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$(\in, \in \vee q)$-Fuzzy Ideals of BG-algebras with respect to t-norm

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Abstract: The aim of this paper is to introduce the concept of $\in, \in \vee q$)-fuzzy ideals of BG-algebra with respect to t-norm and derive some interesting result.

Keywords: BG-algebra, Fuzzy ideal, $(\in, \in \vee q)$-Fuzzy ideal, $(\in, \in \vee q)$-T-Fuzzy ideal, homomorphism.

1 Introduction

The name triangular norm, or simply t-norm originated from the study of generalized triangle inequalities for statistical metric spaces, hence the name triangular norm or simply t-norm. The name first appeared in a paper entitled statistical metrics [13] that was published on 27th october in 1942. A t-norm was supposed to act on the values of two distribution functions, hence on the unit square. Here is the original definition by Menger, a real-valued function T defined on a unit square is called a t-norm in the sense of Menger if $(i) 0 \leq T(\alpha, \beta) \leq 1$ (ii) T is non-decreasing in either variable (iii) $T(\alpha, \beta) = T(\beta, \alpha)$ (iv) $T(1,1) = 1$ (v) If $\alpha > 0$ then $T(\alpha, 1) > 0$. The real starting point of t-norms came in 1960, when Berthold Schweizer and Abe Sklar, (two students of Menger) published their paper, statistical metric spaces [19] After a very short time, Schweizer and Sklar [20] introduced several basic notions and properties. Namely, they introduced triangular conorms (briefly t-conorms) as a dual concept of t-norms. For a given t-norm T, its dual t-conorm S is defined by $S(a,b) = 1 - T(1-a, 1-b)$. They pointed out that the boundary condition is the only difference between the t-norm and t-conorm axioms. The last substantial step in the foundation of t-norms and t-conorms was given in 1965 by Ling [12]. Among other things, she recognised that continuous t-norms and t-conorms form a topological semigroup on [0,1]. She preserved the semigroup theory notation and hence she introduced Archimedean and nilpotent t-norms (and t-conorms). In order to formulate the triangle inequality property in a probabilistic metric space and following the ideas of Menger [13], Schweizer and Sklar [21] introduced a special class of two-place functions on the unit square, the so-called triangular norms. Together with their duals, the triangular conorms, they have been applied in various mathematical disciplines, such as probabilistic metric spaces [22] , fuzzy set theory, multiple-valued logic, and in the theory of non-additive measures [17]. In recent years, a systematic study concerning the properties and related matters of t-norms have been made by Klement et al. [9,10].

After the introduction of BCK and BCI-algebras in 1966 by Imai and Iseki [5,6]. Neggers and Kim [16] introduced a new algebraic structure called B-algebras, which are related to wide classes of algebras such as BCI/BCK-algebras. In

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[7] Kim and Kim introduced the notion of BG-algebra which is a generalization of B-algebra. The notion of fuzzy subset of a set is introduced by Zadeh [25] in 1965, after that researcher are trying to fuzzify almost every concept of Mathemtics. Fuzzification of subalgebras of BG-algebras was done by Ahn and Lee in [1] and fuzzy BG-ideals of BG-algebras were studied in [15] by Muthuraj et al. Bhakat and Das [2,3] used the relation of "belongs to" and "quasi coincident with" between fuzzy point and fuzzy set to introduce the concept of $(\in, \in \vee q)$-fuzzy subgroup, $(\in, \in \vee q)$-fuzzy subring and $(\in, \in \vee q)$-level subset. Jun introduced (α, β)-fuzzy ideals of BCK/BCI-algebras[23]. In [4] Dhanani and Pawar discussed $(\in, \in \vee q)$-fuzzy ideals of lattice. Further in [11] Larimi generalised $(\in, \in \vee q)$-fuzzy ideals to $(\in, \in \vee q_k)$-fuzzy ideals. Using the concept of t-norm, Kim discussed imaginable T-fuzzy ideals of Γ-Rings in [8] and in [24] Jun and Hong discussed imaginable T-fuzzy subalgebras and imaginable T-fuzzy closed ideals in BCH-algebras. In [18] Senapati et al. studied triangular norm based fuzzy BG-algebras. Now in present paper using the concept of t-norm and $(\in, \in \vee q)$-fuzzy ideals, we introduced the concept of $(\in, \in \vee q)$-fuzzy ideals of BG-algebra with respect to t-norm and obtained some interesting result.

2 Preliminaries

Definition 1. [1] *A BG-algebra is a non-empty set X with a constant* 0 *and a binary operation* $*$ *satisfying the following axioms:*

(i) $x * x = 0$,
(ii) $x * 0 = x$,
(iii) $(x * y) * (0 * y) = x, \forall x, y \in X$. *For simplicity, we also call X a BG-algebra. We can define a partial ordering "* $\leq$ *" on X by* $x \leq y$ *iff* $x * y = 0$.

Definition 2. [1] *A non-empty subset S of a BG-algebra X is called a subalgebra of X if* $x * y \in S$ *for all* $x, y \in S$.

Definition 3. *A triangular norm(t-norm) is a function* $T : [0\ 1] \times [0\ 1] \to [0\ 1]$ *satisfying the following conditions:*

(T1) $T(x, 1) = x, T(0, x) = 0$ *; (boundary conditions)*
(T2) $T(x, y) = T(y, x)$ *; (commutativity)*
(T3) $T(x, T(y, z)) = T(T(x, y), z)$ *; (associativity)*
(T4) $T(x, y) \leq T(z, w)$ *;if* $x \leq z, y \leq w$ *for all* $x, y, z \in [0\ 1]$ *(monotonicity)*

Every t-norm T satisfies $T(x, y) \leq min(x, y) \quad \forall x, y \in [0, 1]$.

Example 1. The four basic t-norms are:

(i) The minimum is given by $T_M(x, y) = min(x, y)$.
(ii) The product is given by $T_P(x, y) = xy$.
(iii) The Lukasiewicz is given by $T_L(x, y) = max(x + y - 1, 0)$.
(iv) The Weakest t-norm (drastic product) is given by

$$T_D(x, y) = \begin{cases} min(x, y), & \text{if} \quad max(x, y) = 1 \\ 0, & \text{otherwise.} \end{cases}$$

Definition 4. *A s-norm S is a function* $S : [0\ 1] \times [0\ 1] \to [0\ 1]$ *satisfying the following conditions:*

(S1) $S(x,1)=1, S(0,x)=x$ *; (boundary conditions)*
(S2) $S(x,y)=S(y,x)$ *; (commutativity)*
(S3) $S(x,S(y,z))=S(S(x,y),z)$ *; (associativity)*
(S4) $S(x,y) \leq S(z,w)$ *;if* $x \leq z, y \leq w$ *for all* $x,y,z \in [0\ 1]$ *(monotonicity) Every s-norm S satisfies* $S(x,y) \geq max(x,y) \quad \forall x,y \in [0,1]$.

