy soft set. Feng et al.[8] presented an adjustable approach to fuzzy soft set based decision making and give some illustrative examples.The notion of intuitionistic fuzzy set was initiated by Atanassov[3] as a generalization of fuzzy set. Combining soft sets with intuitionistic fuzzy sets, Maji et al. [16] introduced intuitionistic fuzzy soft sets, which are rich potentials for solving decision making problems.The notion of the interval-valued intuitionistic fuzzy set was introduced by Atanassov and Gargov[4]. It is characterized by an interval–valued membership degree and an interval-valued non-membership degree.In 2010, Jiang et al. [11] introduced theconcept of interval valued intuitionistic fuzzysoft sets.

The rough set theory proposed by Pawlak[20] provides a systematic method for dealing with vague concepts caused by isdiscernability in situation with incomplete information or a lack of knowledge/data. In general a fuzzy set may be viewed as a class with unsharp boundaries, but a rough set is a coarsely described crisp set[26]. Based on the equivalence relation on the universe of discourse, Dubois andPrade[6] introduced the lower and upper approximation of fuzzy sets in a Pawlak's approximation space [20] and obtained a new notion called rough fuzzy sets which are natural extensions of rough sets. They also introduced the concept of fuzzy rough sets [6]. Feng el al. [9] provided a framework to combine rough sets and soft sets all together, which gives rise to several interesting new concepts such as soft rough sets and rough soft sets. A rough soft set is the approximation of a soft set in a Pawlak approximation space, where as a soft rough set is based on softrough approximationsin a soft approximation space.Feng[10] presented a soft rough set based multi-criteria group decision making scheme.Motivated by Dubois andPrade's original idea about rough fuzzy set,Feng et al. [9] introduced lower and upper soft rough approximations of fuzzy sets in a soft approximation space and obtained a new hybrid model called soft rough fuzzy set. By employing a fuzzy soft set to granulate the universe of discourse, Meng et al. [17] introduced a more general model called soft fuzzy rough set.

The concept of type-2 fuzzy sets was introduced by Zadeh [28] as an extension of the concept of an ordinary fuzzy sets (called type-1 fuzzy sets). Such sets are fuzzy sets whose membership grades themselves are type-1 fuzzy sets. These sets are very useful in circumstances where it is difficult to determine an exact membership function for a fuzzy set. Sahaand Mukherjee [19] introduced the concept of interval valued intuitionistic fuzzy set of type-2 as an extension of fuzzy sets of type-2. The aim of this paper is to provide a framework to combine interval valued intuitionistic fuzzy sets of type-2, rough sets and soft sets all togetherwhich gives rise to several interesting new concepts such as interval valued intuitionistic fuzzy soft sets of type-2, soft rough interval valued intuitionistic fuzzy sets of type-2 and softinterval valued intuitionistic fuzzyroughsets of type-2.

2 Fuzzy sets and their generalizations

This section presents a review of some fundamental notions of fuzzy sets and their generalizations.

The theory of fuzzy sets provides an appropriate framework for representing and processing vague concepts by allowing partial memberships.

Definition 2.1: [27] Let X be a non empty set. Then a *fuzzy set* A on X is a set having the form

$$A = \{(x, \mu_A(x)) : x \in X\}$$

where the function μ_A: X→ [0, 1] is called the membership function and μ_A(x) represents the degree ofmembership of each element $x \in X$.

Definition 2.2: [3] Let X be a non empty set. Then an *intuitionistic fuzzy set (IFS for short)* A is a set having the form

$$A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}$$

where the functions μ_A: X→ [0,1] and γ_A: X→ [0,1] represents the degree of membership and the degree of non-membership respectively of each element $x \in X$ and 0≤ μ_A(x)+γ_A(x)≤ 1 for each $x \in X$.

Definition 2.3: [4] An *interval valued intuitionistic fuzzy set* A over an universe set U is defined as the object of the form

$$A = \{< x, \mu_A(x), \gamma_A(x) >: x \in U\},$$

where μ_A: U→ Int ([0, 1]) and γ_A:U→ Int ([0, 1]) are functions such that the condition: $\forall x \in U, 0 \leq sup\mu_A(x) + sup\gamma_A(x) \leq 1$ is satisfied (where Int[0,1]is the set of all closed sub-intervals of [0,1]).

We denote the class of all interval valued intuitionistic fuzzy sets on U by IVIFS^U.

Let $A, B \in IVIFS^U$. Then

- the *union* of A and B is denoted by $A \cup B$ where $A \cup B = \{(x, [max(inf\mu_A(x), inf\mu_B(x)), max(sup\mu_A(x), sup\mu_B(x))], [min(inf\gamma_A(x), inf\gamma_B(x)), min(sup\gamma_A(x), sup\gamma_B(x))]) : x \in U\}$.
- the *intersection* of A and B is denoted by A∩B where $A \cap B = \{(x, [min(inf\mu_A(x), inf\mu_B(x)), min(sup\mu_A(x), sup\mu_B(x))], [max(inf\gamma_A(x), inf\gamma_B(x)), max(sup\gamma_A(x), sup\gamma_B(x))]) : x \in U\}$.

Atanassov and Gargovshows in [4] that $A \cup B$ and $A \cap B$ are again IVIFSs.

Definition 2.4: [28] A *fuzzyset of type-2* is defined as a fuzzy set whose membership degree is a fuzzy set of type-1.

Thus, a fuzzy set of type-2 , A on an universe U is an object of the form

$$A = \{(x, \mu_A(x)) : x \in U\}$$

where

$$\mu_A(x) = \{(u_i, \mu_{ui}(x)) : u_i \in [0,1]\}$$

for $x \in U$, where $\mu_{ui} : U \to [0,1]$ is a function.

3 Soft sets and their generalizations

In this section we recall some basic notions releavant to soft sets and their generalizations.

Definition 3.1: [18] Let U be an universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subseteq E$. Then the pair (F,A) is called a *soft set* over U, where F is a mapping given by $F : A \to P(U)$.

In other words, a soft set over U is a parameterized family of subsets of U. For $e \in A, F(e)$ may be considered as the set of e-approximate elements of the soft set (F,A).

Definition 3.2: [15] Let U be an universe set and E be a set of parameters. Let I^U be the set of all fuzzy subsets of U and $A \subseteq E$. Then the pair (F,A) is called a *fuzzy soft set* over U, where F is a mapping given by $F : A \to I^U$.

For $e \in A, F(e)$ is a fuzzy subset of U and is called the fuzzy value set of the parameter e. Let us denote $\mu_{F(e)}(x)$ by the membership degree that object x holds parameter e, wheree $\in A$ and $x \in U$. Then F(e) can be written as a fuzzy set such that $F(e) = \{(x, \mu_{F(e)}(x)) : x \in U\}$.

Definition 3.3: [16] Let U be an universe set and E be a set of parameters. Let IF^U be the set of all intuitionistic fuzzy subsets of U and $A \subseteq E$. Then the pair (F, A) is called an *intuitionisticfuzzy soft set* over U, where F is a mapping given by $F : A \to IF^U$.

For $e \in A, F(e)$ is an intuitionistic fuzzy subset of U and is called the intuitionistic fuzzy value set of the parameter e. Let us denote $\mu_{F(e)}(x)$ by the membership degree that object x holds parameter e and $\gamma_{F(e)}(x)$ by the membership degree that object x doesn't hold parameter e, where $e \in A$ and $x \in U$. Then $F(e)$ can be written as an intuitionistic fuzzy set such that

$$F(e) = \{(x, \mu_{F(e)}(x), \gamma_{F(e)}(x)) : x \in U\}.$$

Definition 3.4: [11] Let U be an universe set and E be a set of parameters. Let $IVIFS^U$ be the set of all interval valued intuitionistic fuzzy sets on U and $A \subseteq E$. Then the pair (F,A) is called an *interval valued intuitionisticfuzzy soft set(IVIFSS for short)* over U, where F is a mapping given by $F : A \to IVIFS^U$.

For $e \in A, F(e)$ can be written as an interval valued intuitionistic fuzzy set such that

$$F(e) = \{(x, \mu_{F(e)}x), \gamma_{F(e)}(x)) : x \in U\}$$

where $\mu_{F(e)}(x)$ is the interval valued fuzzy membership degree that object x holds parameter e and $\gamma_{F(e)}(x)$ is the interval valued fuzzy membership degree that object x doesn't hold parameter e.

4 Rough set, soft rough set, rough soft set and their generalizations

This section represents a review of the notions of rough sets, soft rough sets, rough soft sets and their generalizations.

The rough set theory provides a systematic method for dealing with vague concepts caused by isdiscernability in situation with incomplete information or a lack of knowledge.

Definition 4.1: [20] Let R be an equivalence relation on the universal set U.Then the pair (U,R) is called a Pawlak approximation space. An equivalence class of R containing x will be denoted by $[x]_R$. Now for $X \subseteq U$, the lower and upper approximation of X with respect to (U,R) are denoted by respectively R_*X and $R*X$ and are defined by

$$R_*X = \{x \in U : [x]_R \subseteq X\},$$

$$R*X = \{x \in U : [x]_R \cap X \neq \varphi\}.$$

Now if $R_*X = R*X$, then X is called definable in (U,R); otherwise X is called a *rough set.*

Feng et al. **[9]** provided a framework to combine rough sets and soft sets all together, which gives rise to several interesting new concepts such as soft rough sets and rough soft sets.

Definition 4.2: [9] Let $\Theta = (f,A)$ be a soft set over U. The pair $S = (U,\Theta)$ is called a soft approximation space. Based on S, the operators$\underline{apr}_S$ and $\bar{a}\bar{p}\bar{r}_S$ are defined as:

$$\underline{apr}_S(X) = \{u \in U : \exists a \in A(u \in f(a) \subseteq X)\},$$

$$\overline{apr}_S(X) = \{u \in U : \exists a \in A(u \in f(a), f(a) \cap X \neq \varphi)\} \text{ for every } X \subseteq U.$$

The two sets $\underline{apr}_S(X) \& \bar{a}\bar{p}\bar{r}_S(X)$ are called the lower and upper soft rough approximations of X in S respectively. If $\underline{apr}_S(X) = \overline{apr}_S(X)$, then X is said to be soft definable; otherwise X is called a *soft rough set*.

Definition 4.3: [9] Let (U,R) be a Pawlak approximation space and $\Theta = (f,A)$ be a soft set over U. Then the lower and upper rough approximations of $\Theta \in (U,R)$ are denoted by $R_*(\Theta) = (F_*,A)$ and $R^*(\Theta) = (F^*,A)$, respectively, which are soft sets over U defined by:

$$F_*(x) = (F(x)) = \{y \in U : [y]_R \subseteq F(x)\}$$

and

$$F^*(x) = \bar{R}(F(x)) = \{y \in U : [y]_R \cap F(x) \neq \phi\} \text{ for all } x \in U.$$

The operators R_* and R^* are called the lower and upper rough approximation operators on soft sets. If $R_*(\Theta) = R^*(\Theta)$, the soft set Θ is said to be definable;otherwise Θ is called a *roughsoft set*.

