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On dual spacelike curves of constant breadth in dual Lorentzian space $\mathbb{D}_1^3$

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Abstract: In this work, dual spacelike curves of constant breadth are defined in dual Lorentzian space $\mathbb{D}_1^3$. Some characterizations of closed dual spacelike curves of constant breadth are presented in dual Lorentzian space $\mathbb{D}_1^3$. These characterizations are made by obtaining special solutions of differential equations which are related to dual spacelike curves of constant breadth in $\mathbb{D}_1^3$.

Keywords: Dual Lorentzian space, dual spacelike curves, curves of constant breadth, differential equations.

1 Introduction

Dual numbers were introduced by William Kingdon Clifford as a tool for his geometrical investigations. Then dual numbers and vectors were used on line geometry and kinematics by Eduard Study. He devoted a special attention to the representation of oriented lines by dual unit vectors and defined the famous mapping: The set of oriented lines in a three-dimensional Euclidean space $\mathbb{E}^3$ is one to one correspondence with the points of a dual space $\mathbb{D}^3$ of triples of dual numbers [4].

In the classical theory of curves in differential geometry, curves of constant breadth have a long history as a research matter [2], [3], [1]. First it was introduced by Euler in [2]. Then Fujivara obtained a problem to determine whether there exist space curves of constant breadth or not, and he defined the concept ”breadth” for space curves on a surface of constant breadth [3]. Furthermore, Blaschke defined the curve of constant breadth on the sphere [1]. Reuleaux gave a method to obtain these kinds of curves and applied the results he had by using his method, in kinematics and engineering [8].

Some geometric properties of plane curves of constant breadth were given by Köse in [6]. And, in another work of Kose, [5], these properties were studied in the Euclidean 3-space E^3. In Minkowski 3-space as an ambient space, some characterizations of timelike curves of constant breadth were given by Yılmaz and Turgut in [12]. Also, Yılmaz dealt with dual timelike curves of constant breadth in dual Lorentzian space in [11].

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In this paper, we study dual spacelike curves of constant breadth in dual Lorentzian space $\mathbb{D}_1^3$. We give some characterizations of dual spacelike curves of constant breadth in $\mathbb{D}_1^3$.

2 Preliminaries

Let $\mathbb{E}_1^3$ be the three-dimensional Minkowski space, that is, the three-dimensional real vector space $\mathbb{E}^3$ with the metric

$$\langle dx,dx\rangle = dx_1^2 + dx_2^2 - dx_3^2,$$

where (x_1,x_2,x_3) denotes the canonical coordinates in $\mathbb{E}^3$. An arbitrary vector x of $\mathbb{E}_1^3$ is said to be spacelike if $\langle x,x\rangle > 0$ or $x=0$, timelike if $\langle x,x\rangle < 0$ and lightlike or null if $\langle x,x\rangle = 0$ and $x \neq 0$. A timelike or light-like vector in $\mathbb{E}_1^3$ is said to be causal. For $x \in \mathbb{E}_1^3$ the norm is defined by $\|x\| = \sqrt{|\langle x,x\rangle|}$, then the vector x is called a spacelike unit vector if $\langle x,x\rangle = 1$ and a timelike unit vector if $\langle x,x\rangle = -1$. Similarly, a regular curve in $\mathbb{E}_1^3$ can locally be spacelike, timelike or null (lightlike), if all of its velocity vectors are spacelike, timelike or null (lightlike), respectively [7].

Dual numbers are given with the set

$$\mathbb{D} = \{\widehat{x} = x + \varepsilon x^*; x, x^* \in \mathbb{E}\},$$

where the symbol ε designates the dual unit with the property $\varepsilon^2 = 0$ for $\varepsilon \neq 0$. Dual angle is defined as $\widehat{\theta} = \theta + \varepsilon\theta^*$, where θ is the projected angle between two spears and θ^* is the shortest distance between them. The set $\mathbb{D}$ of dual numbers is commutative ring the the operations $+$ and $\cdot$. The set

$$\mathbb{D}^3 = \mathbb{D}\times\mathbb{D}\times\mathbb{D} = \left\{\widehat{\varphi} = \varphi + \varepsilon\varphi^*; \varphi, \varphi^* \in \mathbb{E}^3\right\}$$

is a module over the ring $\mathbb{D}$ [10].

