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Traveling wave solutions of new coupled Konno-Oono equation

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Abstract: In this paper, the recently developed tanh-function method and extended tanh-function method are applied to explore the traveling wave solutions of new coupled Konno-Oono equation. We have successfully derived some explicit traveling wave solutions of new coupled Konno-Oono equation.

Keywords: tanh-function method, extended tanh-function method, new coupled Konno-Oono equation, traveling wave solutions

1 Introduction

Nowadays NLEEs have become most examined subject of all-embracing studies in several branches of nonlinear sciences. A special class of analytical solutions named traveling wave solutions for NLEEs has a lot of importance, because most of the phenomena that arise in mathematical physics and engineering fields can be described by NLEEs. NLEEs are repeatedly used to describe many problems of protein chemistry, chemically reactive materials, in ecology most population models, in physics the heat flow and the wave propagation phenomena, quantum mechanics, fluid mechanics, plasma physics, propagation of shallow water waves, optical fibers, biology, solid state physics, chemical kinematics, geochemistry, meteorology, electricity, and so forth. Therefore, investigation, traveling wave solutions is becoming more and more attractive in nonlinear sciences day by day. However, not all equations posed of these models are solvable. As a result, many new techniques have been successfully developed by diverse groups of mathematicians and physicists, such as: the Exp-function method [1-5], the Hirota's bilinear transformation method [6,7], the Adomian decomposition method [8], The F-expansion method [9], the Auxiliary equation method [10], (G'/G)-expansion method [11-15], the modified simple equation method [16, 17], the tanh-function method [18], the sine-cosine method [19], the improved tan $(\Phi(\xi)/2)$-expansion method [20] and many more.

The objective of this paper is to apply the tanh-function method and extended tanh-function method to construct the exact traveling wave solutions for nonlinear evolution: coupled Konno-Oono equations.

2 Methods

In this section we will discuss the main steps of tanh-function and extended tanh-function methods.

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2.1 The tanh-function method

The tanh-function method summarized by Wazwaz are as follows [21, 22], Suppose a partial differential equation,

$$P(u,u_t,u_x,u_{xx},\ldots)=0. \tag{1}$$

By using the wave variable $\xi = x-\omega t$ where $U=U(\xi)=U(x,t)$, the equation (1) can be transform to the following ODE:

$$Q(U,U^{'},U^{''},U^{'''},\ldots)=0. \tag{2}$$

The equation (2) is then integrated as far as all terms contain derivatives. We introducing a new independent variable;

$$Y=tanh(\xi) \tag{3}$$

Equation (3) leading the change of derivatives:

$$\frac{d}{d\xi}=(1-Y^2)\frac{d}{dY} \tag{4}$$

$$\frac{d^2}{d\xi^2}=\left(1-Y^2\right)(-2Y\frac{d}{dY}+(1-Y^2)\frac{d^2}{dY^2}) \tag{5}$$

$$\frac{d^3}{d\xi^3}=\left(1-Y^2\right)(\left(6Y^2-2\right)\frac{d}{dY}-6Y\left(1-Y^2\right)\frac{d^2}{dY^2}\left(1-Y^2\right)^2\frac{d^3}{dY^3}). \tag{6}$$

The tanh-function method acknowledge the use of the finite expansion

$$U\left(\xi\right)=F\left(Y\right)=\sum_{k=0}^{m}a_kY^k, \tag{7}$$

where m is a positive integer, for this method, that will be determined. Expansion (7) reduces to the standard tanh-function method [23] for; $1\leq k\leq m$.The parameter $m$is comonly attained, as stated before, by balancing the linear terms of the highest order in the resulting equation with the highest order nonlinear terms. If m is not an integer, then a transformation formula should be used to overcome this difficulty. Exchanging (7) into the ODE results an algebraic system of equations in powers of Y that will lead to the determination of the parameters $a_k(k=0,\ldots,m)$, and ω. For showing the competency of the method described in the preceding part, we appear some examples.

