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Soft $b-$compact spaces

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Abstract: In this paper, a new class of generalized soft open sets in soft generalized topological spaces as a generalization of compact spaces, called soft *b*-compact spaces, is introduced and studied. A soft generalized topological space is soft *b*-compact if every soft *b*-open soft cover of F_E contains a finite soft subcover. We characterize soft *b*-compact space and study some of their basic properties.

Keywords: Soft $b-$compactness, soft $b-$closed spaces, soft generalized $b-$ compact.

1 Introduction

Molodtsov [2] generalized with the introduction of soft sets the traditional concept of a set in the classical researches. With the introduction of the applications of soft sets [3], the soft set theory has been the research topic and have received attention gradually [1,17,21,23]. The applications of the soft sets are redetected so as to develop and consolidate this theory, utilizing these new applications; a uni-int decision-making method was established [16]. Numerous notions of general topology were involved in soft sets and then authors developed theories about soft topological spaces. Shabir and Naz [5] mentioned this term to define soft topological space. After that definition, I. Zorlutuna et al. [11], Aygunoglu et al.[9] and Hussain et al. [8] continued to search the properties of soft topological space. They obtained a lot of vital conclusion in soft topological spaces. Chen was the first person who examined weak forms of soft open sets [5]. Chen researched soft semi-open sets in soft topological spaces and investigated some properties of it. Arockiarani and Arokialancy [10] described soft β-open sets and continued to study weak forms of soft open sets in soft topological space. Consequently, Akdag and Ozkan [6] described soft α-open (soft α-closed) sets. Furthermore, Through entrenching the notions of fuzzy sets, numerous applications of the soft set theory have been enlarged and varied [3,13,14,15,18,19,20,21,22,24]. Then, several terms of general topology such as compactness have been referred to soft topology. The concept of compactness is one of the basic and vital concepts of paramount interest for topologists. Zorlutuna et al. first studied the compactness for soft topological spaces [11]. Akdag and Ozkan studied the concept of *b* -open sets in soft settings [7]. The aim of this paper is to mention soft *b*-compact, soft *b*-closed spaces and soft generalized *b*-compact spaces using soft finite intersection property. Some characterization, hereditary property, invariance under mapping for these spaces are investigated.

In our research, in the beginning, we explain some new definitions and vital conclusion under soft set theory because we think these explanations give readers opportunity to understand more easily in subsequent sections. We then give the definitions and basic theories of soft generalized topology. Finally, we introduce the concept of soft *b*-compact spaces and study their basic properties. We also give equivalent conditions for a soft *b*-compact space. We can say that a soft

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b-compact soft generalized topological space gives a parameterized family of b-compact generalized topological spaces in the initial universe.

2 Prelimnaries

Throughout this paper, X will be a nonempty initial universal set and E will be a set of parameters and A be a non-empty subset of E. Let $P(X)$ denote the power set of X and $S(X)$ denote the set of all soft sets over X.

Definition 1. [2] *Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A soft set F_A on the universe X is defined by the set of ordered pairs $F_A = \{(e, f_A(e)) : e \in E, f_A(e) \in P(X)\}$, where $f_A : E \to P(X)$ such that $f_A(e) = \emptyset$ if $e \notin A$. Here, f_A is called an approximate function of the soft set F_A. The value of $f_A(e)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection.*

Definition 2. [3] *Let F_A, $G_B \in S(X)$. Then F_A is said to be a soft subset of G_B, if*

(i) *$A \subset B$, and*
(ii) *$f_A(e) \subset f_B(e)$, for all $e \in A$*

We write $F_A \widetilde{\subset} G_B$. In this case, F_A is said to be a soft subset of G_B and G_B is said to be a soft superset of F_A, $F_A \widetilde{\supset} G_B$.

Definition 3. [3] *Two soft subset F_A and G_B over a common universe set X are said to be a soft equal if $f_A(e) = f_B(e)$, for all $e \in A$ and this relation is denoted by $F_A = G_B$.*

Definition 4. [1] *The complement of a soft set F_A, denoted by $(F_A)^c$, is defined by $(F_A)^c = F_A^c$. $f_A^c : A \to P(X)$ is a mapping given by $f_A^c(e) = X - f_A(e)$, $\forall e \in A$. F_A^c is called the soft complement function of F_A. Clearly, $(F_A^c)^c$ is the same as F_A.*

Definition 5. [4] *The difference of two soft sets F_E and G_E over the common universe X, denoted by $F_E - G_E$ is the soft set H_E where for all $e \in E$, $h_E(e) = f_E(e) - g_E(e)$.*