Example 2. The four basic t-conorm are:

(i) Maximum given by $S_M(x,y)=max(x,y)$.
(ii) Probabilistic sum given by $S_P(x,y)=x+y-xy$.
(iii) The Lukasiewicz is given by $S_L(x,y)=min(x+y,1)$.
(iv) Strongest t-conorm given by

$$S_D(x,y)=\begin{cases} max(x,y), & \text{if} \quad max(x,y)=1 \\ 1, & \text{otherwise.} \end{cases}$$

Definition 5. *Let T be a t-norm. Denote by* δ_p *the set of elements* $x \in [0,1]$ *such that* $T(x,x)=x$*, that is*
$\Delta_T=\{x \in [0,1] : T(x,x)=x\}$
A fuzzy set μ *in X is said to satisfy imaginable property with respect to T if* $Im(\mu) \subseteq \Delta_T$.

Definition 6. *If for two t-norms* T_1 *and* T_2 *the inequality* $T_1(x,y) \leq T_2(x,y)$ *holds for all* $(x,y) \in [0\ 1] \times [0\ 1]$ *then* T_1 *is said to be weaker than* T_2*, and we write in this case* $T_1 \leq T_2$*. We write* $T_1 < T_2$*,whenever* $T_1 \leq T_2$ *and* $T_1 \neq T_2$.

Remark. It is not hard to see that T_D is the weakest t-norm and T_M is the strongest t-norm, that is, for all t-norm T

$$T_D \leq T \leq T_M.$$

We get the following ordering of the four basic t-norms:

$$T_D < T_L < T_P < T_M.$$

Lemma 1. *Let T be a t-norm. Then* $T(T(x,\ y)\ T(z,\ t))=T(T(x,\ z)\ T(y,\ t))$ *for all* $x,\ y,\ z$ *and* $t \in [0,\ 1]$.

Definition 7. *A nonempty subset I of a BG-algebra X is called a BG-ideal of X if*

(i) $0 \in I$,
(ii) $x*y \in I,\ y \in I \Rightarrow x \in I\ \forall\, x,y \in X$.

Definition 8. [15] *A fuzzy set* μ *in X is called a fuzzy BG-ideal of X if it satisfies the following conditions:*

(i) $\mu(0) \geq \mu(x)$,
(ii) $\mu(x) \geq \min\{\mu(x*y),\mu(y)\}\ \forall\, x,y \in X$.

Definition 9. *A fuzzy set* μ *in X is called a T-fuzzy BG-ideal of X if it satisfies the following conditions:*

(i) $\mu(0) \geq \mu(x)$,
(ii) $\mu(x) \geq T\{\mu(x*y),\mu(y)\}\ \forall\, x,y \in X$.

Example 3. Consider a *BG*-algebra $X=\{0,1,2\}$ with the following cayley table:

Table 1: Example of fuzzy BG-ideal.

*	0	1	2
0	0	1	2
1	1	0	1
2	2	2	0

Let $T_L : [0\ 1]\times[0\ 1]\to[0\ 1]$ be functions defined by $T_L(x,y) = max(x+y-1,0) \quad \forall x,y\in[0,\ 1]$. Then T_L is a t-norm. Define a fuzzy set $\mu : X\to[0,1]$ by $\mu(0) = 0.9, \mu(1) = 0.6, \mu(2) = 0.3$. Then it is easy to verify that μ satisfies $\mu(0)\geq\mu(x)$, and $\mu(x)\geq T\{\mu(x*y),\mu(y)\} = max\{\mu(x*y)+\mu(y)-1,0\}\ \forall x,y\in X$. Therefore μ is a T-fuzzy BG-ideal of X.

Example 4. Consider a BG-algebra X as defined in Example 3 and fuzzy set μ defined by $\mu(0) = 0.9, \mu(1) = 0.6, \mu(2) = 0.3$. Then it is easy to verify that μ is a fuzzy BG-ideal of X.

Remark. Every fuzzy BG-ideal of X is a T-fuzzy BG-ideal of X, But the converse is not true as shown in Example below.

Example 5. Consider a BG-algebra X as defined in Example 3 and fuzzy set μ defined on X by $\mu(0) = 0.5, \mu(1) = 0.7, \mu(2) = 0.4$. Then it is easy to verify that μ is a T_L-fuzzy ideal of X, but not a fuzzy ideal of X. Since $\mu(0) = 0.5 \ngeq min\{\mu(0*1),\mu(1)\} = \mu(1) = 0.7$.

Example 6. Consider BG-algebra X as defined in Example 3, fuzzy set μ defined on X by $\mu(0) = 0.5, \mu(1) = 0.7, \mu(2) = 0.4$. and t-norm $T_P : [0\ 1]\times[0\ 1]\to[0\ 1]$ be functions defined by $T_P(x,y) = xy \quad \forall x,y\in[0,\ 1]$. Then it is easy to verify that μ is a T_P-fuzzy ideal of X. But not a fuzzy ideal of X, since $\mu(0) = 0.5 \ngeq min\{\mu(0*1),\mu(1)\} = \mu(1) = 0.7$.

Theorem 1. *Every T_1-fuzzy ideal of X is a T_2-fuzzy ideal of X, where T_1 is stronger than T_2. But the converse is not true as shown in above Examples.*

Proof. Proof is straightforward.

3 $(\in,\in\vee q)$-T-fuzzy ideals of BG-algebra

In what follows, let X denote a BG-algebra unless otherwise stated.

Definition 10. [2,3,14] *A fuzzy set μ of the form*

$$\mu(y) = \begin{cases} t \ \ if\ y = x,\ t\in(0,1] \\ 0 \ \ if\ y\neq x \end{cases}$$

is called a fuzzy point with support x and value t and it is denoted by x_t.

Definition 11. [2,3] *A fuzzy point x_t is said to belong to (respectively be quasi coincident with) a fuzzy set μ written as $x_t\in\mu$ (respectively $x_tq\mu$) if $\mu(x)\geq t$ (respectively $\mu(x)+t>1$). If $x_t\in\mu$ or $x_tq\mu$, then we write $x_t\in\vee q\mu$. (Note $\overline{\in\vee q}$ means $\in\vee q$ does not hold).*

Definition 12. *A fuzzy subset μ of a BG-algebra X is said to be an $(\in,\in\vee q)$-fuzzy ideal of X if* *(i)* $x_t \in \mu \Rightarrow 0_t \in \vee q\mu$

(ii) $(x*y)_t, y_s \in \mu \Rightarrow x_{m(t,s)} \in \vee q\mu\ \forall x,y \in X, \forall s,t \in [0,\ 1]$.

