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Bright and dark soliton solution of the nonlinear partial differential equations system

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Abstract: In this paper, we obtained the 1-soliton solutions of the (3+1)-dimensional Zakharov–Kuznetsov (ZK) equation and the Drinfeld-Sokolov equation system. By using a solitary wave ansatz method, we obtain exact bright and dark soliton solutions for these equations. The parameters of the soliton envelope (amplitude, widths, velocity) are obtained as function of the dependent model coefficients. Note that, it is always useful and desirable to construct exact solutions especially soliton-type envelope for the understanding of most nonlinear physical phenomena.

Keywords: Exact solution, dark and bright soliton, the (3+1)dimensional Zakharov Kuznetsov (ZK) equation, Drinfeld-Sokolov equation system.

1 Introduction

In recent years, nonlinear waves is a very important topic of research in nonlinear dynamics, mathematical physics and various fields [1,2,3]. These wave equations are observed in fluid dynamics, aeronautics, optical fibers, solid state physics, plasma physics, chemical physics, geophysical sciences and geochemistry.

In the past decades, new exact solutions may help to find new phenomena. A variety of powerfull methods, such as the Jacobi elliptic function method [4,5], tanh-sech method [6,7], extended tanh method [8,9], $\left(\frac{G'}{G}\right)$-expansion method [10, 11], homogeneous balance method [12,13], sine-cosine method [14,15,16], first integral method [17,18] and F-expansion method [19,20] were used to obtain the solitary wave solutions.

The solitary wave ansatz method was first proposed by Biswas [21] and Triki et al. [22] is particularly notable in its power and applicability in solving nonlinear problems, and it has been successfully applied to many kinds of nonlinear partial differential equations [23,24]

2 The (3+1)-dimensional Zakharov–Kuznetsov (ZK) Equation

The Zakharov–Kuznetsov (ZK) equation governs the behavior of weakly nonlinear ion-acoustic waves in a plasma comprising cold ions and hot isothermal electrons in the presence of a uniform magnetic field. We considered the following ZK equation given by [25]

$$u_t + auu_x + (u_{xx} + u_{yy} + u_{zz})_x = 0, \tag{1}$$

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which is a typical nonlinear evolution equation, where a is real constants. Wazwaz constructed some new exact solutions by sine-cosine method in [26]. Wu and Fan obtained compactons, solitary patterns, plane periodic and solitary traveling wave solutions for Eq. (1), and their analytic expressions in [27].

Now, the bright soliton solution, dark soliton solution and conservation laws of this equation will be obtained.

2.1 The bright (non-topological) soliton solution:

To obtain the bright soliton solution of equation (1), the solitary wave ansatz admits the use of the assumption [28,29],

$$u(x,y,z,t) = \lambda sech^{p}\tau, \tag{2}$$

where $\tau = \eta(x+y+bz-vt\ )$ which λ, η and v are constant coefficients.Here λ is the amplitude, η and b are the inverse width and v is the velocity of the soliton. The exponents p is unknown at this point and will be determined later. From the ansatz (2), we obtain:

$$u_t = p\lambda\eta v sech^{p}\tau\tanh\tau, \tag{3}$$

$$uu_x = -\lambda^2 p\eta sech^{2p}\tau\tanh\tau, \tag{4}$$

$$u_{xxx} = -p^3\lambda\eta^3 sech^{p}\tau\tanh\tau + p(p+1)(p+2)\lambda\eta^3 sech^{p+2}\tau\tanh\tau, \tag{5}$$

$$u_{yyx} = -p^3\lambda\eta^3 sech^{p}\tau\tanh\tau + p(p+1)(p+2)\lambda\eta^3 sech^{p+2}\tau\tanh\tau, \tag{6}$$

$$u_{zzx} = -p^3\lambda\eta^3 b^2 sech^{p}\tau\tanh\tau + p(p+1)(p+2)\lambda\eta^3 b^2 sech^{p+2}\tau\tanh\tau. \tag{7}$$

