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Bifurcation theory of flexible exchange rates in the new Keynesian model: An Application

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Abstract: Formerly, the fixed exchange rate was identified as the main policy in economy, but in time it has been proved that the applicability of this policy in economy is not possible. With the new Keynesian approach, this situation has led to the formation and development of new monetary policies. The Keynesian model monetary policy in this article has been based on the flexible exchange rate. By applying the bifurcation theory on monetary policy parameters the interrelations among the inflation target value, the output gap and the equilibrium real interest rate were examined.

Keywords: Bifurcation, Hopf bifurcation, new-Keynesian macroeconometrics, stability.

1 Introduction

In this study, The new Keynesian model, which is one of the most important models in macroeconomics, is examined. A new model including the new Keynesian structure is tried to be set up. Hopf bifurcations are shown on the model by using Hopf Bifurcation Theorem. By applying the bifurcation theory on monetary policy parameters the interrelations among the inflation target value, the output gap and the equilibrium real interest rate were examined.

It is observed that strict inflation targeting (price stability) strategy together with the last worldwide financial crisis has intensified contractionary effect of the crisis. This shows that it should be production stability instead of price stability. Therefore, post crisis flexible inflation targeting (flexible price) and outward oriented economy approach strategies are applied. So that effects of crisis are tried to be minimized. Depending on the exchange rate in the open economy inflation targeting strategy was developed [23,24].

In advance of global crisis, it has been thought that flexible exchange rate is what increases the efficiency of monetary policy in outward oriented economies. Based on this view, it has been tried to improve inflation targeting decreasing the effects of economic shocks on markets[31]. It puts forward the question of how to determine the interest rates. Within this scope, different views have been discussed. Barnettimply that if the central bank production hungry that occurred fluctuations consideration for implementing a flexible policy, the gradual determination of the nominal interest rate to ensure the convergence of long-term inflation target for the inflation forecast refers[11,33].

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First of these views is Classical Interest Theory which assumes interest is determined by saving and investment decisions. The second view is Liquidity Preference Theory of Keynes. According to Keynes, interest is determined by reel money supply and reel money demand. More contemporary view than the others in literature is Neo-classical Loanable Funds Theory. This theory is a more advanced version of classical interest theory which argues that interest is determined by loanable fund supply and fund demand. Another theory related to interest rates is Neo-Keynesian Hicks-Hansen Interest Rates Theory. According to this theory, interest rates are determined by both reel and monetary processes.

2 Related Literature

2.1 Interest Rate In The Keynesian Approach: Liquidity Preference Theory

Keynes which is seen as a reward for giving up liquidity, interest rate, money owners, refers to others as a measure of the degree of irritability in the rights fail overs [28].

According to Liquidity Preference Theory used by Keynesian economists to explain interest rate, money demand is one of the factors determining interest rate. Money demand changes related to two factors; level of price and income. Increase in income level increases money demand. There are two reasons for this: First, when income and wealth increases due to economical activities, people want to keep more money as a value depositary. And, when income increases, people will want to do more operations so that they will keep more money. As a result, the more increases income, the more increases money demand. Second factor which affects money demand is changes in price level. When price level increases, it becomes impossible for the people to make the same amount ofreel operation with the money they have so they increase the nominal money demand to the level of which they could buy goods and services previously. Therefore, increase in price level increases money demand [28,30]. As a result, factors leading to changes in money demand are income and price levels. In expansion periods, when income increases (all other things being stable), interest rate will increase. Albeit, when general level of prices increases (money supply and other economical variables are stable), increase rate will increase [34].

Interest rate in this aspect, is not a variable determined by reel processes but becomes a variable determined nominally. Keynes explained the interest rates which have been explained in terms of reel factors before, in terms of monetary factors. In Keynesian analysis interest has two important function[35]; first of it is that it affects making a choice between keeping the savings as money and shift to alternative investment tools. The more is this price, the less is the inflation money demand. The second function of inflation in economy is that it relates the money market with the reel market so that the interest rate determined in monetary zone affects reel sector through investment demand. As a result, in Keynes interest rates analysis, interest rates are accepted as a function of money supply and demand, and is stated that increase in the money supply by monetary policy implementers when the money demand is stable, decreases interest rate[16]. In addition to this, Keynes expresses that there is a minimum level of interest rate which it cannot decrease less than that. This level is ”liquidity trap interest rate” [34,35]. In Keynesian theory, the effect leading to liquidity trap is the investors’ expectation of increase in interest rates.

This effect expresses the original short-term effect of a change in money stock on interest rates. In short, if increasing money supply heads for capital market (bond market), bond demand and bond prices increase and interest rates decrease. This effect is named liquidity and as a result of this effect interest rates decrease. Liquidity Preference Theory of Keynes appear due to this effect [28].

2.2 The relationship between the output gap and Interest rate

According to Keynesian theory, every increase in money supply (with an assumption of no change in liquidity function) will decrease interest rate, and every decrease in money supply will increase interest rate. In other words, there is an inverse proportion between money supply in the control of monetary authority independent Central Bank and interest rates [33]. Related to the volatility in money supply increase results in increase investor's keeping treasury bill risk. This increasing risk increases money demand, increases interest rates, decreases investment; therefore, level of output decreases [36]. Effect of money supply increase on interest rates is actualized by four different macro variable which are liquidity effect, general level of prices effect, income effect and inflationary expectations[22].

