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Intuitionistic Fuzzy Dot d-ideals of d-algebras

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Abstract: The concept of intuitionistic fuzzy dot d-ideals in d-algebra is introduced. Intuitionistic fuzzy dot subalgebra of d-algebra is defined. Some important results in respect of intuitionistic fuzzy d-ideal, intuitionistic fuzzy dot d-ideals are derived. Effect of Attanasov's operators $\Box, \Diamond$ and $F_{\alpha,\beta}$ on intuitionistic fuzzy dot d-ideals are studied. Some properties of intuitionistic fuzzy dot d-ideals under homomorphism are investigated.

Keywords: d-algebra, fuzzy d-ideal, fuzzy dot d-ideal, intuitionistic fuzzy dot subalgebra, intuitionistic fuzzy dot d-ideal.

1 Introduction

The notion of fuzzy subset of a set was introduced by Zadeh [12] in 1965, since then researchers have been engrossed in extending the concepts and results of every concept of mathematics to boarder framework of fuzzy setting. Imai and Iseki [6] introduced BCK-algebras as a generalization of notion of the concept of set theoretic difference and propositional calculus and in the same year Iseki [7] introduced the notion of BCI-algebra which is a generalization of BCK-algebra. Xi[10] applied the concept of fuzzy set to BCK-algebra and introduced fuzzy subalgebra and fuzzy ideals in BCK-algebra. In 1996, Neggers and Kim [9] introduced the class of d-algebras which is a generalisation of BCK-algebras and investigated relation between d-algebras and BCK-algebras. Akram and Dar[1] introduced the concepts fuzzy d-algebra, fuzzy subalgebra and fuzzy d-ideals of d-algebra. The notion of a fuzzy dot subalgebra of d-algebra were introduced in [8] by Kim. Al-Shehrie[2] introduced the notion of fuzzy dot d-ideals of a d-algebra. Yun et al. [11] applied the concept of intuitionistic fuzzy set to d-algebra and defined intuitionistic fuzzy d-algebra and intuitionistic fuzzy topological d-algebra. Here in this paper, we introduced the notion of intuitionistic fuzzy dot subalgebra and intuitionistic fuzzy dot d-ideals of d-algebra and we investigated several interesting properties.

2 Preliminaries

Definition 1. [2] *A d-algebra is a non-empty set X with a constant 0 and a binary operation $*$ satisfying the following axioms:*

(i) $x*x=0$
(ii) $0*x=0$
(iii) $x*y=0$ *and* $y*x=0 \Rightarrow x=y$ *for all* $x,y \in X$.

For brevity we also call X a d-algebra.

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Definition 2. [2] *A non-empty subset S of a d-algebra X is called a subalgebra of X if $x*y \in S$, for all $x,y \in S$.*

Definition 3. [2] *A nonempty subset I of a d-algebra X is called an ideal of X if*

(i) $0 \in I$
(ii) $x*y \in I$ *and* $y \in I \Rightarrow x \in I$
(iii) $x \in I$ *and* $y \in X \Rightarrow x*y \in I$.

Definition 4. [2] *A fuzzy subset μ of X is called a fuzzy dot subalgebra of a d-algebra X if for all $x,y \in X$, $\mu(x*y) \geq \mu(x).\mu(y)$, where . (dot) denotes ordinary multiplication.*

Definition 5. [2] *A fuzzy subset μ of X is called a fuzzy d-ideal of X if it satisfies the following conditions:*

(i) $\mu(0) \geq \mu(x)$
(ii) $\mu(x) \geq min\{\mu(x*y), \mu(y)\}$
(iii) $\mu(x*y) \geq min\{\mu(x), \mu(y)\}$.

Definition 6. [2] *A fuzzy subset μ of X is called a fuzzy dot d-ideal of X if it satisfies the following conditions:*

(i) $\mu(0) \geq \mu(x)$
(ii) $\mu(x) \geq \mu(x*y).\mu(y)$
(iii) $\mu(x*y) \geq \mu(x).\mu(y)$ *for all* $x,y \in X$.

Definition 7. *An intuitionistic fuzzy set (IFS) A of a d-algebra X is an object of the form $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$, where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$. The numbers $\mu_A(x)$ and $\nu_A(x)$ denote respectively the degree of membership and the degree of non-membership of the element x in set A. For the sake of simplicity, we shall use the symbol $A = (\mu_A, \nu_A)$ for the intuitionistic fuzzy set $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$. The function $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ for all $x \in X$. Here $\pi_A(x)$ is called the degree of hesitance of $x \in A$.*

Definition 8. *If $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ and $B = \{< x, \mu_B(x), \nu_B(x) > | x \in X\}$ are any two IFS of a set X, then*
$A \subseteq B$ *if and only if for all* $x \in X, \mu_A(x) \leq \mu_B(x)$ *and* $\nu_A(x) \geq \nu_B(x)$,
$A = B$ *if and only if for all* $x \in X, \mu_A(x) = \mu_B(x)$ *and* $\nu_A(x) = \nu_B(x)$,
$A \cap B = \{< x, (\mu_A \cap \mu_B)(x), (\nu_A \cup \nu_B)(x) > | x \in X\}$,
where $(\mu_A \cap \mu_B)(x) = \min\{\mu_A(x), \mu_B(x)\}$ *and* $(\nu_A \cup \nu_B)(x) = \max\{\nu_A(x), \nu_B(x)\}$,
$A \cup B = \{< x, (\mu_A \cup \mu_B)(x), (\nu_A \cap \nu_B)(x) > | x \in X\}$,
where $(\mu_A \cup \mu_B)(x) = \max\{\mu_A(x), \mu_B(x)\}$ *and* $(\nu_A \cap \nu_B)(x) = \min\{\nu_A(x), \nu_B(x)\}$.

