NEW TRENDS IN MATHEMATICAL SCIENCES
Vol. 2, No. 2, 2014, p87-94
ISSN 2147-5520 - www.ntmsci.com

Infinitely many large energy solutions of nonlinear Schrödinger-Maxwell system

*MOHSEN ALIMOHAMMADY[1], MORTEZA KOOZEHGAR KALLEJI[1]

[1]*Department of Mathematics, Faculty of Mathematical Sciences University of Mazandaran, BABOLSAR 47416-1468, IRAN*

Abstract: This paper deals with the existence of infinitely many large energy solutions for nonlinear Schrödinger-Maxwell system

$$\begin{cases} -\Delta u + V(x)u + \lambda\phi u = |u|^{p-1}u & \text{in } \mathbb{R}^3 \\ -\Delta\phi = u^2 & \text{in } \mathbb{R}^3, \end{cases}$$

We use the Fountain theorem under Cerami conditions 2.2 to find infinitely many large solutions for $p \in (2,6)$ and $\lambda \in \mathbb{R}^+ - \left(\frac{4}{7}, \frac{4}{3}\right)$.

Keywords: Schrödinger-Maxwell equations, variational method, Strongly indefinite functionals, Cerami conditions

1. Introduction

In this paper we are concerned with the existence of infinitely many large energy solutions for the nonlinear Schrödinger-Maxwell system

$$\begin{cases} -\Delta u + V(x)u + \lambda\phi u = |u|^{p-1}u & \text{in } \mathbb{R}^3 \\ -\Delta\phi = u^2 & \text{in } \mathbb{R}^3, \end{cases} \tag{1.1}$$

where $\lambda \in \mathbb{R}^+ - \left(\frac{4}{7}, \frac{4}{3}\right)$ is a parameter, $V \in C(\mathbb{R}^3, \mathbb{R})$ which is satisfied in some suitable conditions and $p \in (2,6)$. In the classical model, the interaction of a charge particle with an electromagnetic field can be described by the nonlinear Schrödinger-Maxwell's equations (see for examples [6, 9] and the references therein for more details on the physical aspects).

More precisely, we use the Fountain theorem under Cerami conditions 2.2 to find infinitely many large solitions for $p \in (2,6)$ and $\lambda \in \mathbb{R}^+ - \left(\frac{4}{7}, \frac{4}{3}\right)$ which is different from obtained results in [1,6]. If we consider $V(x) = 1$, then the system 1.1 reduced to the following system

$$\begin{cases} -\Delta u + u + \lambda\phi u = |u|^{p-1}u & \text{in } \mathbb{R}^3 \\ -\Delta\phi = u^2 & \text{in } \mathbb{R}^3, \end{cases} \tag{1.2}$$

which considered by Jiang et. al, [15], of course in homogeneous case. The problem of finding infinitely many large solutions is a vary classical problem. There is an extensive literature concerning the existence of infinitely many large energy solutions of a plethora of problems via the symmetric Mountain Pass theorem and Fountain theorem [4, 7, 10]. But, the existence of solutions for problem 1.1 has been discussed under different ranges of p, for examples [11, 3] for $p \in [3,5)$, 5 for $p \in (2,5)$ and [1, 2, 17] for $p \in (1,5)$. In particular case, with $V(x) = 1$ and $p \in (2,5)$, Ambrosetti and Ruiz have proved that the system 1.2 has infinitely many solutions for all $\lambda > 0$ [1]. Here, we will show infinitely many large energy solutions for 1.1, where $V \in C(\mathbb{R}^3, \mathbb{R})$ and $p \in (2,6)$, via the Fountain theorem under cerami condition. In recent years, for the potential V, many authors assumed (see for examples [19,18]).

$V*$) $V \in C(\mathbb{R}^3, \mathbb{R})$ and $V(x) \geq M_0 > 0$ and there exists some $M > 0$ such that $\Omega_M \coloneqq \{x \in \mathbb{R}^3 | \, V(x) \leq M\}$ is nonempty and has finite Lebesgue measure.

