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A Fuzzy programming approach for interval multiobjective solid transportation problem

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Abstract: This paper presents a fuzzy programming approach for solving Interval Multiobjective Solid Transportation Problem (IMOSTP). In real world application, IMOSTP appears to be more realistic than a conventional Solid Transportation Problem (STP) as available data is uncertain. In such a problem the solution process is very complex. By applying the order relation on the intervals, it is first transformed into a crisp multiobjective solid transportation problem. After determining the individual optimal solution of each objective, a fuzzy programming approach is constructed to achieve the Pareto optimal solution of IMOSTP. Finally, a numerical example is illustrated to demonstrate the feasibility of the presented solution procedure.

Keywords: Multiobjective programming, transportation problem, interval programming, Fuzzy programming, solid transportation problem.

1 Introduction

In the area of mathematical programming, Transportation Problem (TP) is one of the well-known problems. The common mathematical type of TP can be expressed in the form of a Linear Programming (LP) and can be efficiently solved by applying different traditional mathematical programming methods. Practical procedures for solving TP obtained from simplex algorithm were introduced in 1963 by Dantzig [1].

Many different methods [2,3] have been presented for solving TP. In TP, two types of constraints are investigated to be source constraint and destination constraint. But in practical applications, besides these two constraints, we encounter the third type of constraint, which is known as, transportation constraint or product type constraint. This third constraint is a different type of transportation modes (conveyances) such as trucks, cargo flights, goods trains, ships, etc. In this position, TP converts Solid Transportation Problem (STP). STP was introduced by Shell [4] and Haley [5] proposed the MODI method to solve STP. Afterward, various author's [6,7,8,9,10] have introduced various methods for solving STP in crisp as well as in the uncertain environment.

In order to deal with uncertainty, fuzzy set theory has been extensively employed to illustrate vagueness and impreciseness in a decision process. Fuzzy set theory previously presented by Zadeh [17]. Furthermore, the theory of interval numbers is suited to discuss vagueness and uncertainty. Besides, in most of the mathematical programming problems, the objectives, constraints or the parameters are determined on the basis of certain forecasting by experts with their former practices. In such cases, each experiment can be easily illustrated with interval numbers.

The interval theory was first developed by Moore [11] and redefined by Moore in [12]. Ishibuchi and Tanaka [13]

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presented some order relation between intervals for solving interval linear programming problems by converting those into a crisp multiobjective programming problem. Many authors [14,15,16] considered another order relations between intervals for solving LP.

In this paper, a fuzzy programming approach based on the theory of interval numbers is employed to solve the IMOSTP. In the first step, the considered interval multiobjective solid transportation problem is converted into a crisp MOSTP. Then the best and worst solutions of each objective function are determined. In next step, the method based on fuzzy programming tries to reach the better compromise solution which simultaneously satisfied different objectives. Finally, the results of recommended fuzzy approach are compared with the existing fuzzy approach [18].

The rest of this paper is organized as follows. In section 2 and section 3, some preliminary data about interval programming is presented and an IMOSTP is formulated in section 4.Then a fuzzy programming approach in section 5 is constructed for solving IMOSTP. Finally, a numerical example is given to illustrate the efficiency of the proposed method in Section 6.

2 Preliminaries

In this section, we will give some basic definitions and concepts about the theory of interval numbers and STP.

2.1 Interval Numbers

Definition 1. *(Moore [11]) An interval number is a number whose exact value is unknown, but a range within which the value lies is known. Interval number is a number with both lower and upper bounds* $X \in \left[\underline{x}, \overline{x}\right]$ *where* $\underline{x} \leq \overline{x}$.
The main arithmetic operations on interval numbers can be defined as follows.

Definition 2. *(Moore [11]) Let*

$$\tilde{x}_1 = \left[\underline{x_1}, \overline{x_1}\right] \text{ and } \tilde{x}_2 = \left[\underline{x_2}, \overline{x_2}\right]$$

be two closed interval numbers. The following notations can be satisfied,

$$\tilde{x}_1 + \tilde{x}_2 = \left[\underline{x_1} + \underline{x_2}, \overline{x_1} + \overline{x_2}\right]$$
$$\tilde{x}_1 - \tilde{x}_2 = \left[\underline{x_1} - \overline{x_2}, \overline{x_1} - \underline{x_2}\right]$$
$$\tilde{x}_1 * \tilde{x}_2 = \left[\min\left(\underline{x_1}\underline{x_2}, \underline{x_1}\overline{x_2}, \overline{x_1}\underline{x_2}, \overline{x_1}\overline{x_2}\right), \max\left(\underline{x_1}\underline{x_2}, \underline{x_1}\overline{x_2}, \overline{x_1}\underline{x_2}, \overline{x_1}\overline{x_2}\right)\right]$$
$$\tilde{x}_1 \div \tilde{x}_2 = \left[\underline{x_1}, \overline{x_1}\right] \frac{1}{\left[\overline{x_2}, \underline{x_2}\right]}.$$

When $X \in \left[\underline{x}, \overline{x}\right]$ *is an interval number, its absolute value is the maximum of the absolute value of its endpoints:*

$$|x| = \max\left(\left|\underline{x}\right|, \left|\overline{x}\right|\right).$$

Definition 3. *The center x_c and x_w of an interval number of $X \in \left[\underline{x}, \overline{x}\right]$,*

$$x_c = \frac{1}{2}\left[\underline{x} + \overline{x}\right]$$
$$x_w = \frac{1}{2}\left[\overline{x} - \underline{x}\right].$$

It is easily verifiable that $\overline{x} = x_c + x_w$ and $\underline{x} = x_c - x_w$. Also, Ishibuchi and Tanaka [13] defined the following order relations between intervals.