Substituting Eqs. (2)-(7) into Eq. (1), we get

$$\begin{aligned} &p\lambda\eta v sech^{p}\tau\tanh\tau - a\lambda^2 p\eta sech^{2p}\tau\tanh\tau - p^3\lambda\eta^3 sech^{p}\tau\tanh\tau + p(p+1)(p+2)\lambda\eta^3 sech^{p+2}\tau\tanh\tau \\ &- p^3\lambda\eta^3 sech^{p}\tau\tanh\tau + p(p+1)(p+2)\lambda\eta^3 sech^{p+2}\tau\tanh\tau - p^3\lambda\eta^3 b^2 sech^{p}\tau\tanh\tau \\ &+ p(p+1)(p+2)\lambda\eta^3 b^2 sech^{p+2}\tau\tanh\tau = 0. \end{aligned} \tag{8}$$

Equating the exponents of $sech^{2p}\tau\tanh\tau$ and $sech^{p+2}\tau\tanh\tau$ term in equation (8), one yields

$$2p = p+2, \tag{9}$$

which implies

$$p = 2. \tag{10}$$

We next equate the exponents of $sech^{2p}\tau\tanh\tau$ and $sech^{p+2}\tau\tanh\tau$ terms to zero one gets

$$-a\lambda^2 p\eta + p(p+1)(p+2)\lambda\eta^3 + p(p+1)(p+2)\lambda\eta^3 + p(p+1)(p+2)\lambda\eta^3 b^2 = 0. \tag{11}$$

If we put $p = 2$ and solving the equation (11), the velocity of soliton is given by

$$\lambda = \frac{12\eta^2(2+b^2)}{a}, \quad a \neq 0. \tag{12}$$

By setting the coefficients of $\lim sech^p \tau$ terms in Eq. (8) to zero yields

$$p\lambda\eta v - 2p^3\lambda\eta^3 - p^3\lambda\eta^3 b^2 = 0. \tag{13}$$

If we put $p = 2$ and solving the equation (13)

$$v = 4\eta^2(2+b^2). \tag{14}$$

which gives the velocity of the soliton.

Finally, we get the bright (non topological) soliton solution for the constant coefficient the (3+1)-dimensional ZK equation, when the above expressions of p,v and λ given by Eqs. (10), (12) and (14) are substituted in (2) as :

$$u(x,y,z,t) = \lambda \lim sech^2\eta(x+y+bz-vt). \tag{15}$$

2.2 The dark (topological) soliton solution:

In this section, the ansatz method [30] will be used to carry out the integration of the (3+1)-dimensional ZK equation (1). The search is going to be for a dark soliton solution. We use an ansatz solution of the form [31,32,33]

$$u(x,y,z,t) = \lambda \tanh^p \tau, \tag{16}$$

and

$$\tau = \eta(x+y+bz-vt), \tag{2.17}$$

where λ, b and η are unknown free parameters and v is the velocity of the soliton, respectively, that will be determined.

The exponent p is also unknown.

From Eq. (16), we have:

$$u_t = -p\lambda v\eta \left\{\tanh^{p-1}\tau - \tanh^{p+1}\tau\right\}, \tag{17}$$

$$uu_x = p\lambda^2\eta \left\{\tanh^{2p-1}\tau - \tanh^{2p+1}\tau\right\}, \tag{18}$$

$$u_{xxx} = \lambda p\eta^3 \left\{ \begin{array}{c} (p-1)(p-2)\tanh^{p-3}\tau - \left[2p^2+(p-1)(p-2)\right]\tanh^{p-1}\tau \\ +\left[2p^2+(p+1)(p+2)\right]\tanh^{p+1}\tau - (p+1)(p+2)\tanh^{p+3}\tau \end{array} \right\}, \tag{19}$$

$$u_{yyx} = \lambda p\eta^3 \left\{ \begin{array}{c} (p-1)(p-2)\tanh^{p-3}\tau - \left[2p^2+(p-1)(p-2)\right]\tanh^{p-1}\tau \\ +\left[2p^2+(p+1)(p+2)\right]\tanh^{p+1}\tau - (p+1)(p+2)\tanh^{p+3}\tau \end{array} \right\}, \tag{20}$$