We consider the more general case and weaken the condition of $V*$. We assume

V_1*) $V \in C(\mathbb{R}^3, \mathbb{R})$ and there exists some $M > 0$ such that the set $\Omega_M \coloneqq \{x \in \mathbb{R}^3 | \, V(x) \leq M\}$ is nonempty and has finite Lebesgue measure. Also we suppose that there exists a constant $\theta \geq 1$ such that

$$\theta f_\lambda(u) \geq f_\lambda(tu) \tag{1.3}$$

for all $x \in \mathbb{R}^3$, $u \in \mathbb{R}$ and $t \in [0,1]$, where $f_\lambda(u) = \left(1 - \frac{4}{\lambda(p+1)}\right) \int_{\mathbb{R}^3} |u|^{p+1} dx - \|u\|_E^2$, where $\lambda \in \mathbb{R}^+ - \left(\frac{4}{7}, \frac{4}{3}\right)$. The assumption V_1* implies that the potential V is not periodie and changes sign.

2. Main results

Here, we express Cerami condition which was established by G. Cerami in [12]. To approach the main result, we need the following critical point theorem.

Definition 2.1. Suppose that functional I is C^1 and $c \in \mathbb{R}$, if any sequence $\{u_n\}$ satisfies in $I(u_n) \to c$ and $(1 + \|u_n\|)I'(u_n) \to 0$ has a convergence subsequence, we say the I is said to Cerami condition at the level c.

Theorem 2.2. *(Fountain theorem under Cerami condition) Let X be a Banach space with the norm $\|.\|$ and let X_j be a sequence of subspace of X with $\dim X_j < \infty$ for any $j \in \mathbb{N}$. Further, $X = \overline{\oplus_{j\in\mathbb{N}} X_j}$, the closure of the direct sum of all X_j. Set $W_k = \oplus_{j=0}^{k} X_j$, $Z_k = \oplus_{j=k}^{\infty} X_j$.*

Consider an even functional $I \in C^1(X, \mathbb{R})$, that is *$I(-u) = I(u)$ for any $u \in X$. Suppose that for any $k \in \mathbb{N}$, there exists $\rho_k > r_k > 0$ such that*

I_1) $a_k \coloneqq \max_{u\in W_k, \|u\|=\rho_k} I(u) \leq 0,$

I_2) $b_k \coloneqq \inf_{u\in Z_k, \|u\|=r_k} \; I(u) \to +\infty$ *as* $k \to \infty,$

I_3) *the Cerami condition holds at any level $c > 0$. Then the functional I has an unbounded sequence of critical values.*

Now, our main result is the following:

Theorem 2.3. *Let V_1*, and assumption 1.3 satisfies. Then the system 1.1 has infinitely many solutions $\{(u_k, \phi_k)\}_{k\in\mathbb{N}}$ in $H^1(\mathbb{R}^3) \times D^{1,2}(\mathbb{R}^3)$ satisfying*

$$\frac{1}{2} \int_{\mathbb{R}^3} \left(\left|\nabla_{u_k}\right|^2 + V(x)u_k^2\right) dx - \frac{\lambda}{4} \int_{\mathbb{R}^3} \left|\nabla_{\phi_k}\right|^2 dx + \frac{\lambda}{2} \int_{\mathbb{R}^3} \phi_k u_k^2 dx - \frac{1}{p+1} |u|^{p+1} dx \to +\infty,$$

as $k \to \infty$.

3. Some auxiliary results and notations

In this section we give some notations and definitions on the function product space. We set

$$H^1(\mathbb{R}^3) \coloneqq \{u \in L^2(\mathbb{R}^3) | \nabla_u \in L^2(\mathbb{R}^3)\}, \tag{3.1}$$

endowed with the norm

$$\|u\|_{H^1} \coloneqq \left(\int_{\mathbb{R}^3} |\nabla_u|^2 + u^2 dx \right)^{\frac{1}{2}} \tag{3.2}$$

and we consider the function space

$$D^{1,2}(\mathbb{R}^3) \coloneqq \{u \in L^{2^*}(\mathbb{R}^3) \mid \nabla_u \in L^2(\mathbb{R}^3)\} \tag{3.3}$$

with the norm

$$\|u\|_{D^{1,2}} \coloneqq \left(\int_{\mathbb{R}^3} |\nabla_u|^2 dx\right)^{\frac{1}{2}} \tag{3.4}$$

where $2^* = \frac{2n}{n-2} = 6$. Now, we consider the function space

$$E \coloneqq \left\{u \in H^1(\mathbb{R}^3) \mid \int_{\mathbb{R}^3} |\nabla_u|^2 + u^2 dx < \infty\right\}.$$