The demand for money will vary depending on factors as income and price level. The increase in income increases the demand for money. There are two reasons for this: The second reason is the growth of wealth and increase income while first reason for determining changes in the price level. When the money supply increases, interest rates are expected to fall. Accordingly, the independent variables used in the study theoretically increases the money supply when interest rates are expected to fall.

2.3 The relationship between the level of interest rates and inflation

In Fischer book published in 1936 ”The Purchasing Power of Money” Irving Fischer discussed the relation between interest and inflation [22].

Fisher equation, $i \equiv r + \pi e$ (nominal interest rate = reel interest rate + expected inflation rate) puts forward nominal interest rates of any period is equal to the sum of reel interest rates and expected inflation in the same period. In other words, changes in inflation anticipated in the long term is equal to the changes in nominal interest rates. That's to say, nominal interest rates increase in a one to one correspondence with the increase in inflation, however, it does not affect reel interest rates. This fact is named as Fischer Effect. According to this view known as Fisher Effect, continuous in growth rate of money, first leads to decrease in nominal interest rate, then while outcome and inflation are increasing, interest rates increase progressively [37]. In high inflationist milleu, individuals will ask for higher interest in order to protect their savings from the negative effects of inflation. As a result, increase in inflation will lead to increase in interest rates. In accordance with flexible inflation targeting strategy, policy of central bank determines short term forward interest rate in order to minimize the deviance on output gap and inflation. According to Barnett[11], if the central bank applies a flexible policy considering fluctuations in output gap, it is necessary to determine nominal interest rate in order to provide inflation estimate to converge gradually to long term inflation targeting.

The long-term changes in expected inflation, the nominal interest rate gives rise to an equal change. However, does not affect real interest rates. The reason why there is such a relation between inflation and nominal interest rates, in the long term reel interest rate is being affected by the monetary disparity. Nominal interest rates increases in one to one correspondence with inflation. The anticipated relation between inflation and nominal interest rate is in a positive direction.[17].

2.4 The relationship between interest rate and exchange rate

There is no consensus in the direction of the relation between interest rate and exchange rate in the literature. In an outward oriented economy with free capital movements increase in domestic interest rates will lead to decrease in exchange rate due to the increase in capital inflow which in turn national currency increases in value. In this case it can be noted that there is a negative relation between interest rates and exchange rates. However, since interest rate is one of the money demand determiners, a positive relation may emerge between it and exchange rate. In other words, increase in interest rates may limit the demand for national currency and may lead to increase in inflation rate. In this case, national currency will decrease in value and exchange rate will increase[33]. As a result, there may be both negative and positive relation between interest rate and exchange rate.

2.5 Methods

The strategy in which the least deviation taking place in inflation and outturn gap in the presence of economical shock is optimized Taylor in which macro reserve is included. Taylor Rule is a view put forward by John Taylor in 1993 in relation with the rules to be obeyed by the central banks in monetary policy to be carried out[38]. Main idea of Taylor Rule is central banks' determining short term interest rates based on outturn and inflation. According to Taylor Rule, when a central bank anticipates inflation and outturn to be over its targeting level, it increases interest rates, when it anticipates them under its target level it decreases the interest rates [39]. Taylor states that his rule in accordance with the interest rate tool which he defines as a function of outturn gap and inflation outturn which deviates from the targeted is not only a good definition of monetary policy but also a reasonable policy suggestion [38]. But, in the crisis period Taylor Rule is the outermost strategy to the political activities. In the framework of Taylor Rule, deviation from the outturn gap and inflation gap is quite high. Therefore, Taylor Rule is the strategy in which the loss is the highest. So when conducting monetary policy, central banks should not depend on new Keynesian based Taylor Rule and expand it with macro reserve policy. Otherwise, reel interest rate taken constant in accordance with Taylor Rule will lead the economy low interest rate trap and increase the possibility of crisis in the conditions that financial intermediaries borrow the most and the least[36].

3 Bifurcation

Bifurcation analysis has been widely used to examine and classify the dynamic behavior of a variety of economic models in economic literature.

Grandmont [24] found that the parameter space of even the simplest, classical models is stratified into bifurcation regions. Grandmont was not able to reach conclusions about the policy relevance of his dramatic discovery. As a result, Barnett and He [3,5] investigated a Keynesian structural model and found results supporting Grandmont's conclusions within the parameter space of the Bergstrom-Wymer continuous-time dynamic macroeconometric model of the UK economy. Barnett and He [6,7] chose to continue the investigation of policy-relevant bifurcation by searching the parameter space of the best known of the Euler equations macroeconometric models: the Leeper and Sims [30] model. The results further confirm Grandmont's views.

Barnett and He [4,5] show the existence of a transcritical and Hopf bifurcation for different policy parameters in the dynamic, continuous time macro econometric model of Bergstrom et al. [16]. Furthermore, Barnett and He [6,7,10] find the existence of a singularity induced bifurcation within the empirical parameter space of the Leeper and Sims [30]. Euler equations model for the US economy. Barnett and Duzhak [8,9] recently found the presence of period-doubling and Hopf bifurcation in new Keynesian models.