Definition 4. *(Ishibuchi and Tanaka [13]) Let*

$$\tilde{x} = \left[\underline{x}, \overline{x}\right] \text{ and } \tilde{y} = \left[\underline{y}, \overline{y}\right]$$

are two closed interval numbers and then the order relation $\leq_{LR}$ is defined as,

$$\tilde{x} \leq_{LR} \tilde{y} \Leftrightarrow \underline{x} \leq \underline{y} \text{ and } \overline{x} \leq \overline{y}$$

$$\tilde{x} \leq_{LR} \tilde{y} \Leftrightarrow \underline{x} \leq_{LR} \underline{y} \text{ and } \overline{x} \neq \overline{y}.$$

Definition 5. *(Ishibuchi and Tanaka [14]) The order relation $\leq_{CW}$ between two interval numbers*

$$\tilde{x} = \left[\underline{x}, \overline{x}\right] \text{ and } \tilde{y} = \left[\underline{y}, \overline{y}\right]$$

is defined as,

$$\tilde{x} \leq_{CW} \tilde{y} \Leftrightarrow x_C \leq y_C \text{ and } x_W \leq y_W.$$
$$\tilde{x} \leq_{CW} \tilde{y} \Leftrightarrow \tilde{x} \leq_{CW} \tilde{y} \text{ and } x \neq y.$$

The order relations $\leq_{CW}$ and $\leq_{LR}$ never conflicts with each other. Similarly, Ishibuchi and Tanaka [13] introduced $\leq_{LR}^{*}$ and $\leq_{CW}^{*}$.

2.2 Solid transportation problem

Suppose that there are m sources, n destinations and l conveyances. The idea of the STP is to design a transportation plan so that the transportation cost is minimized. In order to develop the interval programming model for the STP, the following concepts are satisfied,

$i = \{1, 2, ..., m\}$ index of origins.

$j = \{1, 2, ..., n\}$ index of destinations.

$k = \{1, 2, ..., l\}$ index of conveyances or different modes of transportation.

x_{ijk} = amount of k^{th} type of commodity transported from the i^{th} origin to j^{th} destination.

c_{ijk} = the variable cost per unit amount of k^{th} type of commodity transported from the i^{th} origin to j^{th} destination which is independent of the amount of the commodity transported.

a_i the total quantity of k^{th} type of commodity received by j^{th} destination from all the sources.

b_j the total quantity of k^{th} type of commodity available at the i^{th} origin to be supplied to all destinations.

e_k the total quantity of all types of commodities to be supplied from i^{th} origin to be j^{th} destination.

In fact, in a transportation problem, the total amount transported from origin i is no more than a_i. Thus, the first constraint can be formulated as

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \le a_i, i = 1,2,...,m.$$

Also, the total amount transported from m origins should satisfy the demand of destination j. Then the second constraint can be formulated as

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \ge b_j, j = 1,2,...,n.$$

Furthermore, the total amount transported by conveyance k is no more than its transportation capacity. Thus, the last constraints can be given as

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le e_k, k = 1,2,...,l.$$

Then, the mathematical model of a STP can be formulated as follows:

$$\begin{cases} \min \left(\sum\limits_{i=1}^{m} \sum\limits_{j=1}^{n} \sum\limits_{k=1}^{l} c_{ijk} x_{ijk} \right) \\ s.t. \begin{cases} \sum\limits_{j=1}^{n} \sum\limits_{k=1}^{l} x_{ijk} \le a_i, \ i = 1,2,...,m \\ \sum\limits_{i=1}^{m} \sum\limits_{k=1}^{l} x_{ijk} \ge b_j, \ j = 1,2,...,n \\ \sum\limits_{i=1}^{m} \sum\limits_{j=1}^{n} x_{ijk} \le e_k, \ k = 1,2,...,l \\ x_{ijk} \ge 0, \ \forall ijk. \end{cases} \end{cases} \tag{1}$$

In problem (1), we need to minimize the total cost of transportation. We assume that the unit costs, the capacity of each origin, destination and conveyance are all constant and denoted c_{ijk}, a_i, b_j, e_k.