$$u_{zzx}=\lambda p\eta^{3}b^{2}\left\{\begin{array}{c}(p-1)(p-2)\tanh^{p-3}\tau-\left[2p^{2}+(p-1)(p-2)\right]\tanh^{p-1}\tau\\+\left[2p^{2}+(p+1)(p+2)\right]\tanh^{p+1}\tau-(p+1)(p+2)\tanh^{p+3}\tau\end{array}\right\}, \tag{21}$$

where $\tau=\eta(x+y+bz-vt)$. Substituting Eqs. (17)-(21) into Eq. (1), we obtain

$$\begin{aligned}&-p\lambda v\eta\left\{\tanh^{p-1}\tau-\tanh^{p+1}\tau\right\}+ap\lambda^{2}\eta\left\{\tanh^{2p-1}\tau-\tanh^{2p+1}\tau\right\}\\&+2\lambda p\eta^{3}\left\{\begin{array}{c}(p-1)(p-2)\tanh^{p-3}\tau-\left[2p^{2}+(p-1)(p-2)\right]\tanh^{p-1}\tau\\+\left[2p^{2}+(p+1)(p+2)\right]\tanh^{p+1}\tau-(p+1)(p+2)\tanh^{p+3}\tau\end{array}\right\}\\&+\lambda p\eta^{3}b^{2}\left\{\begin{array}{c}(p-1)(p-2)\tanh^{p-3}\tau-\left[2p^{2}+(p-1)(p-2)\right]\tanh^{p-1}\tau\\+\left[2p^{2}+(p+1)(p+2)\right]\tanh^{p+1}\tau-(p+1)(p+2)\tanh^{p+3}\tau\end{array}\right\}\\&=0.\end{aligned} \tag{22}$$

By equating the highest exponents of $\tanh^{2p+1}\tau$ and $\tanh^{p+3}\tau$ terms in Eq (22), one gets:

$$2p+1=p+3, \tag{23}$$

so that

$$p=2. \tag{24}$$

Again from (22), setting the coefficients of $\tanh^{p-1}\tau$ terms to zero yields

$$-p\lambda v\eta-2\lambda\eta^{3}p\left[2p^{2}+(p-1)(p-2)\right]-\lambda\eta^{3}b^{2}p\left[2p^{2}+(p-1)(p-2)\right]=0. \tag{25}$$

If we put $p=2$ and solving the equation in (25), we obtain

$$v=-8\eta^{2}(b^{2}+2). \tag{26}$$

Setting the coefficients of $\tanh^{p+1}\tau$ and $\tanh^{2p-1}\tau$ or $\tanh^{p+3}\tau$ and $\tanh^{2p+1}\tau$ terms in Eq. (22) to zero we get

$$ap\lambda^{2}\eta+p\lambda v\eta+2\lambda p\eta^{3}\left[2p^{2}+(p+1)(p+2)\right]+\lambda p\eta^{3}b^{2}\left[2p^{2}+(p+1)(p+2)\right]=0. \tag{27}$$

If we put $p=2$ and solving the equation in (27), we get

$$\lambda=-\frac{12\eta^{2}(b^{2}+2)}{a},\quad a\neq 0. \tag{28}$$

Lastly, we can determine the dark (topological) soliton solution for the Eq. (1) as

$$u(x,y,z,t)=\lambda\tanh^{2}(\eta(x+y+bz-vt)), \tag{29}$$

where the velocity of the solitons v is given in (26), free parameter λ is given by (28).

3 The Drinfeld-Sokolov (DS) system

Let us second consider the Drinfeld-Sokolov system [34]

$$u_t + (v^2)_x = 0, \tag{30}$$

$$v_t - av_{xxx} + 3bu_x v + 3huv_x = 0, \tag{31}$$

where a, b and h are real constants. This system was introduced by Drinfeld and Sokolov as an example of a system of nonlinear equations possessing Lax pairs of a special form [35]. Wazwaz [36] used the sine-cosine method and the tanh method to stress the power of these methods to nonlinear equations. Bekir [34] successfully used the extended tanh method and the auxiliary equation method [37] establish solitary wave solutions of the Drinfeld-Sokolov equations. Wazwaz [36] also investigated the traveling wave solutions with compact and noncompact structures for the Drinfeld-Sokolov equations and Qi et al. employ the complex method to obtain the general meromorphic solutions of the Drinfeld-Sokolov equations (DS system of equations) [38].