Motivated by Dubois andPrade's original idea about rough fuzzy set,Feng et al. **[9]** introduced lower and upper soft rough approximations of fuzzy sets in a soft approximation space and obtained a new hybrid model called soft rough fuzzy set.

Definition 4.4: [9] Let $\Theta = (f,A)$ be a full soft set over U i.e; $\cup_{a\in A} f(a) = U$ and the pair $S = (U,\Theta)$ be the soft approximation space. Then for a fuzzy set $\lambda \in I^U$, the lower and upper soft rough approximations of λ with respect to S are denoted by $\underline{sap}_S(\lambda)$ and $\bar{s}\bar{a}\bar{p}_S(\lambda)$ respectively, which are fuzzy sets in U given by:

$$\underline{sap}_S(\lambda) = \{(x, \underline{sap}_S(\lambda)(x)) : x \in U\},$$

$$\overline{sap}_S(\lambda) = \{(x, \overline{sap}_S(\lambda)(x)) : x \in U\},$$

where

$$\underline{sap}_S(\lambda)(x) = \wedge\{\mu_\lambda(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}$$

and

$$\overline{sap}_S(\lambda)(x) = \vee\{\mu_\lambda(y) : \exists a \in A(\{x,y\} \subseteq f(a))\} \text{ for every } x \in U.$$

The operators$\underline{sap}_S$ and $\overline{sap}_S$ are called the lower and upper soft rough approximation operators on fuzzy sets. If

$$\underline{sap}_S(\lambda) = \overline{sap}_S(\lambda),$$

then λ is said to be fuzzy soft definable; otherwise λ is called a *soft rough fuzzy set*.

Meng et al. [17] introduced the lower and upper soft fuzzy rough approximations of a fuzzy set by granulating the universe of discourse with the help of a fuzzy soft set and obtained a new model called soft fuzzy rough set.

Definition 4.5: [17] Let $\Theta = (f,A)$ be a fuzzy soft set over U. Then the pair $SF = (U,\Theta)$ is called a soft fuzzy approximation space. Then for a fuzzy set $\lambda \in I^U$, the lower and upper soft fuzzy rough approximations of λ with respect to SF are denoted by $\underline{\mathrm{Apr}}_{\mathrm{SF}}(\lambda)$ and $\overline{Apr}_{SF}(\lambda)$ respectively, which are fuzzy sets in U given by:

$$\underline{\mathrm{Apr}}_{\mathrm{SF}}(\lambda) = \{(x, \underline{\mathrm{Apr}}_{\mathrm{SF}}(\lambda)(x)) : x \in U\},$$

$$\overline{Apr}_{SF}(\lambda) = \{(x, \overline{Apr}_{SF}(\lambda)(x)) : x \in U\}$$

where

$$\underline{\mathrm{Apr}}_{\mathrm{SF}}(\lambda)(x) = \wedge_{a\in A}((1-f(a)(x)) \vee (\wedge_{y\in U}((1-f(a)(y)) \vee \mu_\lambda(y))))$$

and

$$\overline{Apr}_{SF}(\lambda)(x) = \vee_{a\in A}(f(a)(x) \wedge (\vee_{y\in U}(f(a)(y) \wedge \mu_\lambda(y)))) \text{ for every } x \in U \text{ and } \mu_\lambda(y)$$

is the degree of membership of $y \in U$.

The operators $\underline{\mathrm{Apr}}_{\mathrm{SF}}$ and $\overline{Apr}_{SF}$ are called the lower and upper soft fuzzy rough approximation operators on fuzzy sets. If $\underline{\mathrm{Apr}}_{\mathrm{SF}}(\lambda)$=$\overline{Apr}_{SF}(\lambda)$, then λ is said to be soft fuzzy definable; otherwise λ is called a *soft fuzzy rough set*.

5 Interval valued intuitionistic fuzzy sets of type-2

The concept of interval valued intuitionistic fuzzy sets of type-2 was introduced by Saha and Mukherjee[19],as an extension of the concept of fuzzy sets of type-2.

Definition 5.1: [9] An *intervalvalued intuitionistic fuzzy set of type-2* is defined as an interval valued intuitionistic fuzzy set whose interval valued fuzzy membership degree as well as interval valued fuzzy non-membership degree are both interval valued intuitionistic fuzzy set of type-1. Thus, an interval valued intuitionistic fuzzy set of type-2 A on an universe U is an object of the form

$$A = \{< x, \mu_A(x), \gamma_A(x) >: x \in U\}$$

where

$$\mu_A(x) = \{< u_i(x), \mu_{ui}(x), \gamma_{ui}(x) >: i = 1, 2, \ldots, n\} \text{ and } \gamma_A(x) = \{< v_j(x), \mu_{vj}(x), \gamma_{vj}(x) >: j = 1, 2, \ldots, m\} \text{ for } x \in U,$$

where $u_i(x), v_j(x) \in Int([0,1])$ and μ_{ui}, γ_{ui}, $\mu_{vj}, \gamma_{vj} : U \to Int([0,1])$ are functions such that $x \in U, 0 \le sup\mu_{ui}(x) + sup\gamma_{ui}(x) \le 1 \& 0 \le sup\mu_{vj}(x) + sup\gamma_{vj}(x) \le 1$ for each i and j.

The set of all interval valued intuitionistic fuzzy sets of type-2 on U is denoted by $IVIFS_2(U)$.

Definition 5.2: An *interval-valued intuitionistic fuzzy null set of type-2* over an universe U is denoted by $\Phi*$ and is defined by $\Phi* = \{< x, \{< u_i(x), [0,0], [1,1] >: i = 1, 2, \ldots, n\}, \{< v_j(x), [0,0], [1,1] >: j = 1, 2, \ldots, m\} : x \in U\}$, where for $x \in U, u_i(x) = [0,0]$ and $v_j(x) = [0,0]$ for each i and j.

Definition 5.3: An *interval-valued intuitionistic fuzzy universal set of type-2* over an universe U is denoted by $U*$ and is defined by $U* = \{< x, \{< u_i(x), [1,1], [0,0] >: i = 1, 2, \ldots, n\}, \{< v_j(x), [1,1], [0,0] >: j = 1, 2, \ldots, m\} >: x \in U\}$, where for $x \in U, u_i(x) = [1,1] and v_j(x) = [1,1]$ for each i and j.

Definition 5.4: Let $\Gamma, \Psi \in IVIFS_2(U)$ be defined by

$$\Gamma = \{< x, \mu_\Gamma(x), \gamma_\Gamma(x) >: x \in U\} \text{ and } \Psi = \{< x, \mu_\Psi(x), \gamma_\Psi(x) >: x \in U\}, \text{ where for } x \in U,$$
$$\mu_\Gamma(x) = \{< u_i(x), \mu_{ui}(x), \gamma_{ui}(x) >: i = 1, 2, ..., n\},$$
$$\gamma_\Gamma(x) = \{< v_j(x), \mu_{vj}(x), \gamma_{vj}(x) >: j = 1, 2, ..., m\},$$
$$\mu_\Psi(x) = \{< w_i(x), \mu_{wi}(x), \gamma_{wi}(x) >: i = 1, 2, ..., n\},$$
$$\gamma_\Psi(x) = \{< t_j(x), \mu_{tj}(x), \gamma_{tj}(x) >: j = 1, 2, ..., m\}.$$

Then Γ is called a *restricted subset* of Ψ, denoted by $\Gamma \subseteq_2 \Psi$, if for all $x \in U$, $infu_i(x) \le infw_i(x)$, $supu_i(x) \le supw_i(x)$, $infv_j(x) \le inft_j(x)$, $supv_j(x) \le supt_j(x)$,$inf\mu_{ui}(x) \le inf\mu_{wi}(x)$, $sup\mu_{ui}(x) \le sup\mu_{wi}(x)$, $inf\gamma_{ui}(x) \ge inf\gamma_{wi}(x)$, $sup\gamma_{ui}(x) \ge sup\gamma_{wi}(x)$, $inf\mu_{vj}(x) \le inf\mu_{tj}(x)$, $sup\mu_{vj}(x) \le sup\mu_{tj}(x)$, $inf\gamma_{vj}(x) \ge inf\gamma_{tj}(x)$, $sup\gamma_{vj}(x) \ge sup\gamma_{tj}(x)$.

Definition 5.5: Let Γ, $\Psi \in IVIFS_2(U)$ be defined by

$$\Gamma = \{< x, \mu_\Gamma(x), \gamma_\Gamma(x) >: x \in U\} \text{ and } \Psi = \{< x, \mu_\Psi(x), \gamma_\Psi(x) >: x \in U\}, \text{where for } x \in U,$$

$$\mu_\Gamma(x) = \{< u_i(x), \mu_{ui}(x), \gamma_{ui}(x) >: i = 1, 2, ..., n\},$$

$$\gamma_\Gamma(x) = \{< v_j(x), \mu_{vj}(x), \gamma_{vj}(x) >: j = 1, 2, ..., m\},$$

$$\mu_\Psi(x) = \{< w_i(x), \mu_{wi}(x), \gamma_{wi}(x) >: i = 1, 2, ..., n\},$$

$$\gamma_\Psi(x) = \{< t_j(x), \mu_{tj}(x), \gamma_{tj}(x) >: j = 1, 2, ..., m\}.$$

Then

- Their *restricted union* is denoted by $\Gamma \cup_2 \Psi$ and is defined by $\Gamma \cup_2 \Psi = \{< x, \{< u_i(x) \cup w_i(x),\ \mu_{ui}(x) \cup \mu_{wi}(x),$ $\gamma_{ui}(x) \cap \gamma_{wi}(x) >: i = 1,2,...,n\},\ \{< v_j(x) \cup t_j(x), \mu_{vj}(x) \cup \mu_{tj}(x),\ \gamma_{vj}(x) \cap \gamma_{tj}(x) >: j = 1,2,....,m\} >: x \in U\}$.
- Their *restricted intersection* is denoted by $\Gamma \cap_2 \Psi$ and is defined by $\Gamma \cap_2 \Psi = \{< x, \{< u_i(x) \cap w_i(x),\ \mu_{ui}(x) \cap \mu_{wi}(x),$ $\gamma_{ui}(x) \cup \gamma_{wi}(x) >: i = 1,2,...,n\},\ \{< v_j(x) \cap t_j(x), \mu_{vj}(x) \cap \mu_{tj}(x),\ \gamma_{vj}(x) \cup \gamma_{tj}(x) >: j = 1,2,....,m\} >: x \in U\}$.

Clearly for $\Gamma, \Psi \in IVIFS_2(U)$, we have $\Gamma \cup_2 \Psi, \Gamma \cap_2 \Psi \in IVIFS_2(U)$.