Barnett and Duzhak [8,9] analyzed bifurcation using a closed economy new Keynesian model, based on Walsh [39], and found both Hopf and period doubling bifurcations within the parameter space. As observed by Barnett and He [3,5,7] and Barnett and Duzhak [8,9], the existence of bifurcation boundaries in the parameter space indicates the presence of different solution types corresponding to parameter values close to each other, but on different sides of the bifurcation boundary. Barnett et al. [11] found that including industrial organization features into a Zellner's Marshallian [40] macroeconomic model, permitting entry and exit of firms, does not decrease the relevancy of bifurcation phenomena. In Barnett and Eryilmaz [12], we previously analyzed Gali and Monacelli's [23] model, which is an open economy new Keynesian model, and found that introducing parameters related to the open economy structure affects the values of bifurcation parameters and changes the location of bifurcation boundaries.

In Barnett and Eryilmaz [13] examine another mainstream new Keynesian model based on Clarida et al. [20,21], hereafter CGG Model, in the open economy tradition to further explore analytically the possibility of Hopf bifurcations within open economy new Keynesian structures.they investigated the possibility of bifurcations in the open-economy new Keynesian model derived by Walsh [39] based on Clarida et al. [20,21]. They provided the theory needed to implement the numerical search and locate Hopf bifurcation boundaries. Their theoretical results are consistent with prior results from other new Keynesian models in Barnett and Duzhak [8,9], and Barnett and Eryilmaz [12]. Consequently, they show considering different monetary policy rules and incorporating numerical analysis of those models have been extended the frontier of bifurcation analysis of new Keynesian models.

4 Hopf Bifurcation

Definition 1. *A Hopf or Poincare-Andronov-Hopf bifurcation is a local bifurcation in which a fixed point of a dynamical system loses stability as a pair of complex conjugate eigenvalues of liberalization around the fixed point cross the imaginary axis of the complex plane.*

Theorem 1. *Consider the two dimensional system*

$$\begin{aligned} \tfrac{dx}{dt} &= f(x,y,\tau), \\ \tfrac{dy}{dt} &= g(x,y,\tau) \end{aligned} \tag{1}$$

where τ is the parameter and suppose that $(x(\tau),y(\tau))$ is the equilibrium point and $\alpha(\tau)\pm i\beta(\tau)$ are the eigenvalues of the Jacobian matrix which is evaluated at the equilibrium point. In addition let's assume that the change in the stability *of the equilibrium point occurs at* $\tau = \tau*$ *where* $\alpha(\tau*) = 0$. *System (1) is rewritten as follows;*

$$\begin{aligned} \tfrac{dx}{dt} &= a_{11}(\tau)x + a_{12}(\tau)y + f_1(x,y,\tau), \\ \tfrac{dy}{dt} &= a_{21}(\tau)x + a_{22}(\tau)y + g_1(x,y,\tau). \end{aligned} \tag{2}$$

The linearization of the system (1) about the origin is given by $\frac{dX}{dt} = J(\tau)X$, *where* $X = \begin{bmatrix} x \\ y \end{bmatrix}$ *and*

$$J(\tau) = \begin{bmatrix} a_{11}(\tau) & a_{12}(\tau) \\ a_{21}(\tau) & a_{22}(\tau) \end{bmatrix}$$

is the Jacobian matrix evaluated at origin.

Theorem 2. *Let* f_1 *and* g_1, *in system(2) have continuous third order partial derivatives in x and y. Suppose that the origin is an equilibrium point of (2) and that the Jacobian matrix* $J(\tau)$ *as above, is valid for all sufficiently small* $|\tau|$. *Moreover, assume that the eigenvalues of matrix* $J(\tau)$ *are* $\alpha(\tau) \pm i\beta(\tau)$ *where*$\alpha(0) = 0, \beta(0) \neq 0$ *such that the eigenvalues cross the imaginary axis with nonzero speed, i.e.,*

$$\frac{d\alpha}{d\tau}|_{\tau=0} \neq 0.$$

Then in any open set Ucontaining the origin in $\mathbf{R}^2$ *and for any* $\tau_0>0$, *there exists a value* $\overline{\tau}$, $|\overline{\tau}| \langle \tau_0$*such that the system of differential equations (2) has a periodic solution for* $\tau = \overline{\tau}$ *in U [1].*

The Hopf bifurcation theorem was used in a lot of economic analysis. Applications of this theorem are found in, for example, Torre (1977), Benhabib and Nishimura (1979), Semmler (1987), Gabisch and Lorenz(1987), Zhang (1990), Lorenz (1993), Lorenz (1994), Sasakura (1994), Flaschel and Sethi (1996) and Asada (2006). Our system is different from all the models presented by them[16], [5].

Definition 2. *The bifurcation stated in the Hopf bifurcation theorem is called "supercritical" if the equilibrium point (0,0) is asymptotically stable when* $\tau = 0$ *(at the bifurcation point) and it is called "subcritical" if the equilibrium point (0,0) is negatively asymptotically stable (as* $t \to -\infty$*) when* $\tau = 0$.