Then E is a Hilbert space [20] with the inner product

$$(u,v)_E \coloneqq \int_{\mathbb{R}^3} (\nabla_u \nabla_v + V(x)uv)dx \tag{3.5}$$

and $\|u\|_E \coloneqq (u,v)_E^{\frac{1}{2}}$.

Lemma 3.1. [19] *If $V_1{}^*$ holds. Then $E \hookrightarrow L^p(\mathbb{R}^N, \mathbb{R}^2)$ is continuous for $p \in [2, 2^*]$ and $E \hookrightarrow L^p_{loc}(\mathbb{R}^N, \mathbb{R}^2)$ is compact for $p \in [2,2^*)$.*

Remark 3.2. *The system 1.1 is the Euler-Lagrange equations of the functional $J_\lambda : E \times D^{1,2}(\mathbb{R}^3) \to \mathbb{R}$ defind by*

$$J_\lambda(u,\phi) \coloneqq \frac{1}{2}\|u\|_E^2 - \frac{\lambda}{4}\int_{\mathbb{R}^3} |\nabla\phi|^2 dx + \frac{\lambda}{2}\int_{\mathbb{R}^3} \phi u^2 dx - \frac{1}{p+1}\int_{\mathbb{R}^3} |u|^{p+1} dx \tag{3.6}$$

The functional $J_\lambda \in C^1(E \times D^{1,2}(\mathbb{R}^3), \mathbb{R})$ and its critical points are the solutions of system 1.1. It is easy to know that J_λ exhibits a strong indefiniteness, namely it is unbounded both from below and from above on infinitely dimensional subspaces. This indefiniteness can be removed using the reduction method described in [9]. We recall this method. For any $u \in E$, the Lax-Milgram theorem [14] implies there exists a unique $\phi_u \in D^{1,2}(\mathbb{R}^3)$ such that

$$-\Delta\phi_u = u^2$$

in a weak sense. We can write an integral expression for ϕ_u in the form:

$$\phi_u = \frac{1}{4\pi}\int_{\mathbb{R}^3} \frac{u(y)^2}{|x-y|} dy, \tag{3.7}$$

for any $u \in E$.

Lemma 3.3. [13] *for any $u \in E$*

i. $\|\phi_u\|_{D^{1,2}} \leq M_1 \|u\|^2_{L^{\frac{12}{5}}}$, *where M_1 is positive constant which does not depend on u. In particular, there exists a positive constant M_2 such that*

$$\int_{\mathbb{R}^3} \phi_u u^2 dx \leq M_2 \|u\|_E^4; \tag{3.8}$$

ii. $\phi_u \geq 0$.

According to the Lemma 3.3, we define the functional $I_\lambda : E \to \mathbb{R}$ by

$I_\lambda(u) \coloneqq J_\lambda(u, \phi_u)$.

Remark 3.4. *Using the relation* $-\Delta\phi_u = u^2$ *and integration by parts, we can obtain*

$$\int_{\mathbb{R}^3} |\nabla\phi_u|^2 dx = \int_{\mathbb{R}^3} \phi_u u^2 dx.$$

Then, we can consider the functional 3.6 as following

$$I_\lambda(u) = \frac{1}{2}\|u\|_E^2 + \frac{\lambda}{4}\int_{\mathbb{R}^3} \phi_u u^2 dx - \frac{1}{p+1}\int_{\mathbb{R}^3} |u|^{p+1} dx. \tag{3.9}$$

It well-known that I is C^1*-functional with derivative given by*

$$\langle I_\lambda'(u), u\rangle = \int_{\mathbb{R}^3} [\nabla_u \nabla_v + V(x)uv + \phi_u uv - |u|^{p-1}uv] dx \tag{3.10}$$

Now, using the proposition 2.3 in [16] we can consider the following proposition for our functional J_λ:

Proposition 3.5. *The following statements are equivalent:*

i) $(u, \phi) \in E \times D^{1,2}(\mathbb{R}^3)$ *is a critical point* J_λ *i.e.* (u, ϕ) *is a solution of problem 1.1;*

ii) u *is a critical point of* I_λ *and* $\phi_u = \phi$.