Definition 9. [2] *An intuitionistic fuzzy set $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ of d-algebra X is called an intuitionistic fuzzy subalgebra of X if it satisfies the following conditions:*

(i) $\mu_A(x*y) \geq min\{\mu_A(x), \mu(y)\}$
(ii) $\nu_A(x*y) \leq max\{\nu_A(x), \nu_A(y)\}$ *for all* $x,y \in X$.

Definition 10. *An intuitionistic fuzzy set $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ of d-algebra X is called an intuitionistic fuzzy d-ideal of X if it satisfies the following conditions:*

(i) $\mu_A(0) \geq \mu_A(x)$
(ii) $\mu_A(x) \geq min\{\mu_A(x*y), \mu(y)\}$
(iii) $\mu_A(x*y) \geq min\{\mu_A(x), \mu_A(y)\}$ *for all* $x,y \in X$.

(iv) $\nu_A(0) \leq \nu_A(x)$

(v) $\nu_A(x) \leq max\{\nu_A(x*y), \mu(y)\}$

(vi) $\nu_A(x*y) \leq max\{\nu_A(x), \nu_A(y)\}$ *for all* $x, y \in X$.

Definition 11. *For any IFS* $A = \{<x, \mu_A(x), \nu_A(x)> |x \in X\}$ *of* X *and* $\alpha \in [01]$, *the operator* $\Box : IFS(X) \rightarrow IFS(X), \Diamond : IFS(X) \rightarrow IFS(X), D_\alpha : IFS(X) \rightarrow IFS(X)$ *are defined as*

1. $\Box(A) = \{<x, \mu_A(x), 1-\mu_A(x)> |x \in X\}$ *is called necessity operator*
2. $\Diamond(A) = \{<x, 1-\nu_A(x), \nu_A(x)> |x \in X\}$ *is called possibility operator*
3. $D_\alpha(A) = \{<x, \mu_A(x)+\alpha\pi_A(x), \nu_A(x)+(1-\alpha)\pi_A(x)> |x \in X\}$ *is called* α*-Model operator.*

Clearly $\Box(A) \subseteq A \subseteq \Diamond(A)$ *and the equality hold, when A is a fuzzy set also* $D_0(A) = \Box(A)$ *and* $D_1(A) = \Diamond(A)$. *Therefore the* α*-Model operator* $D_\alpha(A)$ *is an extension of necessity operator* $\Box(A)$ *and possibility operator* $\Diamond(A)$.

Definition 12. *For any IFS* $A = \{<x, \mu_A(x), \nu_A(x)> |x \in X\}$ *of* X *and for any* $\alpha, \beta \in [01]$ *such that* $\alpha+\beta \leq 1$, *the* (α, β)*-model operator* $F_{\alpha,\beta} : IFS(X) \rightarrow IFS(X)$ *is defined as* $F_{\alpha,\beta}(A) = \{<x, \mu_A(x)+\alpha\pi_A(x), \nu_A(x)+\beta\pi_A(x)> |x \in X\}$, *where* $\pi_A(x) = 1-\mu_A(x)-\nu_A(x)$ *for all* $x \in X$. *Therefore we can write*
$F_{\alpha,\beta}(A)$ *as* $F_{\alpha,\beta}(A)(x) = (\mu_{F_{\alpha,\beta}(A)}(x), \nu_{F_{\alpha,\beta}(A)}(x))$
where $\mu_{F_{\alpha,\beta}}(x) = \mu_A(x)+\alpha\pi_A(x)$ *and* $\nu_{F_{\alpha,\beta}(A)}(x)) = \nu_A(x)+\beta\pi_A(x)$.
Clearly, $F_{0,1}(A) = \Box(A), F_{1,0}(A) = \Diamond(A)$ *and* $F_{\alpha,1-\alpha}(A) = D_\alpha(A)$.

Remark. If X and Y be two d-algebras, then $X \times X$ is also a d-algebra under the binary operation $'*'$ defined in $X \times X$ by $(x,y)*(p,q) = (x*p, y*q)$ for all $(x,y),(p,q) \in X \times X$.

Definition 13. *Let X and Y be two non empty sets and* $f : X \longrightarrow Y$ *be a mapping. Let A and B be IFS's of X and Y respectively . Then the image of A under the map f is denoted by f(A) and is defined by* $f(A)y = (\mu_{f(A)}y, \nu_{f(A)}y)$, *where*

$$\mu_{f(A)}(y) = \begin{cases} \vee\{\mu_A(x) : x \in f^{-1}(y)\} \\ 0 \quad otherwise \end{cases} \qquad \nu_{f(A)}(y) = \begin{cases} \wedge\{\nu_A(x) : x \in f^{-1}(y)\} \\ 1 \quad otherwise \end{cases}$$

also pre image of B under f is denoted by $f^{-1}(B)$ *and is defined as*

$$f^{-1}(B)(x) = (\mu_{f^{-1}(B)}(x), \nu_{f^{-1}(B)}(x)) = (\mu_B(f(x)), \nu_B(f(x))); \forall x \in X.$$

Remark: Note that $\mu_A(x) \leq \mu_{f(A)}(f(x))$ *and* $\nu_A(x) \geq \nu_{f(A)}(f(x)) \quad \forall x \in X$ *however equality hold when the map f is bijective.*

Definition 14. *Let* $A = <\mu_A, \nu_A>$ *be intuitionistic fuzzy subset of X and* $\alpha, \beta \in [0,1]$ *then* (α, β) *cut set of A is*

$$A_{(\alpha,\beta)} = \{x | x \in X, \mu_A(x) \geq \alpha \quad and \quad \nu_A(x) \leq \beta\}$$

Lemma 1. *If* $a, b, c, d \in [0, 1]$, *then*

–$min\{a,b\} \geq a.b$
–$max\{a,b\} \leq a+b-a.b$
–$min\{a.b, c.d\} \geq min\{a,c\}.min\{b,d\}$.