Theorem 5.6: Let Γ, Ψ, $\Omega \in IVIFS_2(U)$. Then

1. $\Gamma \cup_2 \Phi* = \Phi* \cup_2 \Gamma = \Gamma$.
2. $\Gamma \cap_2 \Phi* = \Phi* \cap_2 \Gamma = \Phi*$.
3. $\Gamma \cup_2 U* = U* \cup_2 \Gamma = U*$.
4. $\Gamma \cap_2 U* = U* \cap_2 \Gamma = \Gamma$.
5. $\Gamma \cup_2 \Psi = \Psi \cup_2 \Gamma$.
6. $\Gamma \cap_2 \Psi = \Psi \cap_2 \Gamma$.
7. $\Gamma \cap_2 (\Psi \cap_2 \Omega) = (\Psi \cap_2 \Gamma) \cap_2 \Omega$.
8. $\Gamma \cup_2 (\Psi \cup_2 \Omega) = \Psi \cup_2 (\Gamma \cup_2 \Omega)$.
9. $\Gamma \cap_2 (\Psi \cup_2 \Omega) = (\Psi \cap_2 \Gamma) \cup_2 (\Gamma \cap_2 \Omega)$.
10. $\Gamma \cup_2 (\Psi \cap_2 \Omega) = (\Psi \cup_2 \Gamma) \cap_2 (\Gamma \cup_2 \Omega)$.

Proof:-Straight forward.

6 Interval valued intuitionistic fuzzy soft sets of type-2

In this section we present the theory of interval valued intuitionistic fuzzy sets of type-2. We also discuss some operations and properties of the interval valued intuitionistic fuzzy sets of type-2.

Definition 6.1: Let U be an universe set and E be a set of parameters. Let $IVIFS_2(U)$ be the set of all interval valued intuitionistic fuzzy sets of type-2 over U and $A \subseteq E$. Then the pair (f,A) is called an *interval valuedintuitionisticfuzzy soft set* of type-2 over U, where f is a mapping given by $f : A \to IVIFS_2(U)$.

Definition 6.2: Let U be an universe set and E be a set of parameters. Let $IVIFS_2(U)$ be the set of all interval valued intuitionistic fuzzy sets of type-2 over U and $A \subseteq E$. Then the pair (f,A) is called an *interval valuedintuitionisticfuzzy empty soft set* of type-2 over U, denoted by Φ_{soft} where $f : A \to IVIFS_2(U)$ is defined by $f(a) = \Phi*$ for each $a \in A$.

Definition 6.3: Let U be an universe set and E be a set of parameters. Let $IVIFS_2(U)$ be the set of all interval valued intuitionistic fuzzy sets of type-2 over U and $A \subseteq E$. Then the pair (f,A) is called an *interval valuedintuitionisticfuzzy universal soft set* of type-2 over U, denoted by U_{soft} where $f : A \to IVIFS_2(U)$ is defined by $f(a) = U*$ for each $a \in A$.

Definition 6.4: Let U be an initial universe and E be a set of parameters. Let (f,A), (g,B) be two *interval valuedintuitionisticfuzzy soft sets* of type-2 over U, where $f : A \to IVIFS_2(U)$ is defined by $f(a) = \{< x, \mu_{f(a)}(x), \gamma_{f(a)}(x) >: x \in U\}$, where $\mu_{f(a)}(x) = \{< u_i(a)(x), \mu_{ui(a)}(x),\ \gamma_{ui(a)}(x) >: i = 1,2,\ldots,n\}$, $\gamma_{f(a)}(x) = \{< v_j(a)(x), \mu_{vj(a)}(x),\ \gamma_{vj(a)}(x) >: j = 1,2,\ldots,m\}$ for $x \in U$ and $g : B \to IVIFS_2(U)$ is defined by $g(b) = \{< x, \mu_{g(b)}(x), \gamma_{g(b)}(x) >: x \in U\}$, where $\mu_{g(b)}(x) = \{< w_i(b)(x), \mu_{wi(b)}(x),\ \gamma_{wi(b)}(x) >: i = 1,2,\ldots,n\}$, $\gamma_{g(b)}(x) = \{< t_j(b)(x), \mu_{tj(b)}(x),\ \gamma_{tj(b)}(x) >: j = 1,2,\ldots,m\}$ for $x \in U$. Then,

1. Their *union*, denoted by $(f,A)\cup\%(g,B)(=(h,C),say)$, is an interval valued intuitionistic fuzzy soft set of type-2 over U, where $C=A\cup B$ and for $c\in C, h:C\rightarrow IVIFS_2(U)$ is defined by

$$h(c)=\{<x,\mu_{h(c)}(x),\gamma_{h(c)}(x)>: x\in U\},\ \text{ where}$$
$$\begin{aligned}\mu_{h(c)}(x)&=\mu_{f(c)}(x),\ \text{ if } \ c\in A-B\\ &=\mu_{g(c)}(x),\ \text{ if } \ c\in B-A\\ &=\{<u_i(c)(x)\cup w_i(c)(x),\mu_{ui(c)}(x)\cup\mu_{wi(c)}(x),\gamma_{ui(c)}(x)\cap\gamma_{wi(c)}(x)>: i=1,2,\ldots,n\}\ \text{ if } \ c\in A\cap B\end{aligned}$$

and

$$\begin{aligned}\gamma_{h(c)}(x))&=\gamma_{f(c)}(x),\ \text{ if } \ c\in A-B\\ &=\gamma_{g(c)}(x),\ \text{ if } \ c\in B-A\\ &=\{<v_j(c)(x)\cup t_j(c)(x),\mu_{vj(c)}(x)\cup\mu_{tj(c)}(x),\gamma_{vj(c)}(x)\cap\gamma_{tj(c)}(x)>: j=1,2,\ldots,m\}\ \text{ if } \ c\in A\cap B.\end{aligned}$$

2. Their *intersection*, denoted by $(f,A)\cap\%(g,B)(=(h,C),say)$, is an interval valued intuitionistic fuzzy soft set of type-2 over U, where $C=A\cup B$ and for $c\in C, h:C\rightarrow IVIFS_2(U)$ is defined by

$$h(c)=\{<x,\mu_{h(c)}(x),\gamma_{h(c)}(x)>: x\in U\},\ \text{ where}$$
$$\begin{aligned}\mu_{h(c)}(x)&=\mu_{f(c)}(x),\ \text{ if } c\in A-B\\ &=\mu_{g(c)}(x),\ \text{ if } c\in B-A\\ &=\{<u_i(c)(x)\cap w_i(c)(x),\mu_{ui(c)}(x)\cap\mu_{wi(c)}(x),\gamma_{ui(c)}(x)\cup\gamma_{wi(c)}(x)>: i=1,2,\ldots,n\}\ \text{ if } c\in A\cap B\end{aligned}$$

and

$$\begin{aligned}\gamma_{h(c)}(x))&=\gamma_{f(c)}(x),\ \text{ if } c\in A-B\\ &=\gamma_{g(c)}(x),\ \text{ if } c\in B-A\\ &=\{<v_j(c)(x)\cap t_j(c)(x),\mu_{vj(c)}(x)\cap\mu_{tj(c)}(x),\gamma_{vj(c)}(x)\cup\gamma_{tj(c)}(x)>: j=1,2,\ldots,m\}\ \text{ if } c\in A\cap B.\end{aligned}$$

Theorem 6.5: Let U be an universe set and $(f,A),(g,B),(h,C)$ be three *interval valuedintuitionisticfuzzysoft sets* of type-2 over U. Then

1. $(f,A)\cup\%(f,A)=(f,A)$.
2. $(f,A)\cap\%(f,A)=(f,A)$.
3. $(f,A)\cup\%\Phi_{soft}=(f,A)=\Phi_{soft}\cup\%(f,A)$, if the set of parameters for Φ_{soft} is A.
4. $(f,A)\cap\%\Phi_{soft}=\Phi_{soft}=\Phi_{soft}\cap\%(f,A)$, if the set of parameters for Φ_{soft} is A.
5. $(f,A)\cup\%U_{soft}=U_{soft}=U_{soft}\cup\%(f,A)$, if the set of parameters for U_{soft} is A.
6. $(f,A)\cap\%U_{soft}=(f,A)=U_{soft}\cap\%(f,A)$, if the set of parameters for U_{soft} is A.
7. $(f,A)\cup\%(g,B)=(g,B)\cup\%(f,A)$.
8. $(f,A)\cap\%(g,B)=(g,B)\cap\%(f,A)$.
9. $(f,A)\cap\%((g,B)\cap\%(h,C))=((f,A)\cap\%(g,B))\cap\%(h,C)$.
10. $(f,A)\cup\%((g,B)\cup\%(h,C))=((f,A)\cup\%(g,B))\cup\%(h,C)$.
11. $(f,A)\cap\%((g,B)\cup\%(h,C))=((f,A)\cap\%(g,B))\cup\%((f,A)\cap\%(h,C))$.
12. $(12)(f,A)\cup\%((g,B)\cap\%(h,C))=((f,A)\cup\%(g,B))\cap\%((f,A)\cup\%(h,C))$.

Proof:(1)-(8) are straight forward.(9) Let $f:A\rightarrow IVIFS_2(U)$ be defined by

$$
\begin{aligned}
f(a) &= \{< x, \mu_{f(a)}(x), \gamma_{f(a)}(x) >: x \in U\}, \text{ where for } a \in A, \\
\mu_{f(a)}(x) &= \{< y_i(a)(x), \mu_{yi(a)}(x), \gamma_{yi(a)}(x) >: y_i(a)(x) \in Int([0,1]), i = 1,2,\ldots,n\}, \\
\gamma_{f(a)}(x) &= \{< z_j(a)(x), \mu_{zj(a)}(x), \gamma_{zj(a)}(x) >: z_j(a)(x) \in Int([0,1]), j = 1,2,\ldots,m\} \text{ for } x \in U,
\end{aligned}
$$

Let $g : B \to IVIFS_2(U)$ be defined by

$$
\begin{aligned}
g(b) &= \{< x, \mu_{g(b)}(x), \gamma_{g(b)}(x) >: x \in U\}, \text{ where for } b \in B, \\
\mu_{g(b)}(x) &= \{< u_i(b)(x), \mu_{ui(b)}(x), \gamma_{ui(b)}(x) >: u_i(b)(x) \in Int([0,1]), i = 1,2,\ldots,n\}, \\
\gamma_{g(b)}(x) &= \{< v_j(b)(x), \mu_{vj(b)}(x), \gamma_{vj(b)}(x) >: v_j(b)(x) \in Int([0,1]), j = 1,2,\ldots,m\} \text{ for } x \in U,
\end{aligned}
$$

Let $h : C \to IVIFS_2(U)$ be defined by

$$
\begin{aligned}
h(c) &= \{< x, \mu_{h(c)}(x), \gamma_{h(c)}(x) >: x \in U\}, \text{ where for } c \in C, \\
\mu_{h(c)}(x) &= \{< w_i(c)(x), \mu_{wi(c)}(x), \gamma_{wi(c)}(x) >: w_i(c)(x) \in Int([0,1]), i = 1,2,\ldots,n\}, \\
\gamma_{h(c)}(x) &= \{< t_j(c)(x), \mu_{tj(c)}(x), \gamma_{tj(c)}(x) >: t_j(c)(x) \in Int([0,1]), j = 1,2,\ldots,m\} \text{ for } x \in U.
\end{aligned}
$$

