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Application of DTM for 2D viscous flow through expanding or contracting gaps with permeable walls

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Abstract: In this study, Differential Transformation Method is used to solve the problem of laminar, isothermal, incompressible and viscous flow in a rectangular domain bounded by two moving porous walls, which enable the fluid to enter or exit during successive expansions or contractions. The concept of this method is briefly introduced, and it's application for this problem is studied. Then, the results are compared with numerical results and the validity of these methods is shown. After this verification, we analyze the effects of some physical applicable parameters to show the efficiency of DTM for this type of problems. Graphical results are presented to investigate the influence of the non-dimensional wall dilation rate (α) and permeation Reynolds number (Re) on the velocity, normal pressure distribution and wall shear stress. The present problem for slowly expanding or contracting walls with weak permeability is a simple model for the transport of biological fluids through contracting or expanding vessels.

Keywords: Permeation Reynolds number; Non-dimensional wall dilation rate; Differential Transformation Method (DTM).

1. Introduction

Studies of fluid transport in biological organisms often concern the flow of a particular fluid inside an expanding or contracting vessel with permeable walls. For a valve vessel exhibiting deformable boundaries, alternating wall contractions produce the effect of a physiological pump. The flow behavior inside the lymphatic exhibits a similar character. In such models, circulation is induced by successive contractions of two thin sheets that cause the downstream convection of the sandwiched fluid. Seepage across permeable walls is clearly important to the mass transfer between blood, air and tissue [1]. Therefore, a substantial amount of research work has been invested in the study of the flow in a rectangular domain bounded by two moving porous walls, which enable the fluid to enter or exit during successive expansions or contractions. Dauenhauer and Majdalani [2] studied the unsteady flow in semi-infinite expanding channels with wall injection. They are characterized by two non-dimensional parameters, the expansion ratio of the wall α and the cross-flow Reynolds number. Majdalani and Zhou [3] studied moderate to large injection and suction driven channel flows with expanding or contracting walls. Using perturbations in cross-flow Reynolds number Re, the resulting equation is solved both numerically and analytically.

One of the semi-exact methods which does not need small parameters is the Differential Transformation Method. Therefore, same as the HAM and the HPM, the DTM can overcome the foregoing restrictions and limitations of perturbation methods. This method constructs an analytical solution in the form of a polynomial. It is different from the traditional higher-order Taylor series method. The Taylor series method is computationally expensive for large orders. The differential transform method is an alternative procedure for obtaining an analytic Taylor series solution of differential equations. The main advantage of this method is that it can be applied directly to nonlinear differential equations without requiring linearization, discretization and therefore, it is not affected by errors associated to discretization. The concept of DTM was first introduced by Zhou [4], who solved linear and nonlinear problems in electrical circuits. Chen and Ho [5] developed this method for partial differential equations and Ayaz [6] applied it to the system of differential equations; this method is very powerful [7]. Recently, several papers have been published about numerical [8-32] and analytical methods [33-56].

In this study, differential transformation method is applied to find the approximate solutions of nonlinear differential equations governing Two-dimensional viscous flow through expanding or contracting gaps with permeable walls and have made a comparison with the Numerical Solution. The forth order Runge-Kutta method has been used and considered as the numerical solution for validity of this method.

2. Flow analysis and mathematical formulation

Consider the laminar, isothermal and incompressible flow in a rectangular domain bounded by two permeable surfaces that enable the fluid to enter or exit during successive expansions or contractions. A schematic diagram of the problem is shown in Fig. 1.

The walls expand or contract uniformly at a time-dependent rate $a^{\bullet}$. At the wall, it is assumed that the fluid inflow velocity V_w is independent of position. The equations of continuity and motion for the unsteady flow are given as follows:

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0, \tag{1}$$

$$\frac{\partial u^*}{\partial t}+u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*}+\upsilon[\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}], \tag{2}$$

$$\frac{\partial v^*}{\partial t}+u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial y^*}+\upsilon[\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}]. \tag{3}$$

In the above equations u^* and v^* indicate the velocity components in the *x* and *y* directions, p^* denotes the dimensional pressure, ρ,υ and *t* are the density, kinematic viscosity and time, respectively. The boundary conditions will be:

$$
\begin{aligned}
&y^* = a(t): \quad && u^* = 0,\ v^* = -V_w = -\frac{\overset{\bullet}{a}}{c}, \\
&y^* = 0: && \frac{\partial u^*}{\partial y^*} = 0, v^* = 0, \\
&x^* = 0: && u^* = 0.
\end{aligned} \tag{4}
$$

where $c = \frac{\overset{\bullet}{a}}{V_w}$ is the wall presence or injection/suction coefficient, that is a measure of wall permeability. The stream function and mean flow vorticity can be introduced by putting:

$$
\begin{aligned}
&u^* = \frac{\partial \psi^*}{\partial y^*}, \quad v^* = \frac{\partial \psi^*}{\partial x^*}, \quad \xi^* = \frac{\partial v^*}{\partial x^*} - \frac{\partial u^*}{\partial y^*} \\
&\frac{\partial \xi^*}{\partial t} + u^* \frac{\partial \xi^*}{\partial x^*} + v^* \frac{\partial \xi^*}{\partial y^*} = \upsilon[\frac{\partial^2 \xi^*}{\partial x^{*2}} + \frac{\partial^2 \xi^*}{\partial y^{*2}}].
\end{aligned} \tag{5}
$$