Definition 6.[4] *Let F_E be a soft set over X and $x \in X$. We say that $x \in F_E$ read as x belongs to the soft set F_E whenever $x \in f_E(e)$ for all $e \in E$.*

Definition 7. [3] *A soft set F_A over X is said to be a null soft set, denoted by $\widetilde{\emptyset}$, if $\forall e \in A$, $f_A(e) = \emptyset$.*

Definition 8. [3] *A soft set F_A over X is called an absolute soft set, denoted by $\widetilde{A}$, if $e \in A$, $f_A(e) = X$.*

If $A = E$, then the A-universal soft set is called a universal soft set, denoted by $\widetilde{X}$. Clearly, $\widetilde{X}^c = \widetilde{\emptyset}$ and $\widetilde{\emptyset}^c = \widetilde{X}$.

Definition 9. [9] *The soft set F_A is called a soft point if there exists a $x \in X$ and $A \subseteq E$ such that $F_A(e) = \{x\}$, for all $e \in A$ and $f_A(e) = \emptyset$, for all $e \in E - A$. A soft point is denoted by F_A^x. The soft point F_E^x is called absolute soft point. A soft point F_A^x is said to belong to a soft set G_B if $x \in G_B(e)$, for each $e \in A$, and symbolically denoted by $F_A^x \widetilde{\in} G_B$.*

Definition 10. [3] *The union of two soft sets of F_A and G_B over the common universe X is the soft set H_C,where $C = A \cup B$ and for all $e \in C$,*

$$h_C(e) = \begin{cases} f_A(e), & \text{if } e \in A - B, \\ g_B(e), & \text{if } e \in B - A, \\ f_A(e) \cup g_B(e), & \text{if } e \in A \cup B. \end{cases}$$

We write $F_A \widetilde{\cup} G_B = H_C$.

Definition 11. [3] *The intersection of two soft sets F_A and G_B over the common universe X is the soft set H_C, where $C = A \cap B$ and for all $e \in C$, $h_C(e) = f_A(e) \cap g_B(e)$.*

This relationship is written as $F_A \widetilde{\cap} G_B = H_C$.

Definition 12. [4] *Let τ be the collection of soft sets over X. Then τ is said to be a soft topology on X if,*

(i) $\widetilde{\emptyset}, \widetilde{X} \widetilde{\in} \tau$,
(ii) *the intersection of any two soft sets in τ belongs to τ,*
(iii) *the union of any number of soft sets in τ belongs to τ.*

The triple (X,τ,E) is called a soft topological space over X. The members of τ are said to be soft open sets.

Definition 13. [8] *Let (X,τ,E) be a soft topological space. A soft set F_A over X is said to be closed soft set in X, if its relative complement F_A^c is an open soft set.*

Definition 14. [4] *Let (X,τ,E) be a soft topological space and $F_E \in S(X)$. The soft closure of F_E, denoted by $cl(F_E)$ is the intersection of all closed soft super sets of F_E i.e $cl(F_E) = \widetilde{\cap}\{H_E : H_E$ is closed soft set and $F_E \widetilde{\subset} H_E\}$.*

Definition 15. [11] *Let (X,τ,E) be a soft topological space and $F_E \in S(X)$. The soft interior of G_E, denoted by $int(G_E)$ is the union of all open soft subsets of G_E i.e $int(G_E) = \widetilde{\cup}\{H_E : H_E$ is an open soft set and $H_E \widetilde{\subset} G_E\}$.*

Definition 16. [11] *The soft set $F_E \in S(X)$ is called a soft point in X if there exist $x \in X$ and $e \in E$ such that $f_E(e) = \{x\}$ and $f_E(e\prime) = \emptyset$ for each $e\prime \in E - \{e\}$, and the soft point F_E is denoted by x_e.*

Definition 17. [11] *The soft point x_e is said to be belonging to the soft set G_A, denoted by $x_e \widetilde{\in} G_A$, if for the element $e \in A$, $f_A(e) \subset g_A(e)$.*

Definition 18. [11] *A soft set G_E in a soft topological space (X,τ,E) is called a soft neighborhood (briefly: nbd) of the soft point $x_e \widetilde{\in} X$ if there exists an open soft set H_E such that $x_e \widetilde{\in} H_E \widetilde{\subset} G_E$. A soft set G_E in a soft topological space (X,τ,E) is called a soft neighborhood of the soft F_E if there exists an open soft set H_E such that $F_E \widetilde{\in} H_E \widetilde{\subset} G_E$.*