3 Interval solid transportation problem

In practical applications, transportation cost, supplies, destinations and conveyances of a transportation plan is uncertain as it depends on various factors such as efficiency of transportation modes, etc. Thus, in such conditions, DM is not sure to make a decision about the transportation plan. Therefore, in problem (1), we assume that all coefficients of problem are independent interval variables. Hence, IMOSTP can be formulated as follows,

$$\begin{aligned} &\min \left(\sum\limits_{i=1}^{m} \sum\limits_{j=1}^{n} \sum\limits_{k=1}^{l} \tilde{c}_{ijk} x_{ijk} \right) \\ &s.t. \begin{cases} \sum\limits_{j=1}^{n} \sum\limits_{k=1}^{l} x_{ijk} \le \tilde{a}_i, i = 1,2,...,m \\ \sum\limits_{i=1}^{m} \sum\limits_{k=1}^{l} x_{ijk} \ge \tilde{b}_j, j = 1,2,...,n \\ \sum\limits_{i=1}^{m} \sum\limits_{j=1}^{n} x_{ijk} \le \tilde{e}_k, k = 1,2,...,l \\ x_{ijk} \ge 0, \forall ijk, \end{cases} \end{aligned} \tag{2}$$

where $\tilde{c}_{ijk}, \tilde{a}_i, \tilde{b}_j, \tilde{e}_k$ are interval coefficients of a STP denoted as $\tilde{c}_{ijk} = \left[\underline{c}_{ijk}, \overline{c}_{ijk}\right]$, $\tilde{a}_i - = \left[\underline{a}_i, \overline{a}_i\right]$, $\tilde{b}_j - = \left[\underline{b}_j, \overline{b}_j\right]$, and $\tilde{e}_k = \left[\underline{e}_k, \overline{e}_k\right]$, respectively.

4 Interval multi objective solid transportation problem

Employing the above assumptions for the STP, the multiobjective model of STP with interval numbers is formulated as follows,

$$\begin{aligned}
&\min Z^1 = \left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} \tilde{c}^1_{ijk} x_{ijk}\right)\\
&\min Z^2 = \left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} \tilde{c}^2_{ijk} x_{ijk}\right)\\
&\ldots\\
&\ldots\\
&\ldots\\
&\min Z^p = \left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} \tilde{c}^p_{ijk} x_{ijk}\right)\\
&s.t. \begin{cases}
\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \le \tilde{a}_i, i = 1,2,\ldots,m\\
\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \ge \tilde{b}_j, j = 1,2,\ldots,n\\
\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le \tilde{e}_k, k = 1,2,\ldots,l\\
x_{ijk} \ge 0, \forall ijk
\end{cases}
\end{aligned} \tag{3}$$

Hossein et al. [19] recommended a solution process to optimize the interval multiobjective programming problems. The method converted an interval linear programming method into two equivalent models for its lower bound and upper bound. Thus, they get different optimal solutions for each objective function. Therefore, this paper formulates a fuzzy programming based on interval arithmetic to optimize the interval multiobjective solid transportation problems, simultaneously. By using the order relation $\le^*_{RC}$ on the interval numbers, the supply and the conveyance constraints of the a STP can be constructed as,

$$\begin{aligned}
&\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \le \overline{a}_i,\ i = 1,2,\ldots,m\\
&\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \le (\tilde{a}_i)_C,\ i = 1,2,\ldots,m
\end{aligned} \tag{4}$$

$$\begin{aligned}
&\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le \overline{e}_k,\ k = 1,2,\ldots,l\\
&\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le (\tilde{e}_k)_C,\ k = 1,2,\ldots,l.
\end{aligned} \tag{5}$$

By using the order relation $\leq^{*}_{LC}$, the demand constraint can be constructed as,

$$\begin{aligned} &\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \geq \underline{b}_j,\ j=1,2,...,n \\ &\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \geq \left(\tilde{b}_j\right)_C,\ j=1,2,...,n. \end{aligned} \tag{6}$$

If the objective is to minimize $Z^r(x)$, $(r=1,2,...,p)$ the solution of model (3) can be found as the set of efficient optimal solutions of the following programming problem subject to supply, demand and conveyance constraints:

$$\min\left(Z^r_C(x), \overline{Z}^r(x)\right), (r=1,2,...,p) \tag{7}$$

where, $\overline{Z}^r(x)$ is the upper bound of objective function with interval numbers and $Z^r_C(x)$ is its center of objective function with interval numbers.

If the original objective is to maximize $Z^r(x), (r=1,2,...,p)$ the solution of model (3) can be found as the set of efficient solutions of the following objective problem:

$$\max\left(\underline{Z}^r(x), Z^r_C(x)\right), (r=1,2,...,p) \tag{8}$$

where, $\underline{Z}^r(x)$ is the upper bound of interval objective function and $Z^r_C(x)$ is its center.