Due to mass conservation, a similar solution can be developed with respect to x^* . Starting with:

$$
\begin{aligned}
&\psi^* = \frac{\nu x^* f^*(y,t)}{a}, \qquad u^* = \frac{\nu x^* f^*{}_y}{a^2}, \quad v^* = \frac{-\nu f^*(y,t)}{a}, \\
&y = \frac{y^*}{a}, \quad f_y^* \equiv \frac{\partial f^*}{\partial y}.
\end{aligned} \tag{6}
$$

Substitution Eq.(6) into Eq. (5) yields:

$$
u^*_{y^*t} + u^* u^*_{y^*x^*} + v^* u^*_{y^*y^*} = \nu u^*_{y^*y^*y^*} \tag{7}
$$

In order to solve Eq. (7), one uses the chain rule to obtain:

$$
f^*_{yyyy} + \alpha(y f^*_{yyy} + 3 f^*_{yy}) + f^* f^*_{yyy} - f^*_y f^*_{yy} - a^2 \upsilon^{-1} f^*_{yyt} = 0, \tag{8}
$$

with the following boundary conditions:

$$
\begin{aligned}
&at\ y = 0: \quad f^* = 0, f^*_{yy} = 0, \\
&at\ y = 1: \quad f^* = \mathrm{Re}, f^*_y = 0,
\end{aligned} \tag{9}
$$

where $\alpha(t) \equiv \frac{a\overset{\bullet}{a}}{\upsilon}$ is the non-dimensional wall dilation rate defined positive for expansion and negative for contraction.

Furthermore, $\mathrm{Re} = \frac{aV_w}{\upsilon}$ is the permeation Reynolds number defined positive for injection and negative for suction through the walls. Eqs. (2.6), (2.8) and (2.9) can be normalized by putting:

$$\psi = \frac{\psi^*}{aa^{\bullet}}, \quad u = \frac{u^*}{a^{\bullet}}, \quad v = \frac{v^*}{a}, \quad f = \frac{f^*}{\mathrm{Re}}, \tag{10}$$

and so:

$$\psi = \frac{xf}{c}, u = \frac{xf'}{c}, v = \frac{-f}{c}, c = \frac{\alpha}{\mathrm{Re}}, \tag{11}$$

$$f^{IV} + \alpha(yf''' + 3f'') + \mathrm{Re}\, f\, f''' - \mathrm{Re}\, f'\, f'' = 0 \tag{12}$$

The boundary conditions (9) will be:

$$\begin{aligned} y = 0: &\qquad f = 0, f'' = 0 \\ y = 1: &\qquad f = 1, f' = 0 \end{aligned} \tag{13}$$

The resulting Eq. (12) is the classic Berman's formula [13], with $\alpha = 0$ (channel with stationary walls).
After the flow field is found, the normal pressure gradient can be obtained by substituting the velocity components into Eqs. (1)_(3). Hence it is:

$$\begin{aligned} p_y &= -[\mathrm{Re}^{-1} f'' + ff' + \alpha\,\mathrm{Re}^{-1}(f + yf')], \\ p &= \frac{p^*}{\rho V_w^2}. \end{aligned} \tag{14}$$

We can determine the normal pressure distribution, if we integrate Eq. (14). Let p_c be the centreline pressure, hence:

$$\int_{p_c}^{p(y)} dp = \int_0^y -[\mathrm{Re}^{-1} f'' + ff' + \alpha\,\mathrm{Re}^{-1}(f + yf')], \tag{15}$$

Then using $ff' = (f^2)'/2$ and $(f + yf') = (yf)'$, the resulting normal pressure drop will be:

$$\Delta p_n = \mathrm{Re}^{-1} f'(0) - [\mathrm{Re}^{-1} f' + \frac{f^2}{2} + \alpha\,\mathrm{Re}^{-1} yf]. \tag{16}$$

Another important quantity is the shear stress. The shear stress can be determined from Newton's law for viscosity:

$$\tau^* = \mu(v^*_{x^*} + u^*_{y^*}) = \frac{\rho v^2 x^* f^{*''}}{a^3}. \tag{17}$$

Introducing the non-dimensional shear stress $\tau = \dfrac{\tau^*}{\rho V_w^2}$ w, we have:

$$\tau = \frac{xf''}{\mathrm{Re}}. \tag{18}$$

3. Differential Transform Method

Basic definitions and operations of differential transformation are introduced as follows.