Let $(g,B) \cap \%(h,C) = (s,D)$ and $D = B \cup C$. Then for $d \in D$, we have,

$$
\begin{aligned}
\mu_{s(d)}(x) &= \mu_{g(d)}(x), \text{ if } d \in B - C, \\
&= \mu_{h(d)}(x), if d \in C - B, \\
&= \{< u_i(d)(x) \cap w_i(d)(x), \mu_{ui(d)}(x) \cap \mu_{wi(d)}(x), \gamma_{ui(d)}(x) \cup \gamma_{wi(d)}(x) >: i = 1,2,\ldots,n\} if d \in B \cap C.
\end{aligned}
$$

and

$$
\begin{aligned}
\gamma_{s(d)}(x)) &= \gamma_{g(d)}(x), \text{ if } d \in B - C, \\
&= \gamma_{h(d)}(x), if d \in C - B, \\
&= \{< v_j(d)(x) \cap t_j(d)(x), \mu_{vj(d)}(x) \cap \mu_{tj(d)}(x), \gamma_{vj(d)}(x) \cup \gamma_{tj(d)}(x) >: j = 1,2,\ldots,m\} \text{ if } d \in B \cap C.
\end{aligned}
$$

Now $(f,A) \cap \%((g,B) \cap \%(h,C)) = (f,A) \cap \%(s,D) = (e,P)$, say, where $P = A \cup D$. Then for $p \in P$ we have,

$$
\begin{aligned}
\mu_{e(p)}(x) &= \mu_{g(p)}(x), if p \in B - C - A, \\
&= \mu_{h(p)}(x), if p \in C - B - A, \\
&= \mu_{f(p)}(x), if p \in A - B - C, \\
&= \{< u_i(p)(x) \cap w_i(p)(x), \mu_{ui(p)}(x) \cap \mu_{wi(p)}(x), \gamma_{ui(p)}(x) \cup \gamma_{wi(p)}(x) >: i = 1,2,\ldots,n\} \text{ if } p \in (B \cap C) - A, \\
&= \{< u_i(p)(x) \cap y_i(p)(x), \mu_{ui(p)}(x) \cap \mu_{yi(p)}(x), \gamma_{ui(p)}(x) \cup \gamma_{yi(p)}(x) >: i = 1,2,\ldots,n\} \text{ if } p \in (A \cap B) - C, \\
&= \{< y_i(p)(x) \cap w_i(p)(x), \mu_{yi(p)}(x) \cap \mu_{wi(p)}(x), \gamma_{yi(p)}(x) \cup \gamma_{wi(p)}(x) >: i = 1,2,\ldots,n\} \text{ if } p \in (A \cap C) - B, \\
&= \{< y_i(p)(x) \cap u_i(p)(x) \cap w_i(p)(x), \mu_{yi(p)}(x) \cap \mu_{ui(p)}(x) \cap \mu_{wi(p)}(x), \gamma_{yi(p)}(x) \cup \gamma_{ui(p)}(x) \cup \gamma_{wi(p)}(x) >: \\
&i = 1,2,\ldots,n\} if p \in A \cap B \cap C.
\end{aligned}
$$

and

$$\begin{aligned}
\gamma_{e(p)}(x) &= \gamma_{g(p)}(x), \text{ if } p \in B-C-A,\\
&= \gamma_{h(p)}(x), \text{ if } p \in C-B-A,\\
&= \gamma_{f(p)}(x), \text{ if } p \in A-B-C,\\
&= \{< v_j(p)(x)\cap t_j(p)(x), \mu_{vj(p)}(x)\cap\mu_{tj(p)}(x), \gamma_{vj(p)}(x)\cup\gamma_{tj(p)}(x) >: j=1,2,\ldots,m\} \text{ if } p\in(B\cap C)-A,\\
&= \{< v_j(p)(x)\cap z_j(p)(x), \mu_{vj(p)}(x)\cap\mu_{zj(p)}(x), \gamma_{vj(p)}(x)\cup\gamma_{zj(p)}(x) >: j=1,2,\ldots,m\} \text{ if } p\in(A\cap B)-C,\\
&= \{< z_j(p)(x)\cap t_j(p)(x), \mu_{zj(p)}(x)\cap\mu_{tj(p)}(x), \gamma_{zj(p)}(x)\cup\gamma_{tj(p)}(x) >: j=1,2,\ldots,m\} \text{ if } p\in(A\cap C)-B,\\
&= \{< z_j(p)(x)\cap v_j(p)(x)\cap t_j(p)(x), \mu_{zj(p)}(x)\cap\mu_{vj(p)}(x)\cap\mu_{tj(p)}(x), \gamma_{zj(p)}(x)\cup\gamma_{vj(p)}(x)\cup\gamma_{tj(p)}(x) >:\\
&\quad j=1,2,\ldots,m\} \text{ if } p\in A\cap B\cap C.
\end{aligned}$$

Let $(f,A)\cap\%(g,B)=(l,Q)$ and $Q=A\cup B$. Then for $q\in Q$, we have,

$$\begin{aligned}
\mu_{l(q)}(x) &= \mu_{f(q)}(x), \text{ if } q\in A-B,\\
&= \mu_{g(q)}(x), \text{ if } q\in B-A,\\
&= \{< y_i(q)(x)\cap u_i(q)(x), \mu_{ui(q)}(x)\cap\mu_{ui(q)}(x), \gamma_{yi(q)}(x)\cup\gamma_{ui(q)}(x) >: i=1,2,\ldots,n\} \text{ if } q\in A\cap B,
\end{aligned}$$

and

$$\begin{aligned}
\gamma_{l(q)}(x)) &= \gamma_{f(q)}(x), \text{ if } q\in A-B,\\
&= \gamma_{g(q)}(x), \text{ if } q\in B-A,\\
&= \{< z_j(q)(x)\cap v_j(q)(x), \mu_{zj(q)}(x)\cap\mu_{vj(q)}(x), \gamma_{zj(q)}(x)\cup\gamma_{vj(q)}(x) >: j=1,2,\ldots,m\} \text{ if } q\in A\cap B.
\end{aligned}$$

Now $((f,A)\cap\%(g,B))\cap\%(h,C)=(l,Q)\cap\%(h,C)=(k,R)$, say, where $R=Q\cup C$. Then for $r\in R$ we have,

$$\begin{aligned}
\mu_{k(r)}(x) &= \mu_{g(r)}(x), \text{ if } r\in B-C-A,\\
&= \mu_{h(r)}(x), \text{ if } r\in C-B-A,\\
&= \mu_{f(r)}(x), \text{ if } r\in A-B-C,\\
&= \{< u_i(r)(x)\cap w_i(r)(x), \mu_{ui(r)}(x)\cap\mu_{wi(r)}(x), \gamma_{ui(r)}(x)\cup\gamma_{wi(r)}(x) >: i=1,2,\ldots,n\} \text{ if } r\in(B\cap C)-A,\\
&= \{< u_i(r)(x)\cap y_i(r)(x), \mu_{ui(r)}(x)\cap\mu_{yi(r)}(x), \gamma_{ui(r)}(x)\cup\gamma_{yi(r)}(x) >: i=1,2,\ldots,n\} \text{ if } r\in(A\cap B)-C,\\
&= \{< y_i(r)(x)\cap w_i(r)(x), \mu_{yi(r)}(x)\cap\mu_{wi(r)}(x), \gamma_{yi(r)}(x)\cup\gamma_{wi(r)}(x) >: i=1,2,\ldots,n\} \text{ if } r\in(A\cap C)-B,\\
&= \{< y_i(r)(x)\cap u_i(r)(x)\cap w_i(r)(x), \mu_{yi(r)}(x)\cap\mu_{ui(r)}(x)\cap\mu_{wi(r)}(x), \gamma_{yi(r)}(x)\cup\gamma_{ui(r)}(x)\cup\gamma_{wi(r)}(x) >:\\
&\quad i=1,2,\ldots,n\} \text{ if } r\in A\cap B\cap C,
\end{aligned}$$

and

$$\begin{aligned}
\gamma_{k(r)}(x) &= \gamma_{g(r)}(x), \text{ if } r\in B-C-A,\\
&= \gamma_{h(r)}(x), \text{ if } r\in C-B-A,\\
&= \gamma_{f(r)}(x), \text{ if } r\in A-B-C,\\
&= \{< v_j(r)(x)\cap t_j(r)(x), \mu_{vj(r)}(x)\cap\mu_{tj(r)}(x), \gamma_{vj(r)}(x)\cup\gamma_{tj(r)}(x) >: j=1,2,\ldots,m\} \text{ if } r\in(B\cap C)-A,
\end{aligned}$$

$= \{< v_j(r)(x) \cap z_j(r)(x), \mu_{vj(r)}(x) \cap \mu_{zj(r)}(x), \gamma_{vj(r)}(x) \cup \gamma_{zj(r)}(x) >: j = 1,2,\ldots,m\}$ if $r \in (A \cap B) - C$,
$= \{< z_j(r)(x) \cap t_j(r)(x), \mu_{zj(r)}(x) \cap \mu_{tj(r)}(x), \gamma_{zj(r)}(x) \cup \gamma_{tj(r)}(x) >: j = 1,2,\ldots,m\}$ if $r \in (A \cap C) - B$,
$= \{< z_j(r)(x) \cap v_j(r)(x) \cap t_j(r)(x), \mu_{zj(r)}(x) \cap \mu_{vj(r)}(x) \cap \mu_{tj(r)}(x), \gamma_{zj(r)}(x) \cup \gamma_{vj(r)}(x) \cup \gamma_{tj(r)}(x) >:$
$j = 1,2,\ldots,m\}$ if $r \in A \cap B \cap C$.

Consequently, we get $(f,A) \cap \%((g,B) \cap \%(h,C)) = ((f,A) \cap \%(g,B)) \cap \%(h,C)$.

(10)-(12) can be proved similarly.

7 Soft rough interval valued intuitionistic fuzzy sets of type-2

In this section we shall consider the lower and upper soft rough approximations of an interval valued intuitionistic fuzzy set of type-2 in a soft approximation space and obtain a new hybrid model called soft rough interval valued intuitionistic fuzzy set of type-2.

Definition 7.1: Let us consider an *intervalvalued intuitionistic fuzzy set of type-2*Γ defined by $\Gamma = \{< x, \mu_\Gamma(x), \gamma_\Gamma(x) >: x \in U\}$ where for $x \in U$, $\mu_\Gamma(x) = \{< u_i(x), \mu_{ui}(x), \gamma_{ui}(x) >: i = 1,2,\ldots,n\}$ and $\gamma_\Gamma(x) = \{< v_j(x), \mu_{vj}(x), \gamma_{vj}(x) >: j = 1,2,\ldots,m\}$ where $u_i(x), v_j(x) \in Int([0,1])$ and $\mu_{ui}, \gamma_{ui}, \mu_{vj}, \gamma_{vj} : U \to Int([0,1])$ are functions such that for $x \in U, 0 \leq sup\mu_{ui}(x) + sup\gamma_{ui}(x) \leq 1$ and $0 \leq sup\mu_{vj}(x) + sup\gamma_{vj}(x) \leq 1$ for each i, j.