The construction method, the objective function of IMOSTP (3) is transformed into deterministic objective functions. Then, each objective function optimizes subject to constraints set as a single programming problem, individually. In the other words, for each objective function of IMOSTP (3), the range of objective functions are generated as

$$Z^r{}_{opt} = \left[\underline{Z}^r_{opt}(x), \overline{Z}^r_{opt}(x)\right], (r=1,2,...,p).$$

Definition 6. *The function $Z: R^n \to I, I \in R$ is called a closed and bounded interval function on the R^n and defined as*

$$Z^r(x) = \left[\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} \underline{c}^r_{ijk}x_{ijk}, \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} \overline{c}^r_{ijk}x_{ijk}\right]$$

where

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} \underline{c}^r_{ijk}x_{ijk} \ \text{ and } \ \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} \overline{c}^r_{ijk}x_{ijk}$$

are the lower limit and the upper limit of interval respectively. Then, we have for all $x_{ijk} \in X$, (X is the feasible region of problem)

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} \underline{c}^r_{ijk}x_{ijk} \leq \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} \overline{c}^r_{ijk}x_{ijk}, (r=1,2,...,p).$$

Definition 7. *A transportation plan $x^0 \in X$ is a Pareto optimal solution of interval problem, if and only if there is no other $x \in X$ such that*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} \tilde{c}^r_{ijk}x_{ijk} \leq \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} \tilde{c}^r_{ijk}x^0_{ijk}$$

for all $(r = 1,2,...,p.)$ *and*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{c}^{r}_{ijk}x_{ijk} \prec \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{c}^{r}_{ijk}x^{0}_{ijk}$$

for at least $r = 1,2,...,p$.

5 A Fuzzy programming for IMOSTP

Now, consider the r^th objective function of IMOSTP (3) and then, if objective of IMOSTP (3) is a maximization problem, its membership function using the previous range of optimal objective functions can be determined as follows:

$$\mu^{r}(x) = \begin{cases} 1, & Z^{r}(x) \geq \bar{Z}^{r}_{opt} \\ \dfrac{\bar{Z}^{r}(x) - \underline{Z}^{r}_{opt}}{\bar{Z}^{r}_{opt} - \underline{Z}^{r}_{opt}}, & \bar{Z}^{r}(x) \leq \bar{Z}^{r}_{opt} \end{cases} \tag{9}$$

where the increasing of $\bar{Z}^{r}(x)$ will increase the membership degree $\mu^{r}(x)$.

In the same way, for minimization type objective, the membership function can be determined as follows:

$$\mu^{r}(x) = \begin{cases} 1, & \bar{Z}^{r}(x) \leq \underline{Z}^{r}_{opt} \\ \dfrac{\bar{Z}^{r}_{opt} - Z^{r}(x)}{\bar{Z}^{r}_{opt} - \underline{Z}^{r}_{opt}}, & \underline{Z}^{r}_{opt} \leq \bar{Z}^{r}(x) \end{cases} \tag{10}$$

where the decreasing of $\bar{Z}^{r}(x)$ will increase the membership degree $\mu^{r}(x)$.

After determined the membership function (10), IMOSTP (3) is transformed to the single interval solid transportation problem as follows:

$$\max\left\{\mu^{1},\mu^{2},...,\mu^{p}\right\}$$

$$s.t. \begin{cases} \mu^{r} \leq 1, (r = 1,2,...,p) \\ \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \leq \bar{a}_{i}, i = 1,2,...,m, \\ \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \leq (\tilde{a}_{i})_{C}, i = 1,2,...,m \\ \sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \geq \underline{b}_{j}, j = 1,2,...,n, \\ \sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \geq \left(\tilde{b}_{j}\right)_{C}, j = 1,2,...,n \\ \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \leq \bar{e}_{k}, k = 1,2,...,l, \\ \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \leq (\tilde{e}_{k})_{C}, k = 1,2,...,l \\ x_{ijk} \geq 0, \forall ijk, \end{cases} \tag{11}$$

where $\mu^r,(r=1,2,...,p)$ are interval functions.

This problem is further transformed into the following equivalent form:

$$\max \sum_{r=1}^{k} \mu^r$$
$$s.t. \begin{cases} \mu^r \le 1, (r=1,2,...,p) \\ \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \le \bar{a}_i, i=1,2,...,m, \\ \sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \le (\tilde{a}_i)_C, i=1,2,...,m \\ \sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \ge \underline{b}_j, j=1,2,...,n, \\ \sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \ge \left(\tilde{b}_j\right)_C, j=1,2,...,n \\ \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le \bar{e}_k, k=1,2,...,l, \\ \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le (\tilde{e}_k)_C, k=1,2,...,l \\ x_{ijk} \ge 0, \forall ijk \end{cases} \tag{12}$$

Here, the objective function of the above model is converted to the single objective function using simple weighted sum function $\sum_{r=1}^{k} w^r\mu^r, w^r \in [0,1]$.