Differential transformation of the function $f(\eta)$ is defined as follows:

$$F(k) = \frac{1}{k!}\left[\frac{d^k f(\eta)}{d\eta^k}\right]_{\eta=\eta_0} \tag{19}$$

In (5.1), $f(\eta)$ is the original function and $F(k)$ is the transformed function which is called the T-function (it is also called the spectrum of the $f(\eta)$ at $\eta = \eta_0$, in the k domain). The differential inverse transformation of $F(k)$ is defined as:

$$f(\eta) = \sum_{k=0}^{\infty} F(k)(\eta - \eta_0)^k \tag{20}$$

By Combining (19) and (20) $f(\eta)$ can be obtained:

$$f(\eta) = \sum_{k=0}^{\infty}\left[\frac{d^k f(\eta)}{d\eta^k}\right]_{\eta=\eta_0} \frac{(\eta - \eta_0)^k}{k!} \tag{21}$$

Equation (21) implies that the concept of the differential transformation is derived from Taylor's series expansion, but the method does not evaluate the derivatives symbolically. However, relative derivatives are calculated by an iterative procedure that is described by the transformed equations of the original functions. From the definitions of (19) and (20), it is easily proven that the transformed functions comply with the basic mathematical operations shown in below. In real applications, the function $f(\eta)$ in (21) is expressed by a finite series and can be written as:

$$f(\eta) = \sum_{k=0}^{N} F(k)(\eta - \eta_0)^k \tag{22}$$

Equation (22) implies that $f(\eta) = \sum_{k=N+1}^{\infty}\left(F(k)(\eta - \eta_0)^k\right)$ is negligibly small, where N is series size.

Theorems to be used in the transformation procedure, which can be evaluated from (19) and (20), are given below (Table 1).

4. Solution with Differential Transformation Method

Now Differential Transformation Method into governing equations has been applied. Taking the differential transform of Eqs. (12) and (13) with respect to χ and considering H = 1 gives:

$$
\begin{aligned}
&(k+1)(k+2)(k+3)(k+4)F[k+4]+\alpha\sum_{m=0}^{k}\big(\delta[m](k-m+1)(k-m+2)(k-m+3)F[k-m+3]\big)\\
&+3\alpha\,(k+1)(k+2)F[k+2]+\mathrm{Re}\sum_{m=0}^{k}\big(F[k-m](m+1)(m+2)(m+3)F[m+3]\big)\\
&-\mathrm{Re}\sum_{m=0}^{k}\big((k-m+1)F[k-m+1](m+1)(m+2)F[m+2]\big)=0,
\end{aligned}
\tag{23}
$$

where $\delta[m]=\begin{cases}1 & m=1\\ 0 & m\neq 1\end{cases}.$

$$F[0]=0,\ F[1]=a_0,\ F[2]=0,\ F[3]=a_1 \tag{24}$$

where $F(k)$ are the differential transforms of $f(\eta)$ and a_0, a_1 are constants which can be obtained through boundary condition, Eq.(13). This problem can be solved as followed:

$$
\begin{aligned}
&F[0]=0,\ F[1]=a_0,\ F[2]=0,\ F[3]=a_1,F[4]=F[6]=F[8]=0\\
&F[5]=-\frac{3}{20}\alpha\,a_0\\
&F[7]=\frac{3}{280}\alpha^2a_1+\frac{1}{70}\mathrm{Re}\,a_1^2+\frac{1}{140}\mathrm{Re}\,a_0a_1\,\alpha\\
&F[9]=-\frac{1}{2240}\alpha^3a_1-\frac{1}{560}\alpha\,\mathrm{Re}\,a_1^2-\frac{1}{1120}\mathrm{Re}\,a_0\,a_1\,\alpha^2-\frac{1}{1260}a_0\,a_1\,\alpha^2-\frac{1}{1260}a_0\,\mathrm{Re}\,a_1^2-\frac{1}{2520}\mathrm{Re}\,a_0^2\,\alpha\,a_1
\end{aligned}
\tag{25}
$$

.
.
.

The above process is continuous. By substituting equations (25) into the main equation based on DTM, it can be obtained that the closed form of the solutions is:

$$
\begin{aligned}
F(\eta)=a_0\eta+a_1\eta^3+\left(-\frac{3}{20}\right)\eta^5+\left(\frac{3}{280}\alpha^2a_1+\frac{1}{70}Re\,a_1^2+\frac{1}{140}Re\,a_0a_1\,\alpha\right)\eta^7+\left(-\frac{1}{2240}\alpha^3a_1\left(-\frac{1}{560}\alpha\,Re\,a_1^2\right.\right.\\
\left.-\frac{1}{1120}Re\,a_0\,a_1\,\alpha^2-\frac{1}{1260}a_0\,a_1\,\alpha^2-\frac{1}{1260}a_0\,Re\,a_1^2-\frac{1}{2520}Re\,a_0^2\,\alpha\,a_1\right)\eta^9+\ldots
\end{aligned}
\tag{26}
$$

By substituting the boundary condition from Eq.(13) into Eq.(26) in point $\eta=1$ the values of a_0, a_1 can be obtained.