Now let $\Theta = (f,A)$ be a full soft set over U i.e; $\cup_{a \in A} f(a) = U$ and $S = (U, \Theta)$ be the soft approximation space. Then the lower and upper soft rough approximations of Γ with respect to S are denoted by $\downarrow {}_2saps_S(\Gamma)$ and $\uparrow {}_2saps_S(\Gamma)$ respectively, which are interval valued intuitionistic fuzzy sets of type-2 in U given by:

$\downarrow {}_2saps_S(\Gamma) = \{< x, \{< u_i(x), [\wedge\{inf\mu_{ui}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \wedge\{sup\mu_{ui}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}], [\vee\{inf\gamma_{ui}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \vee\{sup\gamma_{ui}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}] >: i = 1,2,\ldots,n\}, \{< v_j(x), [\wedge\{inf\mu_{vj}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \wedge\{sup\mu_{vj}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}], [\vee\{inf\gamma_{vj}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \vee\{sup\gamma_{vj}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}] >: j = 1,2,\ldots,m\} >: x \in U\}$, $\uparrow {}_2saps_S(\Gamma) = \{< x, \{< u_i(x), [\vee\{inf\mu_{ui}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \vee\{sup\mu_{ui}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}], [\wedge\{inf\gamma_{ui}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \wedge\{sup\gamma_{ui}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}] >: i = 1,2,\ldots,n\}, \{< v_j(x), [\vee\{inf\mu_{vj}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \vee\{sup\mu_{vj}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}], [\wedge\{inf\gamma_{vj}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \wedge\{sup\gamma_{vj}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}] >: j = 1,2,\ldots,m\} >: x \in U\}$.

The operators and $\downarrow {}_2saps_S$ and $\uparrow {}_2saps_S$ are called the lower and upper soft rough approximation operators on interval valued intuitionistic fuzzy sets of type-2. If $\downarrow {}_2saps_S(\Gamma) = \uparrow {}_2saps_S(\Gamma)$, then Γ is said to be soft definable; otherwise Γ is called a*soft rough interval valued intuitionistic fuzzy set of type-2*.

Example 7.2: Let $U = \{1,2\}$ and $\Gamma \in IVIFS_2(U)$ be defined by $\Gamma = \{< x, \mu_\Gamma(x), \gamma_\Gamma(x) >: x \in U\}$ where for $x \in U, \mu_\Gamma(x) = \{< u_i(x), \mu_{ui}(x), \gamma_{ui}(x) >: i = 1,2\}$ and $\gamma_\Gamma(x) = \{< v_j(x), \mu_{vj}(x), \gamma_{vj}(x) >: j = 1,2\}$, Let,

$$\mu_\Gamma(1) = \{< [0.0,0.1], [0.3,0.4], [0.4,0.5] >, < [0.2,0.5], [0.2,0.4], [0.3,0.5] >\},$$
$$\mu_\Gamma(2) = \{< [0.1,0.2], [0.1,0.2], [0.5,0.7] >, < [0.1,0.3], [0.2,0.4], [0.3,0.4] >\},$$

$$\gamma_\Gamma(1) = \{< [0.1,0.2],[0.1,0.3],[0.3,0.4] >,< [0.0,0.1],[0.1,0.2],[0.5,0.6] >\},$$
$$\gamma_\Gamma(2) = \{< [0.3,0.4],[0.6,0.7],[0.1,0.2] >,< [0.2,0.3],[0.4,0.6],[0.2,0.3] >\}.$$

Now let $\Theta = (f,A)$ be a full soft set over U, where $A = \{a,b\}$ and $f : A \to P(U)$ be defined as $f(a) = \{1,2\}, f(b) = \{2\}$. Then,

$$\downarrow_2 saps_S(\Gamma) = \{< 1,\{< [0.0,0.1],[0.1,0.2],[0.5,0.7] >,< [0.2,0.5],[0.2,0.4],[0.3,0.5] >\},$$
$$\{< [0.1,0.2],[0.1,0.3],[0.3,0.4] >,< [0.0,0.1],[0.1,0.2],[0.5,0.6] >\} >,< 2,\{< [0.1,0.2],[0.1,0.2],[0.5,0.7] >,$$
$$< [0.1,0.3],[0.2,0.4],[0.3,0.5] >\},\{< [0.3,0.4],[0.1,0.3],[0.3,0.4] >,< [0.2,0.3],[0.1,0.2],[0.5,0.6] >\} >\},$$
$$\uparrow_2 saps_S(\Gamma) = \{< 1,\{< [0.0,0.1],[0.3,0.4],[0.4,0.5] >,< [0.2,0.5],[0.2,0.4],[0.3,0.4] >\},$$
$$\{< [0.1,0.2],[0.6,0.7],[0.1,0.2] >,< [0.0,0.1],[0.4,0.6],[0.2,0.3] >\} >,< 2,\{< [0.1,0.2],[0.3,0.4],[0.4,0.5] >,$$
$$< [0.1,0.3],[0.2,0.4],[0.3,0.4] >\},\{< [0.3,0.4],[0.6,0.7],[0.1,0.2] >,< [0.2,0.3],[0.4,0.6],[0.2,0.3] >\} >\}.$$

Since $\downarrow_2 saps_S(\Gamma) \neq \uparrow_2 saps_S(\Gamma)$, then Γ a soft rough interval valued intuitionistic fuzzy set of type-2.

Theorem 7.3: Let $\Theta = (f,A)$ be a full soft set over U and $S = (U,\Theta)$ be the soft approximation space. Let $\Gamma,\Psi \in IVIFS_2(U)$ where Γ and Ψ have been defined in 5.5. Then,

1. $\downarrow_2 saps_S(\Phi*) = \Phi* = \uparrow_2 saps_S(\Phi*)$,
2. $\downarrow_2 saps_S(U*) = U* = \uparrow_2 saps_S(U*)$.
3. $\Gamma \subseteq_2 \Psi \Rightarrow \downarrow_2 saps_S(\Gamma) \subseteq_2 \downarrow_2 saps_S(\Psi)$,
4. $\Gamma \subseteq_2 \Psi \Rightarrow \uparrow_2 saps_S(\Gamma) \subseteq_2 \uparrow_2 saps_S(\Psi)$.
5. $\downarrow_2 saps_S(\Gamma \cap_2 \Psi) \subseteq \downarrow_2 saps_S(\Gamma) \cap_2 \downarrow_2 saps_S(\Psi)$,
6. $\uparrow_2 saps_S(\Gamma \cap_2 \Psi) \subseteq \uparrow_2 saps_S(\Gamma) \cap_2 \uparrow_2 saps_S(\Psi)$,
7. $\downarrow_2 saps_S(\Gamma) \cup_2 \downarrow_2 saps_S(\Psi) \subseteq \downarrow_2 saps_S(\Gamma \cup_2 \Psi)$,
8. $\uparrow_2 saps_S(\Gamma) \cup_2 \uparrow_2 saps_S(\Psi) \subseteq \uparrow_2 saps_S(\Gamma \cup_2 \Psi)$.

Proof:-(1)-(4) are straight forward. (5) We have,

$$\downarrow_2 saps_S(\Gamma \cap_2 \Psi) = \{x,\{< u_i(x) \cap w_i(x),[\wedge\{inf(\mu_{ui}(y) \cap \mu_{wi}(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\},$$
$$\wedge\{sup(\mu_{ui}(y) \cap \mu_{wi}(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\}],[\vee\{inf(\gamma_{ui}(y) \cup \gamma_{wi}(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\},$$
$$\vee\{sup(\gamma_{ui}(y) \cup \gamma_{wi}(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\}] >: i = 1,2,...,n\},$$
$$\{< v_j(x) \cap t_j(x),[\wedge\{inf(\mu_{vj}(y) \cap \mu_{tj}(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\},$$
$$\wedge\{sup(\mu_{vj}(y) \cap \mu_{tj}(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\}],[\vee\{inf(\gamma_{vj}(y) \cup \gamma_{tj}(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\},$$
$$\vee\{sup(\gamma_{vj}(y) \cup \gamma_{tj}(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\}] >: j = 1,2,...,m\} >: x \in U\}.$$

Also we have,

$$\downarrow_2 saps_S(\Gamma) \cap_2 \downarrow_2 saps_S(\Psi) = \{< x,\{< u_i(x),[\wedge\{inf\mu_{ui}(y) : \exists a \in A(\{x,y\}$$
$$\subseteq f(a))\},\wedge\{sup\mu_{ui}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}],[\vee\{inf\gamma_{ui}(y) : \exists$$
$$a \in A(\{x,y\} \subseteq f(a))\},\vee\{sup\gamma_{ui}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}] >: i = 1,2,\ldots,n\},\{< v_j(x),$$
$$[\wedge\{inf\mu_{vj}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\},\wedge\{sup\mu_{vj}(y) : \exists a \in A(\{x,y\}$$
$$\subseteq f(a))\}],[\vee\{inf\gamma_{vj}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\},\vee\{sup\gamma_{vj}(y) : \exists a \in A(\{x,y\}$$

$$
\begin{aligned}
&\subseteq f(a))\}] >: j = 1,2,\ldots,m\} >: x \in U\} \cap_2 \{< x, \{< w_i(x), [\wedge\{inf\mu_{wi}(y) : \exists a \in A(\{x,y\} \\
&\subseteq f(a))\}, \wedge\{sup\mu_{wi}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}], [\vee\{inf\gamma_{wi}(y) : \exists \\
&a \in A(\{x,y\} \subseteq f(a))\}, \vee\{sup\gamma_{wi}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}] >: i = 1,2,\ldots,n\}, \{< t_j(x), \\
&[\wedge\{inf\mu_{tj}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \wedge\{sup\mu_{tj}(y) : \exists a \in A(\{x,y\} \\
&\subseteq f(a))\}], [\vee\{inf\gamma_{tj}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \vee\{sup\gamma_{tj}(y) : \exists a \in A(\{x,y\} \\
&\subseteq f(a))\}] >: j = 1,2,\ldots,m\} >: x \in U\} \\
&= \{< x, \{< u_i(x) \cap w_i(x), [min(\wedge\{inf\mu_{ui}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \\
&\wedge\{inf\mu_{wi}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}), min(\wedge\{sup\mu_{ui}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \\
&\wedge\{sup\mu_{wi}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\})], [max(\vee\{inf\gamma_{ui}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \\
&\vee\{inf\gamma_{wi}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}), max(\vee\{sup\gamma_{ui}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \vee\{sup\gamma_{wi}(y) : \exists \\
&a \in A(\{x,y\} \subseteq f(a))\})] >: i = 1,2,...,n\}, \{< v_j(x) \cap t_j(x), [min(\wedge\{inf\mu_{vj}(y) : \exists a \in A(\{x,y\} \\
&\subseteq f(a))\}, \wedge\{inf\mu_{tj}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}), min(\wedge\{sup\mu_{vj}(y) : \exists a \in A(\{x,y\} \\
&\subseteq f(a))\}, \wedge\{sup\mu_{tj}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\})], [max(\vee\{inf\gamma_{vj}(y) : \exists a \in A(\{x,y\} \\
&\subseteq f(a))\}, \vee\{inf\gamma_{tj}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}), max(\vee\{sup\gamma_{vj}(y) : \exists a \in A(\{x,y\} \\
&\subseteq f(a))\}, \vee\{sup\gamma_{tj}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\})] >: j = 1,2,...,m\} >: x \in U\}
\end{aligned}
$$