Definition 19. [4] *Let Y be a nonempty subset of X, then $\widetilde{Y}$ denotes the soft set Y_A over X for which $y_A(e) = Y$, for all $e \in A$.*

Definition 20. [4] *Let F_A be a soft set over X and Y be a nonempty subset of X. Then the subsoft set of F_A over Y denoted by ${}^Y F_A$ is defined as ${}^Y f_A(e) = Y \cap f_A(e)$, for each $e \in A$. In other word, ${}^Y F_A = \widetilde{Y} \widetilde{\cap} F_A$.*

Definition 21. [4] *Let (X,τ,A) be a soft topological space over X and Y be a nonempty subset of X. Then $\tau_Y = \left\{ {}^Y F_A \mid F_A \widetilde{\in} \tau \right\}$ is said to be the soft relative topology on Y and (Y,τ_Y,A) is called a sof subspace of (X,τ,A). Throughout the paper, the notations $cl(F_A)$ and $int(F_A)$ will stand respectively for the soft closure and soft interior of a soft set F_A in a soft topology space X.*

Theorem 1. [4] *Let (Y,τ_Y,E) be a soft subspace of soft topological space (X,τ,E) and F_E be a soft set over X, then*

(i) *F_E is soft open in Y if and only if $F_E = \widetilde{Y} \widetilde{\cap} G_E$ for some $G_E \in \tau$.*
(ii) *F_E is soft closed in Y if and only if $F_E = \widetilde{Y} \widetilde{\cap} G_E$ for some soft closed set G_E in X.*

Definition 22. [7] *A soft set F_A in a soft topological space X is called*

(i) *soft b-open (sb-open) set if and only if $F_A \widetilde{\subset} int(cl(F_A)) \widetilde{\cup} cl(int(F_A))$.*
(ii) *soft b-closed (sb-closed) set if and only if $F_A \widetilde{\supset} int(cl(F_A)) \widetilde{\cap} cl(int(F_A))$.*

Definition 23. [7] *Let (X,τ,E) be a soft topological space and F_A be a soft set over X.*

(i) *Soft b-closure of a soft set F_A in X is denoted by*

$$sbcl(F_A) = \widetilde{\cap}\left\{F_E \widetilde{\supset} F_A : F_E \text{ is a soft b-closed set of } X\right\}.$$

(ii) *Soft b-interior of a soft set F_A in X is denoted by $sbint(F_A) = \widetilde{\cup}\{O_A \widetilde{\subset} F_A : O_A$ is a soft b-open set of $X\}$.*

Clearly $sbcl(F_A)$ is the smallest soft b-closed set over X which contains F_A and $sbint(F_A)$ is the largest soft b-open set over X which is contained in F_A.

Definition 24. [7] *A soft mapping $f : X \to Y$ is said to be*

(i) *soft $b-$continuous (briefly $sb-$continuous) if the inverse image of each soft open set of Y is a soft $b-$open set in X.*
(ii) *soft $b-$open if the image of each soft open set of X is soft b -open set in Y.*

Definition 25. *A soft mapping $f : X \to Y$ is said to be*

(i) [7] *soft $b-$irresolute if the inverse image of each soft b-open set of Y is a soft $b-$open set in X.*
(ii) *soft b^*-open if the image of each soft b-open set of X is soft b-open set in Y.*

Definition 26. [12] *A soft set F_A in a soft topology space X is called soft generalized b-closed (briefly sgb-closed) soft set if $sbcl(F_A) \widetilde{\subset} G_B$ whenever $F_A \widetilde{\subset} G_B$ and G_B is soft b-open in X.*

Let (X, τ, E) be a soft topological space. A soft set F_A is called a soft generalized b-open (briefly sg$b$-open) in X if the complement F_A^c is soft gb-closed in X.

Definition 27. [11]*A family Ψ of soft sets has the finite intersection property if the intersection of the members of each finite subfamily of Ψ is not null soft set.*

3 Soft b-compact spaces

The most important of all covering properties is compactness. In section study, we introduce the concept of soft $b-$compactness and study some of its basic properties. We now consider a soft b-compact space constructed around a soft topology.