$$F(1)=a_0+a_1+\left(-\frac{3}{20}\right)+\left(\frac{3}{280}\alpha^2 a_1+\frac{1}{70}Re\,a_1^2+\frac{1}{140}Re\,a_0 a_1\,\alpha\right)+\left(-\frac{1}{2240}\alpha^3 a_1-\frac{1}{560}\alpha\,Re\,a_1^2\right.$$
$$\left.-\frac{1}{1120}Re\,a_0\,a_1\,\alpha^2-\frac{1}{1260}a_0\,a_1\,\alpha^2-\frac{1}{1260}a_0\,Re\,a_1^2-\frac{1}{2520}Re\,a_0^2\,\alpha\,a_1\right)+\ldots=1 \quad (27)$$

$$F'(1)=a_0\eta+3a_1+5\left(-\frac{3}{20}\right)+7\left(\frac{3}{280}\alpha^2 a_1+\frac{1}{70}Re\,a_1^2+\frac{1}{140}Re\,a_0 a_1\,\alpha\right)+9\left(-\frac{1}{2240}\alpha^3 a_1-\frac{1}{560}\alpha\,Re\,a_1^2\right.$$
$$\left.-\frac{1}{1120}Re\,a_0\,a_1\,\alpha^2-\frac{1}{1260}a_0\,a_1\,\alpha^2-\frac{1}{1260}a_0\,Re\,a_1^2-\frac{1}{2520}Re\,a_0^2\,\alpha\,a_1\right)+\ldots=0 \quad (28)$$

By solving Equations (31), (32) gives the values of a_0, a_1. By substituting obtained a_0, a_1 into Eq. (26), it can be obtained the expression of $F(\eta)$.

5. Results and discussion

The objective of the present study was to apply DTM to obtain an explicit analytic solution of laminar, isothermal, incompressible viscous flow in a rectangular domain bounded by two moving porous walls, which enable the fluid to enter or exit during successive expansions or contractions (Fig. 1). Fig. 2 shows the comparison between numerical method and DTM. It verifies that, there is acceptable agreement between the numerical solution obtained by four-order Rung-kutte method and these methods. After this validity, results are given for the velocity profile and normal pressure distribution for various values of permeation Reynolds number and non-dimensional wall dilation rate.

Fig. 3 illustrate the behavior of $f'(y)$ (or uc/x) for different permeation Reynolds number, over a range of non-dimensional wall dilation rate. For every level of injection or suction, in the case of expanding wall, increasing α leads to higher axial velocity near the center and the lower axial velocity near the wall. The reason is that the flow toward the center becomes greater to make up for the space caused by the expansion of the wall and as a result, the axial velocity also becomes greater near the center.

The pressure distribution in the normal direction for various permeation Reynolds numbers over a range of non-dimensional wall dilation rates, are plotted in Fig. 3. Fig. 4 shows that for every level of injection or suction, the absolute pressure change in the normal direction is lowest near the central portion. Furthermore, by increasing non-dimensional wall dilation rates the absolute value of pressure distribution in the normal direction increases.

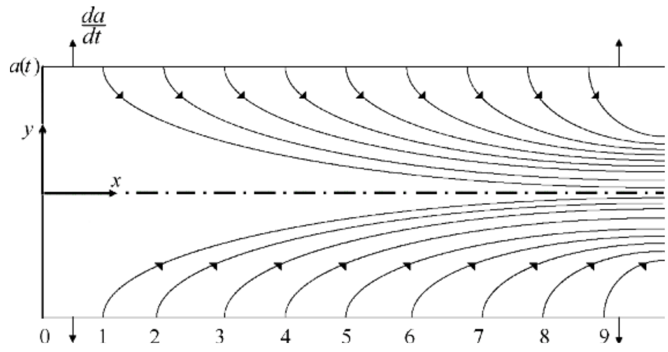


Fig. 1. Two-dimensional domain with expanding or contracting porous walls.

