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Analytical investigation for fluid behavior over a flat plate with oscillating motion and wall transpiration

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Abstract: In this paper, fluid behavior over a flat plate with oscillating motion, starting from rest and wall transpiration is presented. The classical solution of this problem is given by Panton [22] and is found to be an especial case of the solution here presented. The analytical solution is obtained without the use of any special transformations, such as Laplace or Fourier transforms. Three highly accurate and simple semi analytical methods, Variational Iteration Method (VIM), Homotopy Perturbation Method (HPM) and Adomians Decomposition Method (ADM) are used to solve this problem. The results show the effects of suction and injection of the wall on fluid behavior and reveal that VIM, HPM and ADM are very effective and accurate in comparison with the exact solution. A non-dimensional number is used to take in to account the injection or suction of fluid at the wall. This parameter is shown to be of great influence on the proposed velocity solution.

Keywords: Semi Analytical Methods (ADM, HPM, VIM); Oscillating wall; Navier-Stokes equation .

1. Introduction

Nonlinear differential equations are very applicable in engineering sciences like mechanical and chemical engineering, aerospace sciences and so on. There are some methods for solving linear differential equations like Laplace transformation method, Fourier transformation method and variable separation method, but there is no analytical method for solving nonlinear differential equations. In recent years, scientists have presented some new semi-analytical methods for solving nonlinear differential equations which are simple, high accurate and even could solve linear differential equations, for instance Least square method [13-15], Lattice Boltzman method [31-36], Differential transform method [3,6], Homotopy asymptotic method [23,30], Adomian's decomposition method [1,2,9,10], He's HPM [8,9,12,17,18,19,20] and VIM [8,9,10,11,16,21]. One category of nonlinear differential equations which is very practical in mechanical engineering is the Navier-stokes equation that describes motion of fluids. In this paper, we aim to present the basic ideas of VIM and HPM and ADM and then their implementations to Navier-Stokes equations which describe motion of Newtonian viscose fluid flow over an infinite flat plane wall when the wall presents harmonic oscillation in the longitudinal direction is illustrated in Fig.1. This problem has significant application in boundary layer control with important examples in manufacturing techniques, aeronautical systems, mechanical and chemical engineering processes. Almeida cruiz and Ferreira lins [4] calculated the solution of this problem, using the method of separation of variables. Das, S., et.al [5] solved Stokes equation for rotating fluids over an oscillating plate using Laplace transform technic. Erdogan and Imrak [7] calculated the solution of Stokes equations over an oscillating infinite flat plate using the Fourier transform technique. For this problem, three semi-analytical methods which solve the pertinent Stokes problem in a situation that there are wall fluid injection or suction is presented. Then, a comparison between the obtained results and exact solution is offered. [4].

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2. Basic equation

The Navier-Stokes problem considered here, is stated as follows:

Consider a fluid with viscosity ν, initially at rest, occupying a half plane $y \ge 0$ and bounded on the x-axis by an infinite plane wall. At time $t > 0$ the wall moves in x-direction with velocity given by $u_w(t)$. The fluid velocity $u(y,t)$ is described by Navier-Stokes equation, which can be cast as:

$$\frac{\partial u}{\partial t} + V_w \frac{\partial u}{\partial y} - \nu \frac{\partial^2 u}{\partial y^2} = 0 \qquad \text{for } y > 0 \text{ and } t > 0 \tag{1}$$

where V_w is the transpiration velocity. The initial and boundary conditions are:

$$u = u_w(t) = u_0 \exp(i\omega t) \quad \text{at} \quad y = 0 \quad , \quad t > 0 \tag{2}$$

$$u = 0 \quad \text{at} \quad y \to \infty \quad , \quad t > 0 \tag{3}$$

$$u = 0 \quad \text{at} \quad y = 0 \quad , \quad t = 0 \tag{4}$$

where u_0 is the maximum amplitude of wall velocity oscillation, ω is the frequency of the wall velocity and $i = \sqrt{-1}$ is the imaginary constant. Using the wall velocity given in expression (2), the sin and cos oscillations can be treated by taking real and imaginary parts of the velocity field. Eq. (1) and its boundary and initial conditions can be rewritten in the dimensionless forms as:

$$\frac{\partial U}{\partial \tau} + 2\xi \frac{\partial U}{\partial \eta} - \frac{\partial^2 U}{\partial \eta^2} = 0 \qquad \text{for} \quad \eta > 0 \quad \text{and} \quad \tau > 0 \tag{5}$$

$$U = exp(i\tau) \quad \text{at} \quad \eta = 0 \quad , \quad \tau > 0 \tag{6}$$

$$U = 0 \quad \text{at} \quad \eta \to \infty \quad , \quad \tau > 0 \tag{7}$$

$$U = 0 \quad \text{at} \quad \eta = 0 \quad , \quad \tau = 0 \tag{8}$$

where the dimensionless parameters are defined as:

$$U = \frac{u}{u_0}, \qquad \tau = \omega t, \qquad \eta = y\left(\frac{\omega}{\nu}\right)^{1/2}, \qquad \xi = V_w / \sqrt{4\omega\nu} \tag{9}$$

3. Variational Iteration Method

3.1 Principle of method

To illustrate the basic concepts of the VIM [9], we consider the following differential equation:

$$Lu + Nu = g(\mathrm{x}) \tag{10}$$

where L is a linear operator, N a nonlinear operator, and $g(x)$ an inhomogeneous term. According to the VIM, we can construct a correction functional as follows:

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda\{Lu_n(\tau) + Nu_n(\tau) - g(\tau)\}d\tau \tag{11}$$

where λ is a general Lagrangian multiplier [9] which can be identified optimally via the variational theory. The subscript n indicates the n th approximation and u_n is considered as a restricted variation, i.e. $\delta u_n = 0$. The VIM is a powerful tool to search for semi-analytical solutions for various nonlinear problems.

3.2 Application

In order to solve Eq. (5) by using VIM, we construct a correction function, as follow:

$$U_{n+1}(\eta,\tau)=U_n(\eta,\tau)+\int_0^{\tau}\lambda\left[\left(\frac{\partial}{\partial s}U_n(\eta,s)\right)+2\xi\left(\frac{\partial}{\partial \eta}U_n(\eta,s)\right)-\left(\frac{\partial^2}{\partial \eta^2}U_n(\eta,s)\right)\right]ds \tag{12}$$

its stationary conditions can be obtained as follows:

$$\lambda'(\tau)=0 \tag{13}$$

$$1+\lambda(\tau)\big|_{\tau=t}=-1 \tag{14}$$

The lagrangian multiplier can be identified as:

$$\lambda=-1 \tag{15}$$

Substituting Eq. (15) into Eq. (12) results the following iteration formula:

$$U_{n+1}(\eta,\tau)=U_n(\eta,\tau)-\int_0^{\tau}\left[\left(\frac{\partial}{\partial s}U_n(\eta,s)\right)+2\xi\left(\frac{\partial}{\partial \eta}U_n(\eta,s)\right)-\left(\frac{\partial^2}{\partial \eta^2}U_n(\eta,s)\right)\right]ds \tag{16}$$

Now we start with an arbitrary initial approximation as follows:

$$U(\eta,0)=\exp(\eta\left(\xi-\sqrt{\xi^2+i}\right) \tag{17}$$

Higher orders of iterations lead to obtain quasi- exact solution. Using the iteration formula (16) and the initial guess U_0, we have:

$$U_1(\eta,\tau)=\left(1-2\xi\left(\xi-\sqrt{\xi^2+i}\right)\tau+\left(\xi-\sqrt{\xi^2+i}\right)^2\tau\right).\left(\xi-\sqrt{\xi^2+i}\right) \tag{18}$$

the second iteration:

$$U_2(\eta,\tau)=(1-2\xi(\xi-\sqrt{\xi^2+i})\tau+(\xi-\sqrt{\xi^2+i})^2\tau-\frac{1}{2}(2\xi(-2\xi(\xi-\sqrt{\xi^2+i})^2+(\xi-\sqrt{\xi^2+i}) \tag{19}$$
$$2\xi(\xi-\sqrt{\xi^2+i})^3-(\xi-\sqrt{\xi^2+i})^4)\tau^2)\,\mathrm{e}^{\eta(\xi-\sqrt{\xi^2+i})}$$