3 Intuitionistic fuzzy dot d-ideals of d-algebras

Definition 15. *An intuitionistic fuzzy set* $A = \{<x, \mu_A(x), \nu_A(x)> |x \in X\}$ *of d-algebra X is called an intuitionistic fuzzy dot subalgebra of X if it satisfies the following conditions:*

(i) $\mu_A(x*y) \geq \mu_A(x).\mu(y)$
(ii) $\nu_A(x*y) \leq \nu_A(x)+\nu_A(y)-\nu_A(x).\nu_A(y)$ *for all* $x,y \in X$.

Definition 16. *An intuitionistic fuzzy set* $A=\{<x,\mu_A(x),\nu_A(x)>|x \in X\}$ *of d-algebra X is called an intuitionistic fuzzy dot d-ideal of X if it satisfies the following conditions:*

(i) $\mu_A(0) \geq \mu_A(x)$
(ii) $\mu_A(x) \geq \mu_A(x*y).\mu(y)$
(iii) $\mu_A(x*y) \geq \mu_A(x).\mu_A(y)$ *for all* $x,y \in X$.
(iv) $\nu_A(0) \leq \nu_A(x)$
(v) $\nu_A(x) \leq \nu_A(x*y)+\nu_A(y)-\nu_A(x*y).\nu_A(y)$
(vi) $\nu_A(x*y) \leq \nu_A(x)+\nu_A(y)-\nu_A(x).\nu_A(y)$ *for all* $x,y \in X$.

Example 1. Consider d-algebra $X=\{0,a,b,c\}$ with the following cayley table.

*	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	b	0	0
c	c	c	a	0

The intuitionistic fuzzy subset $A=\{<x,\mu_A(x),\nu_A(x)>|x \in X\}$ given by $\mu_A(0)=\mu_A(b)=0.6, \mu_A(a)=\mu_A(c)=0.5$ and $\nu_A(0)=\nu_A(a)=0.3, \nu_A(b)=\nu_A(c)=0.4$ then it is easy to verify that $A=\{<\mu_A(x),\nu_A(x)>\}$ is an intuitionistic fuzzy dot d-ideal of X.

Theorem 1. *Every intuitionistic fuzzy d-ideal of d algebra X is an intuitionistic fuzzy dot d-ideal of X.*

Proof. Let $A=\{<x,\mu_A(x),\nu_A(x)>|x \in X\}$ be an intuitionistic fuzzy d-ideal of X, therefore we have

(i) $\mu_A(0) \geq \mu_A(x)$
(ii) $\mu_A(x) \geq min\{\mu_A(x*y),\mu_A(y)\}$
(iii) $\mu_A(x*y) \geq min\{\mu_A(x),\mu_A(y)\}$ for all $x,y \in X$.
(iv) $\nu_A(0) \leq \nu_A(x)$
(v) $\nu_A(x) \leq max\{\nu_A(x*y),\mu(y)\}$
(vi) $\nu_A(x*y) \leq max\{\nu_A(x),\nu_A(y)\}$ for all $x,y \in X$.

Now $\mu_A(x) \geq min\{\mu_A(x*y),\mu(y)\} \geq \mu_A(x*y).\mu(y)$ and $\mu_A(x*y) \geq min\{\mu_A(x).\mu_A(y)\} \geq \mu_A(x).\mu_A(y)$ for all $x,y \in X$.
Also $\nu_A(x) \leq max\{\nu_A(x*y),\mu(y)\} \leq \nu_A(x*y)+\mu_A(y)-\nu_A(x*y).\nu_A(y)$ and $\nu_A(x*y) \leq max\{\nu_A(x),\nu_A(y)\} \leq \nu_A(x)+\mu_A(y)-\nu_A(x).\nu_A(y)$ for all $x,y \in X$.
Hence the proof.

Remark. The converse of Theorem 1 is not true as shown in following Example.

Example 2. Consider d-algebra $X=\{0,a,b\}$ with the following cayley table.

*	0	a	b
0	0	0	0
a	b	0	b
b	a	a	0

The intuitionistic fuzzy subset $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ given by $\mu_A(0) = 0.6, \mu_A(a) = 0.4, \mu_A(b) = 0.5$ and $\nu_A(0) = 0.4, \nu_A(a) = 0.6, \nu_A(b) = 0.5$ then $A =< \mu_A, \nu_A >$ is an IF dot d-ideal of X. But $A =< \mu_A, \nu_A >$ is not an IF d-ideal of X. Since $\mu_A(a) = 0.4 \ngeq min\{\mu_A(a*b), \mu_A(b)\} = min\{\mu_A(b), \mu_A(b)\} = 0.5$.

Theorem 2. *Every intuitionistic fuzzy dot d-ideal of a d algebra X is an intuitionistic fuzzy dot subalgebra of X.*

*Remark.*The converse of Theorem 2 is not true as shown in following Example.

Example 3. Consider d-algebra X as in Example 2 and intuitionistic fuzzy subset $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ given by $\mu_A(0) = 0.3, \mu_A(a) = 0.5, \mu_A(b) = 0.4$ and $\nu_A(0) = 0.4, \nu_A(a) = 0.5, \nu_A(b) = 0.3$ then it can be easily verified that $A =< \mu_A, \nu_A >$ is an IF dot subalgebra of X . But $A =< \mu_A, \nu_A >$ is not an IF dot d ideal X. Since $\mu_A(0) = 0.3 \ngeq \mu_A(a) = 0.5$ and $\nu_A(0) = 0.4 \nleq \mu_A(b) = 0.3$.

Lemma 2. *If* $a, b, p, q \in [0\,1]$, *then*
$max(a + b - a.b, p + q - p.q < max(a, p) + max(b, q) - max(a, p).max(b, q) < 1$ *and the equality holds only when* $a = b = p = q = 1$.