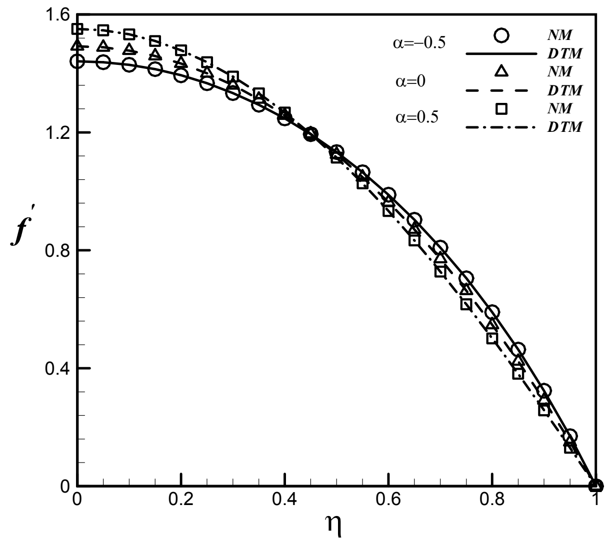
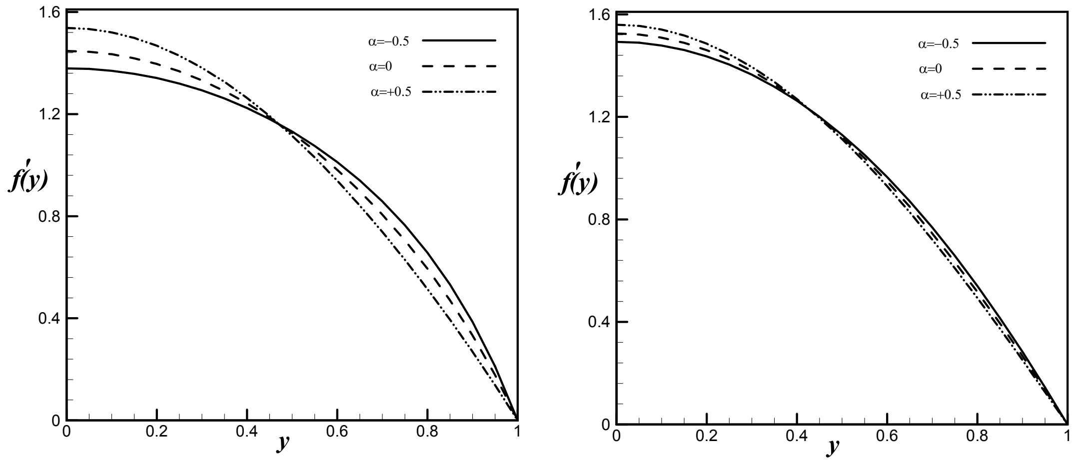
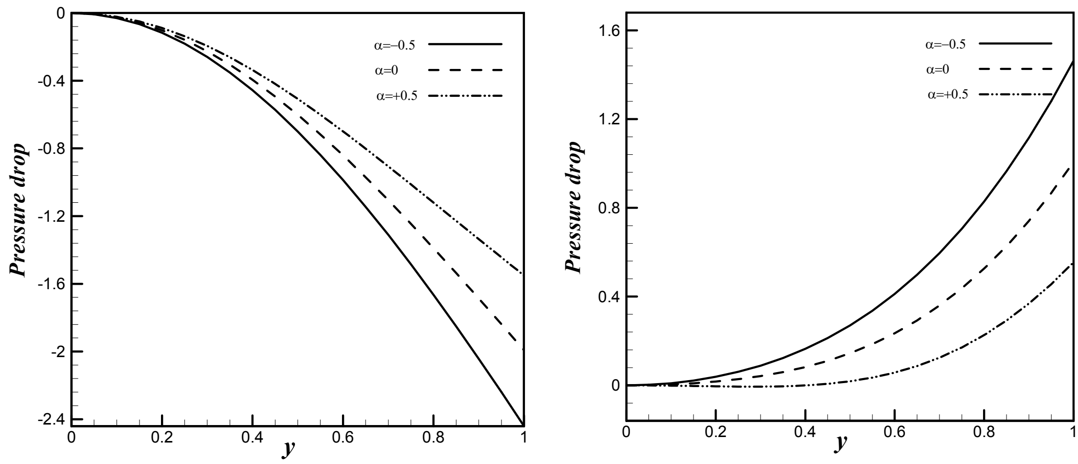


Fig. 2. Comparison between numerical method and DTM solutions for $f(y)$ when $\mathrm{Re}=-1$.



(a) (b)

Fig. 3. $f'(y)$ changes shown over a range of α at (a) $\mathrm{Re} = -5$ (b) $\mathrm{Re} = 5$



(a) (b)

Fig. 4. The pressure drop in the normal direction (Δp_n) changes shown over a range of α at (a) $\mathrm{Re} = -1$ (b) $\mathrm{Re} = 1$.

Table1. Some of the basic operations of Differential Transformation Method

Original function	**Transformed function**
$f(\eta)=\alpha g(\eta)\pm\beta h(\eta)$	$F[k]=\alpha G[k]\pm\beta H[k]$
$f(\eta)=\dfrac{d^n g(\eta)}{d\eta^n}$	$F[k]=\dfrac{(k+n)!}{k!}G[k+n]$
$f(\eta)=g(\eta)h(\eta)$	$F[k]=\sum_{m-0}^{k}F[m]H[k-m]$
$f(\tau)=\sin(\varpi\eta+\alpha)$	$F[k]=\dfrac{\varpi^k}{k!}\sin(\dfrac{\pi k}{2}+\alpha)$
$f(\tau)=\cos(\varpi\eta+\alpha)$	$F[k]=\dfrac{\varpi^k}{k!}\cos(\dfrac{\pi k}{2}+\alpha)$
$f(\eta)=e^{\lambda\eta}$	$F[k]=\dfrac{\lambda^k}{k!}$
$F(\eta)=(1+\eta)^m$	$F[k]=\dfrac{m(m-1)...(m-k+1)}{k!}$
$f(\eta)=\eta^m$	$F[k]=\delta(k-m)=\begin{cases}1\ ,k=m\\0\ ,k\neq m\end{cases}$

6. Conclusion

In this research, the DTM was successfully applied to find the analytical solution for laminar, isothermal, incompressible and viscous flow in a rectangular domain bounded by two moving porous walls, which enable the fluid to enter or exit during successive expansions or contractions. The results show that for every level of injection or suction, in the case of expanding wall, increasing α leads to higher axial velocity near the center and the lower axial velocity near the wall.