Definition 8.*(Dalman et. al. [9]) Assume that the constraint set of a fuzzy programming model (12) is X. A $x^0 \in X$ is an efficient to the fuzzy problem (12) if there does not exist other solution $x \in X$ such that $\mu^r(x) \ge \mu^r(x_0)$ and $\mu^k(x) \ge \mu^k(x_0)$ at least one k.*

6 A Numerical example

In order to show the applications of the models, we apply the interval transportation model to a multiobjective coal transportation problem and give an optimal transportation plan. For the convenience of description, we summarize the problem as follows. Suppose that there are two coal mines to supply the coal for three cities, and two kinds of conveyances are available to be determined i.e. train and cargo ship. Here, the decision maker should make a transportation plan for the next month such that the transportation cost minimized, simultaneously. To illustrate the recommended solution procedures for an MOSTP, let us consider the following data;

Interval transportation cost for 1.st objective $\tilde{c}^1_{ijk} = \left[\underline{c}^1_{ijk}, \bar{c}^1_{ijk}\right]$;

$$\underline{c}^1_{ijk} = \begin{bmatrix} \underline{c}^{(1)}_{111} & \underline{c}^{(1)}_{112} & \underline{c}^{(1)}_{121} \\ \underline{c}^{(1)}_{122} & \underline{c}^{(1)}_{131} & \underline{c}^{(1)}_{132} \\ \underline{c}^{(1)}_{211} & \underline{c}^{(1)}_{212} & \underline{c}^{(1)}_{221} \\ \underline{c}^{(1)}_{222} & \underline{c}^{(1)}_{231} & \underline{c}^{(1)}_{232} \end{bmatrix} = \begin{bmatrix} 13/2 & 10 & 5 \\ 7 & 11 & 8 \\ 9 & 21/2 & 13/2 \\ 7 & 12 & 15 \end{bmatrix}, \bar{c}^1_{ijk} = \begin{bmatrix} \bar{c}^{(1)}_{111} & \bar{c}^{(1)}_{112} & \bar{c}^{(1)}_{121} \\ \bar{c}^{(1)}_{122} & \bar{c}^{(1)}_{131} & \bar{c}^{(1)}_{132} \\ \bar{c}^{(1)}_{211} & \bar{c}^{(1)}_{212} & \bar{c}^{(1)}_{221} \\ \bar{c}^{(1)}_{222} & \bar{c}^{(1)}_{231} & \bar{c}^{(1)}_{232} \end{bmatrix} = \begin{bmatrix} 10 & 14 & 10 \\ 11 & 15 & 13 \\ 14 & 14 & 17/2 \\ 11 & 33/2 & 17 \end{bmatrix},$$

Interval transportation cost for 2.st objective $\tilde{c}^2_{ijk} = \left[\underline{c}^2_{ijk}, \overline{c}^2_{ijk}\right]$;

$$\underline{c}^2_{ijk} = \begin{bmatrix} \underline{c}^{(2)}_{111} & \underline{c}^{(2)}_{112} & \underline{c}^{(2)}_{121} \\ \underline{c}^{(2)}_{122} & \underline{c}^{(2)}_{131} & \underline{c}^{(2)}_{132} \\ \underline{c}^{(2)}_{211} & \underline{c}^{(2)}_{212} & \underline{c}^{(2)}_{221} \\ \underline{c}^{(2)}_{222} & \underline{c}^{(2)}_{231} & \underline{c}^{(2)}_{232} \end{bmatrix} = \begin{bmatrix} 19/2 & 12 & 13/2 \\ 13/2 & 21/2 & 27/2 \\ 12 & 15 & 8 \\ 10 & 13 & 27/2 \end{bmatrix}, \overline{c}^2_{ijk} = \begin{bmatrix} \overline{c}^{(2)}_{111} & \overline{c}^{(2)}_{112} & \overline{c}^{(2)}_{121} \\ \overline{c}^{(2)}_{122} & \overline{c}^{(2)}_{131} & \overline{c}^{(2)}_{132} \\ \overline{c}^{(2)}_{211} & \overline{c}^{(2)}_{212} & \overline{c}^{(2)}_{221} \\ \overline{c}^{(2)}_{222} & \overline{c}^{(2)}_{231} & \overline{c}^{(2)}_{232} \end{bmatrix} = \begin{bmatrix} 25/2 & 29/2 & 11 \\ 10 & 12 & 14 \\ 13 & 19 & 13 \\ 27/2 & 17 & 31/2 \end{bmatrix},$$

The following notations $\tilde{a}_i$, $\tilde{b}_j$ and $\tilde{e}_k$ for each $i = 1,2,..,m; j = 1,2,..,n;\ k = 1,2,..,l$ are used to express the interval supply capacities, the interval demands, and the interval transportation capacities, respectively.

$$\tilde{a}_1 = [\frac{45}{2}, 27],\ \tilde{a}_2 = [30,36] \rightarrow\ interval parameters of supplies,$$
$$\tilde{b}_1 = [15, \frac{41}{2}],\ \tilde{b}_2 = [\frac{37}{2}, \frac{47}{2}],\ \tilde{b}_3 = [\frac{27}{2}, \frac{39}{2}] \rightarrow\ interval parameters of demands,$$
$$\tilde{e}_1 = [\frac{95}{2}, 52],\ \tilde{e}_2 = [52, \frac{115}{2}] \rightarrow\ interval parameters of transportation capacities.$$