Theorem 3. *If* $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ *and* $B = \{< x, \mu_B(x), \nu_B(x) > | x \in X\}$ *are two intuitionistic fuzzy dot d-ideals of X d-algebra X, Then* $(A \cap B)$ *is also an intuitionistic fuzzy dot d-ideal of X.*

Proof. We have $A \cap B = \{< x, (\mu_A \cap \mu_B)(x), (\nu_A \cup \nu_B)(x) > | x \in X\}$,
where $(\mu_A \cap \mu_B)(x) = \min\{\mu_A(x), \mu_B(x)\}$ and $(\nu_A \cup \nu_B)(x) = \max\{\nu_A(x), \nu_B(x)\}$
Let $x, y \in X$, then

$$\begin{aligned}
\text{(i)}(\mu_A \cap \mu_B)(0) &= \min\{\mu_A(0), \mu_B(0)\} \\
&= \min\{\mu_A(0), \mu_B(0)\} \\
&\geq \min\{\mu_A(x), \mu_B(x)\} \\
&= (\mu_A \cap \mu_B)(x) \\
\Rightarrow (\mu_A \cap \mu_B)(0) &\geq (\mu_A \cap \mu_B)(x) \\
\text{(ii)}(\nu_A \cup \nu_B)(0) &= \max\{\nu_A(0), \nu_B(0)\} \\
&= \max\{\nu_A(0), \nu_B(0)\} \\
&\leq \max\{\nu_A(x), \nu_B(x)\} \\
&= (\nu_A \cup \nu_B)(x) \\
\Rightarrow (\nu_A \cup \nu_B)(0) &\leq (\nu_A \cup \nu_B)(x) \\
\text{(iii)}(\mu_A \cap \mu_B)(x) &= \min\{\mu_A(x), \mu_B(x)\} \\
&\geq \min\{\mu_A(x*y).\mu_A(y), \mu_B(x*y).\mu_B(y)\} \\
&\geq \min\{\mu_A(x*y), \mu_B(x*y)\}.min\{\mu_A(y), \mu_B(y)\}\} \\
&= (\mu_A \cap \mu_B)(x*y).(\mu_A \cap \mu_B)(y)\}\} \\
\Rightarrow (\mu_A \cap \mu_B)(x) &\geq (\mu_A \cap \mu_B)(x*y).(\mu_A \cap \mu_B)(y)\}\}.
\end{aligned}$$

(iv) $(\nu_A \cup \nu_B)(x)$

$= \max\{\nu_A(x), \nu_B(x)\}$

$\leq \max\{\nu_A(x*y)+\nu_A(y)-\nu_A(x*y).\nu_A(y), \nu_B(x*y)+\nu_B(y)-\nu_B(x*y).\nu_B(y)\}$

$\leq max\{\nu_A(x*y), \nu_B(x*y)\}+max\{\nu_A(y), \nu_B(y)\}-max\{\nu_A(x*y), \nu_B(x*y)\}.max\{\nu_A(y), \nu_B(y)\}$. By Lemma 2

$= (\nu_A \cup \nu_B)(x*y)+(\nu_A \cup \nu_B)(y)-(\nu_A \cup \nu_B)(x*y).(\nu_A \cup \nu_B)(y)$.

$\Rightarrow (\nu_A \cup \nu_B)(x) \leq (\nu_A \cup \nu_B)(x*y)+(\nu_A \cup \nu_B)(y)-(\nu_A \cup \nu_B)(x*y).(\nu_A \cup \nu_B)(y)$.

(v)$(\mu_A \cap \mu_B)(x*y) = \min\{\mu_A(x*y), \mu_B(x*y)\}$

$\geq min\{\mu_A(x).\mu_A(y)\}, \mu_B(x).\mu_B(y)\}$

$\geq min\{\mu_A(x), \mu_B(x)\}.min\{\mu_A(y), \mu_B(y)\}$

$= (\mu_A \cap \mu_B)(x)\}.(\mu_A \cup \mu_B)(y)\}$

$\Rightarrow (\mu_A \cap \mu_B)(x*y) \geq (\mu_A \cap \mu_B)(x).(\mu_A \cap \mu_B)(y)\}$.

(vi) $(\nu_A \cup \nu_B)(x*y)$

$= \max\{\nu_A(x*y), \nu_B(x*y)\}$

$\leq \max\{\nu_A(x)+\nu_A(y)-\nu_A(x).\nu_A(y), \nu_B(x)+\nu_B(y)-\nu_B(x).\nu_B(y)\}$

$\leq max\{\nu_A(x), \nu_B(x)\}+max\{\nu_A(y), \nu_B(y)\}-max\{\nu_A(x), \nu_B(x)\}.max\{\nu_A(y), \nu_B(y)\}$. By Lemma 2

$= (\nu_A \cup \nu_B)(x)+(\nu_A \cup \nu_B)(y)-(\nu_A \cup \nu_B)(x).(\nu_A \cup \nu_B)(y)$.

$\Rightarrow (\nu_A \cup \nu_B)(x*y) \leq (\nu_A \cup \nu_B)(x)+(\nu_A \cup \nu_B)(y)-(\nu_A \cup \nu_B)(x).(\nu_A \cup \nu_B)(y)$.

Hence $(A \cap B)$ is also an intuitionistic fuzzy dot d-ideal of X.