the third iteration:

$$U_3(\eta,\tau)=(1-2\xi(\xi-\sqrt{\xi^2+i})\tau+(\xi-\sqrt{\xi^2+i})^2\tau-\frac{1}{2}(2\xi(\xi-\sqrt{\xi^2+i})^2+(\xi-\sqrt{\xi^2+i})^3)+$$
$$2\xi(\xi-\sqrt{\xi^2+i})^3-(\xi-\sqrt{\xi^2+i})^4)\tau^2-\frac{1}{3}(2\xi(-\xi(-2\xi(\xi-\sqrt{\xi^2+i})^3+(\xi-\sqrt{\xi^2+i})^4)- \tag{20}$$
$$\xi(\xi-\sqrt{\xi^2+i})^4+\frac{1}{2}(\xi-\sqrt{\xi^2+i})^5)+\xi(-2\xi(\xi-\sqrt{\xi^2+i})^4+(\xi-\sqrt{\xi^2+i})^5)+\xi(\xi-\sqrt{\xi^2+i})^5-$$
$$\frac{1}{2}(\xi-\sqrt{\xi^2+i})^6)\tau^3))e^{\eta(\xi-\sqrt{\xi^2+i})}$$

In the same manner, the rest of the components of the iteration formula can be obtained which shows that $14th$ iterations converged to the exact solution. Therefore we have:

$$U(\eta,\tau)=U_{14}(\eta,\tau) \tag{21}$$

4. Homotopy Perturbation Method

4.1 Principle of method

To illustrate the basic concepts of this method [9, 24, 26, 27, 37], we consider the following nonlinear differential equation:

$$A(u)-f(r)=0, \qquad r\in\Omega \tag{22}$$

with the boundary condition of:

$$B\left(u,\frac{\partial u}{\partial n}\right)=0 \tag{23}$$

where $A(u)$ is defined as follows:

$$A(u)=L(u)+N(u) \tag{24}$$

Homotopy perturbation structure is shown as:

$$H(v,p)=L(v)-L(u_0)+pL(u_0)+p\left[N(v)-f(r)\right]=0 \tag{25}$$

or

$$H(v,p)=(1-p)\left[L(v)-L(u_0)\right]+p\left[A(v)-f(r)\right]=0 \tag{26}$$

where:

$$v(r,p):\ \Omega\times[0,1]\to R. \tag{27}$$

Obviously, considering Eqs. (25) and (26) we have:

$$H(v,0)=L(v)-L(u_0)=0 \qquad\qquad H(v,1)=A(v)-f(r)=0 \tag{28}$$

where $p\in[0,1]$ is an embedding parameter and u_0 is the first approximation that satisfies the boundary condition. According to HPM, the approximation solution of Eq. (28) can be expressed as a power series of p -terms:

$$v=v_0+pv_1+p^2v_2+\ldots, \tag{29}$$

and the best approximation is:

$$u=\lim_{p\to 1}v=v_0+v_1+v_2+\ldots, \tag{30}$$

4.2 Application

A Homotopy perturbation method can be constructed as follow:

$$H(U,p)=$$
$$(1-p)\left(\frac{\partial}{\partial\tau}U(\eta,\tau)-\frac{\partial}{\partial\tau}U_0(\eta,\tau)\right)+p\left(\frac{\partial}{\partial\tau}U(\eta,\tau)+2\xi\left(\frac{\partial}{\partial\eta}U(\eta,\tau)\right)-\left(\frac{\partial^2}{\partial\eta^2}U(\eta,\tau)\right)\right) \tag{31}$$

we consider $U(\eta,\tau)$ as:

$$U(\eta,\tau)=U_0(\eta,\tau)+pU_1(\eta,\tau)+p^2U_2(\eta,\tau)+p^3U_3(\eta,\tau)+\ldots, \tag{32}$$