Appliying the above information, the IMOSTP can be formulated as follows:

$$\begin{aligned} &\min Z^1(x) = \sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2}\left[\underline{c}^1_{ijk}, \overline{c}^1_{ijk}\right]x_{ijk} \\ &\min Z^2(x) = \sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2}\left[\underline{c}^2_{ijk}, \overline{c}^2_{ijk}\right]x_{ijk} \\ &s.t. \begin{cases} \sum_{j=1}^{3}\sum_{k=1}^{2} x_{1jk} \le \tilde{a}_1 = [\frac{45}{2}, 27], (j = 1,2,3), (k = 1,2) \\ \sum_{j=1}^{3}\sum_{k=1}^{2} x_{2jk} \le \tilde{a}_2 = [30,36],\ (j = 1,2,3), (k = 1,2), \\ \sum_{i=1}^{2}\sum_{k=1}^{2} x_{i1k} \ge \tilde{b}_1 = [15, \frac{41}{2}],\ (i = 1,2), (k = 1,2) \\ \sum_{i=1}^{2}\sum_{k=1}^{2} x_{i2k} \ge \tilde{b}_2 = [\frac{37}{2}, \frac{47}{2}], (i = 1,2), (k = 1,2) \\ \sum_{i=1}^{2}\sum_{k=1}^{2} x_{i3k} \ge \tilde{b}_3 = [\frac{27}{2}, \frac{39}{2}], (i = 1,2), (k = 1,2) \\ \sum_{i=1}^{2}\sum_{j=1}^{3} x_{ij1} \le \tilde{e}_1 = [\frac{95}{2}, 52], (i = 1,2), (j = 1,2,3) \\ \sum_{i=1}^{2}\sum_{j=1}^{3} x_{ij2} \le \tilde{e}_2 = [52, \frac{115}{2}], (i = 1,2), (j = 1,2,3) \\ x_{ijk} \ge 0, i = 1,2., j = 1,2,3, k = 1,2. \end{cases} \end{aligned} \quad (13)$$

According to minimization type (7), the above problem transformed into the following equivalent deterministic MOSTP as follows,

$$\min \overline{Z}^1(x) = \sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2} \overline{c}^1_{ijk} x_{ijk},$$
$$\min Z^1_C(x) = \sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2} \left(\frac{\underline{c}^1_{ijk}+\overline{c}^1_{ijk}}{2}\right) x_{ijk},$$
$$\min \overline{Z}^2(x) = \sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2} \overline{c}^2_{ijk} x_{ijk},$$
$$\min Z^2_C(x) = \sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2} \left(\frac{\underline{c}^2_{ijk}+\overline{c}^2_{ijk}}{2}\right) x_{ijk}$$
$$s.t \begin{cases} \sum_{j=1}^{3}\sum_{k=1}^{2} x_{1jk} \le \overline{a}_1 = 27, & \sum_{j=1}^{3}\sum_{k=1}^{2} x_{1jk} \le \tilde{a}_{1C} = 99/2, \\ \sum_{j=1}^{3}\sum_{k=1}^{2} x_{2jk} \le \overline{a}_2 = 36, & \sum_{j=1}^{3}\sum_{k=1}^{2} x_{2jk} \le \tilde{a}_{2C} = 33, \\ \sum_{i=1}^{2}\sum_{k=1}^{2} x_{i1k} \ge \underline{b}_1 = 15, & \sum_{i=1}^{2}\sum_{k=1}^{2} x_{i1k} \ge \tilde{b}_{1C} = 71/2, \\ \sum_{i=1}^{2}\sum_{k=1}^{2} x_{i2k} \ge \underline{b}_2 = 37/2, & \sum_{i=1}^{2}\sum_{k=1}^{2} x_{i2k} \ge \tilde{b}_{2C} = 42/2, \\ \sum_{i=1}^{2}\sum_{k=1}^{2} x_{i3k} \ge \underline{b}_3 = 27/2, & \sum_{i=1}^{2}\sum_{k=1}^{2} x_{i3k} \ge \tilde{b}_{3C} = 33, \\ \sum_{i=1}^{2}\sum_{j=1}^{3} x_{ij1} \le \overline{e}_1 = 52, & \sum_{i=1}^{2}\sum_{j=1}^{3} x_{ij1} \le \tilde{e}_{1C} = 199/2, \\ \sum_{i=1}^{2}\sum_{j=1}^{3} x_{ij2} \le \overline{e}_2 = 115/2, & \sum_{i=1}^{2}\sum_{j=1}^{3} x_{ij2} \le \tilde{e}_{2C} = 219/2. \\ x_{ijk} \ge 0, i = 1,2., j = 1,2,3., k = 1,2. \end{cases} \quad (14)$$

Solving $minZ^1_C$; $min\overline{Z}^1$; $min\overline{Z}^2$ and $minZ^2_C$ problems as a single programming problem and then the optimal results in the first step are determined as $\min Z^1_C = 468.4375; \min \overline{Z}^1 = 603.75$ and $\min Z^2_C = 609.4375$; $\min \overline{Z}^2 = 677$.