Theorem 4. *IF $C_{\alpha,\beta}(A) = \{x \in X \mid \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta\}$ is an ideal of X, then $A =< \mu_A, \nu_A >$ is an intuitionistic fuzzy dot d-ideal of X, where $\alpha, \beta \in [0, 1]$.*

Proof. Let $C_{\alpha,\beta}(A)$ is an ideal. To prove $A =< \mu_A, \nu_A >$ is an intuitionistic fuzzy dot d-ideal of X. In view of Theorem 1 it is enough to show that $A =< \mu_A, \nu_A >$ is an intuitionistic fuzzy d-ideal of X.
Let $x \in X$ such that $\mu_A(x) = \alpha$. Also since $0 \in C_{\alpha,\beta}(A)$.
Therefore $\mu_A(0) \geq \alpha = \mu_A(x)$.
Let $x*y, y \in X$, such that $\mu_A(x*y) = \alpha, \mu_A(y) = \gamma$ where $\alpha \leq \gamma$.
Then $(x*y), y \in C_{\alpha,\beta}(A)$. Since $C_{\alpha,\beta}(A)$ is an ideal.
Therefore $x \in C_{\alpha,\beta}(A)$, which implies
$\mu_A(x) \geq \alpha = min\{\alpha, \gamma\} = min\{\mu_A(x*y), \mu_A(y)\}$.
Again let $x, y \in X$ such that $\mu_A(x) = \alpha, \mu_A(y) = \gamma$ where $\alpha \leq \gamma$ Then $x, y \in C_{\alpha,\beta}(A)$ Since $C_{\alpha,\beta}(A)$ is an ideal Therefore $x*y \in C_{\alpha,\beta}(A)$, which implies
$\mu_A(x*y) \geq \alpha = min\{\alpha, \gamma\} = min\{\mu_A(x), \mu_A(y)\}$.
Similarly we can prove $\nu_A(0) \leq \nu_A(x)$
$\nu_A(x) \leq max\{\nu_A(x*y), \nu_A(y)\}$ and
$\nu_A(x*y) \leq max\{\nu_A(x), \nu_A(y)\}$.

Theorem 5. *If* $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ *is an intuitionistic fuzzy dot d-ideals of d-algebra X, Then* $\Box A$ *and* $\Diamond A$ *is also an intuitionistic fuzzy dot d-ideal of X.*

Proof. We have $\Box A = \{< x, \mu_A(x), \overline{\mu_A}(x) > | x \in X\}$,
$\Diamond A = \{< x, \overline{\nu_A}(x), \nu_A(x) > | x \in X\}$,
To prove $\Box A$ is an intuitionistic fuzzy dot d-ideal of X. Let $x, y \in X$, then

$$\begin{aligned}
\mu_A(0) &\geq \mu_A(x)\\
\therefore 1-\mu_A(0) &\leq 1-\mu_A(x)\\
\overline{\mu_A}(0) &\leq \overline{\mu_A}(x)\\
\text{and,} \qquad \mu_A(x) &\geq \mu_A(x*y).\mu_A(y)\\
\overline{\mu_A}(x) &= 1-\mu_A(x)\\
&\leq 1-\mu_A(x*y).\mu_A(y)\\
&= 1-\mu_A(x*y)+1-\mu_A(y)-(1-\mu_A(x*y)).(1-\mu_A(y)).\\
&= \overline{\mu_A}(x*y)+\overline{\mu_A}(y)-\overline{\mu_A}(x*y).\overline{\mu_A}(y)\\
\text{Again here,} \qquad \mu_A(x*y) &\geq \mu_A(x).\mu_A(y)\\
\overline{\mu_A}(x*y) &= 1-\mu_A(x*y)\\
&\leq 1-\mu_A(x).\mu_A(y)\\
&= 1-\mu_A(x)+1-\mu_A(y)-(1-\mu_A(x).(1-\mu_A(y))\\
&= \overline{\mu_A}(x)+\overline{\mu_A}(y)-\overline{\mu_A}(x).\overline{\mu_A}(y).
\end{aligned}$$

To prove $\Diamond A$ is an intuitionistic fuzzy dot d-ideal of X. Let $x, y \in X$, then

$$\begin{aligned}
\nu_A(0) &\leq \nu_A(x)\\
\therefore 1-\nu_A(0) &\geq 1-\nu_A(x)\\
\overline{\nu_A}(0) &\geq \overline{\nu_A}(x).\\
\text{Here,} \qquad \nu_A(x) &\leq \nu_A(x*y)+\nu_A(y)-\nu_A(x*y).\nu_A(y)\\
\overline{\nu_A}(x) &= 1-\nu_A(x)\\
&\geq 1-\nu_A(x*y)-\nu_A(y)+\nu_A(x*y).\nu_A(y)\\
&= (1-\nu_A(x*y)).(1-\nu_A(y))\\
&= \overline{\nu_A}(x*y).\overline{\nu_A}(y).\\
\text{Again here,} \qquad \nu_A(x*y) &\leq \nu_A(x)+\nu_A(y)-\nu_A(x).\nu_A(y)\\
\overline{\nu_A}(x*y) &= 1-\nu_A(x*y)\\
&\geq 1-\nu_A(x)-\nu_A(y)+\nu_A(x).\nu_A(y)\\
&= (1-\nu_A(x)).(1-\nu_A(y))\\
&= \overline{\nu_A}(x).\overline{\nu_A}(y).
\end{aligned}$$

Remark. The converse of Theorem 5 is not true as shown in following Example.

Example 4. Consider d-algebra X as in Example 2 and an intuitionistic fuzzy subset $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ given by $\mu_A(0) = 0.6, \mu_A(a) = 0.5, \mu_A(b) = 0.4$ and $\nu_A(0) = 0.2, \nu_A(a) = 0.3, \nu_A(b) = 0.5$ then it can be easily verified that $A = \{< \mu_A(x), \nu_A(x) >\}$ is not an IF dot d-ideal of X. Since $\nu_A(b) = 0.5 \not\leq \nu_A(b*0) + \nu_A(0) - \nu_A(b*0).\nu_A(0) = \nu_A(a) + \nu_A(0) - \nu_A(a).\nu_A(0) = 0.2 + 0.3 - 0.06 = 0.44$. But $\Box A = \{< \mu_A(x), \overline{\mu_A}(x) >\}$ is given by $\mu_A(0) = 0.6, \mu_A(a) = 0.5, \mu_A(b) = 0.4$ and $\overline{\mu_A}(0) = 0.4, \overline{\mu_A}(a) = 0.5, \overline{\mu_A}(b) = 0.6$ is an IF dot d ideal X.