Definition 13. [2,3] *A fuzzy point x_t is said to belong to (respectively be quasi coincident with) a fuzzy set μ with respect to t-norm T written as $x_t \in \mu$ (respectively $x_t q\mu$) if $\mu(x) \geq t$ (respectively $\mu(x)+t > 2T(1,\frac{1}{2})$. If $x_t \in \mu$ or $x_t q\mu$, then we write $x_t \in \vee q\mu$. (Note $\overline{\in \vee q}$ means $\in \vee q$ does not hold).*

Definition 14. *A fuzzy subset μ of a BG-algebra X is said to be an $(\in, \in \vee q)$-T-fuzzy ideal of X if*

(i) $x_t \in \mu \Rightarrow 0_t \in \vee q\mu$

(ii) $(x*y)_t, y_s \in \mu \Rightarrow x_{T(t,s)} \in \vee q\mu,\ \forall x,y \in X, \forall s,t \in [0,\ 1]$.

Example 7. Consider a *BG*-algebra $X = \{0,1,2,3\}$ with the following cayley table:

Table 2: Example of $(\in, \in \vee q)$-T-fuzzy ideal.

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	1

(i) Consider a fuzzy set μ_1 defined on X by $\mu_1(0) = \mu_1(1) = \mu_1(2) = 0.8, \mu_1(3) = 0.6$. Then it is easy to verify that μ_1 is an $(\in, \in \vee q)$-fuzzy ideal of X. But it is not a fuzzy *BG*-ideal of X. Since $\mu_1(3) = 0.6 \ngeq min\{\mu_1(3*1), \mu_1(2)\} = \mu_1(2) = 0.8$.

(ii) Again consider a fuzzy set μ_2 defined on X by $\mu_2(0) = \mu_2(1) = \mu_2(2) = 0.4, \mu_2(3) = 0.3$. Then it is easy to verify that μ_2 is an $(\in, \in \vee q)$-T-fuzzy ideal of X with respect to t-norm T_L. Since for all $x \in X$ it satisfies $x_t \in \mu_2 \Rightarrow 0_t \in \vee q\mu_2$ and $(x*y)_t, y_s \in \mu \Rightarrow x_{max(t+s-1,0)} \in \vee q\mu_2$. But it is not an $(\in, \in \vee q)$-fuzzy ideal of X. Since $(3*1)_{0.4}, 2_{0.4} \in \mu_2$, but $3_{0.4}\overline{(\in, \in \vee q)}\mu_2$.

(iii) Again consider a fuzzy set μ_3 defined on X by $\mu_3(0) = \mu_3(1) = \mu_3(2) = 0.9, \mu_3(3) = 0.6$. Then it is easy to verify that μ_3 is an $(\in, \in \vee q)$-T-fuzzy ideal of X with respect to t-norm T_P. Since for all $x \in X$ it satisfies $x_t \in \mu_3 \Rightarrow 0_t \in \vee q\mu_3$ and $(x*y)_t, y_s \in \mu_3 \Rightarrow x_{t.s} \in \vee q\mu_3$.

(iv) Again consider a fuzzy set μ_4 defined on X by $\mu_4(0) = \mu_4(1) = \mu_4(2) = 0.9, \mu_4(3) = 0.4$. Then it is easy to verify that μ_3 is an $(\in, \in \vee q)$-T-fuzzy ideal of X with respect to t-norm T_L. But it is not an $(\in, \in \vee q)$-T-fuzzy ideal of X with respect to t-norm T_P. Since $\mu_4(3) = 0.3 \ngeq T_P\{\mu_4(3*1).\mu_4(2), 0.5\} = \frac{\mu_4(3*1).\mu_4(2)}{2} = 0.405$.

Theorem 2. *If a fuzzy subset μ of a BG-algebra X is an $(\in, \in \vee q)$-T-fuzzy ideal of X iff*

(i) $\mu(0) \geq T\{\mu(x), T(1,\frac{1}{2})\}$

(ii) $\mu(x) \geq T\{T(\mu(x*y),\mu(y)), T(1,\frac{1}{2})\}$.

Proof. (i) Suppose μ is an $(\in,\in\vee q)$-T-fuzzy ideal of X. Assume that (i) is not valid, then there exists some $x\in X$ such that $\mu(0)<T\{\mu(x),T(1,\frac{1}{2})\}$, Choose a real number t such that

$$\mu(0)<t<T\{\mu(x),T(1,\frac{1}{2})\}. \quad (1)$$
$$\Rightarrow \mu(x)>t$$
$$\Rightarrow x_t\in\mu$$
$$\Rightarrow 0_t\in\vee q\mu \text{ [Since } \mu \text{is an } (\in,\in\vee q)-\text{T}-\text{fuzzy ideal of } X.]$$
$$\Rightarrow 0_t\in\mu \text{ or } 0_t q\mu$$
$$\Rightarrow \mu(0)\geq t \text{ or } \mu(0)+t>2T(1,\frac{1}{2})$$
$$\Rightarrow \mu(0)\geq t \text{ or } 2T(1,\frac{1}{2})<\mu(0)+t<t+t=2t \text{ by (1)}$$
$$\Rightarrow \mu(0)\geq t \text{ or } T(1,\frac{1}{2})<t \text{ which contradicts (1).} \quad (2)$$

Hence (i) is valid.

(ii) Assume that (ii) is not valid, then there exists some $x,y\in X$ such that $\mu(x)<T\{T(\mu(x*y),\mu(y)),T(1,\frac{1}{2})\}$ Choose a real number t such that

$$\mu(x)<t<T\{T(\mu(x*y),\mu(y)),T(1,\frac{1}{2})\}$$
$$\Rightarrow \mu(x)<t<T\{T(\mu(x*y),\mu(y)),T(1,\frac{1}{2})\}\leq min\{min(\mu(x*y),\mu(y)),min(1,\frac{1}{2})\}. \quad (3)$$
$$\Rightarrow \mu(y)>t \text{ and } \mu(x*y)>t$$
$$\Rightarrow (x*y)_t,y_t\in\mu$$
$$\Rightarrow x_t\in\vee q\mu \text{ [Since } \mu \text{ is an } (\in,\in\vee q)-\text{T}-\text{fuzzy ideal of X.]}$$
$$\Rightarrow x_t\in\mu \text{ or } x_t q\mu$$
$$\Rightarrow \mu(x)\geq t \text{ or } \mu(x)+t>2T(1,\frac{1}{2})$$
$$\Rightarrow \mu(x)\geq t \text{ or } 2T(1,\frac{1}{2})<\mu(x)+t<t+t=2t \text{ by (3)}$$
$$\Rightarrow \mu(x)\geq t \text{ or } T(1,\frac{1}{2})<t \text{ whichcontradicts (3).}$$

Hence (ii) is valid.