Thus, the range of interval objective values in the original problem are calculates as $\min Z^1_{opt} = [468.4375, 603.75]$ and $\min Z^2_{opt} = [609.4375, 677]$. According to minimization type of the membership function of each objective function can be determined as follows,

$$\mu^1(x) = \begin{cases} 1, & Z^1(x) \le 468.4375 \\ \frac{603.75 - Z^1(x)}{603.75 - 468.4375}, & 468.4375 \le Z^1(x) \end{cases}$$

and

$$\mu^2(x) = \begin{cases} 1, & Z^2(x) \le 609.437 \\ \frac{677 - Z^2(x)}{677 - 609.4375}, & 609.4375 \le Z^2(x). \end{cases}$$

The problem based on the model (11) is transformed into an interval linear programming problem as follows,

$$\max\left\{\frac{603.75-\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2}\left[\underline{c}_{ijk}^{1},\overline{c}_{ijk}^{1}\right]x_{ijk}}{603.75-468.4375},\ \frac{677-\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2}\left[\underline{c}_{ijk}^{2},\overline{c}_{ijk}^{2}\right]x_{ijk}}{677-609.4375}\right\}$$

$$s.t.\begin{cases}\dfrac{603.75-\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2}\left[\underline{c}_{ijk}^{1},\overline{c}_{ijk}^{1}\right]x_{ijk}}{603.75-468.4375}\leq 1,\\ \dfrac{677-\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2}\left[\underline{c}_{ijk}^{2},\overline{c}_{ijk}^{2}\right]x_{ijk}}{677-609.4375}\leq 1,\\ \sum_{j=1}^{3}\sum_{k=1}^{2}x_{1jk}\leq\overline{a}_{1}=27,\quad \sum_{j=1}^{3}\sum_{k=1}^{2}x_{1jk}\leq\tilde{a}_{1C}=99/2,\\ \sum_{j=1}^{3}\sum_{k=1}^{2}x_{2jk}\leq\overline{a}_{2}=36,\quad \sum_{j=1}^{3}\sum_{k=1}^{2}x_{2jk}\leq\tilde{a}_{2C}=33\\ \sum_{i=1}^{2}\sum_{k=1}^{2}x_{i1k}\geq\underline{b}_{1}=15,\quad \sum_{i=1}^{2}\sum_{k=1}^{2}x_{i1k}\geq\tilde{b}_{1C}=71/2,\\ \sum_{i=1}^{2}\sum_{k=1}^{2}x_{i2k}\geq\underline{b}_{2}=37/2,\quad \sum_{i=1}^{2}\sum_{k=1}^{2}x_{i2k}\geq\tilde{b}_{2C}=42/2\\ \sum_{i=1}^{2}\sum_{k=1}^{2}x_{i3k}\geq\underline{b}_{3}=27/2,\quad \sum_{i=1}^{2}\sum_{k=1}^{2}x_{i3k}\geq\tilde{b}_{3C}=33\\ \sum_{i=1}^{2}\sum_{j=1}^{3}x_{ij1}\leq\overline{e}_{1}=52,\quad \sum_{i=1}^{2}\sum_{j=1}^{3}x_{ij1}\leq\tilde{e}_{1C}=199/2,\\ \sum_{i=1}^{2}\sum_{j=1}^{3}x_{ij2}\leq\overline{e}_{2}=115/2,\quad \sum_{i=1}^{2}\sum_{j=1}^{3}x_{ij2}\leq\tilde{e}_{2C}=219/2.\\ x_{ijk}\geq 0, i=1,2., j=1,2,3., k=1,2.\end{cases}\tag{15}$$

Then, this problem is transformed to the following crisp linear programming problem based on model (12) and model (8).

$$\max\left(\begin{array}{l}\dfrac{603.75-\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2}\underline{c}_{ijk}^{1}x_{ijk}}{603.75-468.4375}+\dfrac{603.75-\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2}\left(\frac{\underline{c}_{ijk}^{1}+\overline{c}_{ijk}^{1}}{2}\right)x_{ijk}}{603.75-468.4375}\\ +\dfrac{677-\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2}\underline{c}_{ijk}^{2}x_{ijk}}{677-609.4375}+\dfrac{603.75-\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2}\left(\frac{\underline{c}_{ijk}^{2}+\overline{c}_{ijk}^{2}}{2}\right)x_{ijk}}{603.75-468.4375}\end{array}\right)$$