Again consider an intuitionistic fuzzy subset $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ given by $\mu_A(0) = 0.6, \mu_A(a) = 0.5, \mu_A(b) = 0.1$ and $\nu_A(0) = 0.4, \nu_A(a) = 0.5, \nu_A(b) = 0.6$ then it can be easily verified that $A = \{< \mu_A(x), \nu_A(x) >\}$ is not an IF dot d-ideal of X. Since $\mu_A(b) = 0.1 \not\geq \mu_A(b*0).\mu_A(0) = \mu_A(a).\mu_A(0) = 0.5.0.6 = 0.3$. But $\Diamond A = \{< \overline{\nu_A}(x), \nu_A(x) >\}$ is given by $\overline{\nu_A}(0) = 0.6, \overline{\nu_A}(a) = 0.5, \overline{\nu_A}(b) = 0.4$ and $\nu_A(0) = 0.4, \nu_A(a) = 0.5, \nu_A(b) = 0.6$ is an IF dot d ideal X.

Theorem 6. *If $A =< \mu_A, \nu_A >$ is an IF d-ideal of d-algebra X, then $F_{\alpha,\beta}(A)$ is also an IF d- ideal X.*

Proof. Let $x \in X$, then $F_{\alpha,\beta}(x) = (\mu_{F_{\alpha,\beta}(A)}(x), \nu_{F_{\alpha,\beta}(A)}(x))$ where $\mu_{F_{\alpha,\beta}(A)}(x) = \mu_A(x) + \alpha\pi_A(x)$ and $\nu_{F_{\alpha,\beta}(A)}(x) = \nu_A(x) + \beta\pi_A(x)$. Now

$$\begin{aligned}
\mu_{F_{\alpha,\beta}(A)}(0) &= \mu_A(0) + \alpha\pi_A(0) \\
&= \mu_A(0) + \alpha(1 - \mu_A(0) - \nu_A(0)) \\
&= \alpha + (1-\alpha)\mu_A(0) - \alpha\nu_A(0) \\
&\geq \alpha + (1-\alpha)\mu_A(x) - \alpha\nu_A(x) \\
&= \mu_A(x) + \alpha(1 - \mu_A(x) - \nu_A(x)) \\
&= \mu_A(x) + \alpha\pi_A(x) = \mu_{F_{\alpha,\beta}(A)}(x).
\end{aligned}$$

$$\therefore \mu_{F_{\alpha,\beta}(A)}(0) \geq \mu_{F_{\alpha,\beta}(A)}(x)$$

Similarly we can prove

$$\nu_{F_{\alpha,\beta}(A)}(0) \leq \nu_{F_{\alpha,\beta}(A)}(x).$$

$$\begin{aligned}
&\mu_{F_{\alpha,\beta}(A)}(x) \\
&= \mu_A(x) + \alpha\pi_A(x) \\
&= \mu_A(x) + \alpha(1 - \mu_A(x) - \nu_A(x)) \\
&= \alpha + (1-\alpha)\mu_A(x) - \alpha\nu_A(x) \\
&\geq \alpha + (1-\alpha)min(\mu_A((x*y), \mu_A(y)) - \alpha max(\nu_A((x*y), \nu_A(y)) \\
&\geq \alpha\{1 - max(\nu_A((x*y), \nu_A(y))\} + (1-\alpha)min(\mu_A((x*y), \mu_A(y)) \\
&\geq \alpha min(1 - \nu_A((x*y), 1 - \nu_A(y))\} + (1-\alpha)min(\mu_A((x*y), \mu_A(y)) \\
&\geq min\{\alpha(1 - \nu_A((x*y)) + (1-\alpha)\mu_A((x*y), \alpha(1 - \nu_A(y)) + (1-\alpha)\mu_A(y)\} \\
&\geq min\{\mu_A((x*y) + \alpha(1 - \mu_A((x*y) - \nu_A((x*y)), \mu_A(y) + \alpha(1 - \mu_A(y) - \nu_A(y))\} \\
&\geq min\{\mu_{F_{\alpha,\beta}(A)}((x*y), \mu_{F_{\alpha,\beta}(A)}(y)\}.
\end{aligned}$$

$$\therefore \mu_{F_{\alpha,\beta}(A)}(x) \ \geq min\{\mu_{F_{\alpha,\beta}(A)}((x*y),\mu_{F_{\alpha,\beta}(A)}(y)\}$$

Similarly we can prove

$$\nu_{F_{\alpha,\beta}(A)}(x) \ \leq max\{\nu_{F_{\alpha,\beta}(A)}((x*y),\nu_{F_{\alpha,\beta}(A)}(y)\}.$$

$$\begin{aligned}
&\mu_{F_{\alpha,\beta}(A)}(x*y)\\
&= \mu_A(x*y)+\alpha\pi_A(x*y)\\
&= \mu_A(x*y)+\alpha(1-\mu_A(x*y)-\nu_A(x*y))\\
&= \alpha+(1-\alpha)\mu_A(x*y)-\alpha\nu_A(x*y)\\
&\geq \alpha+(1-\alpha)min(\mu_A(x),\mu_A(y))-\alpha max(\nu_A(x),\nu_A(y))\\
&\geq \alpha\{1-max(\nu_A(x),\nu_A(y))\}+(1-\alpha)min(\mu_A(x),\mu_A(y))\\
&\geq \alpha min(1-\nu_A(x),1-\nu_A(y))\}+(1-\alpha)min(\mu_A(x),\mu_A(y))\\
&\geq min\{\alpha(1-\nu_A(x))+(1-\alpha)\mu_A(x),\alpha(1-\nu_A(y))+(1-\alpha)\mu_A(y)\}\\
&\geq min\{\mu_A(x)+\alpha(1-\mu_A(x)-\nu_A(x)),\mu_A(y)+\alpha(1-\mu_A(y)-\nu_A(y))\}\\
&\geq min\{\mu_{F_{\alpha,\beta}(A)}(x),\mu_{F_{\alpha,\beta}(A)}(y)\}.
\end{aligned}$$

$$\therefore \mu_{F_{\alpha,\beta}(A)}(x*y) \ \geq min\{\mu_{F_{\alpha,\beta}(A)}(x),\mu_{F_{\alpha,\beta}(A)}(y)\}$$

Similarly we can prove

$$\nu_{F_{\alpha,\beta}(A)}(x*y) \ \leq max\{\nu_{F_{\alpha,\beta}(A)}(x),\nu_{F_{\alpha,\beta}(A)}(y)\}.$$

Hence $F_{\alpha,\beta}(A)$ is an IF d-ideal of BG-algebra X.