In a supercritical Hopf bifurcation, the limit cycle grows out of the equilibrium point. In other words, right at the parameters of the Hopf bifurcation, the limit cycle has zero amplitude, and this amplitude grows as the parameters move further in to the limit cycle[3],[4].

However in a supercritical Hopf bifurcation, there is an unstable limit cycle surrounding the equilibrium point, and a stable limit cycle surrounding that. The unstable limit cycle shrinks down to the equilibrium point, which becomes unstable in the process.

Let $f(0) = 0$, for the dynamical system

$$x' = f(x), \; x \in R^n \tag{3}$$

and let the eigenvalues of the Jacobien matrix be $\lambda_1, \lambda_2, \cdots, \lambda_n$. Suppose that, the real parts of the eigenvalues are zero and if not, suppose there are n_+ numbers of eigenvalues with $Re\lambda > 0$, n_0 number of eigenvalues with $Re\lambda = 0$ and n_-

numbers of eigenvalues with $Re\lambda < 0$. Let T^c be the eigenspace on imaginary axis corresponding to n_0 eigenvalues. The eigenvalues on the imaginary axis ($Re\lambda = 0$) are called the critical eigenvalues as on the eigenspace T^c. And suppose the function φ^t denote the flow corresponding to the equation (3). With these assumption, we state the Center Manifold theorem as follows.

Theorem 3. *(Center Manifold) Let $f(0) = 0$, for the dynamical system*

$$x' = f(x), \; x \in R^n \tag{4}$$

and let the eigenvalues of the Jacobien matrix be $\lambda_1, \lambda_2, \cdots, \lambda_n$. Suppose that, the real parts of the eigenvalues are zero and if not, suppose there are n_+ numbers of eigenvalues with $Re\lambda > 0$, n_0 number of eigenvalues with $Re\lambda = 0$ and n_- numbers of eigenvalues with $Re\lambda < 0$. Let T^c be the eigenspace on imaginary axis corresponding to n_0 eigenvalues. The eigenvalues on the imaginary axis ($Re\lambda = 0$) are called the critical eigenvalues as on the eigenspace T^c. And suppose the function φ^t denote the flow corresponding to the equation (3). With these assumption, we state the Center Manifold theorem as follows;

There exists a locally invariant C^∞ center manifold $W^c_{loc}(0)$ such that

$$W^c_{loc}(0) = \{(x,y) : y = h(x); |x| < \delta, \; h(0) = 0; DJ(0) = 0\}$$

Such that the dynamics of the system $x' = A^c x + r_1(x,y)$, $y' = A^s y + r_2(x,y)$, (where A^c and A^s are the blocks in the canonical form whose diagonals contain the eigenvalues with $Re\lambda = 0$ and $Re\lambda < 0$; respectively) restricted to the center manifold are given by $x' = A^c(x) + r_1(x, h(x))$. And the manifold W^c_{loc} is called center manifold [18].

4.1 Center Manifold reduction for Hopf bifurcation

The aim of this section is to give a formal framework for the analytical bifurcation analysis of Hopf bifurcations in delay differential equations

$$x' = f(x, \tau), \; x \varepsilon R^3 \tag{5}$$

with a single fixed time delay τ to be chosen as a bifurcation parameter. Characteristic equations of the delay differential equation form (4) are often studied in order to understand changes in the local stability of equilibria of certain delay differential equations. It is therefore important the determine the values of the delay at which there are roots with zero part. We give a general formalization of these calculations and determine closed form algebraic equations where the stability and amplitude of periodic solutions close to bifurcation can be calculated.

We shall determine the direction of Hopf bifurcation and the stability of the bifurcating periodic solutions by applying the normal form theory and the center manifold theorem by Hassard et al., [26], and we assume that the three dimensional system of delay differential equations (4) undergoes Hopf bifurcations at the positive equilibrium $(X_0^*, \pi_{0,}^* r_0^*)$ at $\tau = \tau_k$, and $i\omega_1$ is the corresponding purely imaginary root of the characteristic equation at the positive equilibrium $(X_0^*, \pi_{0,}^* r_0^*)$. For the sake of simplicity, we use the notation $i\omega$ for $i\omega_1$.

5 Model

Not all but some of the differential equations are bound to parameters. In relation with the values of these parameters, qualitative behavior of system analysis can be quite different. This leads to showing different features in orbital portrait of the cycles and equilibrium points on the ground in autonomous systems. Briefly, when a parameter changes in the system, there may be an equilibrium point drift, spiral congestion or relaxation or cycle shrink. In addition to this, an equilibrium point may suddenly disappear and a second equilibrium point may appear or a steady equilibrium point may become unsteady and exit the limit cycle. In this situation, we can tell that when there is a change in orbital structure with the change of parameters in ADD systems, system has got bifurcation. In the course of time, bifurcation analysis is proved that to be a strong method in understanding the different features of different differential systems. Especially, in order to create required qualifications at bifurcation diagram, the important problem of determining the parameters of system model is an attractive reason. Besides it enables deciding about the qualifications of conditions where this mathematical equation is valid. Equilibrium values of the model are obtained and the steadiness of those are discussed. In the model, total income does not exceed employment income or does not drop below income level of zero capital. Similarly, factors such as medium term economical bifurcations for capital formation (in the long term economic growth and population growth) are ignored. Let's assume that in the medium term analysis, total income and capital values as the changes of them in the area limited by their maximums and minimums. Keynesian Economy estimates in relation with endowment pace, investment and saving, especially on the differences of them. Before the global crisis, in the open economies where capital movements are free the strategy which increases the efficiency monetary policy is accepted to be the inflation targeting strategy of having flexible rate regime within, avoiding the shocks and decreasing macroeconomic variables without any collapse. Together with this view, opinions are formed how to determine interest rates.