Theorem 3. *A fuzzy subset μ of a BG-algebra X is a T-fuzzy ideal of X iff μ is an $(\in,\in)$-T-fuzzy ideal of X.*

Proof. Let μ be a T-fuzzy ideal of X. Then

$$\mu(0)\geq\mu(x) \quad (4)$$

$$\mu(x)\geq T\{\mu(x*y),\mu(y)\} \quad (5)$$

to prove that μ is an $(\in,\in)$-T-fuzzy ideal of X. Let $x\in X$ such that $x_t\in\mu$, where $t\in(0,1)$. Then $\mu(x)\geq t$. Now (4) $\Rightarrow \mu(0)\geq\mu(x)\geq t\Rightarrow 0_t\in\mu$, Let $x,y\in X$ such that $(x*y)_t,y_s\in\mu$, where $t,s\in(0,1)$. Then $\mu(x*y)\geq t,\mu(y)\geq s$. Now

(5) $\Rightarrow \mu(x) \geq T\{\mu(x*y), \mu(y)\} \geq T\{t,s\} = T(t,s) \Rightarrow x_{T(t,s)} \in \mu$, Therefore, μ is an $(\in,\in)$-T-fuzzy ideal of X.

Conversely, let μ be an $(\in,\in)$-T-fuzzy ideal of X. To prove that μ is a T-fuzzy ideal of X. Let $x \in X$ and $t = \mu(x)$. Then $\mu(x) \geq t \Rightarrow x_t \in \mu \Rightarrow 0_t \in \mu$ [since μ is an $(\in,\in)$-T-fuzzy ideal of X]

$$\Rightarrow \mu(0) \geq t = \mu(x). \tag{6}$$

Again let $x,y \in X$ and $t = \mu(x*y), s = \mu(y)$. Then $\mu(x*y) \geq t, \mu(y) \geq s \Rightarrow (x*y)_t \in \mu, y_s \in \mu$
$\Rightarrow x_{T(t,s)} \in \mu$ [since μ is an $(\in,\in)$-T-fuzzy ideal of X] $\Rightarrow \mu(x) \geq T(t,s)$

$$\Rightarrow \mu(x) \geq T\{\mu(x*y), \mu(y)\}. \tag{7}$$

Hence (6) and (7)implies μ is a T-fuzzy ideal of X.

Definition 15. *Let I be an ideal of X and let μ be a fuzzy set of X such that*
(i) $\mu(x) = 0$ for all $x \in X \setminus I$,
(ii) $\mu(x) \geq T(1,\frac{1}{2})$ for all $x \in I$.
Then μ is a $(q,\in\vee q)$-T-fuzzy ideal of X.

Proof. Let $x \in X$ and $t \in (0,1]$ be such that $x_t q\mu$. Then we get $\mu(x)+t > 2T(1,\frac{1}{2})$. Since I is an ideal therefore $0 \in I$, i. e., $\mu(0) \geq T(1,\frac{1}{2})$ Now if $T(1,\frac{1}{2}) \geq t$ then $\mu(0) \geq T(1,\frac{1}{2}) \geq t$ which implies $0_t \in \mu$. If $t > T(1,\frac{1}{2})$ then $\mu(0)+t > 2T(1,\frac{1}{2})$ and so $0_t q\mu$. Hence $0_t \in \vee q\mu$. Again let $x,y \in X$ and $t,s \in (0 1]$ be such that $(x*y)_t q\mu$ and$x_s q\mu$. Then we get that $\mu(x*y)+t > 2T(1,\frac{1}{2})$ and $\mu(y)+s > 2T(1,\frac{1}{2})$. We can conclude that $x \in X$, since in otherwise $x \in X \setminus I$, and therefore $t > 2T(1,\frac{1}{2})$ *or* $s > 2T(1,\frac{1}{2})$ which is a contradiction. If $T(t,s) > T(1,\frac{1}{2})$, then $\mu(x)+T(t,s) > 2T(1,\frac{1}{2})$ and so $x_{T(t,s)} q\mu$. If $T(t,s) \leq T(1,\frac{1}{2})$, then $\mu(x) \geq T(t,s)$ and thus $x_{T(t,s)} \in \mu$.. Hence $x_{T(t,s)} \in \vee q\mu$.

Definition 16. *Let λ and μ be two fuzzy sets in X, then their intersection $'\cap'$ and union $'\cup'$ with respect to t-norm T and s-norm S is defined by $\lambda \cap \mu(x) = T\{\lambda(x), \mu(x)\}$ and $\lambda \cup \mu(x) = S\{\lambda(x), \mu(x)\}$.*

Theorem 4. *If λ and μ be two $(\in,\in\vee q)$-T-fuzzy ideals of X, then $\lambda \cap \mu$ is also an $(\in,\in\vee q)$-T-fuzzy ideal of X.*

Proof. Here λ, μ both are $(\in,\in\vee q)$-T-fuzzy ideals of X. Therefore

$$\lambda(0) \geq T\{\lambda(x), T(1,\frac{1}{2})\} \tag{8}$$

$$\lambda(x) \geq T\{T(\lambda(x*y), \lambda(y)), T(1,\frac{1}{2})\} \tag{9}$$

$$\mu(0) \geq T\{\mu(x), T(1,\frac{1}{2})\} \tag{10}$$

$$\mu(x) \geq T\{T(\mu(x*y), \mu(y)), T(1,\frac{1}{2})\}. \tag{11}$$

Now

$$\begin{aligned}\lambda\cap\mu(0) =&T\{\lambda(0),\mu(0)\}\\ \geq&T\{T\{\lambda(x),T(1,\frac{1}{2})\},T\{\mu(x),T(1,\frac{1}{2})\}\} \quad \text{by}(8),(10)\\ =&T\{T(\lambda(x),\mu(x))T(1,\frac{1}{2})\} \quad \text{By Lemma 1}\\ =&T\{\lambda\cap\mu(x),T(1,\frac{1}{2})\}\end{aligned}$$

$$\begin{aligned}\lambda\cap\mu(x) =&T\{\lambda(x),\mu(x)\}\\ \geq&T\{T\{T(\lambda(x*y),\lambda(y)),T(1,\frac{1}{2})\},T\{T(\mu(x*y),\mu(y)),T(1,\frac{1}{2})\}\} \quad \text{by}(9),(11)\\ =&T\{T\{T(\lambda(x*y),\mu(x*y)),T(\lambda(y),\mu(y))\},T(1,\frac{1}{2})\} \quad \text{By Lemma 1}\\ =&T\{T\{\lambda\cap\mu(x*y),\lambda\cap\mu(y)\},T(1,\frac{1}{2})\}.\end{aligned}$$