7. References

[1] Chang ,H.N. and J.S. Ha, J.K. Park, I.H. Kim, H.D. Shin, 1989. Velocity field of pulsatile flow in a porous tube, J. Biomech, 22 :1257_1262.

[2] Dauenhauer, E.C. and J. Majdalani, 1999. Unsteady flows in semi-infinite expanding channels with wall injection, in: AIAA paper, 99_3523.
[3] Majdalani, J. and C. Zhou, 2003. Moderate-to-large injection and suction driven channel flows with expanding or contracting walls, ZAMM Z. Angew. Math. Mech. 83:181_196.
[4] K. Zhou, Differential transformation and its applications for electrical circuits, Huazhong Univ.Press,Wuhan,China, 1986.
[5] C.K. Chen, S.H. Ho, Solving partial differential equations by two dimensional differential transform method, Appl. Math. Comput. 106 (1999) 171–179.
[6] F. Ayaz, Solutions of the systems of differential equations by differential transform method, Appl. Math. Comput. 147 (2004) 547–567.
[7] IH. Abdel-Halim Hassan, Comparison differential transformation technique with Adomian decomposition method for linear and nonlinear initial value problems, Chaos Solitons and Fractals 36 (2008) 53–65.
[8] M. Sheikholeslami, M. Gorji-Bandpy, D.D. Ganji, Soheil Soleimani, Natural convection heat transfer in a nanofluid filled inclined L-shaped enclosure, IJST, Transactions of Mechanical Engineering, 38 (2014) 217-226.
[9] M. Sheikholeslami, M. Gorji-Bandpy, D.D. Ganji, P. Rana, Soheil Soleimani, Magnetohydrodynamic free convection of Al2O3-water nanofluid considering Thermophoresis and Brownian motion effects, Computers & Fluids 94 (2014) 147–160.
[10] M. Sheikholeslami, M. Gorji-Bandpy, D.D. Ganji, Soheil Soleimani, Thermal management for free convection of nanofluid using two phase model, Journal of Molecular Liquids 194 (2014) 179–187.
[11] M. Sheikholeslami, M. Gorji-Bandpy, D.D. Ganji, Soheil Soleimani, Heat flux boundary condition for nanofluid filled enclosure in presence of magnetic field, Journal of Molecular Liquids,193 (2014) 174-184.
[12] M. Sheikholeslami, D.D. Ganji, M. Gorji-Bandpy, Soheil Soleimani, Magnetic field effect on nanofluid flow and heat transfer using KKL model, Journal of the Taiwan Institute of Chemical Engineers 45 (2014) 795–807
[13] M. Sheikholeslami, M. Gorji-Bandpy, Soheil Soleimani, Two phase simulation of nanofluid flow and heat transfer using heatline analysis, International Communications in Heat and Mass Transfer 47 (2013) 73–81
[14] M. Sheikholeslami, M. Gorji Bandpy, R. Ellahi, Mohsan Hassan , Soheil Soleimani, Effects of MHD on Cu-water nanofluid flow and heat transfer by means of CVFEM, Journal of Magnetism and Magnetic Materials 349 (2014)188–200.
[15] M. Sheikholeslami, M. Gorji-Bandpy, I. Pop, Soheil Soleimani, Numerical study of natural convection between a circular enclosure and a sinusoidal cylinder using control volume based finite element method, International Journal of Thermal Sciences 72 (2013) 147-158.
[16] M. Sheikholeslami, M. Gorji-Bandpy, D.D. Ganji, Soheil Soleimani, Natural convection heat transfer in a cavity with sinusoidal wall filled with CuO-water nanofluid in presence of magnetic field, Journal of the Taiwan Institute of Chemical Engineers 45 (2014) 40–49.
[17] M. Sheikholeslami, I. Hashim and Soheil Soleimani, Numerical Investigation of the Effect of Magnetic Field on Natural Convection in a Curved-Shape Enclosure, Hindawi Publishing Corporation Mathematical Problems in Engineering Volume 2013, Article ID 831725, 11 pages http://dx.doi.org/10.1155/2013/831725
[18] M. Sheikholeslami, M. Gorji-Bandpy, D.D. Ganji, Soheil Soleimani, Effect of a magnetic field on natural convection in an inclined half-annulus enclosure filled with Cu–water nanofluid using CVFEM, Advanced Powder Technology 24 (2013) 980–991.
[19]M. Sheikholeslami, M. Gorji-Bandpy, D. D. Ganji, Soheil Soleimani, MHD natural convection in a nanofluid filled inclined enclosure with sinusoidal wall using CVFEM, Neural Comput & Applic (2014) 24:873–882.