Using differential equations which include parameters are common choice for modeling in dynamical systems by scientists. Furthermore studying on solutions and their behavior that depends on a parameter are crucial to create a balance in the economic structure, their studies show that it yields results closer to the truth with time delays models. It has shown that the analysis of dynamic stability of the positive equilibrium point of the system with delay parameter plays an important role.

Moreover, our concern is in the medium run economic fluctuations (not the long run economic growth) and any factors for economic growth (e.g., population growth and technical progress) are ignored[35]. Thus, it is reasonable to do that economic factors vary within the bounded regions, i.e., that they have their upper limits(maximums) and lower limits (minimums).

In this section, we will consider flexible exchange rate macroeconomic system which may be regarded as a new Keynes model. The system is composed of the following equations:

Consider the following system with discrete time delay τ;

$$\frac{dx(t)}{dt} = r_1 \cdot x(t) - \varepsilon\pi(t) \cdot x(t)$$

$$\frac{d\pi(t)}{dt} = r_2 \cdot \pi(t) - \theta \frac{\pi(t) \cdot r(t)}{x(t-1)}$$

$$\frac{dr(t)}{dt} = \alpha \cdot r(t) - \alpha \pi(t) \tag{6}$$

where, α is a positive parameter and $r_1, r_2, \theta, \varepsilon$ are nonnegative parameters. Let x stand for the time derivative of a real variable x , i.e., $x \equiv dx/dt$ for t ≥ 0 [5].

In system (5), X and π are, respectively, the output gap function and the expected rate of inflation function. r is an interest rate function. The application of the Hopf bifurcation theorem our system for now that we will present some numerical simulations by using computer algebra system [32]. We simulate the system (5) by choosing the parameters $r_1 = 0.45$, $r_2 = 0.1$, $\theta = 0.05$, $\varepsilon = 0.03$, and $\alpha = 1, i.e.$, [19]. We consider the system (5) which has only one positive equilibrium $(X_0^*, \pi_{0,}^* r_0^*)$=$(7.5, 15, 15)$.

In computer simulations, the initial conditions are taken as (X_0,π_0,r_0)= (50, 25, 25) and computer algebra system delay differential equations (DDE) solver is used to simulate the system (5).

In what follows, we will investigate some properties of a periodic orbit generated by the Hopf bifurcation.According to Guckenheimer and Holmes, there are two types in Hopf bifurcations: subcritical and supercritical. In the case of the former type, the periodic orbit generated by the bifurcation is unstable, while in the case of the latter type, it is stable[25].

6 Conclusion

As a result in this article, bifurcations for new-Keynesian model are brought out the light. Bifurcation analysis is indicated as a strong tool to understand financial market behavior. Lately, interest policy of financial environments are moved to new-Keynesian models. Finally, we insist on the importance of Keynesian approach on money and finance in non-linear macroeconomics. Therefore, non-linear inequality in new-Keynesian system, a flexible interest rate is formulated and inflexible interest rate is removed. Results of the study related to macroeconomic factors' effect on determining the interest rates while showing consistency with some of the studies done in the literature, differentiate from some others. In the study it is determined that whether factors determining the interest rates such as reel exchange rate, money supply and inflation rate affect the interest rate or not, whether there is a long term relation between dependent variable interest rate and independent variables. In this study, monthly data of the 2004:1- 2014:2 period are used and interest rate is dealt together with reel exchange rate, money supply and inflation variables.

In the study, when there is an increase money supply in Turkey, interest rates decrease. The finding of the study about the effects of money supply increase on interest rates supports Keynesian interest theory and known as ”liquidity effect”. Similarly, a finding as any increase in exchange rate increases interest rate is in consistence with the theory by affirming the result which exists in the literature saying that ”in a country which applies flexible exchange rate, any increase in flexible exchange rate, increases national income by increasing net export at first, then money demand increase in relation with income increase, and as a result it leads to increase in interest rates. Likewise, in the 2004-2014 period analyzed, parallel with the continuous drop prices in Turkey, interest rates continuously dropped. However, since the

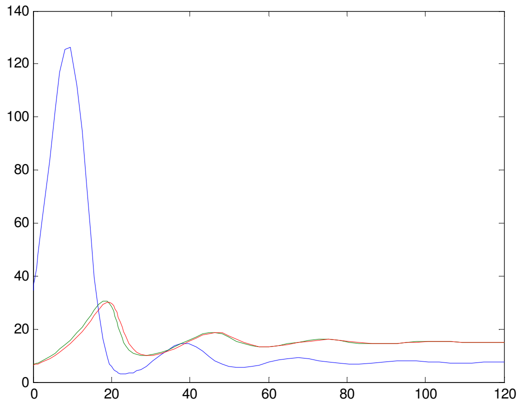

Fig. 1: Central bank data 2008-2009. Hopf-bifurcation for $t = 0.45$; for $t_1 > o.45$ unstable and for $t_2 < 0.45$ subcritical-hopf-bifurcation.