Definition 28. *A collection $\{(G_E)_i : i \in I\}$ of soft b-open sets in a soft topological space (X, τ, E) is called a soft b-open cover of a soft set F_E if $F_E \widetilde{\subset} \widetilde{\cup}\{(G_E)_i : i \in I\}$ holds. If $F_E = \widetilde{X}$, then the collection $\{(G_E)_i : i \in I\}$ is said to be soft b-open covering of (X, τ, E). A finite subfamily of a soft b-open cover $\{(G_E)_i : i \in I\}$ of X is called a finite subcover of $\{(G_E)_i : i \in I\}$, if it is also a soft b-open cover of X.*

Definition 29. *A soft topological space (X, τ, E) is called a soft b-compact space if every soft b-open cover of X has a finite subcover.*

Definition 30. *A soft subset F_E of a soft topological space (X, τ, E) is called soft b-compact in X provided for every collection $\{(G_E)_i : i \in I\}$ of soft b-open sets of X such that $F_E \widetilde{\subset} \widetilde{\cup}\{(G_E)_i : i \in I\}$, there exists a finite subset I_0 of I such that $F_E \widetilde{\subset} \widetilde{\cup}\{(G_E)_i : i \in I_0\}$.*

Definition 31. *A soft topological space (X, τ, E) is called soft b-space if every soft b-open set of X is soft open set in X.*

The following three results immediately from the above definitions.

Corollary 1. *If (X, τ, E) is a soft b-compact space and soft b-space, then X is soft compact space.*

Proof. Let $\{(F_E)_i : i \in I\}$ be a soft open cover of X. Since any soft open set is soft b-open set, $\{(F_E)_i : i \in I\}$ is a soft b-open cover of X. Since X is soft b-compact space and soft b-space, there exists a finite subset I_0 of I such that $X \widetilde{\subset} \widetilde{\cup}\{(F_E)_i : i \in I\}$. Hence X is soft compact space.

Corollary 2. *If $f : X \to Y$ is a soft b-continuous function and soft b-space, then f is soft continuous function.*

Proof. Take a soft open set $\{(G_K)_i : i \in I\}$ of Y. For f is soft b-continuous function, $\{f^{-1}((G_K)_i) : i \in I\}$ is a soft b-open set of X and for X is soft b-space, $\{f^{-1}((G_K)_i) : i \in I\}$ forms a soft open set of X. Hence f is a soft continuous function.

Corollary 3. *Let (X,τ,E) be a soft topological space. If (X,τ_e) is a soft b -compact space, for each $e \in E$, then (X,τ,E) is a soft b-compact space.*

Proof. Let $E = \{e_1,e_2,...,e_n\}$ be a parameter set and (X,τ_e) is soft b-compact space, for every $i = \overline{1,n}$. Let $\{(F_E)_i : i \in I\}$ be a soft b-open cover of X. For, $\widetilde{\cup}_{i\in I}(F_E)_i(e) = \widetilde{X}$, for every $e \in E$, and (X,τ_e) is soft b-compact space, there is a finite subset I_0 of I that $\widetilde{\cup}_{i\in I_0}(F_E)_i(e) = \widetilde{X}$. So, $\widetilde{\cup}_{i\in I_0}(F_E)_i(e) = \widetilde{X}$. Hence $\{(F_E)_i : i \in I_0\}$ is a finite subcover of $\{(F_E)_i : i \in I\}$. Hence (X,τ,E) is soft b-compact space.

Theorem 2. *A soft topological space (X,τ,E) is soft b-compact space if and only if for every family $\{(F_E)_i : i \in I\}$ of soft b-closed sets of X having the finite intersection property, $\widetilde{\cap}_{i\in I}(F_E)_i \neq \Phi$.*

Proof. Let $\{(F_E)_i : i \in I\}$ be a family of soft b-closed sets with the finite intersection property. Assume that $\widetilde{\cap}_{i\in I}(F_E)_i = \Phi$. Then $\widetilde{\cup}_{i\in I}(F_E)_i^c = \widetilde{X}$. For $\{(F_E)_i^c : i \in I\}$ is a collection of soft b-open sets covering X, then from the definition of soft b-compactness of X it follows that there exists a finite subset $I_0 \subset I$ such that $\widetilde{\cup}_{i\in I_0}(F_E)_i^c = \widetilde{X}$. Then $\widetilde{\cap}_{i\in I_0}(F_E)_i = \Phi$, which gives a contradictions. Therefore $\widetilde{\cap}_{i\in I}(F_E)_i \neq \Phi$.