Now we have

$$inf(\mu_{ui}(y) \cap \mu_{wi}(y)) = inf([min(inf\mu_{ui}(y), inf\mu_{wi}(y)), min(sup\mu_{ui}(y), sup\mu_{wi}(y))]) = min(inf\mu_{ui}(y), inf\mu_{wi}(y))$$

Therefore,

$$inf(\mu_{ui}(y) \cap \mu_{wi}(y)) \leq inf\mu_{ui}(y) \text{ and } inf(\mu_{ui}(y) \cap \mu_{wi}(y)) \leq inf\mu_{wi}(y).$$

So,

$$
\begin{aligned}
&\wedge\{inf(\mu_{ui}(y) \cap \mu_{wi}(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\} \leq min(\wedge\{inf\mu_{ui}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \\
&\wedge\{inf\mu_{wi}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}).
\end{aligned}
$$

Similarly it can be shown that,

$$
\begin{aligned}
&\wedge\{sup(\mu_{ui}(y) \cap \mu_{wi}(y)) : \exists a \in A(\{x,y\} \subseteq f(a))\} \leq min(\wedge\{sup\mu_{ui}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}, \\
&\wedge\{sup\mu_{wi}(y) : \exists a \in A(\{x,y\} \subseteq f(a))\}).
\end{aligned}
$$

Also

$$inf(\gamma_{ui}(y) \cup \gamma_{wi}(y)) = inf([max(inf\gamma_{ui}(y), inf\gamma_{wi}(y)), max(sup\gamma_{ui}(y), sup\gamma_{wi}(y))]) = max(inf\gamma_{ui}(y), inf\gamma_{wi}(y))$$

Therefore, $inf(\gamma_{ui}(y)\cup\gamma_{wi}(y))\geq inf\gamma_{ui}(y)$, and $inf(\gamma_{ui}(y)\cup\gamma_{wi}(y))\geq inf\gamma_{wi}(y)$. So,

$$\vee\{inf(\gamma_{ui}(y)\cup\gamma_{wi}(y)):\exists a\in A(\{x,y\}\subseteq f(a))\}\geq max(\vee\{inf\gamma_{ui}(y):\exists a\in A(\{x,y\}\subseteq f(a))\},\\ \vee\{inf\gamma_{wi}(y):\exists a\in A(\{x,y\}\subseteq f(a))\}).$$

Similarly it can be shown that,

$$\vee\{sup(\gamma_{ui}(y)\cup\gamma_{wi}(y)):\exists a\in A(\{x,y\}\subseteq f(a))\}\\ \geq max(\vee\{sup\gamma_{ui}(y):\exists a\in A(\{x,y\}\subseteq f(a))\},\vee\{sup\gamma_{wi}(y):\exists a\in A(\{x,y\}\subseteq f(a))\}).$$

In a similar way we can prove the followings:

$$\wedge\{inf(\mu_{vj}(y)\cap\mu_{tj}(y)):\exists a\in A(\{x,y\}\subseteq f(a))\}\leq min(\wedge\{inf\mu_{vj}(y):\exists a\in A(\{x,y\}\subseteq f(a))\},\\ \wedge\{inf\mu_{tj}(y):\exists a\in A(\{x,y\}\subseteq f(a))\}),\\ \wedge\{sup(\mu_{vj}(y)\cap\mu_{tj}(y)):\exists a\in A(\{x,y\}\subseteq f(a))\}\leq min(\wedge\{sup\mu_{vj}(y):\exists a\in A(\{x,y\}\subseteq f(a))\},\\ \wedge\{sup\mu_{tj}(y):\exists a\in A(\{x,y\}\subseteq f(a))\}),\vee\{inf(\gamma_{vj}(y)\cup\gamma_{tj}(y)):\exists a\in A(\{x,y\}\subseteq f(a))\}\\ \geq max(\vee\{inf\gamma_{vj}(y):\exists a\in A(\{x,y\}\subseteq f(a))\},\vee\{inf\gamma_{tj}(y):\exists a\in A(\{x,y\}\subseteq f(a))\}),\\ \vee\{sup(\gamma_{vj}(y)\cup\gamma_{tj}(y)):\exists a\in A(\{x,y\}\subseteq f(a))\}\\ \geq max(\vee\{sup\gamma_{vj}(y):\exists a\in A(\{x,y\}\subseteq f(a))\},\vee\{sup\gamma_{tj}(y):\exists a\in A(\{x,y\}\subseteq f(a))\}).$$

Consequently, $\downarrow_2 saps_S(\Gamma\cap_2\Psi)\subseteq_2\downarrow_2 saps_S(\Gamma)\cap_2\downarrow_2 saps_S(\Psi)$. (vi)-(viii)can be proved similarly.

8 Soft interval valued intuitionistic fuzzy rough sets of type-2

In this section we use an interval valued intuitionistic fuzzy soft set of type-2 to granulate the universe of discourse and obtain a new hybrid model called soft interval valued intuitionistic fuzzy rough set of type-2 which can be seen as an extension of soft rough interval valued intuitionistic fuzzy set of type-2.

Definition 8.1: Let $\Omega=(f,A)$ be an interval valued intuitionistic fuzzy soft set of type-2 over U, where $f:A\rightarrow IVIFS_2(U)$ is defined by

$$f(a)=\{<x,\mu_{f(a)}(x),\gamma_{f(a)}(x)>:x\in U\},\text{ where for }x\in U$$

$$\mu_{f(a)}(x)=\{<u_i(a)(x),\mu_{ui(a)}(x),\gamma_{ui(a)}(x)>:i=1,2,\ldots,n\}$$

and

$$\gamma_{f(a)}(x)=\{<v_j(a)(x),\mu_{vj(a)}(x),\gamma_{vj(a)}(x)>:j=1,2,\ldots,m\}.$$

Then the pair $SIVIF_2=(U,\Omega)$ is called a soft interval valued intuitionistic fuzzytype-2 approximation space. Let $\Gamma\in IVIFS_2(U)$ be defined by $\Gamma=\{<x,\mu_\Gamma(x),\gamma_\Gamma(x)>:x\in U\}$ where for $x\in U$,

$$\mu_\Gamma(x)=\{<u_i*(x),\mu_{ui}*(x),\gamma_{ui}*(x)>:i=1,2,\ldots,n\}and\gamma_\Gamma(x)=\{<v_j*(x),\mu_{vj}*(x),\gamma_{vj}*(x)>:j=1,2,\ldots,m\}.$$

Then the lower and upper soft interval valued intuitionistic fuzzy rough approximations of $\Gamma w.r.t SIVIF_2$ are denoted respectively by $\downarrow_2 Apr_{SIVIF2}(\Gamma)$ and $\uparrow_2 Apr_{SIVIF2}(\Gamma)$ which are interval valued intuitionistic fuzzy sets of type-2 in U, given by

$$\downarrow {}_2Apr_{SIVIF2}(\Gamma) = \{< x, \mu_{\downarrow 2AprSIVIF2(\Gamma)}(x), \gamma_{\downarrow 2AprSIVIF2(\Gamma)}(x) >: x \in U\}$$
$$\uparrow {}_2Apr_{SIVIF2}(\Gamma) = \{< x, \mu_{\uparrow 2AprSIVIF2(\Gamma)}(x), \gamma_{\uparrow 2AprSIVIF2(\Gamma)}(x) >: x \in U\}$$

where for $x \in U$,

$\mu_{\downarrow 2AprSIVIF2(\Gamma)}(x) = \{< u_i*(x), [\wedge_{a\in A}(inf\mu_{ui(a)}(x) \wedge inf\mu_{ui}*(x)), \wedge_{a\in A}(sup\mu_{ui(a)}(x) \wedge sup\mu_{ui}*(x))],$
$[\wedge_{a\in A}(inf\gamma_{ui(a)}(x) \vee inf\gamma_{ui}*(x)), \wedge_{a\in A}(sup\gamma_{ui(a)}(x) \vee sup\gamma_{ui}*(x))] >: i = 1,2,\ldots\ldots,n\}$
$\gamma_{\downarrow 2AprSIVIF2(\Gamma)}(x) = \{< v_j*(x), [\wedge_{a\in A}(inf\mu_{vj(a)}(x) \wedge inf\mu_{vj}*(x)), \wedge_{a\in A}(sup\mu_{vj(a)}(x) \wedge sup\mu_{vj}*(x))],$
$[\wedge_{a\in A}(inf\gamma_{vj(a)}(x) \vee inf\gamma_{vj}*(x)), \wedge_{a\in A}(sup\gamma_{vj(a)}(x) \vee sup\gamma_{vj}*(x))] >: j = 1,2,\ldots..,m\}$
$\mu_{\uparrow 2AprSIVIF2(\Gamma)}(x) = \{< u_i*(x), [\wedge_{a\in A}(inf\mu_{ui(a)}(x) \vee inf\mu_{ui}*(x)), \wedge_{a\in A}(sup\mu_{ui(a)}(x) \vee sup\mu_{ui}*(x))],$
$[\wedge_{a\in A}(inf\gamma_{ui(a)}(x) \wedge inf\gamma_{ui}*(x)), \wedge_{a\in A}(sup\gamma_{ui(a)}(x) \wedge sup\gamma_{ui}*(x))] >: i = 1,2,\ldots\ldots,n\} \gamma_{\uparrow 2AprSIVIF2(\Gamma)}(x) = \{< v_j*(x),$
$[\wedge_{a\in A}(inf\mu_{vj(a)}(x) \vee inf\mu_{vj}*(x)), \wedge_{a\in A}(sup\mu_{vj(a)}(x) \vee sup\mu_{vj}*(x))],$
$[\wedge_{a\in A}(inf\gamma_{vj(a)}(x) \wedge inf\gamma_{vj}*(x)), \wedge_{a\in A}(sup\gamma_{vj(a)}(x) \wedge sup\gamma_{vj}*(x))] >: j = 1,2,\ldots..,m\}$

If $\downarrow {}_2Apr_{SIVIF2}(\Gamma) = \uparrow {}_2Apr_{SIVIF2}(\Gamma)$, then Γ is called soft interval valued intuitionistic fuzzy type-2 definable; otherwise Γ is called a soft interval valued intuitionistic fuzzy rough set of type-2.