[20] M. Sheikholeslami , M. Gorji-Bandpy, D.D. Ganji, Soheil Soleimani, S.M. Seyyedi, Natural convection of nanofluids in an enclosure between a circular and a sinusoidal cylinder in the presence of magnetic field, International Communications in Heat and Mass Transfer 39 (2012) 1435–1443.
[21] Soheil Soleimani, M. Sheikholeslami, D.D. Ganji and M. Gorji-Bandpay, Natural convection heat transfer in a nanofluid filled semi-annulus enclosure, International Communications in Heat and Mass Transfer 39 (2012) 565–574.
[22] M. Sheikholeslami, M. Gorji-Bandpay, D.D. Ganji, Investigation of nanofluid flow and heat transfer in presence of magnetic field using KKL model, Arabian Journal for Science and Engineering, 39(6) 2014 5007-5016.
[23] Mohsen Sheikholeslami, Mofid Gorji Bandpy, R. Ellahi, A. Zeeshan, Simulation of MHD CuO–water nanofluid flow and convective heat transfer considering Lorentz forces, Journal of Magnetism and Magnetic Materials 369 (2014) 69–80.
[24] M. Sheikholeslami, M. Gorji-Bandpay, D.D. Ganji, Magnetic field effects on natural convection around a horizontal circular cylinder inside a square enclosure filled with nanofluid, International Communications in Heat and Mass Transfer 39 (2012) 978–986.
[25] M. Sheikholeslami, M. Gorji-Bandpy, D.D. Ganji, MHD free convection in an eccentric semi-annulus filled with nanofluid, Journal of the Taiwan Institute of Chemical Engineers 45 (2014) 1204–1216.
[26] Mohsen Sheikholeslami, Mofid Gorji-Bandpy, Free convection of ferrofluid in a cavity heated from below in the presence of an external magnetic field, Powder Technology 256 (2014) 490–498
[27] M. Sheikholeslami, M. Gorji-Bandpy, D.D. Ganji, Lattice Boltzmann method for MHD natural convection heat transfer using nanofluid, Powder Technology 254 (2014) 82-93.
[28] M. Sheikholeslami, M. Gorji-Bandpy, D. D. Ganji, Numerical investigation of MHD effects on Al2O3-water nanofluid flow and heat transfer in a semi-annulus enclosure using LBM, Energy 60 (2013) 501-510
[29]M. Sheikholeslami, M. Gorji-Bandpy, S.M. Seyyedi, D.D. Ganji, Houman B. Rokni, Soheil Soleimani, Application of LBM in simulation of natural convection in a nanofluid filled square cavity with curve boundaries, Powder Technology 247 (2013) 87–94.
[30]M. Sheikholeslami, M. Gorji-Bandpy, D.D. Ganji, Natural convection in a nanofluid filled concentric annulus between an outer square cylinder and an inner elliptic cylinder, Scientia Iranica, Transaction B: Mechanical Engineering. (2013) 20(4), 1241-1253.
[31] M. Sheikholeslami, M. Gorji-Bandpy, G. Domairry, Free convection of nanofluid filled enclosure using lattice Boltzmann method (LBM), Appl. Math. Mech. -Engl. Ed., 34(7), (2013) 1–15.
[32]Hamid Reza Ashorynejad, Abdulmajeed A. Mohamad, Mohsen Sheikholeslami, Magnetic field effects on natural convection flow of a nanofluid in a horizontal cylindrical annulus using Lattice Boltzmann method, International Journal of Thermal Sciences 64 (2013) 240-250.
[33] M. Sheikholeslami, D.D. Ganji, Heated permeable stretching surface in a porous medium using Nanofluids, Journal of Applied Fluid Mechanics, 7(3) (2014) 535-542.
[34] M. Sheikholeslami, D.D. Ganji, Three dimensional heat and mass transfer in a rotating system using nanofluid, Powder Technology 253 (2014) 789–796.
[35] Mohammad Hatami, Mohsen Sheikholeslami, M. Hosseini, Davood Domiri Ganji, Analytical investigation of MHD nanofluid flow in non-parallel walls, Journal of Molecular Liquids 194 (2014) 251–259.
[36] M. Sheikholeslami, D.D. Ganji, Magnetohydrodynamic flow in a permeable channel filled with nanofluid, Scientia Iranica B (2014) 21(1), 203-212 .
[37] M. Hatami, M. Sheikholeslami, D.D. Ganji, Nanofluid flow and heat transfer in an asymmetric porous channel with expanding or contracting wall, Journal of Molecular Liquids 195 (2014) 230–239.