Subject to the initial condition:

$$U(\eta,0)=\exp\left(\eta\left(\xi-\sqrt{\xi^2+i}\right)\right) \tag{33}$$

Substituting Eq. (32) into Eq. (31) and after some simplification and rearranging based on powers of p -terms we have:

$$p^0: \quad \frac{\partial}{\partial \tau} U_0(\eta,\tau) = 0 \tag{34}$$

the initial condition is defined as:

$$\tau = 0\ , \quad U_0(\eta,\tau) = \exp\left(\eta\left(\xi - \sqrt{\xi^2 + i}\right)\right) \tag{35}$$

in the same way we have:

$$p^1: \ 2\xi\left(\frac{\partial}{\partial \eta} U_0(\eta,\tau)\right) - \left(\frac{\partial^2}{\partial \eta^2} U_0(\eta,\tau)\right) + \left(\frac{\partial}{\partial \tau} U_1(\eta,\tau)\right) = 0 \tag{36}$$

$$\tau = 0\ , \quad U_1(\eta,\tau) = 0 \tag{37}$$

$$p^2: \ 2\xi\left(\frac{\partial}{\partial \eta} U_1(\eta,\tau)\right) - \left(\frac{\partial^2}{\partial \eta^2} U_1(\eta,\tau)\right) + \left(\frac{\partial}{\partial \tau} U_2(\eta,\tau)\right) = 0 \tag{38}$$

$$\tau = 0\ , \quad U_2(\eta,\tau) = 0 \tag{39}$$

Solving Eq. (34) to Eq. (39) results, U_0, U_1, U_2 as follows:

$$U_0(\eta,\tau) = \exp\left(\eta\left(\xi - \sqrt{\xi^2 + i}\right)\right) \tag{40}$$

$$U_1(\eta,\tau) = iexp\left(\eta\left(\xi - \sqrt{\xi^2 + i}\right)\right)\tau \tag{41}$$

$$U_2(\eta,\tau) = -\frac{1}{2} exp\left(\eta\left(\xi - \sqrt{\xi^2 + i}\right)\right)\tau^2 \tag{42}$$

In the same manner we obtained $U_3, U_4, \dots$, then the solution, when $p \to 1$ will be as follows:

$$U(\eta,\tau) = U_1(\eta,\tau) + U_2(\eta,\tau) + U_3(\eta,\tau) + \dots, \tag{43}$$

We ultimately obtain the solution as follows:

$$U(\eta,\tau) = \exp\left(\eta\left(\xi - \sqrt{\xi^2 + i}\right)\right)\left[1 + i\,\tau - \frac{\tau^2}{2} + \dots\right] = \exp\left(i\,\tau + \eta\left(\xi - \sqrt{\xi^2 + i}\right)\right) \tag{44}$$

as you can see, the obtained solution is very close to the exact solution [9].

5. Adomian's Decomposition Method

5.1 Principle of method

Let us discuss a brief outline of the Adomian Decomposition Method (ADM) [9, 25, 28].So, we consider a general nonlinear equation in the form of:

$$Lu + Ru + Nu = g \tag{45}$$

where L is the highest order derivative which is assumed to be easily invertible, R the linear differential operator of less order than L, Nu presents the nonlinear terms and g is the source term. Applying the inverse operator L^{-1} to the both sides of Eq. (45), and using the given conditions we obtain:

$$u = f(x) - L^{-1}(Ru) - L^{-1}(Nu) \tag{46}$$

where the function $f(x)$ represents the terms arising from integration the source term $g(x)$, by using the given conditions for nonlinear differential equations, the nonlinear operator $\mathrm{Nu}=\mathrm{F}(\mathrm{u})$ is represented by an infinite of the so-called Adomian polynomials:

$$F(u)=\sum_{m=0}^{\infty}A_m \tag{47}$$

The polynomials $\mathrm{A_m}$ are generated for all kinds of nonlinearity so A_0 depends only on u_0, A_1 depends on u_0 and u_1, and so on. The Adomian polynomials introduced above show that the sum of subscripts of the components of u for each term of A_m is equal to n. The Adomian method defines the solution $u(x)$ by the series:

$$u=\sum_{m=0}^{\infty}u_m \tag{48}$$

In the case of $F(u)$, the infinite series is a Taylor expansion about u_0, as follows:

$$F(u)=F(u_0)+F'(u_0)(u-u_0)+F''(u_0)\frac{(u-u_0)^2}{2!}+F'''(u_0)\frac{(u-u_0)^3}{3!}+\ldots \tag{49}$$