For any $\widehat{a} = a + \xi a^*$, $\widehat{b} = b + \xi b^* \in \mathbb{D}^3$, if the Lorentzian inner product of $\widehat{a}$ and $\widehat{b}$ is defined by

$$<\widehat{a},\widehat{b}> = <a,b> + \varepsilon(<a^*,b> + <a,b^*>),$$

then the dual space $\mathbb{D}^3$ together with this Lorentzian inner product is called the dual Lorentzian space and denoted by $\mathbb{D}_1^3$ [9]. For $\widehat{\varphi} \neq 0$, the norm $\|\widehat{\varphi}\|$ of $\widehat{\varphi}$ is defined by

$$\|\widehat{\varphi}\| = \sqrt{<\widehat{\varphi},\widehat{\varphi}>}.$$

A dual vector $\widehat{\omega} = \omega + \varepsilon\omega^*$ is called dual spacelike vector if $\langle\widehat{\omega},\widehat{\omega}\rangle > 0$ or $\widehat{\omega} = 0$, dual timelike vector if $\langle\widehat{\omega},\widehat{\omega}\rangle < 0$ and dual null (lightlike) vector if $\langle\widehat{\omega},\widehat{\omega}\rangle = 0$ for $\widehat{\omega} \neq 0$. Therefore, an arbitrary dual curve which is a differential mapping onto $\mathbb{D}_1^3$, can locally be dual spacelike, dual timelike or dual null if its velocity vector is dual spacelike, dual timelike or dual null, respectively. Also, for the dual vectors $\widehat{a},\widehat{b} \in \mathbb{D}_1^3$, Lorentzian vector product of these dual vectors is defined by

$$\widehat{a}\times\widehat{b} = a\times b + \varepsilon(a^*\times b + a\times b^*),$$

where $a\times b$ is the classical cross product according to the signature $(+,+,-)$ [9].

Let $\widehat{\varphi}: I \subset R \to \mathbb{D}_1^3$ be a dual spacelike curve with the arc-length parameter s. Then the unit tangent vector is defined $\dot{\widehat{\varphi}} = \widehat{t}$ and the principal normal is $\widehat{n} = \dfrac{\dot{\widehat{t}}}{\widehat{\kappa}}$ where $\widehat{\kappa}$ is never pure dual curvature. The function $\widehat{\kappa} = \left\| \dot{\widehat{t}} \right\| = \kappa + \varepsilon \kappa^*$ is called dual curvature of the dual curve $\widehat{\varphi}$. Then the binormal vector of $\widehat{\varphi}$ is given by the dual vector $\widehat{b} = \widehat{t} \times \widehat{n}$. Hence, the triple $\{\widehat{t}, \widehat{n}, \widehat{b}\}$ is called Frenet trihedra at the point $\widehat{\varphi}(s)$. The Frenet derivative formulas of a dual spacelike curve $\widehat{\varphi}$ is given as

$$\begin{aligned} \widehat{t}^{\prime} &= \widehat{\kappa}\widehat{n}, \\ \widehat{n}^{\prime} &= -\varepsilon\widehat{\kappa}\widehat{t} + \widehat{\tau}\widehat{b}, \\ \widehat{b}^{\prime} &= \widehat{\tau}\widehat{n}, \end{aligned} \tag{1}$$

where $\widehat{\tau} = \tau + \varepsilon \tau^*$ is the dual torsion of the dual spacelike curve $\widehat{\varphi}$. Here we assume that the dual torsion $\widehat{\tau}$ is never pure dual one [9].

3 Dual spacelike curves of constant breadth in $\mathbb{D}_1^3$

In this section, we give some characterizations of dual spacelike curve of constant breadth in the dual Lorentzian space $\mathbb{D}_1^3$. First, we give the definition of dual spacelike curve of constant breadth in $\mathbb{D}_1^3$.

Definition 1. Let (C_1) be a dual spacelike curve with position vector $\widehat{\varphi} = \widehat{\varphi}(s)$ in $\mathbb{D}_1^3$. If (C) has parallel tangents in opposite directions at corresponding points $\widehat{\varphi}(s)$ and $\widehat{\alpha}(s_\alpha)$ and the distance between these points is always constant, then (C_1) is called a dual spacelike curve of constant breadth. Moreover, a pair of dual spacelike curves (C_1) and (C_2) for which the tangents at the corresponding points $\widehat{\varphi}(s)$ and $\widehat{\alpha}(s_\alpha)$, respectively, are parallel and in opposite directions, and the distance between these points is always constant are called a spacelike dual curve pair of constant breadth.