Theorem 7. *If $A =< \mu_A,\nu_A >$ is an IF d-ideal of d-algebra X, then $F_{\alpha,\beta}(A)$ is also an IF dot d- ideal X.*

Proof. It follows from Theorem 6 and Theorem 1.

Remark. The converse of Theorem 6 is not true as shown in following Example.

Example 5. Consider d-algebra X as in Example 2 and intuitionistic fuzzy subset $A = \{< x,\mu_A(x),\nu_A(x) > |x \in X\}$ given by $\mu_A(0) = 0.55,\mu_A(a) = 0.6,\mu_A(b) = 0.5$ and $\nu_A(0) = 0.3,\nu_A(a) = 0.4,\nu_A(b) = 0.35$ then A is not an IF d- ideal of X, since $\mu_A(0) = 0.55 \ngeq \mu_A(a) = 0.6$. But $\pi_A(0) = 0.1,\pi_A(a) = 0.0,\pi_A(b) = 0.15$, take $\alpha = 0.6,\beta = 0.3$ then $\mu_{F_{0.6,0.3}(A)}(0) = 0.61,\mu_{F_{0.6,0.3}(A)}(a) = 0.6,\mu_{F_{0.6,0.3}(A)}(b) = 0.59$ and $\nu_{F_{0.6,0.3}(A)}(0) = 0.33,\nu_{F_{0.6,0.3}(A)}(a) = 0.4,\nu_{F_{0.6,0.3}(A)}(b) = 0.395$. Then it is easy to verify that $F_{0.6,0.3}(A)$ is also an IF dot d- ideal X.

4 Homomorphism of d-algebras and intuitionistic fuzzy dot d-ideals

Definition 17. *Let X and X' be two d-algebras, then a mapping $f : X \to X'$is said to be homomorphism if $f(x*y) = f(x)*f(y)\forall x,y \in X$.*

Theorem 8. *Let X and X' be two d-algebras and $f : X \to X'$ be a homomorphism. Then $f(0) = 0'$.*

Proof. Let $x \in X$ therefore $f(x) \in X'$. Now $f(0) = f(x*x) = f(x)*f(x) = 0*0 = 0'$.

Theorem 9. *Let $f : X \to X'$ be an onto homomorphism of d-algebras, Let A be an intuitionistic fuzzy dot d-ideal of X', then the pre-image $f^{-1}(A)$ of A under f is an intuitionistic fuzzy dot d-ideal of X.*

Proof. $f^{-1}(A)$ is defined as

$$f^{-1}(A)(x) = f^{-1}(\mu_A, \nu_A)(x) = (f^{-1}(\mu_A), f^{-1}(\nu_A))(x) = (f^{-1}(\mu_A)(x), f^{-1}(\nu_A)(x)) = ((\mu_A)f(x), (\nu_A)f(x)) \forall x \in X.$$

$$\begin{aligned}
f^{-1}(\mu_A)(0) &= \mu_A f(0)\\
&\geq \mu_A f(x)\\
&= f^{-1}(\mu_A)(x).\\
\text{Therefore,}\quad f^{-1}(\mu_A)(0) &\geq f^{-1}(\mu_A)(x)\\
f^{-1}(\mu_A)(x) &= \mu_A f(x)\\
&\geq \mu_A(f(x)*f(y)).\mu_A f(y)\\
&= \mu_A f(x*y).\mu_A f(y)\\
&= f^{-1}(\mu_A)(x*y).f^{-1}(\mu_A)(y).\\
\text{Therefore,}\quad f^{-1}(\mu_A)(x) &\geq f^{-1}(\mu_A)(x*y).f^{-1}(\mu_A)(y)\\
f^{-1}(\mu_A)(x*y) &= \mu_A f(x*y)\\
&\geq \mu_A(f(x)).\mu_A f(y)\\
&= \mu_A f(x).\mu_A f(y)\\
&= f^{-1}(\mu_A)(x).f^{-1}(\mu_A)(y).\\
\text{Therefore,}\quad f^{-1}(\mu_A)(x*y) &\geq f^{-1}(\mu_A)(x).f^{-1}(\mu_A)(y)\\
\text{Also}\quad f^{-1}(\nu_A)(0) &= \nu_A f(0)\\
&\leq \nu_A f(x)\\
&= f^{-1}(\nu_A)(x).\\
\text{Therefore,}\quad f^{-1}(\nu_A)(0) &\leq f^{-1}(\nu_A)(x)\\
f^{-1}(\nu_A)(x) &= \nu_A f(x)\\
&\leq \nu_A(f(x)*f(y)) + \nu_A f(y) - \nu_A(f(x)*f(y)).\nu_A f(y)\\
&= \nu_A(f(x*y)) + \nu_A f(y) - \nu_A(f(x*y)).\nu_A f(y)\\
&= f^{-1}(\nu_A)(x*y) + f^{-1}(\nu_A)(y) - f^{-1}(\nu_A)(x*y).f^{-1}(\nu_A)(y).\\
\text{Therefore,}\quad f^{-1}(\nu_A)(x) &\leq f^{-1}(\nu_A)(x*y) + f^{-1}(\nu_A)(y) - f^{-1}(\nu_A)(x*y).f^{-1}(\nu_A)(y)\\
f^{-1}(\nu_A)(x*y) &= \nu_A f(x*y)\\
&\leq \nu_A f(x) + \nu_A f(y) - \nu_A f(x).\nu_A f(y)\\
&= f^{-1}(\nu_A)(x) + f^{-1}(\nu_A)(y) - f^{-1}(\nu_A)(x).f^{-1}(\nu_A)(y)\\
f^{-1}(\nu_A)(x*y)) &\leq f^{-1}(\nu_A)(x) + f^{-1}(\nu_A)(y) - f^{-1}(\nu_A)(x).f^{-1}(\nu_A)(y).
\end{aligned}$$

Hence $f^{-1}(A)$ is an intuitionistic fuzzy dot d-ideal of X.