By rewriting Eq. (48) as $\mathrm{u}-\mathrm{u}_0=\mathrm{u}_1+\mathrm{u}_2+\mathrm{u}_3+\ldots$, substituting it into Eq. (49) and then equating the two expressions for $F(u)$ which are found in Eq. (49) and Eq. (47), defines formulas for the Adomian polynomials in the form of:

$$F(u)=A_1+A_2+\ldots=F(u_0)+F'(u_0)(u_1+u_2+\ldots)+F''(u_0)\frac{(u_1+u_2+\ldots)^2}{2!}+\ldots \tag{50}$$

By equating terms in Eq. (50), the first few Adomians polynomials $\mathrm{A}_0,\mathrm{A}_1,\mathrm{A}_2,\mathrm{A}_3$ and A_4 are given:

$$A_0=F(u_0) \tag{51}$$

$$A_1=u_1F'(u_0) \tag{52}$$

$$A_2=u_2F'(u_0)+\frac{1}{2!}u_1^2F''(u_0) \tag{53}$$

$$A_3=u_3F'(u_0)+u_1u_2F''(u_0)+\frac{1}{3!}u_1^3F'''(u_0) \tag{54}$$

$$A_4=u_4F'(u_0)+\left(\frac{1}{2!}u_2^2+u_1u_3\right)F''(u_0)+\frac{1}{2!}u_1^2u_2F'''(u_0)+\frac{1}{4!}u_1^4F^{(iv)}(u_0) \tag{55}$$

Now that the A_m are known, Eq. (47) can be substituted in Eq. (46) to specify the terms in the expansion for the solution of Eq. (48).

5.2 Application

To illustrate the basic concepts of the Adomian's decomposition method for solving the Eq. (5), first we rewrite it in the following operator form:

$$L_\tau U(\eta,\tau)=L_{\eta\eta}U(\eta,\tau)-2\xi L_\eta U(\eta,\tau) \tag{56}$$

where the notations:

$$L_\tau = \frac{\partial}{\partial \tau} \quad , \quad L_\eta = \frac{\partial}{\partial \eta} \quad , \quad L_{\eta\eta} = \frac{\partial^2}{\partial \eta^2} \tag{57}$$

Assuming L_τ is invertible; hence the inverse operator L_τ^{-1} is given by:

$$L_\tau^{-1} = \int_0^\tau (.) d\tau \tag{58}$$

Operating with the inverse operator on both sides of Eq. (56), we obtain:

$$U(\eta,\tau) = U(\eta,0) + L_\tau^{-1}\left(L_{\eta\eta} U(\eta,\tau) - 2\xi L_\eta U(\eta,\tau)\right) \tag{59}$$

Adomian method defines the solution $U(\eta,\tau)$ by the decomposition series:

$$U(\eta,\tau) = \sum_{n=0}^{\infty} U_n(\eta,\tau) \tag{60}$$

Substituting Eq. (60) into Eq. (59) yields:

$$\sum_{n=0}^{\infty} U_n(\eta,\tau) = U(\eta,0) + L_\tau^{-1}\left(L_{\eta\eta}\sum_{n=0}^{\infty} U_n(\eta,\tau) - 2\xi L_\eta \sum_{n=0}^{\infty} U_n(\eta,\tau)\right) \tag{61}$$

To determine the components of $U_n(\eta,\tau)$, Adomian decomposition method uses the recursive relation:

$$U_0(\eta,\tau) = U(\eta,0) \tag{62}$$

$$U_{n+1}(\eta,\tau) = L_\tau^{-1}\left(L_{\eta\eta} U_n(\eta,\tau) - 2\xi L_\eta U_n(\eta,\tau)\right) \tag{63}$$

Initial approximate solution is:

$$U(\eta,0) = \exp\left(\eta\left(\xi - \sqrt{\xi^2 + i}\right)\right) \tag{64}$$