Let $\widehat{\varphi} = \widehat{\varphi}(s)$ be a simple closed dual curve in $\mathbb{D}_1^3$. We consider a dual spacelike curve in the class Γ as in [3] having parallel tangents $\widehat{t}$ and $\widehat{t}_\alpha$ in opposite directions at the opposite points $\widehat{\varphi}$ and $\widehat{\alpha}$ of the curve. A simple closed dual curve of constant breadth having parallel tangents in opposite directions at opposite points can be represented with respect to dual Frenet frame by the equation

$$\widehat{\alpha} = \widehat{\varphi} + \widehat{\gamma}\widehat{t} + \widehat{\delta}\widehat{n} + \widehat{\lambda}\widehat{b}, \tag{2}$$

where $\widehat{\gamma}, \widehat{\delta}$ and $\widehat{\lambda}$ are arbitrary functions of s. Differentiating both sides of (2), we get

$$\frac{d\widehat{\alpha}}{ds_\alpha}\frac{ds_\alpha}{ds} = \left(\frac{d\widehat{\gamma}}{ds} - \varepsilon\widehat{\delta}\widehat{\kappa} + 1\right)\widehat{t} + \left(\widehat{\gamma}\widehat{\kappa} + \frac{d\widehat{\delta}}{ds} + \widehat{\lambda}\widehat{\tau}\right)\widehat{n} + \left(\widehat{\delta}\widehat{\tau} + \frac{d\widehat{\lambda}}{ds}\right)\widehat{b}, \tag{3}$$

and we know that $\widehat{t}_\alpha = -\widehat{t}$. Considering this, we have the following system of equations

$$\begin{aligned} \frac{d\widehat{\gamma}}{ds} &= \varepsilon\widehat{\delta}\widehat{\kappa} - 1 - \frac{ds_\alpha}{ds}, \\ \frac{d\widehat{\delta}}{ds} &= -\widehat{\gamma}\widehat{\kappa} - \widehat{\lambda}\widehat{\tau}, \\ \frac{d\widehat{\lambda}}{ds} &= -\widehat{\delta}\widehat{\tau}. \end{aligned} \tag{4}$$

If we call $\widehat{\theta}$ as the angle between the tangent of the curve C at point $\widehat{\varphi}$ with a given direction and consider $\frac{d\widehat{\theta}}{ds} = \widehat{\kappa} = \frac{1}{\widehat{\rho}}$ and $\frac{d\widehat{\theta}}{ds_\alpha} = \widehat{\kappa}^* = \frac{1}{\widehat{\rho}^*}$, we have (4) as follow;

$$\begin{aligned}
\frac{d\widehat{\gamma}}{d\widehat{\theta}} &= \varepsilon\widehat{\delta}\widehat{\rho}\widehat{\kappa} - f(\widehat{\theta}),\\
\frac{d\widehat{\delta}}{d\widehat{\theta}} &= -\widehat{\gamma} - \widehat{\lambda}\widehat{\tau}\widehat{\rho},\\
\frac{d\widehat{\lambda}}{d\widehat{\theta}} &= -\widehat{\delta}\widehat{\rho}\widehat{\tau},
\end{aligned} \tag{5}$$

where $f(\widehat{\theta}) = \widehat{\rho} + \widehat{\rho}^*$.Using the system of ordinary differential equations (5), we have the following dual third order differential equation with respect to $\widehat{\gamma}$ as;

$$\frac{d^3\widehat{\gamma}}{d\widehat{\theta}^3} + \varepsilon\frac{d\widehat{\gamma}}{d\widehat{\theta}} + \varepsilon\frac{d\widehat{\lambda}}{d\widehat{\theta}}\widehat{\rho}\widehat{k_2} + \varepsilon\widehat{\lambda}\frac{d\widehat{\rho}}{d\widehat{\theta}} + \widehat{k_2} = 0. \tag{6}$$

We can give the following corollary.

Corollary 1. The dual differential equation of third order in (6) is a characterization of the simple closed dual spacelike curve $\widehat{\alpha}$.