[38] M. Hatami, M. Sheikholeslami, G. Domairry, High Accuracy Analysis for Motion of a Spherical Particle in Plane Couette Fluid Flow by Multi-step Differential Transformation Method, Powder Technology (2014), doi: 10.1016/j.powtec.2014.02.057
[39] M. Sheikholeslami, D.D. Ganji, Numerical investigation for two phase modeling of nanofluid in a rotating system with permeable sheet, Journal of Molecular Liquids 194 (2014) 13-19.
[40] M. Sheikholeslami, M. Hatami, D.D.Ganji, Micropolar fluid flow and heat transfer in a permeable channel using analytical method, Journal of Molecular Liquids 194 (2014) 30–36.
[41] M. Hatami, M. Sheikholeslami, D.D. Ganji, Laminar flow and heat transfer of nanofluid between contracting and rotating disks by least square method, Powder Technology 253 (2014) 769–779.
[42] M. Sheikholeslami, M. Hatami, D.D. Ganji, Nanofluid flow and heat transfer in a rotating system in the presence of a magnetic field, Journal of Molecular Liquids 190 (2014) 112–120.
[43] M. Sheikholeslami, F. Bani Sheykholeslami, S. Khoshhal, H. Mola-Abasia, D. D. Ganji, Houman B. Rokni, Effect of magnetic field on Cu–water nanofluid heat transfer using GMDH-type neural network, Neural Comput & Applic, DOI 10.1007/s00521-013-1459-y.
[44] M. Sheikholeslami, R. Ellahi, H. R. Ashorynejad, G. Domairry, and T. Hayat, Effects of Heat Transfer in Flow of Nanofluids Over a Permeable Stretching Wall in a Porous Medium, Journal of Computational and Theoretical Nanoscience, Vol. 11, 1–11, 2014.
[45] M. Sheikholeslami, M. Hatami, D. D. Ganji, Analytical investigation of MHD nanofluid flow in a Semi-Porous Channel, Powder Technology 246 (2013) 327–336
[46] M. Sheikholeslami, D.D. Ganji, H.R. Ashorynejad, Investigation of squeezing unsteady nanofluid flow using ADM, Powder Technology 239 (2013) 259–265.
[47] M. Sheikholeslami, D.D. Ganji, Heat transfer of Cu-water nanofluid flow between parallel plates, Powder Technology 235 (2013) 873–879.
[48]H. R. Ashorynejad, M. Sheikholeslami,I. Pop, D. D. Ganji, Nanofluid flow and heat transfer due to a stretching cylinder in the presence of magnetic field, Heat Mass Transfer 49 (2013) 427–436.
[49]Davood Domairry, Mohsen Sheikholeslami, Hamid Reza Ashorynejad,Rama Subba Reddy Gorla and Mostafa Khani, Natural convection flow of a non-Newtonian nanofluid between two vertical flat plates, Proc IMechE Part N:J Nanoengineering and Nanosystems225(3) 115–122©IMechE 2012. DOI: 10.1177/1740349911433468.
[50]M. Sheikholeslami, D. D. Ganji, H. R. Ashorynejad, Houman B. Rokni, Analytical investigation of Jeffery-Hamel flow with high magnetic field and nano particle by Adomian decomposition method, Appl. Math. Mech.-Engl. Ed., 33(1), 1553–1564 (2012).
[51] M. Sheikholeslami,H. R. Ashorynejad, G. Domairry and I. Hashim, Flow and Heat Transfer of Cu-Water Nanofluid between a Stretching Sheet and a Porous Surface in a Rotating System, Hindawi Publishing Corporation Journal of Applied Mathematics Volume 2012, Article ID 421320, 19 pages ,doi:10.1155/2012/421320.
[52] M. Sheikholeslami, H. R. Ashorynejad,D. D. Ganji and A. Kolahdooz, Investigation of Rotating MHD Viscous Flow and Heat Transfer between Stretching and Porous Surfaces Using Analytical Method, Hindawi Publishing Corporation Mathematical Problems in Engineering Volume 2011, Article ID 258734, 17 pages,doi:10.1155/2011/258734.
[53] M. Sheikholeslami, H.R. Ashorynejad, D.D. Ganji, A. Yıldırım, Homotopy perturbation method for three-dimensional problem of condensation film on inclined rotating disk, Scientia Iranica B (2012) 19 (3), 437–442

[54] Mohsen Sheikholeslami, Hamid Reza Ashorynejad ,Davood Domairry, Ishak Hashim, Investigation of the Laminar Viscous Flow in a Semi-Porous Channel in the Presence of Uniform Magnetic Field using Optimal Homotopy Asymptotic Method, Sains Malaysiana 41(10)(2012): 1177–1229.
[55] M. Sheikholeslami, H.R. Ashorynejad, A. Barari, Soheil Soleimani, Investigation of heat and mass
transfer of rotating MHD viscous flow between a stretching sheet and a porous surface. Engineering Computations, 30(3) (2013) 357-378.
[56] M. Sheikholeslami , D.D. Ganji , Houman B. Rokni ,Nanofluid Flow in a Semi-Porous Channel in the Presence of Uniform Magnetic Field, Ije Transactions C: Aspects Vol. 26, No. 6, (June 2013) 653-662.