Proof. It follows using the remark 3.2 and theorem 2.3 in [9].

4. Proof of main theorem

We take an orthogonal basis $\{e_j\}$ of product space $X \coloneqq E$ and we define $W_k \coloneqq span\{e_j\}_{j=1,\dots,k}$, $Z_k \coloneqq W_k^\perp$.

Lemma 4.1. [13] *for any* $p \in [2, 2^*)$ $\beta_k \coloneqq \sup_{u \in Z_k, \|u\| = 1} \|u\|_{Lp} \to 0$, *as* $k \to \infty$.

Now, we prove that the functional $I_\lambda : E \to \mathbb{R}$ satisfies the Cerami condition.

Proposition 4.2. *Under the assumption 1.3, the functional* $I_\lambda(u)$ *satisfies the Cerami condition at any positive level.*

Proof. We suppose that $\{u_n\}$ is the Cerami sequence, that is for some $c \in \mathbb{R}^+$,

$$I_\lambda(u_n) = \frac{1}{2}\|u\|_E^2 + \frac{\lambda}{4}\int_{\mathbb{R}^3} \phi_{u_n} u_n^2 dx - \frac{1}{p+1}\int_{\mathbb{R}^3} |u_n|^{p+1} dx \to c \tag{4.1}$$

as $n \to \infty$ and

$$(1 + \|u_n\|_E) I_\lambda'(u_n) \to 0 \tag{4.2}$$

as $n \to \infty$. From relations 4.1 and 4.2 for n large enough,

$$1 + c \geq I_\lambda(u_n) - \frac{\lambda}{4}\langle I_\lambda'(u_n), u_n\rangle = \frac{1}{2}\|u_n\|_E^2 + \int_{\mathbb{R}^3} \phi_{u_n} u_n^2 dx -$$

$$\frac{1}{p+1}\int_{\mathbb{R}^3} |u_n|^{p+1} dx - \frac{\lambda}{4}\left[\int_{\mathbb{R}^3} \left(\nabla_{u_n}\nabla_{u_n} + V(x)u_n^2 + \phi_{u_n}u_n^2 - |u_n|^{p-1}u_n^2\right) dx\right].$$

Then,

$$1 + c \geq \left(\frac{1}{2} - \frac{\lambda}{4}\right) \|u_n\|_E^2 - \left(\frac{1}{p+1} - \frac{\lambda}{4}\right) \int_{\mathbb{R}^3} |u_n|^{p+1} dx. \tag{4.3}$$

We show that $\{u_n\}$ is bounded sequence. Otherwise, there exists a subsequence of $\{u_n\}$ satisfying $\|u_n\|_E \to \infty$ as $n \to \infty$. Then we consider $\omega_n = \frac{u_n}{\|u_n\|_E} \in E$, so the sequence ω_n is bounded. Up to a subsequence, for some $\omega \in E$,

$$\omega_n \rightharpoonup \omega$$

in E,

$$\omega_n \to \omega \ in \ L^t(\mathbb{R}^3) \ \forall t \in [2, 2^*)$$

and

$$\omega_n(x) \to \omega(x) \ \ a.e. \ in \ \ \mathbb{R}^3. \tag{4.4}$$

Now, we consider two cases. In first case suppose that $\omega \neq 0$ in E. Dividing by $\|u_n\|_E^2$ in both sides of relation 4.1 and by lemma 3.3 we can get

$$\frac{1}{p+1} \int_{\mathbb{R}^3} \frac{|u_n|^{p+1}}{|u_n|^2} dx = 1 + \frac{\int_{\mathbb{R}^3} \phi_{u_n} u_n^2 dx}{\|u_n\|_E^2} + \mathcal{O}(\|u_n\|_E^2) \leq M_3 < \infty \tag{4.5}$$