Example 8.2: Let $\Omega = (f,A)$ be an interval valued intuitionistic fuzzy soft set of type-2 over U, where $U = \{1,2\}, A = \{a,b\}$ and $f : A \to IVIFS_2(U)$ is defined by

$$f(a) = \{< 1, \mu_{f(a)}(1), \gamma_{f(a)}(1) >, < 2, \mu_{f(a)}(2), \gamma_{f(a)}(2) >\}$$

and

$$f(b) = \{< 1, \mu_{f(b)}(1), \gamma_{f(b)}(1) >, < 2, \mu_{f(b)}(2), \gamma_{f(b)}(2) >\},$$

where

$$\mu_{f(a)}(1) = \{< [0.1,0.2],[0.4,0.5],[0.2,0.3] >, < [0.3,0.5],[0.1,0.4],[0.5,0.6] >\},$$
$$\mu_{f(a)}(2) = \{< [0.7,0.8],[0.1,0.2],[0.3,0.4] >, < [0.5,0.6],[0.3,0.6],[0.2,0.4] >\},$$
$$\gamma_{f(a)}(1) = \{< [0.4,0.5],[0.5,0.6],[0.2,0.3] >, < [0.3,0.4],[0.2,0.6],[0.1,0.3] >\},$$
$$\gamma_{f(a)}(2) = \{< [0.3,0.5],[0.2,0.4],[0.2,0.4] >, < [0.1,0.4],[0.5,0.6],[0.2,0.3] >\},$$
$$\mu_{f(b)}(1) = \{< [0.7,0.9],[0.1,0.2],[0.7,0.8] >, < [0.1,0.3],[0.4,0.6],[0.1,0.2] >\},$$
$$\mu_{f(b)}(2) = \{< [0.5,0.7],[0.5,0.7],[0.1,0.2] >, < [0.3,0.6],[0.2,0.4],[0.3,0.5] >\},$$
$$\gamma_{f(b)}(1) = \{< [0.4,0.6],[0.3,0.4],[0.2,0.6] >, < [0.5,0.6],[0.2,0.8],[0.1,0.2] >\},$$
$$\gamma_{f(b)}(2) = \{< [0.3,0.7],[0.1,0.3],[0.1,0.5] >, < [0.2,0.5],[0.2,0.5],[0.3,0.4] >\}.$$

Let $\Gamma \in IVIFS_2(U)$ be defined by $\Gamma = \{< 1, \mu_\Gamma(1), \gamma_\Gamma(1) >, < 2, \mu_\Gamma(2), \gamma_\Gamma(2) >\}$ where

$$\mu_\Gamma(1) = \{< [0.2,0.3],[0.4,0.6],[0.1,0.3] >, < [0.2,0.6],[0.1,0.2],[0.6,0.7] >\},$$
$$\mu_\Gamma(2) = \{< [0.5,0.7],[0.2,0.4],[0.3,0.5] >, < [0.1,0.2],[0.4,0.6],[0.2,0.4] >\},$$

$$\gamma_{\Gamma}(1) = \{< [0.6,0.8],[0.1,0.2],[0.3,0.6] >,< [0.4,0.5],[0.2,0.5],[0.3,0.4] >\},$$
$$\gamma_{\Gamma}(2) = \{< [0.4,0.5],[0.5,0.6],[0.1,0.2] >,< [0.6,0.7],[0.5,0.7],[0.1,0.2] >\}.$$

Then,

$$\downarrow_2 Apr_{SIVIF2}(\Gamma) = \{< 1,\mu_{\downarrow 2AprSIVIF2(\Gamma)}(1),\gamma_{\downarrow 2AprSIVIF2(\Gamma)}(1) >,< 2,\mu_{\downarrow 2AprSIVIF2(\Gamma)}(2),\gamma_{\downarrow 2AprSIVIF2(\Gamma)}(2) >\},$$
$$\uparrow_2 Apr_{SIVIF2}(\Gamma) = \{< 1,\mu_{\uparrow 2AprSIVIF2(\Gamma)}(1),\gamma_{\uparrow 2AprSIVIF2(\Gamma)}(1) >,< 2,\mu_{\uparrow 2AprSIVIF2(\Gamma)}(2),\gamma_{\uparrow 2AprSIVIF2(\Gamma)}(2) >\},$$

where

$$\mu_{\downarrow 2AprSIVIF2(\Gamma)}(1) = \{< [0.2,0.3],[0.1,0.2],[0.2,0.3] >,< [0.2,0.6],[0.1,0.2],[0.6,0.7] >\},$$
$$\mu_{\downarrow 2AprSIVIF2(\Gamma)}(2) = \{< [0.5,0.7],[0.1,0.2],[0.1,0.2] >,< [0.1,0.2],[0.2,0.4],[0.2,0.4] >\},$$
$$\gamma_{\downarrow 2AprSIVIF2(\Gamma)}(1) = \{< [0.6,0.8],[0.1,0.2],[0.3,0.6] >,< [0.4,0.5],[0.2,0.5],[0.3,0.4] >\},$$
$$\gamma_{\downarrow 2AprSIVIF2(\Gamma)}(2) = \{< [0.4,0.5],[0.1,0.3],[0.1,0.4] >,< [0.6,0.7],[0.2,0.5],[0.2,0.3] >\},$$
$$\mu_{\uparrow 2AprSIVIF2(\Gamma)}(1) = \{< [0.2,0.3],[0.4,0.6],[0.1,0.3] >,< [0.2,0.6],[0.1,0.4],[0.1,0.2] >\},$$
$$\mu_{\uparrow 2AprSIVIF2(\Gamma)}(2) = \{< [0.5,0.7],[0.2,0.4],[0.3,0.5] >,< [0.1,0.2],[0.4,0.6],[0.2,0.4] >\},$$
$$\gamma_{\uparrow 2AprSIVIF2(\Gamma)}(1) = \{< [0.6,0.8],[0.3,0.4],[0.2,0.3] >,< [0.4,0.5],[0.2,0.6],[0.1,0.2] >\},$$
$$\gamma_{\uparrow 2AprSIVIF2(\Gamma)}(2) = \{< [0.4,0.5],[0.5,0.6],[0.1,0.2] >,< [0.6,0.7],[0.5,0.7],[0.1,0.2] >\}.$$

Then clearly $\downarrow_2 Apr_{SIVIF2}(\Gamma) \neq \uparrow_2 Apr_{SIVIF2}(\Gamma)$. So Γ is a soft interval valued intuitionistic fuzzy rough set of type-2 over U.

Theorem 8.3: Let $\Omega = (f,A)$ be an interval valued intuitionistic fuzzy universal soft set of type-2 over U and $SIVIF_2 = (U,\Omega)$ soft interval valued intuitionistic fuzzy type-2 approximation space. Then for $\Gamma,\Psi \in IVIFS_2(U)$, we have

1. $\downarrow_2 Apr_{SIVIF2}(\Phi*) = \Phi*$
2. $\uparrow_2 Apr_{SIVIF2}(U*) = U*$
3. $\Gamma \subseteq_2 \Psi \Rightarrow \downarrow_2 Apr_{SIVIF2}(\Gamma) \subseteq_2 \downarrow_2 Apr_{SIVIF2}(\Psi)$
4. $\Gamma \subseteq_2 \Psi \Rightarrow \uparrow_2 Apr_{SIVIF2}(\Gamma) \subseteq_2 \uparrow_2 Apr_{SIVIF2}(\Psi)$
5. $\downarrow_2 Apr_{SIVIF2}(\Gamma \cap_2 \Psi) \subseteq_2 \downarrow_2 Apr_{SIVIF2}(\Gamma) \cap_2 \downarrow_2 Apr_{SIVIF2}(\Psi)$
6. $\uparrow_2 Apr_{SIVIF2}(\Gamma \cap_2 \Psi) \subseteq_2 \uparrow_2 Apr_{SIVIF2}(\Gamma) \cap_2 \uparrow_2 Apr_{SIVIF2}(\Psi)$
7. $\downarrow_2 Apr_{SIVIF2}(\Gamma) \cup_2 \downarrow_2 Apr_{SIVIF2}(\Psi) \subseteq_2 \downarrow_2 Apr_{SIVIF2}(\Gamma \cup_2 \Psi)$
8. $\uparrow_2 Apr_{SIVIF2}(\Gamma) \cup_2 \uparrow_2 Apr_{SIVIF2}(\Psi) \subseteq_2 \uparrow_2 Apr_{SIVIF2}(\Gamma \cup_2 \Psi)$.

Proof:(1)-(4) are straight forward.(5) Let Γ and Ψ be defined as follows: $\Gamma = \{< x,\mu_{\Gamma}(x),\gamma_{\Gamma}(x) >: x \in U\}$ where for $x \in U$,

$$\mu_{\Gamma}(x) = \{< u_i*(x),\mu_{ui}*(x),\gamma_{ui}*(x) >: i = 1,2,\ldots,n\}$$

and

$$\gamma_{\Gamma}(x) = \{< v_j*(x),\mu_{vj}*(x),\gamma_{vj}*(x) >: j = 1,2,\ldots,m\}$$

and

$$\Psi = \{< x,\mu_{\Psi}(x),\gamma_{\Psi}(x) >: x \in U\}$$

where for $x \in U$,

$$\mu_{\Psi}(x) = \{< w_i * (x), \mu_{wi} * (x), \gamma_{wi} * (x) >: i = 1, 2, \ldots, n\}$$

and

$$\gamma_{\Psi}(x) = \{< t_j * (x), \mu_{tj} * (x), \gamma_{tj} * (x) >: j = 1, 2, \ldots, m\}.$$

Let $f : A \to IVIFS_2(U)$ be defined by

$$f(a) = \{< x, \mu_{f(a)}(x), \gamma_{f(a)}(x) >: x \in U\},$$

wherefor $x \in U$

$$\mu_{f(a)}(x) = \{< u_i(a)(x), \mu_{ui(a)}(x), \gamma_{ui(a)}(x) >: i = 1, 2, \ldots, n\}$$

and

$$\gamma_{f(a)}(x) = \{< v_j(a)(x), \mu_{vj(a)}(x), \gamma_{vj(a)}(x) >: j = 1, 2, \ldots, m\}.$$

Then

$$\Gamma \cap_2 \Psi = \{< x, \{< u_i * (x) \cap w_i * (x), \mu_{ui} * (x) \cap \mu_{wi} * (x), \gamma_{ui} * (x) \cup \gamma_{wi} * (x) >: i = 1, 2, ..., n\},$$

$$\{< v_j * (x) \cap t_j * (x), \mu_{vj} * (x) \cap \mu_{tj} * (x), \gamma_{vj} * (x) \cup \gamma_{tj} * (x) >: j = 1, 2, ..., m\} >: x \in U\}.$$