2.2 The extended tanh-function method

The extended tanh-function method admits the use of the finite expansion [21,22],

$$U(\xi)=F(Y)=\sum_{k=0}^{m}a_kY^k+\sum_{k=1}^{m}b_kY^{(-k)}; \tag{8}$$

where m is a positive integer, for this method, that will be determined. Expansion (8) reduces to the standard tanh-function method for $b_k=0;1\leq k\leq m$. The parameter $m$is comonly attained, as stated before, by balancing the linear terms of the highest order in the resulting equation with the highest order nonlinear terms. If m is not an integer, then a transformation formula should be used to overcome this difficulty. Exchanging (8) into the ODE results an algebraic system of equations in powers of Y that will lead to the determination of the parameters $a_k\left(k=0,\ldots,m\right),b_k\left(k=1,\ldots,m\right)$ and ω. For showing the competency of the method described in the preceding part, we appeared some examples.

3 Applications

In this section we will exert the tanh-function and extended tanh-function methods to find the exact traveling wave solutions then the solitary wave solutions of the new coupled Konno-Oono equations [24] of the form,

$$u_{xt}-2uv=0, v_t+2uu_x=0. \tag{9}$$

We consider the traveling wave transformation equation

$$u(\xi)=u(x,t), v(\xi)=v(x,t), \xi=x-\omega t \tag{10}$$

The traveling transformation Eq. (10) transforms Eq. (9) into the following ODEs,

$$-\omega u''-2uv=0 \tag{11}$$
$$-\omega v'+2uu'=0. \tag{12}$$

Now integrating Eq. (12) with respect to ξ, we obtain

$$v=\frac{1}{\omega}\left(u^2+d\right), \tag{13}$$

where d is an integral constant. Substituting Eq. (13) into the Eq. (11), we attain

$$\omega^2 u''+2ud+2u^3=0. \tag{14}$$

Now the balancing the highest order derivative u'' and the nonlinear term u^3 in Eq. (14), we obtain $m=1$.

3.1 The tanh-function method

In this subsection we will apply the tanh-function method to find the exact traveling wave solutions of Konno-Oono equation.

For $m=1$, the finite expansion (7) of tanh-function method admits

$$u(x,t)=a_0+a_1Y. \tag{15}$$

Now, Substituting Eq. (15) into Eq. (14), and equating the coefficients of the powers of Y then we obtain a system of algebraic equations:

$$2\omega^2 a_1+2a_1^3=0,$$
$$6a_0a_1^2=0,$$
$$6a_0^2a_1+2da_1-2?^2a_1=0.$$
$$2a_0^3+2da_0=0.$$

By solving these algebraic equations for a_0, $a_1$and ωthen we obtain following set of solution:

$$\omega=\pm\sqrt{d}, a_0=0, a_1=\pm I\sqrt{d}.$$

Solving this set of solution, we required the following solutions:

$$u(x,t) = \pm I\sqrt{d}\tanh\left(x \pm \sqrt{d}t\right). \tag{16}$$

$$v(x,t) = \pm\sqrt{d}\tanh^2\left(x \pm \sqrt{d}t\right). \tag{17}$$

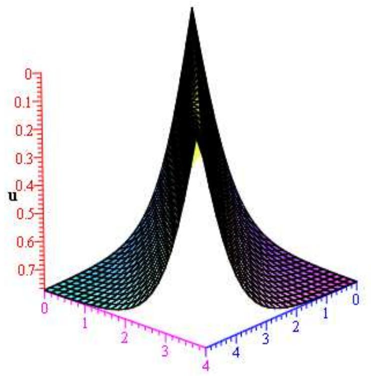

Fig. 1: Bell shaped profile of Eq. (16) with $d = 0.60$.

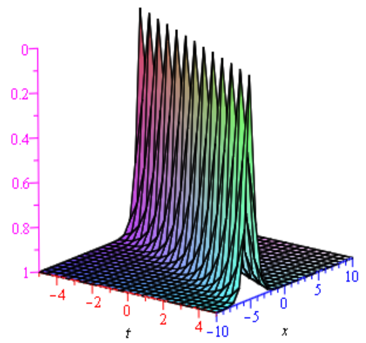

Fig. 2: Profile of Eq.(17) with $d = 1$.