Conversely, let $\{(F_E)_i : i \in I\}$ be a family of soft b-open sets covering X. Suppose that for every finite subset $I_0 \subset I$, we have $\widetilde{\cup}_{i\in I_0}(F_E)_i \neq \widetilde{X}$. Then $\widetilde{\cap}_{i\in I_0}(F_E)_i^c \neq \Phi$. Hence $\{(F_E)_i^c : i \in I\}$ gratify the finite interesection property. Then by definition we have $\widetilde{\cap}_{i\in I}(F_E)_i^c \neq \Phi$ which implies $\widetilde{\cup}_{i\in I_0}(F_E)_i \neq \widetilde{X}$ and this contradicts that $\{(F_E)_i : i \in I\}$ is a soft b-cover of X. Thus X is soft b-compact space.

Theorem 3. *Let $f : X \to Y$ be a soft b-continuous function. If X is soft b-compact space, then the image of X under the f is soft compact.*

Proof. Let $f : X \to Y$ be a soft b-continuous function from a soft b-compact soft topological space (X,τ,E) to (Y,υ,K). Take a soft open cover $\{(G_K)_i : i \in I\}$ of Y. For f is soft b-continuous, $\{f^{-1}((G_K)_i) : i \in I\}$ is a soft b-open cover of X and for X is soft b-compact, there exists a finite subset I_0 of I such that $\{f^{-1}((G_K)_i) : i \in I_0\}$ forms a soft b-open cover of X. Thus, $\{(G_K)_i : i \in I_0\}$ forms a finite soft open cover of Y.

Theorem 4. *A soft topological space (X,τ,E) is soft b-compact space if and only if every family Ψ of soft sets with the finite intersection property, $\widetilde{\cap}_{G_E\in\Psi} sbcl(G_E) \neq \Phi$.*

Proof. Let (X,τ,E) be soft b-compact space and if possible let $\widetilde{\cap}_{G_E\in\Psi} sbcl(G_E) = \Phi$ for some family Ψ of soft sets with the finite intersection property. Thus, $\widetilde{\cup}_{G_E\in\Psi}(sbclG_E)^c = \widetilde{X} \Rightarrow \Upsilon = \{(sbcl(G_E))^c : G_E \in \Psi\}$ is a soft b-open cover of X. Then by soft b-compactness of X, at least one a finite subcover ω of Υ. i.e., $\widetilde{\cup}_{G_E\in\omega}(sbcl(G_E))^c = \widetilde{X} \Rightarrow \widetilde{\cup}_{G_E\in\omega}G_E^c = \widetilde{X} \Rightarrow \widetilde{\cap}_{G_E\in\omega}G_E = \Phi$, a contradiction. Hence $\widetilde{\cap}_{G_E\in\Psi} sbcl(G_E) \neq \Phi$.

Conversely, we have $\widetilde{\cap}_{G_E\in\Psi} sbcl(G_E) \neq \Phi$, for every collection Ψ of soft sets with finite intersection property. Suppose (X,τ,E) is not soft b-compact space. Then at least one a collection of soft b-open soft sets covering X without a finite subcover. Hence for every finite subfamily ω of Υ we have $\widetilde{\cup}_{G_E\in\omega}G_E \neq \widetilde{X} \Rightarrow \widetilde{\cap}_{G_E\in\omega}G_E^c \neq \Phi \Rightarrow \{G_E^c | G_E \in \Upsilon\}$ is a family of soft sets with finite intersection property. Now $\widetilde{\cup}_{G_E\in\Upsilon}(G_E) = \widetilde{X} \Rightarrow \widetilde{\cap}_{G_E\in\Upsilon}(G_E^c) = \Phi \Rightarrow \widetilde{\cap}_{G_E\in\Upsilon} sbcl(G_E^c) = \Phi$, a contradiction.

Theorem 5. *Let f be a soft b^*-continuous function carrying the soft b-compact space (X,τ,E) onto the soft topological space (Y,υ,K). Then (Y,υ,K) is soft b-compact space.*

Proof. Let $\{(G_K)_i : i \in I\}$ be a soft b-open cover of Y. Then $\{f^{-1}((G_K)_i) : i \in I\}$ is a cover of X. For f is soft $b-$irresolute, $f^{-1}((G_K)_i)$ is soft b-open set, and hence $\{f^{-1}((G_K)_i) : i \in I\}$ is a soft b-open cover of X. Since X is soft b-compact space, there exists a finite subset $I_0 \subset I$ such that $X \widetilde{\subset} \widetilde{\cup}\{f^{-1}((G_K)_i) : i \in I_0\}$. Thus $f(X) \widetilde{\subset} f(\widetilde{\cup}\{f^{-1}((G_K)_i) : i \in I_0\}) = \widetilde{\cup}\{f(f^{-1}((G_K)_i)) : i \in I_0\} = \widetilde{\cup}\{(G_K)_i : i \in I_0\}$. Since f is surjective, $Y = f(X) \widetilde{\subset} \widetilde{\cup}\{(G_E)_i : i \in I_0\}$. Hence Y is soft b-compact space.