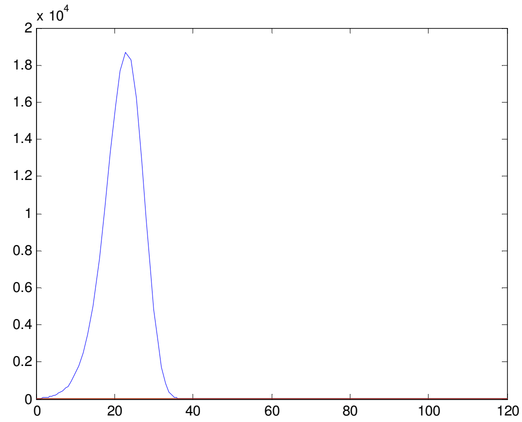

Fig. 2: Central bank data 2009-2010 Hopf-bifurcation (2008 global cris), $t = 0.0015$, for $t_1 > 0.0015$ unstable.

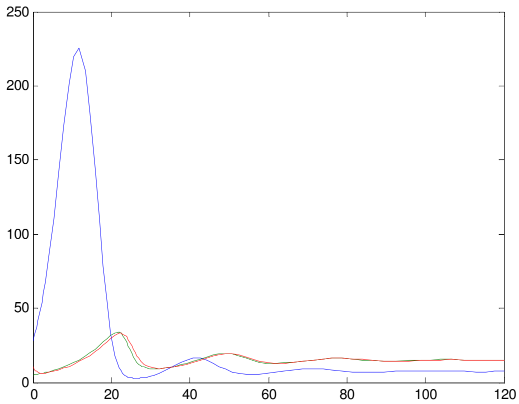

Fig. 3: Central bank data 2010-2011 Hopf-bifurcation, for $t = 0.2$, for $t_1 > 0.2$ unstable and $t_2 < 0.2$ subcritical-hopf.

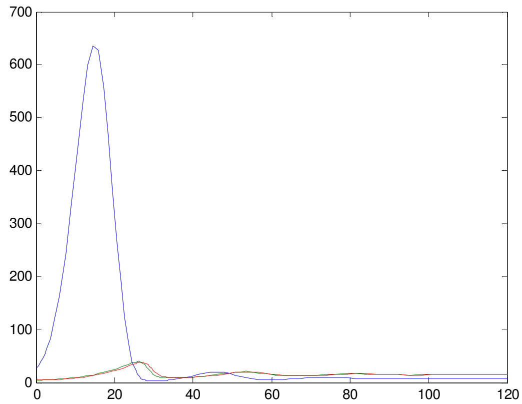

Fig. 4: Central bank data 2010-2012 Hopf-bifurcation, for $t = 0.0023$, for $t_1 > 0.0023$ unstable and $t_2 < 0.0023$ subcritical-hopf.

decrease in general level of prices is more than the decrease in interest rates, results are not up to the expectations.

This situation shows that reel positive interest rate decreases in the signified period. In fact, real positive interest in 2004 is more than the level in 2014. As a result, when the costs on production and employment of the crisis are considered, the most efficient monetary policy to be applied before, while and after crisis is the flexible inflation targeting which is used correctly and uses all the information about financial conditions. It is known that change in exchange rate is in converse interaction with the interest rate. However, monetary policy may react unexpectedly in the presence of crisis and because of this there is no right to be defended in the monetary framework. Monetary framework can change according to the macroeconomic conditions of the countries and their position in international economy. In this context, new-Keynesian monetary policy may not give desired and expected results in outward oriented, small developing countries like Turkey.