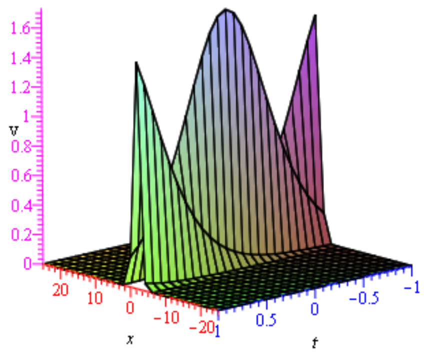

Fig. 3: Profile of Eq. (16) with with $d = 3$, $-25 \le x \le 25, -1 \le t \le 1$.

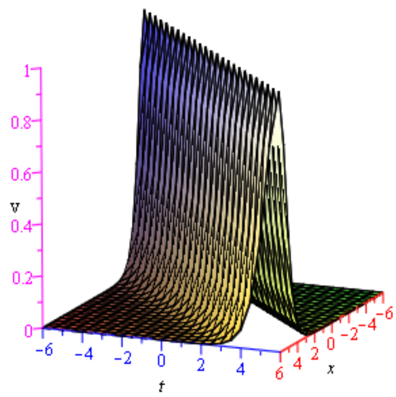

Fig. 4: Bell shaped profile of (17) with $d = 1$, $-6 \le x \le 6, -6 \le t \le 6$.

3.2 The extended tanh-function method

In this subsection we will apply the extended tanh-function method to find the exact traveling wave solutions of Konno-Oono equation for the value of $m = 1$, the finite expansion of extended tanh-function method admits,

$$u(x,t) = a_0 + a_1 Y + \frac{b_1}{Y}. \tag{18}$$

Now, Substituting Eq. (18) into Eq. (14), and equating the coefficients of the powers Y then we obtain a system of algebraic equations:

$$\begin{aligned}
&2a_1^3 + 52\omega^2 a_1 = 0\\
&6a_0 a_1^2 = 0\\
&6a_1^2 b_1 + 2da_1 - 2\omega^2 a_1 + 6a_0^2 a_1 = 0\\
&2da_0 + 2a_0^3 + 12a_0 a_1 b_1 = 0\\
&6a_0^2 b_1 - 2\omega^2 b_1 + 2db_1 + 6a_1^2 b_1^2 = 0\\
&6a_0 b_1^2 = 0\\
&2\omega^2 b_1 + 2b_1^3 = 0.
\end{aligned}$$

Solving these algebraic equations for a_0, a_1, a_2 , b_1 and ω then we obtain following sets of solutions:

Set-1 $\omega = \pm\sqrt{d}, a_0 = a_1 = 0, b_1 = \pm I\sqrt{d}$
Set-2 $\omega = \pm\sqrt{d}, a_0 = b_1 = 0, a_1 = \pm I\sqrt{d}$
Set-3 $\omega = \pm\frac{1}{2}\sqrt{d}, a_0 = 0, a_1 = \pm\frac{1}{2}I\sqrt{d}, b_1 = \pm\frac{1}{2}I\sqrt{d}$
Set-4 $\omega = \pm\frac{1}{\sqrt{2}}I\sqrt{d}, a_0 = 0, a_1 = \pm\frac{1}{\sqrt{2}}\sqrt{d}, b_1 = \mp\frac{1}{\sqrt{2}}\sqrt{d}.$

By solving these sets of equations we required the following solutions:

$$u_1 = \pm I\sqrt{d}\coth\left(x \mp \sqrt{d}t\right) \tag{19}$$

$$v_1 = \mp\sqrt{d}\left(coth^2\left(x \mp \sqrt{d}t\right) - 1\right) \tag{20}$$

$$u_2 = \pm I\sqrt{d}\tanh\left(x \mp \sqrt{d}t\right) \tag{21}$$

$$v_2 = \mp\sqrt{d}\left(tanh^2\left(x \mp \sqrt{d}t\right) - 1\right) \tag{22}$$

$$u_3 = \pm\frac{1}{2}I\sqrt{d}\left(tanh\left(x \mp \frac{1}{2}\sqrt{d}t\right) \pm coth\left(x \mp \frac{1}{2}\sqrt{d}t\right)\right) \tag{23}$$