Theorem 5. *If $\{\mu_i|i\in\wedge\}$ be a family of $(\in,\in\vee q)$-T-fuzzy ideals of X, then $\cap_{i\in\wedge}\mu_i$ is an $(\in,\in\vee q)$-T-fuzzy ideal of X.*

Proof. Here by Theorem 2 we have, forall $i\in\wedge$

$$\begin{aligned}\mu_i(0) \geq&T\{\mu_i(x),T(1,\frac{1}{2})\}\\ \mu_i(x) \geq&T\{T(\mu_i(x*y),\mu_i(y)),T(1,\frac{1}{2})\}.\end{aligned}$$

Therefore taking infimum with respect to t-norm we get.

$$\begin{aligned}\mu(0)=\inf_{i\in\wedge}\mu_i(0)\geq&\inf_{i\in\wedge}T\{\mu_i(x),T(1,\frac{1}{2})\}\\ \geq&T\{\inf_{i\in\wedge}\mu_i(x),T(1,\frac{1}{2})\}\\ \geq&T\{\mu(x),T(1,\frac{1}{2})\},\end{aligned}$$

and

$$\begin{aligned}\mu(0)=\inf_{i\in\wedge}\mu_i(x)\geq&\inf_{i\in\wedge}T\{T(\mu_i(x*y),\mu_i(y)),T(1,\frac{1}{2})\}\\ \geq&\inf_{i\in\wedge}T\{T(\mu_i(x*y),\mu_i(y)),T(1,\frac{1}{2})\}\\ \geq&T\{T(\inf_{i\in\wedge}\mu_i(x*y),\inf_{i\in\wedge}\mu_i(y)),T(1,\frac{1}{2})\}\\ \geq&T\{T(\mu(x*y),\mu(y)),T(1,\frac{1}{2})\}.\end{aligned}$$

Hence by Theorem 2 $\cap_{i\in\wedge}\mu_i$ is an $(\in,\in\vee q)$-T-fuzzy ideal of X.

Theorem 6. *Union of two $(\in,\in\vee q)$-T-fuzzy ideals of X, may not be an $(\in,\in\vee q)$-T-fuzzy ideal of X.*

Proof. Consider BG-algebra X as defined in Example 7. Now consider two fuzzy sets λ and μ defined by $\lambda(0) = \lambda(1) = \lambda(2) = 0.4, \lambda(3) = 0.3$. and $\mu(0) = \mu(1) = \mu(2) = 0.5, \mu(3) = 0.3$. Then it is easy to verify that both λ and μ are $(\in, \in \vee q)$-T-fuzzy ideal of X with respect to t-norm T_L. But the dual (t-conorm) of t norm T_L is S_L where $S_L(a,b) = 1 - T_L(1-a, 1-b) = 1 - max(1-a+1-b-1, 0) = min(a+b, 1)$ Now $\lambda \cup \mu(0) = \lambda \cup \mu(1) = \lambda \cup \mu(2) = S_L(0.4, 0.5) = 0.9, \lambda \cup \mu(3) = S_L(0.3, 0.3) = 0.6$. Now $(3*1)_{0.9}, 1_{0.9} \in (\lambda \cup \mu)$ But $3_{T_L(0.9,0.9)} = 3_{0.8} \notin (\lambda \cup \mu)$. Hence $(\lambda \cup \mu)$ is not an $(\in, \in \vee q)$-T-fuzzy ideal of X.

Theorem 7. *If μ is a (q,q)-T-fuzzy ideal of X then it is also an $(\in,\in)$-T-fuzzy ideal of X.*

Proof. Let μ be a (q,q)-T-fuzzy ideal of X. Let $x \in X$ such that $x_t \in \mu$ then $\mu(x) \geq t$

$$\Rightarrow \mu(x) + \delta > t$$
$$\Rightarrow \mu(x) + \delta - t + 2T(1, \frac{1}{2}) > 2T(1, \frac{1}{2})$$
$$\Rightarrow (x)_{\delta - t + 2T(1,\frac{1}{2})} q\mu$$
$$\Rightarrow (0)_{\delta - t + 2T(1,\frac{1}{2})} q\mu \text{ [Since } \mu \text{ be a (q,q)} - \text{T} - \text{fuzzy ideal of X.]}$$
$$\Rightarrow \mu(0) + \delta - t + 2T(1, \frac{1}{2}) > 2T(1, \frac{1}{2})$$
$$\Rightarrow \mu(0) + \delta > t$$
$$\Rightarrow \mu(0) \geq t$$
$$\Rightarrow 0_t \in \mu.$$

Therefore $x_t \in \mu \Rightarrow 0_t \in \mu$. Again let $x, y \in X$ such that $(x*y)_t, y_s \in \mu$.

$$\Rightarrow \mu(x*y) \geq t, \mu(y) \geq s$$
$$\Rightarrow \mu(x*y) + \delta > t, \mu(y) + \delta > s$$
$$\Rightarrow \mu(x*y) + \delta - t + 2T(1, \frac{1}{2}) > 2T(1, \frac{1}{2}), \mu(y) + \delta - s + 2T(1, \frac{1}{2}) > 2T(1, \frac{1}{2})$$
$$\Rightarrow (x*y)_{\delta - t + 2T(1,\frac{1}{2})} q\mu \text{ and } (y)_{\delta - s + 2T(1,\frac{1}{2})} q\mu$$
$$\Rightarrow (x)_{T(\delta - t + 2T(1,\frac{1}{2}), \delta - s + 2T(1,\frac{1}{2}))} q\mu$$
$$\Rightarrow \mu(x) + T\{\delta - t + 2T(1, \frac{1}{2}), \delta - s + 2T(1, \frac{1}{2})\} > 2T(1, \frac{1}{2})$$
$$\Rightarrow \mu(x) + T\{\delta - t + 2T(1, \frac{1}{2}), \delta - s + 2T(1, \frac{1}{2})\} > \mu(x) + \delta - S\{t,s\} + 2T(1, \frac{1}{2}) > 2T(1, \frac{1}{2})$$
$$\Rightarrow \mu(x) + \delta - S\{t,s\} + 2T(1, \frac{1}{2}) > 2T(1, \frac{1}{2})$$
$$\Rightarrow \mu(x) + \delta > S\{t,s\}$$
$$\Rightarrow \mu(x) \geq S\{t,s\}$$
$$\Rightarrow \mu(x) \geq S\{t,s\} \geq T\{t,s\}$$
$$\Rightarrow \mu(x) \geq T\{t,s\}$$
$$\Rightarrow x_{T\{t,s\}} \in \mu,$$
$$i.e., \ (x*y)_t, y_s \in \mu, \Rightarrow x_{T\{t,s\}} \in \mu.$$

Hence μ is an $(\in,\in)$-T-fuzzy ideal of X.