$$
s.t. \begin{cases} \dfrac{603.75-\sum\limits_{i=1}^{2}\sum\limits_{j=1}^{3}\sum\limits_{k=1}^{2}\overline{c}^{1}_{ijk}x_{ijk}}{603.75-468.4375}\leq 1, \\ \dfrac{603.75-\sum\limits_{i=1}^{2}\sum\limits_{j=1}^{3}\sum\limits_{k=1}^{2}\left(\frac{\underline{c}^{1}_{ijk}+\overline{c}^{1}_{ijk}}{2}\right)x_{ijk}}{603.75-468.4375}\leq 1, \\ \dfrac{677-\sum\limits_{i=1}^{2}\sum\limits_{j=1}^{3}\sum\limits_{k=1}^{2}\overline{c}^{2}_{ijk}x_{ijk}}{677-609.4375}\leq 1, \\ \dfrac{603.75-\sum\limits_{i=1}^{2}\sum\limits_{j=1}^{3}\sum\limits_{k=1}^{2}\left(\frac{\underline{c}^{2}_{ijk}+\overline{c}^{2}_{ijk}}{2}\right)x_{ijk}}{603.75-468.4375}\leq 1, \\ \sum\limits_{j=1}^{3}\sum\limits_{k=1}^{2}x_{1jk}\leq\overline{a}_{1}=27, \quad \sum\limits_{j=1}^{3}\sum\limits_{k=1}^{2}x_{1jk}\leq\tilde{a}_{1C}=99/2, \\ \sum\limits_{j=1}^{3}\sum\limits_{k=1}^{2}x_{2jk}\leq\overline{a}_{2}=36, \quad \sum\limits_{j=1}^{3}\sum\limits_{k=1}^{2}x_{2jk}\leq\tilde{a}_{2C}=33 \\ \sum\limits_{i=1}^{2}\sum\limits_{k=1}^{2}x_{i1k}\geq\underline{b}_{1}=15, \quad \sum\limits_{i=1}^{2}\sum\limits_{k=1}^{2}x_{i1k}\geq\tilde{b}_{1C}=71/2, \\ \sum\limits_{i=1}^{2}\sum\limits_{k=1}^{2}x_{i2k}\geq\underline{b}_{2}=37/2, \quad \sum\limits_{i=1}^{2}\sum\limits_{k=1}^{2}x_{i2k}\geq\tilde{b}_{2C}=42/2 \\ \sum\limits_{i=1}^{2}\sum\limits_{k=1}^{2}x_{i3k}\geq\underline{b}_{3}=27/2, \quad \sum\limits_{i=1}^{2}\sum\limits_{k=1}^{2}x_{i3k}\geq\tilde{b}_{3C}=33 \\ \sum\limits_{i=1}^{2}\sum\limits_{j=1}^{3}x_{ij1}\leq\overline{e}_{1}=52, \quad \sum\limits_{i=1}^{2}\sum\limits_{j=1}^{3}x_{ij1}\leq\tilde{e}_{1C}=199/2, \\ \sum\limits_{i=1}^{2}\sum\limits_{j=1}^{3}x_{ij2}\leq\overline{e}_{2}=115/2, \quad \sum\limits_{i=1}^{2}\sum\limits_{j=1}^{3}x_{ij2}\leq\tilde{e}_{2C}=219/2. \\ x_{ijk}\geq 0, i=1,2., j=1,2,3., k=1,2. \end{cases} \tag{16}
$$

Applying the weighted sum method, this equivalent model can be easily solved by Maple 18 optimization toolbox and then, the following results are obtained;

$$
x_{ijk}=\begin{bmatrix} x_{111} & x_{112} & x_{121} \\ x_{122} & x_{131} & x_{132} \\ x_{211} & x_{212} & x_{221} \\ x_{222} & x_{231} & x_{232} \end{bmatrix}=\begin{bmatrix} 8.25 & 0 & 0 \\ 0 & 0 & 16.5 \\ 9.5 & 0 & 21 \\ 0 & 0 & 0 \end{bmatrix}
$$

The optimal ranges of each objective function in the IMOSTP are determined as $\min Z^{1}_{opt}=[407.625, 608.500]$ and $\min Z^{2}_{opt}=[583.125, 730.625]$.

There are several effective fuzzy methods for solving multiobjective programming problems. By using fuzzy method of paper [18], the results are as:

$$x_{ijk} = \begin{bmatrix} x_{111} & x_{112} & x_{121} \\ x_{122} & x_{131} & x_{132} \\ x_{211} & x_{212} & x_{221} \\ x_{222} & x_{231} & x_{232} \end{bmatrix} = \begin{bmatrix} 4.714 & 0 & 0 \\ 3.536 & 0 & 16.5 \\ 13.036 & 0 & 17.464 \\ 0 & 0 & 0 \end{bmatrix},$$

Moreover, The optimal ranges of each objective function is as follows; $\min Z^1_{opt} = [418.232, 631.482]$ and $\min Z^2_{opt} = [586.661, 721.786]$.

Hence, the above results mean that the presented solution procedures can be considered as an efficient approach for solving problems of this type.

7 Conclusions

In this paper, a fuzzy programming approach presented for solving Interval Multiobjective Solid Transportation Problem (IMOSTP). Usually, the decision maker encounters inadequate information to define a precise value in a practical application. In this case, the decision maker has to estimate these inadequate values. And also, the uncertainty of interval numbers is a strong characteristic for expressing of realistic systems and provide a powerful structure for representing the system's information as interval numbers, in place of crisp numbers. Also, this structure presents flexibility in expressing the uncertain systems. To reduce the complexities arising from uncertainty, the order relation between intervals is applied and therefore the multiobjective model based on the interval numbers is transformed to the crisp multiobjective programming problem. Then maximizing the sum of membership functions associated with different interval objectives, single objective programming problem obtained. Thus it is solved easily by Maple 18 optimization toolbox. So the efficiency of the presented procedures is proved.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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