Substituting Eq. (64) into Eq. (63), results U_1, U_2, U_3 as follows:

$$U_1(\eta,\tau) = (1 - 2\xi(\xi - \sqrt{\xi^2+i})\tau + (\xi - \sqrt{\xi^2+i})^2\tau).(\xi - \sqrt{\xi^2+i}) \tag{65}$$

$$U_2(\eta,\tau) = \frac{1}{2}((\xi - \sqrt{\xi^2+i})^4 - 2\xi(\xi - \sqrt{\xi^2+i})^3 - 2\xi((\xi - \sqrt{\xi^2+i})^3 - \\ 2\xi(\xi - \sqrt{\xi^2+i})^2))\tau^2 e^{\eta(\xi - \sqrt{\xi^2+i})} \tag{66}$$

$$U_3(\eta,\tau) = \frac{1}{3}(\frac{1}{2}(\xi - \sqrt{\xi^2+i})^6 - \xi(\xi - \sqrt{\xi^2+i})^5 - \xi((\xi - \sqrt{\xi^2+i})^5 - 2\xi(\xi - \sqrt{\xi^2+i})^4) - \\ \xi(((\xi - \sqrt{\xi^2+i})^5 - 2\xi(\xi - \sqrt{\xi^2+i})^4 - 2\xi((\xi - \sqrt{\xi^2+i})^4 - 2\xi(\xi - \sqrt{\xi^2+i})^3)))\tau^3 e^{\eta(\xi - \sqrt{\xi^2+i})} \tag{67}$$

As the same manner, we determined $U_4(\eta,\tau)$,$U_5(\eta,\tau)$,... which leads to the solution as follows:

$$U(\eta,\tau) = U_1(\eta,\tau) + U_2(\eta,\tau) + U_3(\eta,\tau) + \ldots \tag{68}$$

6. Conclusion

In this paper, we consider navier-stokes equations for fluid behavior over an oscillating plate with wall transpiration for finding analytical solutions via VIM, HPM and ADM. Fig. 2 shows comparison between VIM and the exact solution in horizontal velocity profile for the sine oscillation (in this case, we take the imaginary part of the expression) when the transpiration parameter is $\xi = 0.5$. Fig. 3 shows a comparison between VIM and the exact solution in horizontal velocity profile for different values of the transpiration parameter, for the sine excitation. A very good agreement between the result of VIM and the exact solution was observed. Fig. 4 and 5 indicate that the difference among ADM and the exact solution are negligible and ADM was converged to the exact solution by increasing iterations.

Fig. 6 confirms the validity of the HPM. As can be seen, HPM completely is similar to exact solution. It may be concluded that HPM methodology is very efficient technique in finding analytical solutions for a wide variety of linear and nonlinear problems. In this article we use the maple package to calculate the functions obtained from VIM, HPM and ADM.

Fig. 2 and 4 illustrate the positive transpiration parameter (injection) which increases the horizontal velocity and negative transpiration parameter (suction), then Fig. 6 has reverse effect since the momentum transmitted to the fluid by the wall which is sucked away.

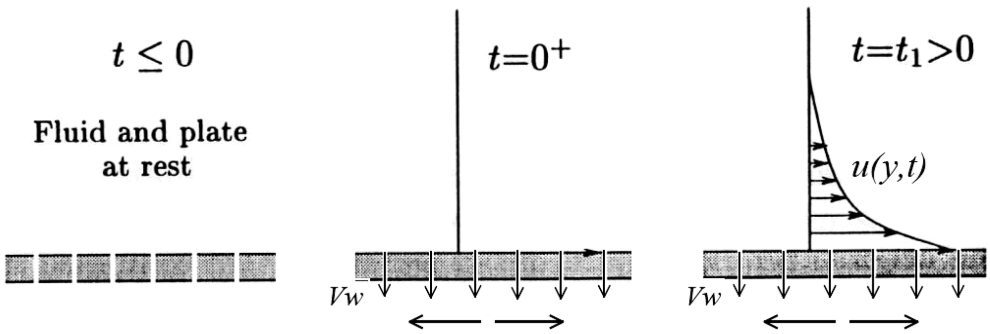

Fig. 1. A flat plate with oscillating motion, starting from rest, and wall transpiration.