We have

$$\begin{aligned}
\downarrow_2 Apr_{SIVIF2}(\Gamma \cap_2 \Psi) = \{&< x, \{< u_i * (x) \cap w_i * (x), [\wedge_{a\in A}(inf\mu_{ui(a)}(x) \wedge inf(\mu_{ui} * (x) \cap \mu_{wi} * (x))), \\
&\wedge_{a\in A}(sup\mu_{ui(a)}(x) \wedge sup(\mu_{ui} * (x) \cap \mu_{wi} * (x)))], [\wedge_{a\in A}(inf\gamma_{ui(a)}(x) \vee inf(\gamma_{ui} * (x) \cup \gamma_{wi} * (x))), \\
&\wedge_{a\in A}(sup\gamma_{ui(a)}(x) \vee sup(\gamma_{ui} * (x) \cup \gamma_{wi} * (x)))] >: i = 1, 2, ..., n\}, \{< v_j * (x) \cap t_j * (x), \\
&[\wedge_{a\in A}(inf\mu_{vj(a)}(x) \wedge inf(\mu_{vj} * (x) \cap \mu_{tj} * (x))), \wedge_{a\in A}(sup\mu_{vj(a)}(x) \wedge sup(\mu_{vj} * (x) \cap \mu_{tj} * (x)))], \\
&[\wedge_{a\in A}(inf\gamma_{vj(a)}(x) \vee inf(\gamma_{vj} * (x) \cup \gamma_{tj} * (x))), \wedge_{a\in A}(sup\gamma_{vj(a)}(x) \\
&\vee sup(\gamma_{vj} * (x) \cup \gamma_{tj} * (x)))] >: j = 1, 2, ..., m\} >: x \in U\} \\
=\{&< x, \{< u_i * (x) \cap w_i * (x), [\wedge_{a\in A}(inf\mu_{ui(a)}(x) \wedge min(inf\mu_{ui} * (x), inf\mu_{wi} * (x))), \\
&\wedge_{a\in A}(sup\mu_{ui(a)}(x) \wedge min(sup\mu_{ui} * (x), sup\mu_{wi} * (x)))], [\wedge_{a\in A}(inf\gamma_{ui(a)}(x) \\
&\vee max(inf\gamma_{ui} * (x), inf\gamma_{wi} * (x))), \wedge_{a\in A}(sup\gamma_{ui(a)}(x) \\
&\vee max(sup\gamma_{ui} * (x), sup\gamma_{wi} * (x)))] >: i = 1, 2, ..., n\}, \\
&\{< v_j * (x) \cap t_j * (x), [\wedge_{a\in A}(inf\mu_{vj(a)}(x) \wedge min(inf\mu_{vj} * (x), inf\mu_{tj} * (x))), \wedge_{a\in A}(sup\mu_{vj(a)}(x) \\
&\wedge min(sup\mu_{vj} * (x), sup\mu_{tj} * (x)))], [\wedge_{a\in A}(inf\gamma_{ui(a)}(x) \vee max(inf\gamma_{vj} * (x) \cup inf\gamma_{tj} * (x)), \\
&\wedge_{a\in A}(sup\gamma_{ui(a)}(x) \vee max(sup\gamma_{vj} * (x), sup\gamma_{tj} * (x)))] >: j = 1, 2, ..., m\} >: x \in U\}
\end{aligned}$$

Again we have,

$$\begin{aligned}
&\downarrow_2 Apr_{SIVIF2}(\Gamma) \cap_2 \downarrow_2 Apr_{SIVIF2}(\Psi) \\
=&\{< x, \{< u_i * (x), [\wedge_{a\in A}(inf\mu_{ui(a)}(x) \wedge inf\mu_{ui} * (x)), \wedge_{a\in A}(sup\mu_{ui(a)}(x) \wedge sup\mu_{ui} * (x))], \\
&[\wedge_{a\in A}(inf\gamma_{ui(a)}(x) \vee inf\gamma_{ui} * (x)), \wedge_{a\in A}(sup\gamma_{ui(a)}(x) \vee sup\gamma_{ui} * (x))] >: i = 1, 2, \ldots\ldots, n\}, \{< v_j * (x),
\end{aligned}$$

$$[\wedge_{a\in A}(inf\mu_{vj(a)}(x)\wedge inf\mu_{vj}*(x)),\wedge_{a\in A}(sup\mu_{vj(a)}(x)\wedge sup\mu_{vj}*(x))],$$
$$[\wedge_{a\in A}(inf\gamma_{vj(a)}(x)\vee inf\gamma_{vj}*(x)),\wedge_{a\in A}(sup\gamma_{vj(a)}(x)\vee sup\gamma_{vj}*(x))]>: j=1,2,\ldots..,m\}>: x\in U\}\cap_2\{<x,\{<w_i*(x),$$
$$[\wedge_{a\in A}(inf\mu_{ui(a)}(x)\wedge inf\mu_{wi}*(x)),\wedge_{a\in A}(sup\mu_{ui(a)}(x)\wedge sup\mu_{wi}*(x))],$$
$$[\wedge_{a\in A}(inf\gamma_{ui(a)}(x)\vee inf\gamma_{wi}*(x)),\wedge_{a\in A}(sup\gamma_{ui(a)}(x)\vee sup\gamma_{wi}*(x))]>: i=1,2,\ldots\ldots,n\},\{<t_j*(x),$$
$$[\wedge_{a\in A}(inf\mu_{vj(a)}(x)\wedge inf\mu_{tj}*(x)),\wedge_{a\in A}(sup\mu_{vj(a)}(x)\wedge sup\mu_{tj}*(x))],[\wedge_{a\in A}(inf\gamma_{vj(a)}(x)\vee inf\gamma_{tj}*(x)),$$
$$\wedge_{a\in A}(sup\gamma_{vj(a)}(x)\vee sup\gamma_{tj}*(x))]>: j=1,2,\ldots..,m\}>: x\in U\}$$
$$=\{<x,\{<u_i*(x)\cap w_i*(x),[min(\wedge_{a\in A}(inf\mu_{ui(a)}(x)\wedge inf\mu_{ui}*(x)),\wedge_{a\in A}(inf\mu_{ui(a)}(x)\wedge inf\mu_{wi}*(x))),$$
$$min(\wedge_{a\in A}(sup\mu_{ui(a)}(x)\wedge sup\mu_{ui}*(x)),\wedge_{a\in A}(sup\mu_{ui(a)}(x)\wedge sup\mu_{wi}*(x)))],$$
$$[max(\wedge_{a\in A}(inf\gamma_{ui(a)}(x)\vee inf\gamma_{ui}*(x)),\wedge_{a\in A}(inf\gamma_{ui(a)}(x)\vee inf\gamma_{wi}*(x))),$$
$$max(\wedge_{a\in A}(sup\gamma_{ui(a)}(x)\vee sup\gamma_{ui}*(x)),\wedge_{a\in A}(sup\gamma_{ui(a)}(x)\vee sup\gamma_{wi}*(x)))]>: i=1,2,\ldots\ldots,n\},\{<v_j*(x)\cap t_j*(x),$$
$$[min(\wedge_{a\in A}(inf\mu_{vj(a)}(x)\wedge inf\mu_{vj}*(x)),\wedge_{a\in A}(inf\mu_{vj(a)}(x)\wedge inf\mu_{tj}*(x))),$$
$$min(\wedge_{a\in A}(sup\mu_{vj(a)}(x)\wedge sup\mu_{vj}*(x)),\wedge_{a\in A}(sup\mu_{vj(a)}(x)$$
$$\wedge sup\mu_{tj}*(x)))],[max(\wedge_{a\in A}(inf\gamma_{vj(a)}(x)\vee inf\gamma_{vj}*(x)),\wedge_{a\in A}(inf\gamma_{vj(a)}(x)$$
$$\vee inf\gamma_{tj}*(x))),max(\wedge_{a\in A}(sup\gamma_{vj(a)}(x)\vee sup\gamma_{vj}*(x)),$$
$$\wedge_{a\in A}(sup\gamma_{vj(a)}(x)\vee sup\gamma_{tj}*(x)))]>: j=1,2,\ldots..,m\}>: x\in U\}$$

Nowit can be shown that

$$\wedge_{a\in A}(inf\mu_{ui(a)}(x)\wedge min(inf\mu_{ui}*(x),inf\mu_{wi}*(x)))\leq min(\wedge_{a\in A}(inf\mu_{ui(a)}(x)\wedge inf\mu_{ui}*(x)),\wedge_{a\in A}(inf\mu_{ui(a)}(x)$$
$$\wedge inf\mu_{wi}*(x))),\wedge_{a\in A}(sup\mu_{ui(a)}(x)\wedge min(sup\mu_{ui}*(x),sup\mu_{wi}*(x)))\leq min(\wedge_{a\in A}(sup\mu_{ui(a)}(x)$$
$$\wedge sup\mu_{ui}*(x)),\wedge_{a\in A}(sup\mu_{ui(a)}(x)\wedge sup\mu_{wi}*(x))),\wedge_{a\in A}(inf\gamma_{ui(a)}(x)$$
$$\vee max(inf\gamma_{ui}*(x),inf\gamma_{wi}*(x)))\geq max(\wedge_{a\in A}(inf\gamma_{ui(a)}(x)\vee inf\gamma_{ui}*(x)),$$
$$\wedge_{a\in A}(inf\gamma_{ui(a)}(x)\vee inf\gamma_{wi}*(x))),\wedge_{a\in A}(sup\gamma_{ui(a)}(x)\vee max(sup\gamma_{ui}*(x),sup\gamma_{wi}*(x)))$$
$$\geq max(\wedge_{a\in A}(sup\gamma_{ui(a)}(x)\vee sup\gamma_{ui}*(x)),\wedge_{a\in A}(sup\gamma_{ui(a)}(x)\vee sup\gamma_{wi}*(x))),$$
$$\wedge_{a\in A}(inf\mu_{ui(a)}(x)\wedge min(inf\mu_{vj}*(x),inf\mu_{tj}*(x))\leq min(\wedge_{a\in A}(inf\mu_{vj(a)}(x)\wedge inf\mu_{vj}*(x)),$$
$$\wedge_{a\in A}(inf\mu_{vj(a)}(x)\wedge inf\mu_{tj}*(x))),\wedge_{a\in A}(sup\mu_{ui(a)}(x)\wedge min(sup\mu_{vj}*(x),$$
$$sup\mu_{tj}*(x))\leq min(\wedge_{a\in A}(sup\mu_{vj(a)}(x)\wedge sup\mu_{vj}*(x)),\wedge_{a\in A}(sup\mu_{vj(a)}(x)\wedge sup\mu_{tj}*(x))),$$
$$\wedge_{a\in A}(inf\gamma_{ui(a)}(x)\vee max(inf\gamma_{vj}*(x),inf\gamma_{tj}*(x)))\geq max(\wedge_{a\in A}(inf\gamma_{vj(a)}(x)\vee inf\gamma_{vj}*(x)),$$
$$\wedge_{a\in A}(inf\gamma_{vj(a)}(x)\vee inf\gamma_{tj}*(x))),\wedge_{a\in A}(sup\gamma_{ui(a)}(x)\vee max(sup\gamma_{vj}*(x),sup\gamma_{tj}*(x)))$$
$$\geq max(\wedge_{a\in A}(sup\gamma_{vj(a)}(x)\vee sup\gamma_{vj}*(x)),\wedge_{a\in A}(sup\gamma_{vj(a)}(x)\vee sup\gamma_{tj}*(x))).$$

Using the above results we get, $\downarrow_2 Apr_{SIVIF2}(\Gamma\cap_2\Psi)\subseteq_2\downarrow_2 Apr_{SIVIF2}(\Gamma)\cap_2\downarrow_2 Apr_{SIVIF2}(\Psi)$. (6)-(8) can be proved similarly.

9 Conclusion

Inthis paper we have investigated the problem of combining interval valued intuitionistic fuzzy set of type-2 with soft sets and rough setsand we obtain three different types of hybrid models, namely-interval valued intuitionistic fuzzy soft sets of type-2, soft rough interval valued intuitionistic fuzzy sets of type-2 and soft interval valued intuitionistic fuzzy rough sets of type-2.We have also investigated some basic properties of these new hybridizations.

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