Since position vector of a simple closed dual spacelike curve can be determined by solution of (6), let us investigate solution of the equation (6) in a special case. Let $\widehat{\kappa}, \widehat{\kappa}^*$ and $\widehat{\tau}, \widehat{\tau}^*$ be constants. Then the equation (6) has the form

$$\frac{d^3\widehat{\gamma}}{d\widehat{\theta}^3} + (\varepsilon + \frac{\widehat{\tau}^2}{\widehat{\kappa}^2})\frac{d\widehat{\gamma}}{d\widehat{\theta}} + \frac{\widehat{\tau}^2}{\widehat{\kappa}^2} f(\widehat{\theta}) = 0. \tag{7}$$

Solution of equation (7) yields the components

$$\begin{aligned}
\widehat{\gamma} &= A + B\cos(\widehat{l}\widehat{\theta}) + C\sin(\widehat{l}\widehat{\theta}) - \int \frac{F(\widehat{\theta})}{\widehat{l}^2} d\widehat{\theta} + \int \frac{F(\widehat{\theta})\cos(\widehat{l}\widehat{\theta})}{\widehat{l}^2} d\widehat{\theta}\\
&\quad + \int \frac{F(\widehat{\theta})\sin(\widehat{l}\widehat{\theta})}{\widehat{l}^2} d\widehat{\theta},\\
\widehat{\delta} &= \frac{1}{\varepsilon}\{-B\widehat{l}\sin(\widehat{l}\widehat{\theta}) + C\widehat{l}\cos(\widehat{l}\widehat{\theta}) + \frac{F(\widehat{\theta})}{\widehat{l}^2}[\cos(\widehat{l}\widehat{\theta}) + \sin(\widehat{l}\widehat{\theta}) - 1] + f(\widehat{\theta})\},\\
\widehat{\lambda} &= -\frac{1}{\varepsilon}\int\{-B\widehat{l}\sin(\widehat{l}\widehat{\theta}) + C\widehat{l}\cos(\widehat{l}\widehat{\theta}) + \frac{F(\widehat{\theta})}{\widehat{l}^2}[\cos(\widehat{l}\widehat{\theta}) + \sin(\widehat{l}\widehat{\theta}) - 1]\\
&\quad + f(\widehat{\theta})\}\varepsilon\frac{\widehat{\tau}}{\widehat{k}} d\widehat{\theta},
\end{aligned} \tag{8}$$

where $\widehat{l} = \varepsilon + \frac{\widehat{\tau}^2}{\widehat{\kappa}^2}, F(\widehat{\theta}) = \frac{\widehat{\tau}^2}{\widehat{\kappa}^2} f(\widehat{\theta})$. We can give the following corollary.

Corollary 2. Position vector of a simple dual spacelike closed curve with constant dual curvature and constant dual torsion are obtained in terms of the values of $\widehat{\gamma}$, $\widehat{\delta}$ and $\widehat{\lambda}$ in the equation (8).

If the distance between opposite points of $\widehat{\varphi}$ and $\widehat{\alpha}$ is constant, then we can write that

$$\|\widehat{\alpha}-\widehat{\varphi}\| = \widehat{\gamma}^2 + \varepsilon(\widehat{\delta}^2 - \widehat{\lambda}^2) = \text{constant}. \tag{9}$$

Differentiating (9) with respect to $\widehat{\theta}$

$$\widehat{\gamma}\frac{d\widehat{\gamma}}{d\widehat{\theta}} + \varepsilon\widehat{\delta}\frac{d\widehat{\delta}}{d\widehat{\theta}} - \varepsilon\widehat{\lambda}\frac{d\widehat{\lambda}}{d\widehat{\theta}} = 0. \tag{10}$$

By virtue of (5), the differential equation (10) yields

$$\widehat{\gamma}(\frac{d\widehat{\gamma}}{d\widehat{\theta}} - \varepsilon\widehat{\delta}) = 0. \tag{11}$$

There are two cases for the equation (11), we study these cases as follows.