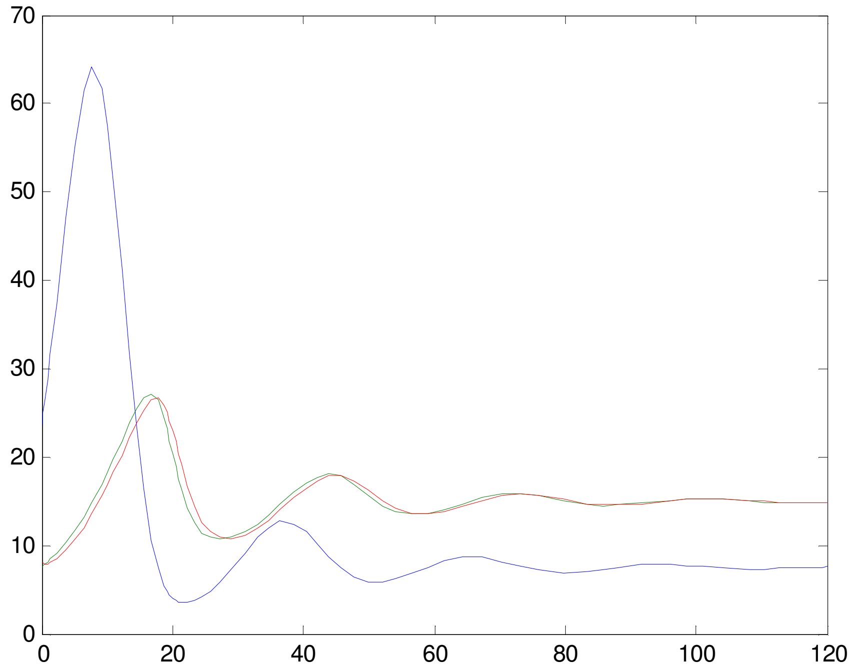

Fig. 5. Central bank data 2013-2014 Hopf-bifurcation, for $t = 1.012$, for $t_1 > 1.012$ unstable and $t_2 < 1.012$ subcritical-hopf.

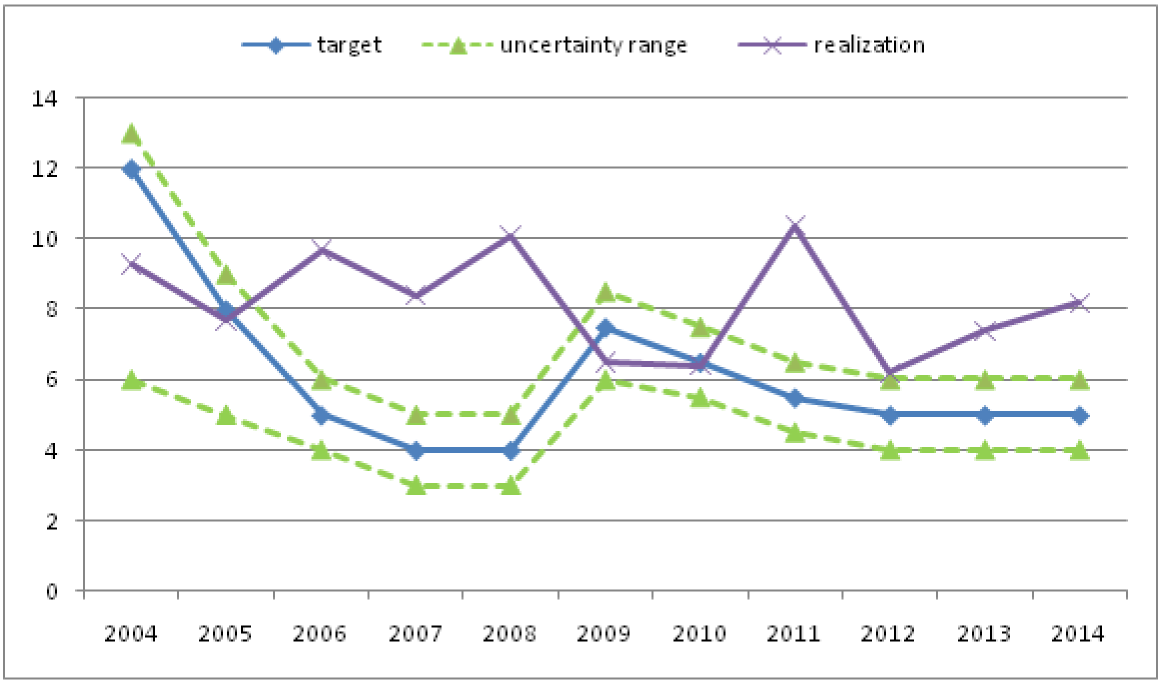

Fig. 6. The level of success of inflation targeting

References

[1] Allen L.J.S., An Introduction to Mathematical Biology, 2007.

[2] Asada, T., 2006. Stabilization policy in a Keynes-Goodwin model with debt accumulation. Structural Change and Economic Dynamics 17, 466-485.

[3] Barnett, W.A., He, Y., 1999.Stability analysis of continuous time macroeconometricsystems. Studies in Nonlinear Dynamics and Econometrics 3 (4), 169-188.

[4] Barnett, W.A., 2000. Perspective on the current state of macroeconomic theory. In: Barnett, W.A., Serletis, A. (Eds.), The Theory of Monetary Aggregation. North-Holland, pp. 600-601.

[5] Barnett, W.A., He, Y., 2002. Stabilization policy as bifurcation selection: Would stabilization policy work if the economy really were unstable? Macroeconomic Dynamics 6 (5), 713-747.

[6] Barnett, W.A., He, Y., 2004. Bifurcations in macroeconomic models. In: Dowrick, S., Pitchford, S.T. (Eds.), Economic Growth and Macroeconomic Dynamics: Recent Developments in Economic Theory. Cambridge University Press, pp. 95-112.

[7] Barnett, W.A., He, Y., 2006. Singularity bifurcations. Journal of Macroeconomics 28 (1), 5-22.

[8] Barnett, W.A., Duzhak, E., 2008. Non-robust dynamic inferences from macroeconometric models: Bifurcation stratification of confidence regions. Physica A 387, 3817-3825.

[9] Barnett, W.A., Duzhak, E., 2010. Empirical assessment of bifurcation regions within new Keynesian models. Economic Theory 45, 99-128.