3.1 Bright (non-topological) soliton solution

In order to seek soliton solutions to (30) and (31), the starting assumptions are

$$u(x,t) = A_1 sech^p \{\eta (x - vt)\} \tag{32}$$

and

$$v(x,t) = A_2 sech^r \{\eta (x - vt)\} \tag{33}$$

which A_1, A_2, η and v are constant coefficients.Here A_1, A_2 are the amplitude, η is the inverse width and v is the velocity of the soliton. The exponents p is unknown at this point and will be determined later. From (32) and (33), it is possible to obtain:

$$pA_1 \eta v sech^p \tau \tanh\tau - 2rA_2^2 \eta sech^{2r} \tau \tanh\tau = 0, \tag{34}$$

and

$$\begin{aligned} & rA_2 \eta v sech^r \tau \tanh\tau + ar^3 A_2 \eta^3 sech^r \tau \tanh\tau \\ & - r(r+1)(r+2)aA_2 \eta^3 sech^{r+2} \tau \tanh\tau \\ & - 3bpA_1 A_2 \eta sech^{p+r} \tau \tanh\tau \\ & - 3hrA_1 A_2 \eta sech^{p+r} \tau \tanh\tau \\ & = 0. \end{aligned} \tag{35}$$

Equating the exponents of $sech^p \tau \tanh\tau$ and $sech^{2r} \tau \tanh\tau$ terms in Eq. (34), one obtains

$$p = 2r, \tag{36}$$

and theirs coefficients gives

$$pA_1\eta v - 2rA_2^2\eta = 0, \tag{37}$$

then we get,

$$v = \frac{A_2^2}{A_1}. \tag{38}$$

Similarly, equating the exponents of $sech^{r+2}\tau\tanh\tau$ and $sech^{p+r}\tau\tanh\tau$ terms in Eq. (35), it is possible to obtain

$$p = 2, \tag{39}$$

so that from (36)

$$r = 1. \tag{40}$$

Setting the coefficients of $sech^{r}\tau\tanh\tau$ terms to zero we get

$$rA_2\eta v + ar^3A_2\eta^3 = 0. \tag{41}$$

By using (38) and some calculating, we have

$$\eta = \pm\frac{A_2}{r\sqrt{-aA_1}}, \tag{42}$$

which gives the inverse width of the soliton. Additionally, it is necessary to have

$$aA_1 < 0. \tag{43}$$

By setting the coefficients of $sech^{p+r}\tau\tanh\tau$ and $sech^{r+2}\tau\tanh\tau$ terms to zero one gets

$$-r(r+1)(r+2)aA_2\eta^3 - 3bpA_1A_2\eta - 3hrA_1A_2\eta = 0, \tag{44}$$

and using Eqs. (39), (40) and (42) that gives

$$A_1 = \pm\frac{2A_2}{\sqrt{4b+2h}}, \tag{45}$$

which provided that $4b+2h>0$.

Therefore the 1-soliton solution of the Drinfeld-Sokolov system is given by

$$u(x,t) = A_1 sech^2\eta\,(x-vt)\,, \tag{46}$$

and

$$v(x,t) = A_2 sech\eta\,(x-vt)\,, \tag{47}$$

where the soliton velocity, inverse width and amplitude are given by (38), (42) and (45). respectively.

3.2 Dark (topological) soliton solution

For a dark (topological) 1-soliton solution, the starting hypothesis is given by

$$u(x,t) = A_1 \tanh^p \{\eta (x - vt)\}, \tag{48}$$

and

$$v(x,t) = A_2 \tanh^r \{\eta (x - vt)\}. \tag{49}$$

Here in (48) and (49), A_1, A_2 and η are the free parameters for topological solitons and v is the velocity of the soliton. Similar approach like section 2, by using (48) and (49) we get

$$pv\eta A_1\{\tanh^{p+1}\tau - \tanh^{p-1}\tau\} + 2r\eta A_2^2\{\tanh^{2r-1}\tau - \tanh^{2r+1}\tau\} = 0, \tag{50}$$