$$v_3 = \mp\frac{1}{2}\sqrt{d}\left(\left(\tanh\left(x \mp \frac{1}{2}\sqrt{d}t\right) \pm \coth\left(x \mp \frac{1}{2}\sqrt{d}t\right)\right)^2 - 4\right) \tag{24}$$

$$u_4 = \pm\frac{1}{\sqrt{2}}\sqrt{d}\left(\tanh\left(x \mp \frac{1}{\sqrt{2}}I\sqrt{d}t\right) \mp \coth\left(x \mp \frac{1}{\sqrt{2}}I\sqrt{d}t\right)\right) \tag{25}$$

$$v_4 = \mp\frac{1}{\sqrt{2}}I\sqrt{d}\left(\left(\tanh\left(x \mp \frac{1}{\sqrt{2}}I\sqrt{d}t\right) \mp \coth\left(x \mp \frac{1}{\sqrt{2}}I\sqrt{d}t\right)\right)^2 + 2\right). \tag{26}$$

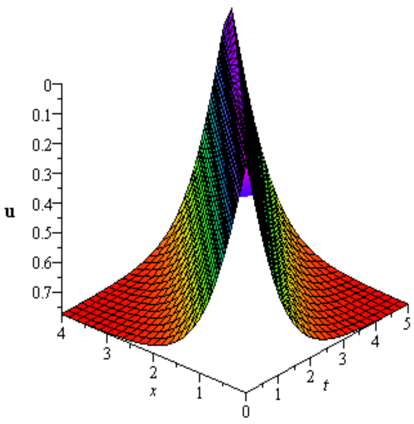

Fig. 5: Bell shaped wave of Eq. (21) with $d = 0.60$, $0 \leq x \leq 4, 0 \leq t \leq 5$.

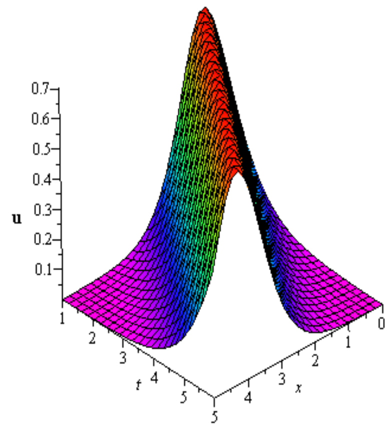

Fig. 6: Bell shaped soliton, graph of Eq.(22) with $d = 0.50$, $0 \leq x \leq 5, 1 \leq t \leq 6$.

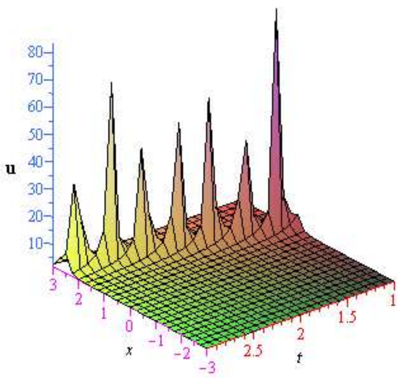

Fig. 7: Modulus plot of Eq. (23) with $d = 3$, $-3 \leq x \leq 3, 1 \leq t \leq 3$.

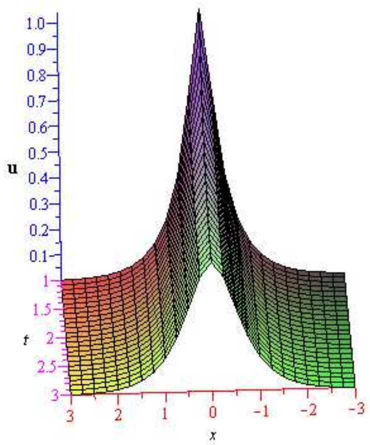

Fig. 8: Modulus plot of Eq. (25) with $d = 1/8$, $-3 \leq x \leq 3, 1 \leq t \leq 3$.

4 Conclusions

The tanh-function method and extended tanh-function method have been applied successfully to construct the exact traveling wave solutions of new coupled Konno-Oono Equation. The traveling wave transformation formulae have used to find the solutions. We have emphasized in this work that this relevant transformation is powerful and can be effectively used to discuss nonlinear evolution equations and related models in scientific fields.

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