[10] Barnett, W.A., He, Y., 2010. Existence of singularity bifurcation in an Euler-equations model of the United States economy: Grandmont was right. Economic Modelling 27 (6), 1345-1354.

[11] Barnett, W.A., Banerjee, S., Duzhak, E., Gopalan, R., 2011. Bifurcation analysis of Zellner's Marshallian macro model. Journal of Economic Dynamics and Control 9 (35), 1577-1585.

[12] Barnett, W.A., Eryilmaz, U., 2012. An Analytical and Numerical Search for Bifurcations in Open Economy new Keynesian models. University of Kansas working paper, Lawrence, Kansas.

[13] Barnett, W.A., Eryilmaz, U., 2013. Hopf bifurcation in the Clarida, Gali, and Gertler model. Economic Modelling 31, 401-404.

[14] Benhabib, J., Day, R.H., 1982. A characterization of erratic dynamics in the overlapping generations model. Journal of Economic Dynamics and Control 4, 37-55.

[15] Benhabib, J., Nishimura, K., 1979. The Hopf bifurcation and the existence and stability of closed orbits in multisector models of optimal economic growth. Journal of Economic Theory 21 (3), 421-444.

[16] Bergstrom, A.R., Nowman, K.B., Wymer, C.R., 1992. Gaussian estimation of a second order continuous time macroeconometric model of the UK. Economic Modelling 9 (4), 313-351.

[17] Brzoza–Brzezina, M. & Cuaresma, J.C., 2008. Mr. Wicksell and the Global Economy: What Drives Real Interest Rates, Oesterreichische National Bank Working Paper, No.139, Wien-Austria.

[18] Celik C., 2008. The stability and Hopf bifurcation for a predator-prey system with time delay,Chaos, Solitons & Fractals (37), 87-99.

[19] Celik C., 2011. Dynamical Behavior of a Ratio Dependent Predator-Prey System with Distributed Delay, Discrete and Continuous Dynamical Systems, Series B (16),719-738.

[20] Clarida, R., Gali, J., Gertler, M., 2001. Optimal monetary policy in open versus closed economies. The American Economic Review 91 (2), 248-252 (May).

[21] Clarida, R., Gali, J., Gertler, M., 2002. A simple framework for international monetary policy analysis. Journal of Monetary Economics 49 (5), 879-904 (Elsevier, July).

[22] Fischer, S., 1972. Keynes-Wicksell and neoclassical models of money andgrowth. American Economic Review 62 (5), 880-890.

[23] Gali, J., Monacelli, T., 2005. Monetary policy and exchange rate volatility in a small open economy. Review of Economic Studies 72 (3) (July).

[24] Grandmont, J.-M., 1985. On endogenous competitive business cycles. Econometrica 53 (5), 995-1045.

[25] Guckenheimer, J., Holmes, P., 1983. Nonlinear Oscillations, Dynamical Sys-tems and Bifurcations of Vector Fields. Springer-Verlag, New York.

[26] Hassard.B.D.,Kazarinoff N. D., Wan Y-H.,1981. Theory and Applications of Hopf Bifurcation, Cambridge University Press, Cambridge.

[27] He, X., Liao, M. & Xu, C. , 2011. Stability and Hopf Bifurcation analysis for a Lotka- Volterra predator-prey models with two delays, Int. J. Appl. Math. Comput., pp.97- 107.

[28] Keynes, J.M., 1936. The General Theory of Employment.Interest andMoney. Macmillan, London.

[29] Kuznetsov, Y.A., 2004. Elements of Applied Bifurcation Theory, third ed., Springer.

[30] Leeper, E., Sims, C., 1994. Toward a modern macro model usable for policy analysis. NBER Macroeconomic Annual 5, 81-117.

[31] Li, Y., Wu, Y. ,Yau, X. & Zhou, X. , 2009. Stability and Hopf Bifurcation analysis on a two neuron network with discrete and distributed delays, Chaos, Solitons & Fractals, pp.1493-1505.

[32] Lynch, S., 2004. Dynamical Systems With Applications Using Matlab, Birkhauser, Boston.

[33] Melvin, M.,1997. International Money and Finance, New York, Addison-Wesley Pub.

[34] Mishkin F.S., 2002, The Role of Output in the Conduct of Monetary Policy, NBER Working Paper9291.

[35] Murakami H., 2014. Keynesian Systems with Rigidity and Flexibility of Prices and Inflation-Deflation Expectations, Structural Change and Economic Dynamics 30, 68-85.

[36] Sarı, A., 2009.The growth in money supply, interest rate volatility and the effects of exchange rate volatility: the case of Turkey. University of Afyon Kocatepe academic journal, 11(2): 19-36.

[37] Şimşek M. & Kadılar, C., 2006.With Turkish Data test on The Fisher Effect, Universityof Doğuş academic journal, 7(1): 99-111.

[38] Taylor, J., 1993. Macroeconomic Policy in a World Economy: From Economic Design to Practical Operation. Norton, N.Y.

[39] Walsh, E.C., 2003. Monetary Theory and Policy, Second edition. MIT Press, Cambridge,MA.

[40] Zellner, A., Israilevich, G., 2005. Marshallian macroeconomic model: a progress report. Macroeconomic Dynamics 9 (2), 220-243.