Theorem 6. *Let F_E be a soft b-closed subset of a soft b-compact space X. Then F_E is also soft b-compact in X.*

Proof. Let F_E be any soft b-closed subset of X and $\{(G_E)_i : i \in I\}$ be a soft b-open cover of X. For F_E^c is soft b-open, $\{(G_E)_i : i \in I\} \widetilde{\cup} F_E^c$ is a soft b-open cover of$X$. For X is soft b-compact space, there exists a finite subset $I_0 \subset I$ such that $X \widetilde{\subset} \{(G_E)_i : i \in I_0\} \widetilde{\cup} F_E^c$. But F_E and F_E^c are disjoint, hence $F_E \widetilde{\subset} \{(G_E)_i : i \in I_0\}$. Therefore F_E is soft b-compact in X.

Theorem 7. *Soft b-closed subspace of a soft b-compact space is soft b-compact.*

Proof. Let Y a soft b-closed subspace of a soft b-compact space (X, τ, E) and $\{(G_B)_i : i \in I\}$ be a soft b-open cover of Y. For each $(G_B)_i$, at least one a soft b-open soft set G_E of X such that $G_B = G_E \widetilde{\cap} Y$. Then the family $\{(G_B)_i : i \in I\} \widetilde{\cup} (X - Y)$ is a soft b-open cover of X, which has a finite subcover. So $\{(G_B)_i : i \in I\}$ has a finite subfamily to cover Y. Hence Y is soft b-compact space.

Theorem 8. *Let F_A and G_B be soft subsets of a soft topological space (X, τ, E) such that F_A is soft b-compact in X and G_B is soft b-closed set in X. Then $F_A \widetilde{\cap} G_B$ is soft b-compact in X.*

Proof. Let $\{(G_E)_i : i \in I\}$ be a cover of $F_A \widetilde{\cap} G_B$ consisting of soft b-open subsets of X. Since G_B^c is a soft b-open set, $\{(G_E)_i : i \in I\} \widetilde{\cup} G_B^c$ is a soft b-open cover of F_A. Since F_A is soft b-compact in X, there exists a finite subset $I_0 \subset I$ such that $F_A \widetilde{\subset} \{(G_E)_i : i \in I_0\} \widetilde{\cup} G_B^c$. Therefore $F_A \widetilde{\cap} G_B \widetilde{\subset} \{(G_E)_i : i \in I_0\}$. Hence $F_A \widetilde{\cap} G_B$ is soft b-compact in X.

Theorem 9. *Let $f : X \to Y$ be a soft b-continuous function, soft b-open and injective mapping. If a soft subset G_E of Y is soft b-compact in Y, then $f^{-1}(G_B)$ is soft b-compact in X.*

Proof. Let $\{(H_C)_i : i \in I\}$ be a soft b-open cover of $f^{-1}(G_B)$ in X. Then $f^{-1}(G_B) \widetilde{\subset} \widetilde{\cup}\{(H_C)_i : i \; inI\}$ and hence $G_B \widetilde{\subset} \widetilde{\cup} f(f^{-1}(G_B)) \widetilde{\subset} f(\widetilde{\cup}\{(H_C)_i : i \in I\}) = \widetilde{\cup}\{f(H_C)_i : i \in I\}$. Since G_B is soft b-compact in Y, there is a finite subset $I_0 \subset I$ such that $G_B \widetilde{\subset} \widetilde{\cup}\{f(H_C)_i : i \in I_0\}$. So $f^{-1}(G_B) \widetilde{\subset} f^{-1}(\widetilde{\cup}\{f((H_C)_i : i \in I_0\}) = \widetilde{\cup}\{f^{-1}(f(H_C)_i) : i \in I_0\} = \widetilde{\cup}\{(H_C)_i : i \in I_0\}$. The proof is completed.

In the following theorem it is shown that image of a soft b-compact space under a soft $b-$irresolute mapping is soft b-compact.