Case 1. If $\widehat{\gamma} = 0$, then we have the components $\widehat{\delta}$ and $\widehat{\lambda}$ as

$$\widehat{\delta} = \varepsilon f(\widehat{\theta}),\ \widehat{\lambda} = -\varepsilon\frac{df(\widehat{\theta})}{d\widehat{\theta}}\frac{\widehat{\kappa}}{\widehat{\tau}}. \tag{12}$$

Using the values of (12) in (2), we have the following invariant of dual spacelike curves of constant breadth as

$$\widehat{\alpha} = \widehat{\varphi} + +\varepsilon f(\widehat{\theta})\widehat{n} - \varepsilon\frac{df(\widehat{\theta})}{d\widehat{\theta}}\frac{\widehat{\kappa}}{\widehat{\tau}}\widehat{b}. \tag{13}$$

Case 2. If $\dfrac{d\widehat{\gamma}}{d\widehat{\theta}} = \varepsilon\widehat{\delta}$, that is, $f(\widehat{\theta}) = 0$, then we have a relation among radii of curvatures as

$$\frac{1}{\widehat{\kappa}} + \frac{1}{\widehat{\kappa}^*} = 0. \tag{14}$$

Using other components, we easily have a third order differential equation of variable coefficient with respect to $\widehat{\gamma}$ as

$$\frac{d^3\widehat{\gamma}}{d\widehat{\theta}^3} + \varepsilon\frac{d\widehat{\gamma}}{d\widehat{\theta}} - \varepsilon\frac{\widehat{\tau}^2}{\widehat{\kappa}^2} = 0. \tag{15}$$

This equation is a characterization for the components. However, its general solution of has not been found. Due to this, we investigate its solutions in special cases.

Let us suppose $\widehat{\varphi}$ is a dual spacelike curve which has constant dual curvatures, in this case, we rewrite the equation (15) as

$$\frac{d^3\widehat{\gamma}}{d\widehat{\theta}^3} + (\varepsilon - \frac{\widehat{\tau}^2}{\widehat{\kappa}^2})\frac{d\widehat{\gamma}}{d\widehat{\theta}} = 0. \tag{16}$$

The general solution of (16) depends on the character of $\varepsilon - \dfrac{\widehat{\tau}^2}{\widehat{\kappa}^2}$. Therefore, we distinguish the following cases.

Case 2.1. If $\kappa = \tau$ and $\kappa^* = \tau^*$, then we get

$$\frac{d^3\widehat{\gamma}}{d\widehat{\theta}^3}+(\varepsilon-1)\frac{d\widehat{\gamma}}{d\widehat{\theta}}=0. \tag{17}$$

The general solution of (17) depends on the value of ε.

Case 2.1.1. If $\varepsilon=1$, then, by means of solution of (16), we find the following components:

$$\begin{aligned}
\widehat{\gamma}&=\widehat{c}_1+\widehat{c}_2\widehat{\theta}+\widehat{c}_3\widehat{\theta}^2,\\
\widehat{\delta}&=\frac{1}{\varepsilon}(\widehat{c}_2+2\widehat{c}_3\widehat{\theta}),\\
\widehat{\lambda}&=-\frac{1}{\varepsilon}\int((\widehat{c}_2+2\widehat{c}_3)\frac{\widehat{\kappa}}{\widehat{\tau}})d\widehat{\theta}.
\end{aligned} \tag{18}$$

Case 2.1.2. If $\varepsilon=-1$, then, by means of solution of (16), we have the components:

$$\begin{aligned}
\widehat{\gamma}&=\widehat{c}_1+\widehat{c}_2e^{\sqrt{2}\widehat{\theta}}+\widehat{c}_3e^{-\sqrt{2}\widehat{\theta}},\\
\widehat{\delta}&=\frac{1}{\varepsilon}(\sqrt{2}\widehat{c}_2e^{\sqrt{2}\widehat{\theta}}-\sqrt{2}\widehat{c}_3e^{-\sqrt{2}\widehat{\theta}}+f(\widehat{\theta})),\\
\widehat{\lambda}&=-\frac{1}{\varepsilon}\int[\sqrt{2}\widehat{c}_2e^{\sqrt{2}\widehat{\theta}}-\sqrt{2}\widehat{c}_3e^{-\sqrt{2}\widehat{\theta}}+f(\widehat{\theta})]\frac{\widehat{\tau}}{\kappa}d\widehat{\theta}.
\end{aligned} \tag{19}$$