Theorem 8. *A fuzzy subset μ of BG- algebra X is an $(\in,\in\vee q)$-T-fuzzy ideal of X and $\mu(x)<T(1,\frac{1}{2})\,\forall x\in X$ then μ is also an $(\in,\in)$-T-fuzzy ideal of X.*

Proof. Let μ be an $(\in,\in\vee q)$-T-fuzzy ideal of X and $\mu(x)<T(1,\frac{1}{2})\,\forall x\in X$. Let $x_t\in\mu\Rightarrow\mu(x)\geq t$
$\Rightarrow t\leq\mu(x)<T(1,\frac{1}{2})$ and also $\mu(0)<T(1,\frac{1}{2})$
$\mu(0)+t<T(1,\frac{1}{2})+T(1,\frac{1}{2})=2T(1,\frac{1}{2})\Rightarrow\mu(0)+t<2T(1,\frac{1}{2})\Rightarrow\mu(0)+t\not>2T(1,\frac{1}{2})\Rightarrow 0_t\bar{q}\mu$. Therefore $x_t\in\mu\Rightarrow 0_t\bar{q}\mu$
Since μ be an $(\in,\in\vee q)$-T-fuzzy ideal of X, therefore we must have $x_t\in\mu\Rightarrow 0_t\in\mu$. Again let $(x*y)_t,y_s\in\mu$,
$\Rightarrow t\leq\mu(x*y)<T(1,\frac{1}{2})$ and $s\leq\mu(y)<T(1,\frac{1}{2})$
$\Rightarrow T(t,s)<T(1,\frac{1}{2})$ and also $\mu(x)+T(t,s)<T(1,\frac{1}{2})+T(1,\frac{1}{2})=2T(1,\frac{1}{2})$.
Since μ is an $(\in,\in\vee q)$-T-fuzzy ideal of X i.e., $\mu(x)\geq T(t,s)$ or $\mu(x)+T(t,s)>2T(1,\frac{1}{2})$. So we must have $\mu(x)\geq T(t,s)$ i.e., $x_{T(t,s)}\in\mu$. Hence μ is an $(\in,\in)$-T-fuzzy ideal of X.

Theorem 9. *A fuzzy set μ in X is an $(\in,\in\vee q)$-T-fuzzy ideal of X if and only if the level set $\mu_t=\{x\in X|\mu(x)\geq t\}$ is a ideal of X for all $t\in(0,T(1,\frac{1}{2}))$‘ and $\mu_t\neq\phi$.*

Proof. Assume that μ be an $(\in,\in\vee q)$-T-fuzzy ideal of X and $t\in(0,T(1,\frac{1}{2})]$. Let $x\in X$ such that $x\in\mu_t$. Therefore $\mu(x)\geq t$. Now by the Theorem 2,

$$\mu(0)\geq T\{\mu(x),T(1,\frac{1}{2})\}\geq T\{t,T(1,\frac{1}{2})\}=t$$
$$\Rightarrow\mu(0)\geq t$$
$$\Rightarrow 0\in\mu_t.$$

Again let $x,y\in X$ such that $x*y,y\in\mu_t$. Therefore $\mu(x*y)\geq t,\mu(y)\geq t$. Again by the Theorem 2,

$$\mu(x)\geq T\{\mu(x*y),\mu(y),T(1,\frac{1}{2})\}\geq T\{t,t,T(1,\frac{1}{2})\}=t$$
$$\Rightarrow\mu(x)\geq t\Rightarrow x\in\mu_t.$$

Therefore $x*y,y\in\mu_t\Rightarrow x\in\mu_t$. Hence μ_t is a ideal of X.

Conversely, Suppose that μ be a fuzzy set in X and $\mu_t=\{x\in X|\mu(x)\geq t\}$ is an ideal of X for all $t\in(0,T(1,\frac{1}{2})]$ To prove μ is an $(\in,\in\vee q)$-T-fuzzy ideal of X. Suppose μ is not an $(\in,\in\vee q)$-T-fuzzy ideal of X. The there exists some $x,y\in X$ such that at least one of $\mu(0)<T\{\mu(x),T(1,\frac{1}{2})\}$ and $\mu(x)<T\{\mu(x*y),\mu(y),T(1,\frac{1}{2})\}$ hold . Suppose $\mu(0)<T\{\mu(x),T(1,\frac{1}{2})\}$ holds. Let

$$t=\frac{\mu(0)+T\{\mu(x),T(1,\frac{1}{2})\}}{2},$$

then $t\in(0,T(1,\frac{1}{2}))$ and

$$\mu(0)<t<T\{\mu(x),T(1,\frac{1}{2})\}. \tag{12}$$

Since μ_t is an ideal, therefore $0\in\mu_t$ i.e., $\mu(0)>t$ which contradicts (12) Again if $\mu(x)<T\{T(\mu(x*y),\mu(y)),T(1,\frac{1}{2})\}$ holds. Let

$$t=\frac{\mu(x)+T\{T(\mu(x*y),\mu(y)),T(1,\frac{1}{2})\}}{2}$$

then $t \in (0, T(1, \frac{1}{2}))$ and

$$\mu(x) < t < T\{T(\mu(x*y), \mu(y)), T(1, \frac{1}{2})\}\} < min\{min(\mu(x*y), \mu(y)), min(1, \frac{1}{2})\}. \tag{13}$$

$$\begin{aligned} &\Rightarrow \mu(x*y), \mu(y) > t \\ &\Rightarrow (x*y), y \in \mu_t \\ &\Rightarrow x \in \mu_t \\ &\Rightarrow \mu(x) > t \text{ whichcontradicts}(13). \end{aligned}$$

Therefore we must have $\mu(x) \geq T\{\mu(x*y), \mu(y), T(1, \frac{1}{2})\}$ consequently μ is an $(\in, \in \vee q)$-T- fuzzy ideal of X.

Theorem 10. *Let A be a subset of BG-algebra X. Consider the fuzzy set μ_A in X defined by*

$$\mu_A(x) = \begin{cases} 1, & if \quad x \in A \\ 0, & otherwise \end{cases} .$$

Then A is an ideal of X iff μ_A is an $(\in, \in \vee q)$-T-fuzzy ideal of X.