Theorem 10. *If a function $f : X \to Y$ is soft $b-$irresolute and F_E is soft b-compact relative to X, then the image $f(F_E)$ is soft b-compact in Y.*

Proof. Let $\{(G_E)_i : i \in I\}$ be a soft b-open cover of $f(F_E)$ in Y. For f is soft $b-$irresolute function, $\{f^{-1}(G_E)_i : i \in I\}$ is soft b-open cover of F_E in X. For F_E is soft b-compact relative to X, there is a finite subset $I_0 \subset I$ such that $F_E \widetilde{\subset} \widetilde{\cup}\{f^{-1}(G_E)_i) : i \in I_0\}$. Thus $f(F_E) \widetilde{\subset} f(\widetilde{\cup}\{f^{-1}((G_E)_i) : i \in I_0\}$, and hence $f(F_E) \widetilde{\subset} \widetilde{\cup}\{(G_E)_i : i \in I_0\}$. Therefore $f(F_E)$ is soft b-compact in Y.

The pre image of a soft b-compact space under soft b^*-open bijective mapping is soft b-compact space.

Theorem 11. *If a function $f : X \to Y$ is soft b^*-open bijective mapping and Y be soft b-compact space, then X is soft b-compact space.*

Proof. Let $\{(F_E)_i : i \in I\}$ be a collection of soft b-open covering of X. Then let $\{f((F_E)_i) : i \in I\}$ be a soft b-open cover is a collection of soft b-open sets covering Y. For Y is soft b-compact space, by definition there exist a finite family $I_0 \subset I$ such that $\{f((F_E)_i) : i \in I_0\}$ covers Y. Also since f is bijective we have $X = f^{-1}(Y) = f^{-1}(f(\widetilde{\cup}_{i \in I_0}(F_E)_i)) = \widetilde{\cup}_{i \in I_0}(F_E)_i$. Thus X issoft b-compact space.

4 Soft b-closed spaces

Definition 32. *A soft topological space X is said to be soft b-closed if and only if for every family $\{(G_E)_i : i \in I\}$ of soft b-open set such that $\widetilde{\cup}_{i\in I}(G_E)_i = \widetilde{X}$ there is a finite subfamily $I_0 \subset I$ such that $\widetilde{\cup}_{i\in I_0} sbcl(G_E)_i = \widetilde{X}$.*

Definition 33. *A soft set F_E in a soft topological space X is said to be soft b-closed relative to X if and only if for every family $\{(G_E)_i : i \in I\}$ of soft b-open set such that $\widetilde{\cup}_{i\in I}(G_E)_i = F_E$ there is a finite subfamily $I_0 \subset I$ such that $\widetilde{\cup}_{i\in I_0} sbcl(G_E)_i = F_E$.*

Remark. Every soft b-compact space is soft b-closed, but the converse is not true.

Theorem 12. *A soft topological space X is soft b-closed if and only if for every soft finite intersection property Ψ in X, $\widetilde{\cap}_{G_B\in\Psi} sbcl(G_B) \neq \Phi$.*

Proof. Let $\{(F_E)_i : i \in I\}$ be a soft b-open set cover of X and let for every finite collection of $\{(F_E)_i : i \in I\}$, $\widetilde{\cup}_{i\in I_0}(F_{A_i}) \widetilde{\subset} \widetilde{X}$ for some $i \in I_0$. Then $\widetilde{\cap}_{i\in I_0}(G_B)_i^c \widetilde{\supset} \Phi$ for some $i \in I_0$. Thus $\{(sbcl(F_{A_i}))^c : i \in I)\} = \Psi$ forms a soft b-open finite intersection property in X. For $\{(F_E)_i : i \in I\}$ is a soft b-open set cover of X, then $\widetilde{\cap}_{i\in I}(F_{A_i}) = \Phi$ which implies $\widetilde{\cap}_{i\in I} sbcl(sbcl(G_B))^c = \Phi$, which is a contradiction. Then every soft b-open $\{(F_E)_i : i \in I\}$ of X has a finite subfamily I_0 such that $\widetilde{\cup}_{i\in I_0} sbcl(F_{A_i}) = \widetilde{X}$ for every $i \in I_0$. Hence X is soft b-closed space.