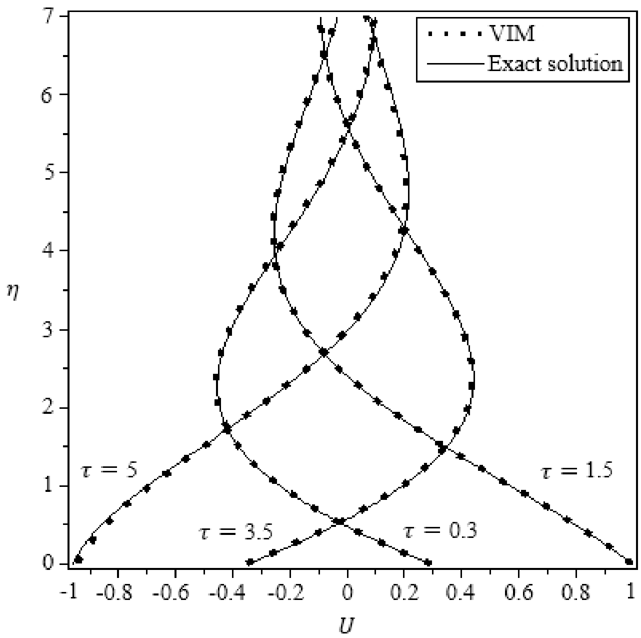

Fig. 2. Comparison of VIM and exact solution in horizontal velocity profile for various values of non-dimensional time τ , using $\xi = 0.5$ (injection) and a sine excitation of wall.

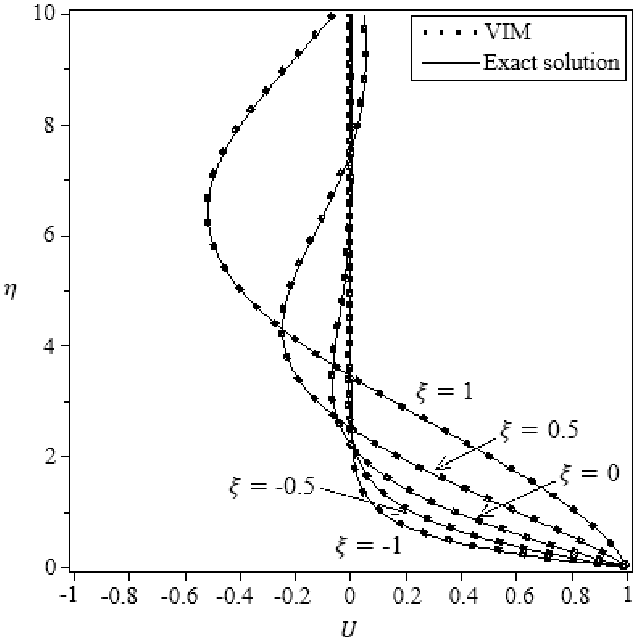

Fig. 3. Comparison of VIM and the exact solution in horizontal velocity profile for non-dimensional time $\tau = \pi / 2$, using various transpiration parameter ξ with a sine excitation of the wall.

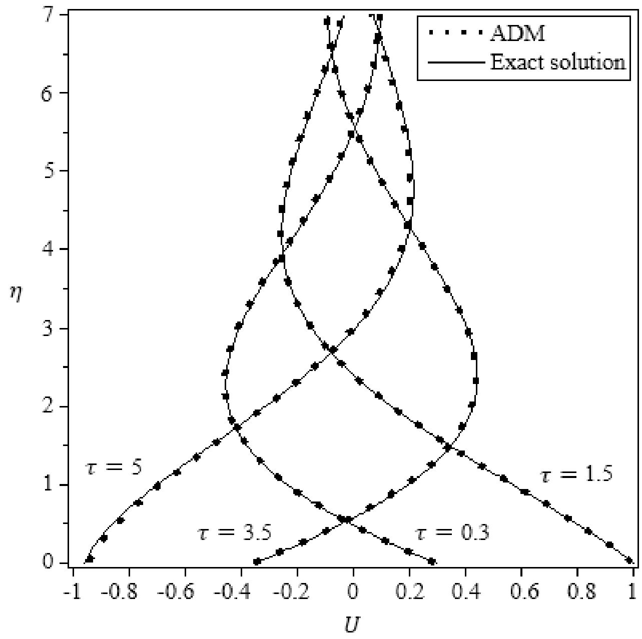

Fig. 4. Comparison of ADM and exact solution in horizontal velocity profile for various values of non-dimensional time τ, using $\xi = 0.5$ (injection) and a sine excitation of wall.