Proof. Let A be an ideal of X. Now $(\mu_A)_t = x \in X | \mu_A(x) \geq t = A$, which is an ideal. Hence by above Theorem μ_A is an $(\in, \in \vee q)$-T-fuzzy ideal of X.

Conversely, assume that μ_A is an $(\in, \in \vee q)$-T-fuzzy ideal of X, to prove A is an ideal of X. Let $x \in A$. Then

$$\mu_A(0) \geq T(\mu_A(x), T(1, \frac{1}{2})) = T(1, T(1, \frac{1}{2})) = T(1, \frac{1}{2}) \Rightarrow \mu_A(0) \geq T(1, \frac{1}{2}) \Rightarrow \mu_A(0) = 1 \Rightarrow 0 \in A.$$

Again let $x*y, y \in A$. Then $\mu_A(x) \geq T(T(\mu_A(x*y), \mu_A(y)), T(1, \frac{1}{2})) = T(T(1,1), T(1, \frac{1}{2})) = T(1, \frac{1}{2}) \Rightarrow \mu_A(x) \geq T(1, \frac{1}{2}) \Rightarrow \mu_A(x) = 1 \Rightarrow x \in A$. Hence A is an ideal of X.

Theorem 11. *Let A be an ideal of X, then for every $t \in (0, T(1, \frac{1}{2})]$ there exists an $(\in, \in \vee q)$-T-fuzzy ideal μ of X, such that $\mu_t = A$.*

Proof. Letμ be a fuzzy set in X defined by

$$\mu(x) = \begin{cases} 1, & \text{if} \quad x \in A \\ s, & \text{otherwise} \end{cases} ,$$

where $s < t \in (0, T(1, \frac{1}{2})]$ Therefore $\mu_t = x \in X | \mu(x) \geq t = A$ and hence μ_t is an ideal. Now if μ is not an $(\in, \in \vee q)$-T-fuzzy ideal of X then there exist some $a, b \in X$ such that at least one of $\mu(0) < T(\mu(a), T(1, \frac{1}{2}))$ and $\mu(a) < T(T(\mu(a*b), \mu(b)), T(1, \frac{1}{2}))$ hold. Suppose $\mu(0) < T(\mu(a), T(1, \frac{1}{2}))$ holds. Then choose a real number $t \in (0, T(1, \frac{1}{2})]$ such that

$$\mu(0) < t < T(\mu(a), T(1, \frac{1}{2})). \tag{14}$$

But $0 \in \mu_t = A$ [since μ_t is ideal] Therefore $\mu(0) = 1 > t$, which contradicts (14) Hence we must have $\mu(0) < T(\mu(x), T(1, \frac{1}{2}))$. Again if $\mu(a) < T(T(\mu(a*b), \mu(b)), T(1, \frac{1}{2}))$ holds then choose a real number $t \in (0, T(1, \frac{1}{2})]$,

$$\mu(a) < t < T(T(\mu(a*b), \mu(b)), T(1, \frac{1}{2})), \tag{15}$$

i.e., $\mu(a*b) > t, \mu(b) > t \Rightarrow a*b \in, b \in \mu_t \Rightarrow a \in \mu_t = A$ [since μ_t is ideal]. Therefore $\mu(a) = 1 > t$, which contradicts (15). Hence we must have $\mu(x) \geq T(T(\mu(x*y), \mu(y)), T(1, \frac{1}{2}))$. Thus μ_A is an $(\in, \in \vee q)$-T-fuzzy ideal of X.

4 Cartesian product of BG-algebras and their $(\in, \in \vee q)$-T-fuzzy ideals

Theorem 12. *Let X ,Y be two BG-algebras. Then their cartesian product $X \times Y = \{(x,y) \mid x \in X, y \in Y\}$ is also a BG-algebra under the binary operation $*$ defined in $X \times Y$ by $(x,y)*(p,q) = (x*p, y*q)$ for all $(x,y),(p,q) \in X \times Y$.*

Proof. Straightforward.

Definition 17. *Let λ and μ be two $(\in, \in \vee q)$-T-fuzzy ideals of BG-algebra X. Then their cartesian product $\lambda \times \mu$ is defined by $(\lambda \times \mu)(x,y) = T\{\lambda(x), \mu(x)\}$ Where $(\lambda \times \mu) : X \times X \to [0,1] \quad \forall x, y \in X$*

Theorem 13. *Let λ and μ be two $(\in, \in \vee q)$-T-fuzzy ideals of a BG-algebra X. Then $\lambda \times \mu$ is also an $(\in, \in \vee q)$-T-fuzzy ideal of $X \times X$.*

Proof. Similar to Theorem 4.

5 Homomorphism of BG-algebras and $(\in, \in \vee q)$-T-fuzzy ideals

Definition 18. *Let X and X' be two BG-algebras, then a mapping $f : X \to X'$is said to be homomorphism if $f(x*y) = f(x)*f(y)\ \forall x, y \in X$.*

Theorem 14. *Let X and X' be two BG-algebras and $f : X \to X'$ be a homomorphism. Then $f(0) = 0'$.*

Proof. We have $f(0) = f(x*x) = f(x)*f(x) = 0'$.

Theorem 15.*Let X and X' be two BG-algebras and $f : X \to X'$ be a homomorphism. If μ be an $(\in, \in \vee q)$-T-fuzzy ideal of X', then $f^{-1}(\mu)$ is an $(\in, \in \vee q)$-T-fuzzy ideal of X.*

Proof. $f^{-1}(\mu)$ is defined as $f^{-1}(\mu)(x) = \mu(f(x))\forall x \in X$. Let μ be an $(\in, \in \vee q)$-T-fuzzy ideal of X' and $x \in X$ such that $x_t \in f^{-1}(\mu)$, then $f^{-1}(\mu)(x) \geq t$.

$$\begin{aligned}
&\Rightarrow \mu f(x) \geq t \\
&\Rightarrow (f(x)_t \in \mu \\
&\Rightarrow 0'_t \in \vee q\mu \text{ [Since } \mu \text{ be an} (\in, \in \vee q) - \text{T} - \text{fuzzy ideal of } X'] \\
&\Rightarrow 0'_t \in \mu \text{ or } 0'_t q\mu \\
&\Rightarrow \mu(0') \geq t \text{ or } \mu(0') + t \geq 2T(1, \frac{1}{2}) \\
&\Rightarrow \mu(f(0)) \geq t \text{ or } \mu(f(0)) + t \geq 2T(1, \frac{1}{2}) \\
&\Rightarrow f^{-1}(\mu)(0) \geq t \text{ or } f^{-1}(\mu)(0) + t \geq 2T(1, \frac{1}{2}) \\
&\Rightarrow 0_t \in f^{-1}(\mu) \text{ or } 0_t q f^{-1}(\mu) \\
&\Rightarrow 0_t \in \vee q f^{-1}(\mu).
\end{aligned}$$