and

$$\begin{aligned}
&rv\eta A_2\{\tanh^{r+1}\tau - \tanh^{r-1}\tau\} - ar(r-1)(r-2)A_2\eta^3\left\{\tanh^{r-3}\tau - \tanh^{r-1}\tau\right\}\\
&-ar(r+1)(r+2)A_2\eta^3\left\{\tanh^{r+1}\tau - \tanh^{r+3}\tau\right\}\\
&-2a(\eta r)^3A_2\left\{\tanh^{r+1}\tau - \tanh^{r-1}\tau\right\} + 3bp\eta A_1A_2\{\tanh^{p+r-1}\tau - \tanh^{p+r+1}\tau\}\\
&+3hr\eta A_1A_2\left\{\tanh^{p+r-1}\tau - \tanh^{p+r+1}\tau\right\}\\
&=0.
\end{aligned} \tag{51}$$

Now from (50), equating the coefficients of $2r+1$ and $p+1$ gives

$$2r+1 = p+1, \tag{52}$$

so that

$$p = 2r. \tag{53}$$

It need to be noted that the same result is yielded when the exponent $2r-1$ and $p-1$ is equated the other. Again from (51) equating the coefficients of $p+r-1$ and $r+1$ gives

$$p+r-1 = r+1, \tag{54}$$

that leads to

$$p = 2. \tag{55}$$

So that from (3.24),

$$r = 1. \tag{56}$$

Setting the coefficients of $\tanh^{p+j}\tau$ and $\tanh^{2r+j}\tau$ in (50), to zero where $j = -1, 1$ gives

$$pvA_1 - 2r\eta A_2^2 = 0, \tag{57}$$

which gives after using (3.26) and (3.27):

$$v = \frac{A_2^2}{A_1}. \tag{58}$$

Again from (51), setting the coefficients of $\tanh^{r+3}\tau$ and $\tanh^{p+r+1}\tau$ terms to zero one obtains:

$$ar(r+1)(r+2)A_2\eta^3 - 3bp\eta A_1A_2 - 3hr\eta A_1A_2 = 0, \tag{59}$$

the latter gives after using (55) and (56):

$$\eta = \pm\sqrt{\frac{A_1(2b+h)}{2a}}. \tag{60}$$

From (60), it is possible to see that the solitons will exist provided

$$A_1(2b+h)a > 0. \tag{61}$$

Finally, setting the coefficients of $\tanh^{r+1}\tau$ and $\tanh^{p+r-1}\tau$ in (51) to zero gives

$$rvA_2 - ar(r+1)(r+2)A_2\eta^3 - 2a(\eta r)^3A_2 + 3bp\eta A_1A_2 + 3hr\eta A_1A_2 = 0, \tag{62}$$

this leads to after using (55), (56), (31) and (60):

$$A_2 = \pm A_1\sqrt{2b+h}, \tag{63}$$

which implies that it is necessary to have $2b+h > 0$. By inserting (63) in (58), it is possible to recover

$$v = A_1(2b+h). \tag{64}$$

Lastly, the dark (topological) soliton solutions for the Drinfeld-Sokolov system is given by

$$u(x,t) = A_1\tanh^2\{\eta(x-vt)\}, \tag{65}$$

and

$$v(x,t) = A_2\tanh\{\eta(x-vt)\}, \tag{66}$$

where the velocity of the solitons v is given in (58) or (64), free parameters η and A_2 are given by (60) and (63) respectively. We see from (60) that the free parameter η is dependent on the other free parameter A_1. Also in (58) the velocity of the soliton v is dependent on the free parameters A_1 and A_2.

4 Conclusions

We have derived the exact bright and dark soliton solutions of the two nonlinear evolution equations namely the Zakharov–Kuznetsov (ZK) equation and the Drinfeld-Sokolov system. This has been realized by using the solitary wave ansatz method. The solitary wave ansatz is used to carry out this derivation. In view of the analysis, we see that the used method is an efficient method of integrability for constructing exact soliton solutions. Conditions for the existence of soliton envelopes have also been reported. To our knowledge, these new solutions have not been reported in former literature, they may be of significant importance for the explanation of some special physical phenomena.

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