Case 2.2. If $\varepsilon-\dfrac{\widehat{\tau}^2}{\widehat{\kappa}^2}>0$, then, by means of solution of (16), we find the components:

$$\begin{aligned}
\widehat{\gamma}&=\widehat{A}\cos[\sqrt{\varepsilon-\frac{\widehat{\tau}^2}{\widehat{\kappa}^2}}]\widehat{\theta}+\widehat{B}\sin[\sqrt{\varepsilon-\frac{\widehat{\tau}^2}{\widehat{\kappa}^2}}]\widehat{\theta},\\
\widehat{\delta}&=\frac{1}{\varepsilon}(\sqrt{\varepsilon-\frac{\widehat{\tau}^2}{\widehat{\kappa}^2}}\{\widehat{A}\sin[\sqrt{\varepsilon-\frac{\widehat{\tau}^2}{\widehat{\kappa}^2}}]\widehat{\theta}+\widehat{B}\cos[\sqrt{\varepsilon-\frac{\widehat{\tau}^2}{\widehat{\kappa}^2}}]\widehat{\theta}\}),\\
\widehat{\lambda}&=-\frac{1}{\varepsilon}\int[\sqrt{\varepsilon-\frac{\widehat{\tau}^2}{\widehat{\kappa}^2}}\frac{\kappa}{\tau}\{\widehat{A}\sin[\sqrt{\varepsilon-\frac{\widehat{\tau}^2}{\widehat{\kappa}^2}}]\widehat{\theta}+\widehat{B}\cos[\sqrt{\varepsilon-\frac{\widehat{\tau}^2}{\widehat{\kappa}^2}}]\widehat{\theta}\}\\
&\quad-(\varepsilon-\frac{\widehat{\tau}^2}{\widehat{\kappa}^2})\{\widehat{A}\cos[\sqrt{\varepsilon-\frac{\widehat{\tau}^2}{\widehat{\kappa}^2}}]\widehat{\theta}+\widehat{B}\sin[\sqrt{\varepsilon-\frac{\widehat{\tau}^2}{\widehat{\kappa}^2}}]\widehat{\theta}\}]d\widehat{\theta}.
\end{aligned} \tag{20}$$

Case 2.3. If $\varepsilon-\dfrac{\widehat{\tau}^2}{\widehat{\kappa}^2}<0$, then, by means of solution of (16), we obtain the components:

$$\begin{aligned}
\widehat{\gamma}&=\widehat{K}e^{-\sqrt{\varepsilon-\frac{\widehat{\tau}^2}{\widehat{\kappa}^2}}\widehat{\theta}}+\widehat{L}e^{\sqrt{\varepsilon-\frac{\widehat{\tau}^2}{\widehat{\kappa}^2}}\widehat{\theta}},\\
\widehat{\delta}&=\frac{1}{\varepsilon}(\sqrt{\varepsilon-\frac{\widehat{\tau}^2}{\widehat{\kappa}^2}}\{\widehat{K}e^{-\sqrt{\varepsilon-\frac{\widehat{\tau}^2}{\widehat{\kappa}^2}}\widehat{\theta}}-\widehat{L}e^{\sqrt{\varepsilon-\frac{\widehat{\tau}^2}{\widehat{\kappa}^2}}\widehat{\theta}}\}),\\
\widehat{\lambda}&=-\frac{1}{\varepsilon}\int(\sqrt{\varepsilon-\frac{\widehat{\tau}^2}{\widehat{\kappa}^2}}\{\widehat{K}e^{-\sqrt{\varepsilon-\frac{\widehat{\tau}^2}{\widehat{\kappa}^2}}\widehat{\theta}}+\widehat{L}e^{\sqrt{\varepsilon-\frac{\widehat{\tau}^2}{\widehat{\kappa}^2}}\widehat{\theta}}\}\frac{\kappa}{\widehat{\tau}})d\widehat{\theta},
\end{aligned} \tag{21}$$

where $\widehat{K},\widehat{L}$ are constants.

4 Conclusion

In the classical theory of curves in differential geometry, curves of constant breadth have a long history as a research matter. In this paper, we give dual spacelike curves of constant breadth in dual Lorentzian 3- space $\mathbb{D}_1^3$. These characterizations are made by obtaining special solutions of differential equations which are related to dual spacelike curves of constant breadth in $\mathbb{D}_1^3$.

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