Therefore $x_t \in f^{-1}(\mu) \Rightarrow 0_t \in \vee q f^{-1}(\mu)$. Again let $x, y \in X$ such that $(x*y)_t, y_s \in f^{-1}(\mu)$, then $f^{-1}(\mu)(x*y) \geq t$ and $f^{-1}(\mu)(y) \geq s$,

$$\begin{aligned}
&\Rightarrow \mu f(x*y) \geq t \text{ and } \mu f(y) \geq s \\
&\Rightarrow [f(x*y)]_t \in \mu \text{ and } [f(y]_s \in \mu \\
&\Rightarrow [f(x)*f(y)]_t \in \mu \text{ and } [f(y)]_s \in \mu \\
&\Rightarrow [f(x)]_{T(t,s)} \in \vee q\mu \\
&\Rightarrow [f(x)]_{T(t,s)} \in \mu \text{ or } [f(x)]_{T(t,s)} q\mu \\
&\Rightarrow \mu(f(x)) \geq T(t,s) \text{ or } \mu(f(x)) + T(t,s) \geq 2T(1, \frac{1}{2}) \\
&\Rightarrow f^{-1}(\mu)(x) \geq T(t,s) \text{ or } f^{-1}(\mu)(x) + T(t,s) \geq 2T(1, \frac{1}{2}) \\
&\Rightarrow x_{T(t,s)} \in f^{-1}(\mu) \text{ or } x_{T(t,s)} q f^{-1}(\mu) \\
&\Rightarrow x_{T(t,s)} \in \vee q f^{-1}(\mu).
\end{aligned}$$

Therefore $(x*y)_t, y_s \in f^{-1}(\mu) \Rightarrow x_{T(t,s)} \in \vee q f^{-1}(\mu)$.

Theorem 16. *Let X and X' be two BG-algebras and $f : X \to X'$ be an onto homomorphism. If μ be a fuzzy subset of X' such that $f^{-1}(\mu)$ is an $(\in, \in \vee q)$-T-fuzzy ideal of X, then μ is also an $(\in, \in \vee q)$-T-fuzzy ideal of X'.*

Proof. Let $x' \in X'$ such that $x'_t \in \mu$ where $t \in [0\,1]$, then $\mu(x') \geq t$ since f is onto so there exists $x \in X$ such that $f(x) = x'$. Now $\mu(x') \geq t \;\Rightarrow\; \mu(f(x)) \geq t$.

$$\begin{aligned}
&\Rightarrow f^{-1}(\mu)(x) \geq t \\
&\Rightarrow x_t \in f^{-1}(\mu) \\
&\Rightarrow 0_t \in \vee q f^{-1}(\mu) \text{ [since } f^{-1}(\mu) \text{ is an } (\in, \in \vee q) - T - \text{fuzzy ideal of} X],
\end{aligned}$$

$$\Rightarrow 0_t \in f^{-1}(\mu) \text{ or } 0_t q f^{-1}(\mu),$$
$$\Rightarrow f^{-1}(\mu)(0) \geq t \text{ or } f^{-1}(\mu)(0) + t > 2T(1,\frac{1}{2})$$
$$\Rightarrow \mu(f(0)) \geq t \text{ or } \mu(f(0)) + t > 2T(1,\frac{1}{2})$$
$$\Rightarrow \mu(0') \geq t \text{ or } \mu(0') + t > 2T(1,\frac{1}{2})$$
$$\Rightarrow 0'_t \in \mu \text{ or } 0'_t q\mu$$
$$\Rightarrow 0'_t \in \vee q\mu.$$

Therefore $x'_t \in \mu \Rightarrow 0'_t \in \vee q\mu$. Again let $x', y' \in X'$ such that $(x' * y')_t, y'_s \in \mu$ where $t, s \in [01]$, then $\mu(x' * y') \geq t, \mu(y') \geq t$ since f is onto so there exists $x, y \in X$, such that $f(x) = x', f(y) = y'$ also f is homomorphism so $f(x * y) = f(x) * f(y) = x' * y'$. Now $\mu(x' * y') \geq t$ and $\mu(y') \geq s$,

$$\Rightarrow \mu(f(x) * f(y)) \geq t \text{ and } \mu(f(y)) \geq s$$
$$\Rightarrow \mu(f(x * y)) \geq t \text{ and } \mu(f(y)) \geq s \text{ [Since } f \text{ is homomorphism]}$$
$$\Rightarrow f^{-1}(\mu)(x * y) \geq t \text{ and } f^{-1}(\mu)(y) \geq s$$
$$\Rightarrow (x * y)_t \in f^{-1}(\mu) \text{ and } y_s \in f^{-1}(\mu),$$
$$\Rightarrow x_{T(t,s)} \in \vee q f^{-1}(\mu) \text{ [Since } f^{-1}(\mu) \text{ is an} (\in,\in\vee q) - \mathrm{T} - \text{fuzzy ideal of X}],$$
$$\Rightarrow f^{-1}(\mu)(x) \geq T(t,s) \text{ or } f^{-1}(\mu)(x) + T(t,s) > 2T(1,\frac{1}{2})$$
$$\Rightarrow \mu(f(x)) \geq T(t,s) \text{ or } \mu(f(x)) + T(t,s) > 2T(1,\frac{1}{2})$$
$$\Rightarrow \mu(x') \geq T(t,s) \text{ or } \mu(x') + T(t,s) > 2T(1,\frac{1}{2})$$
$$\Rightarrow x'_{T(t,s)} \in \mu \text{ or } x'_{T(t,s)} q\mu$$
$$\Rightarrow x'_{T(t,s)} \in \vee q\mu.$$

Therefore $(x' * y')_t, y'_s \in \mu \Rightarrow x'_t \in \vee q\mu$. Hence μ is an $(\in,\in\vee q)$-T-fuzzy ideal of X'.

6 Conclusions

In this paper, we have studied $(\in,\in\vee q)$-T-fuzzy ideal of *BG*-algebra and obtained some interesting results. Choosing different t-norm T we can obtain different types of fuzzy ideals in BG-algebra. We can consider $(\in,\in\vee q)$-T-fuzzy ideal is the generalised form of fuzzy ideal. By using same idea, we can define $(\in,\in\vee q)$-T-fuzzy ideal in other algebric systems also.

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