Theorem 10. *An onto homomorphic image of an intuitionistic fuzzy dot d-ideal with the sup & inf property is an intuitionistic fuzzy dot d-ideal.*

Proof. Let $f : X \to X'$ be an onto homomorphism of d-algebras, $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ be an intuitionistic fuzzy dot d-ideal of X, and $f(A)$ be the image of A under f. To prove $f(A)$ is an intuitionistic fuzzy dot d-ideal of X'

$$f(\mu)(0') = \sup_{z\in f^{-1}(0')} \mu(z) = \mu(0) \geq \mu(x) \quad \forall x \in X$$

$$\Rightarrow f(\mu)(0') \geq \sup_{z\in f^{-1}(x')} \mu(z) = f(\mu)(x') \quad \forall x' \in X'.$$

Let $x_2, y_2 \in X'$ be any elements, as f is onto, there exists unique $x_1, y_1 \in X$ such that $f(x_1) = x_2, f(y_1) = y_2$, and $f(x_1 * y_1) = f(x_1) * f(y_1) = x_2 * y_2$. Now $f(A)(x_2) = (\mu_{f(A)}(x_2), \nu_{f(A)}(x_2))$. But $\mu_{f(A)}(x_2) = \sup_{z\in f^{-1}(x_2)} \mu_A(z) = \mu_A(t)$, where $f(t) = x_2 = f(x_1) \Rightarrow t = x_1$.

$$\begin{aligned}
\mu_{f(A)}(x_2) = \mu_A(t) &= \mu_A(x_1) \\
&\geq \mu_A(x_1 * y_1).\mu_A(y_1) \\
&\geq \mu_{f(A)} f(x_1 * y_1).\mu_{f(A)} f(y_1) \\
&= \mu_{f(A)}(x_2 * y_2).\mu_{f(A)} y_2. \\
\mu_{f(A)}(x_2 * y_2) &= \sup_{z\in f^{-1}(x_2*y_2)} \mu_A(z) \\
&= \mu_A(t), \quad \text{where} f(t) = x_2 * y_2 = f(x_1) * f(y_1) = f(x_1 * y_1) \\
&= \mu_A(x_1 * y_1) \\
&\geq \mu_A(x_1).\mu_A(y_1) \\
&\geq \mu_{f(A)} f(x_1).\mu_{f(A)} f(y_1) \\
&= \mu_{f(A)}(x_2).\mu_{f(A)}(y_2).
\end{aligned}$$

$$\begin{aligned}
&f(\nu)(0') = \inf_{z\in f^{-1}(0')} \nu(z) = \nu(0) \leq \nu(x) \quad \forall x \in X \\
&\Rightarrow f(\nu)(0') \leq \inf_{z\in f^{-1}(x')} \nu(z) = f(\nu)(x') \quad \forall x' \in X'.
\end{aligned}$$

$$\begin{aligned}
\nu_{f(A)}(x_2) &= \inf_{z\in f^{-1}(x_2)} \nu_A(z) \\
&= \nu_A(t), \quad \text{where} f(t) = x_2 = f(x_1) \\
&= \nu_A(x_1) \\
&\leq \nu_A(x_1 * y_1) + \nu_A(y_1) - \nu_A(x_1 * y_1).\nu_A(y_1) \\
&\leq \nu_{f(A)} f(x_1 * y_1) + \nu_{f(A)}(y_1) - \nu_{f(A)}(x_1 * y_1).\nu_{f(A)}(y_1) \\
&= \nu_{f(A)}(x_2 * y_2) + \nu_{f(A)}(y_2) - \nu_{f(A)}(x_2 * y_2).\nu_{f(A)}(y_1).
\end{aligned}$$

$$\begin{aligned}
\nu_{f(A)}(x_2 * y_2) &= \inf_{z\in f^{-1}(x_2*y_2)} \nu_A(z) \\
&= \nu_A(t), \quad \text{where} f(t) = x_2 * y_2 = f(x_1) * f(y_1) = f(x_1 * y_1) \\
&= \nu_A(x_1 * y_1) \\
&\leq \nu_A(x_1) + \mu_A(y_1) - \nu_A(x_1).\nu_A(y_1) \\
&\leq \nu_{f(A)} f(x_1) + \nu_{f(A)} f(y_1) - \nu_{f(A)} f(x_1).\nu_{f(A)} f(y_1) \\
&= \nu_{f(A)}(x_2) + \nu_{f(A)}(y_2) - \nu_{f(A)}(x_2).\nu_{f(A)} f((y_2)).
\end{aligned}$$

Hence from above $f(A)$ is an intuitionistic fuzzy dot d-ideal of X.

5 Conclusions

In this paper, we have studied intuitionistic fuzzy dot d-ideals of d-algebras and obtained some interesting results. We have shown that intersection of any two intuitionistic fuzzy dot d-ideals is a intuitionistic fuzzy dot d-ideal. Intuitionistic fuzzy dot d-ideals is invarient under model operators $\Box, \Diamond$ and $F_{\alpha,\beta}$. By using same idea, we can define intuitionistic fuzzy dot d-ideal in other algebric systems such as BCK/BCI/BG/BF-algebras etc.

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