Conversely, assume there exists a soft b-open finite intersection property Ψ in X such that $\widetilde{\cap}_{G_B\in\Psi} sbcl(G_B) = \Phi$. That implies $\widetilde{\cup}_{G_B\in\Psi}(sbcl(G_B))^c = \widetilde{X}$ for every $i \in I$ and hence $\{(F_E)_i : i \in I\} = \{sbcl(G_B) : G_B \in \Psi\}$ is a soft b-open set cover of X. For X is soft b-closed, by definition $\{(F_E)_i : i \in I\}$ has a finite subfamily I_0 such that $\widetilde{\cup}_{i\in I_0} sbcl(sbclG_B)^c = \widetilde{X}$ for every $i \in I_0$, and hence $\widetilde{\cap}_{i\in I_0}(sbcl(sbclG_B)^c)^c = \Phi$. Thus $\widetilde{\cap}_{G_B\in I_0} G_B = \Phi$ is a contradiction. Therefore $\widetilde{\cap}_{G_B\in\Psi} sbcl(G_B) \neq \Phi$.

Theorem 13. *Let $f : X \to Y$ be a soft $b-$irresolute surjection. If X is soft b-closed space, then Y is soft b-closed space.*

Proof. Let $\{(F_E)_i : i \in I\}$ be a soft b-open cover of Y. Since f is soft $b-$irresolute, $\{f^{-1}(F_E)_i : i \in I\}$ is soft b-open cover of X. By hypothesis, there exists a finite subset I_0 of Ψ such that $\widetilde{\cup}_{i\in I_0} sbcl(f^{-1}(F_E)_i) = X$. For f is surjective and by theorem $Y = f(X) = f(\widetilde{\cup}_{i\in I_0} sbcl(f^{-1}(F_E)_i)) \widetilde{\subset} \widetilde{\cup}_{i\in I_0} sbcl(f(f^{-1}(F_E)_i = \widetilde{\cup}_{i\in I_0} sbcl(F_E)_i))$. Hence Y is soft b closed space.

5 Soft generalized b-compact spaces

We shall define the concept of soft generalized b-compact spaces.

Definition 34. *A collection $\{(F_E)_i : i \in I\}$ of soft generalized b-open sets in X is called soft generalized b-open cover of a soft set G_B in X if $G_B \widetilde{\subset} \widetilde{\cup}_{i\in I}(F_E)_i$.*

Definition 35. *A topological space X is called soft generalized b-compact if every soft generalized b-open cover of X has a finite subcover.*

Definition 36. *A soft set F_E in X is said to be soft generalized b-compact relative to X if for every collection $\{(F_E)_i : i \in I\}$ of soft generalized b-open sets of X such that $F_E \widetilde{\subset} \widetilde{\cup}_{i\in I}(F_E)_i$, there exists a finite subset I_0 of I such that $F_E \widetilde{\subset} \widetilde{\cup}_{i\in I_0}(F_E)_i$.*

Definition 37. *A soft set F_E of X is said to be soft generalized b-compact if F_E is soft generalized b-compact relative to X.*

Theorem 14. *Let X be soft generalized b-compact and F_E be soft generalized b-closed set in X. Then F_E is soft generalized b-compact.*

Proof. Let F_E be any soft generalized b-closed subset of X and $\{(G_E)_i : i \in I\}$ be a soft generalized b-open cover of X. Since F_E^c is soft generalized b-open, $\{(G_E)_i : i \in I\}\tilde{\cup}F_E^c$ is a soft generalized b-open cover of X. For X is soft generalized b-compact space, there exists a finite subset $I_0 \subset I$ such that $X\tilde{\subset}\{(G_E)_i : i \in I\}\tilde{\cup}F_E^c$. But F_E and F_E^c are disjoint, hence $F_E\tilde{\cup}\{(G_E)_i : i \in I_0\}$. Therefore F_E is soft generalized b-compact in X.

Theorem 15. *A soft generalized b-continuous image of a soft generalized b-compact space is soft compact space.*

Proof. Let $f : X \to Y$ be a soft generalized b-continuous function from a soft generalized b-compact space (X,τ,E) to (Y,υ,K). Take a soft open cover $\{(G_K)_i : i \in I\}$ of Y. For f is soft generalized b-continuous, $\{f^{-1}((G_K)_i) : i \in I\}$ is a soft generalized b-open cover of X and for X is soft generalized b-compact, there exists a finite subset I_0 of I such that $\{f^{-1}((G_K)_i) : i \in I_0\}$ forms a soft generalized b-open cover of X. Thus $\{(G_K)_i : i \in I_0\}$ forms a finite soft open cover of Y.

6 Conclusion

It is an interesting exercise to work on soft b-compact spaces and soft b-closed spaces. Similarly other forms of generalized open set can be applied to define different forms of compact spaces and closed spaces.

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