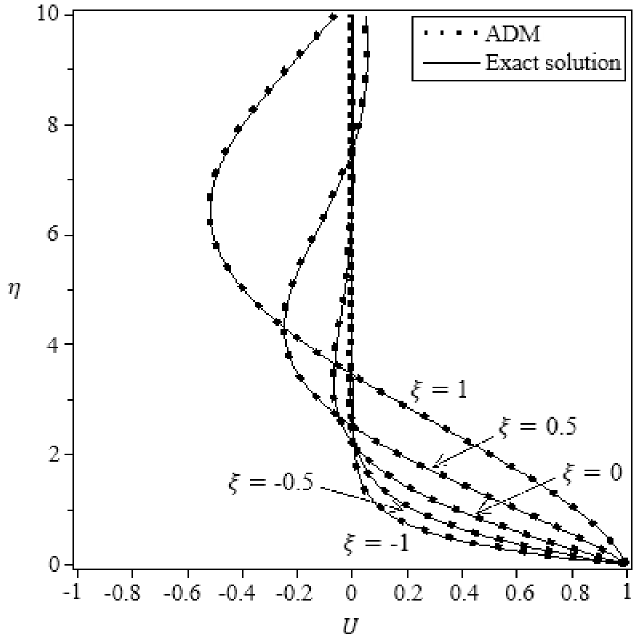

Fig. 5. Comparison of ADM and exact solution in horizontal velocity profile for non-dimensional time $\tau = \pi / 2$, using various transpiration parameter ξ with a sine excitation of the wall.

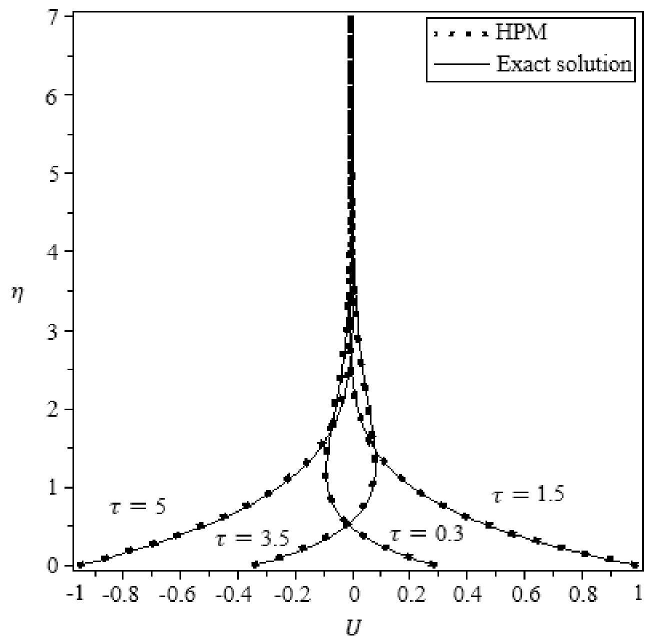

Fig. 6. Comparison of HPM and the exact solution in horizontal velocity profile for various values of non-dimensional time τ, using $\xi = -0.5$ (suction) and a sine excitation of wall.

Nomenclature			
u	fluid velocity	ξ	transpiration parameter
t	Time	η	dimensionless axial parameter
V_w	transpiration velocity	VIM	variational iteration method
y	axial coordinate	λ	general lagrangian multiplier
ν	kinematic viscosity	HPM	homotopy perturbation method
u_0	maximum amplitude of wall velocity	p	embedding parameter
ω	frequency of wall the wall velocity	ADM	adomian's decomposition method
U	dimensionless velocity	Ω	domain
τ	dimensionless time	n	number of iteration

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