where M_3 is a positive constant. We consider,

$$\Omega \coloneqq \{x \in \mathbb{R}^3 \mid \omega(x) \neq 0\},$$

then for all $x \in \Omega$ and $p \in (2, \infty)$

$$\frac{|u_n|^{p+1}}{\|u_n\|_E^2} = \frac{|u_n|^{p+1}}{|u_n|^2} \omega_n(x)^2 \to +\infty \tag{4.6}$$

as $n \to \infty$. Since $meas(\Omega) > 0$, using Fatou's lemma,

$$\frac{1}{p+1} \int_{\mathbb{R}^3} \frac{|u_n|^{p+1}}{\|u_n\|_E^2} dx \to +\infty \tag{4.7}$$

as $n \to \infty$. This is contradiction with relation 4.5. In second case, suppose that $\omega(x) = 0$, then we define a sequence, $t_n \in \mathbb{R}$ as

$$I_\lambda(t_n u_n) = \max_{t \in [0,1]} I_\lambda(t u_n).$$

For any positive m, we set $\overline{\omega}_n = \sqrt{4m} \frac{u_n}{\|u_n\|_E} = \sqrt{4m}\omega_n$. Hence, by relation 4.5 and for n large enough,

$$\begin{aligned} I_\lambda(t_n u_n) \geq I_\lambda(\overline{\omega}_n) &= \frac{1}{2} \|\overline{\omega}_n\|_E^2 + \frac{\lambda}{4} \int_{\mathbb{R}^3} \phi_{\overline{\omega}_n} \overline{\omega}_n^2 dx - \frac{1}{p+1} \int_{\mathbb{R}^3} |\overline{\omega}_n|^{p+1} dx \\ &= 2m + \frac{\lambda}{4} \int_{\mathbb{R}^3} \phi_{\overline{\omega}_n} \overline{\omega}_n^2 dx - \frac{1}{p+1} \int_{\mathbb{R}^3} |\overline{\omega}_n|^{p+1} dx \geq m. \end{aligned} \tag{4.8}$$

Therefore, $\lim_{n \to \infty} I_\lambda(t_n u_n) = +\infty$ by relation 4.8. Since $I_\lambda(0) = 0$ and $I_\lambda(u_n) \to c$ then for $t_n \in (0,1)$ and n large enough, we obtain that

$$\int_{\mathbb{R}^3} \left(\nabla_{t_n u_n} \nabla_{t_n u_n} + V(x) t_n u_n t_n u_n + \phi_{t_n u_n} t_n u_n t_n u_n - |t_n u_n|^{p-1} t_n u_n t_n u_n\right) dx = \langle I'_\lambda(t_n u_n), t_n u_n\rangle$$

$$= t_n \frac{d}{dt}|_{t=t_n} I_\lambda(t u_n) = 0.$$

Hence, by assumption 1.3,

$$I_\lambda(u_n) - \frac{\lambda}{4}\langle I'_\lambda(u_n), u_n\rangle = \frac{2-\lambda}{4}\|u_n\|_E^2 - \frac{4-\lambda(p+1)}{4(p+1)} \int_{\mathbb{R}^3} |u_n|^{p+1} dx =$$

$$\frac{1}{2}\|u_n\|_E^2 - \frac{1}{p+1}\int_{\mathbb{R}^3} |u_n|^{p+1} dx - \frac{\lambda}{4}\|u_n\|_E^2 + \frac{\lambda}{4}\int_{\mathbb{R}^3} |u_n|^{p+1} dx =$$

$$\frac{1}{2}\|u_n\|_E^2 + \frac{\lambda}{4}\left[1 - \frac{4}{\lambda(p+1)}\int_{\mathbb{R}^3} |u_n|^{p+1} dx - \|u_n\|_E^2\right] =$$

$$\frac{1}{2}\|u_n\|_E^2 + \frac{\lambda}{4} f_\lambda(u_n) \geq \frac{1}{2\theta}\|t_n u_n\|_E^2 + \frac{\lambda}{4\theta} f_\lambda(t_n u_n) =$$

$$\frac{1}{\theta} I_\lambda(t_n u_n) - \frac{\lambda}{4\theta}\langle I'_\lambda(t_n u_n), t_n u_n\rangle \to \infty,$$

as $n \to \infty$. This contradicts relation 4.3. Therefore, $\{u_n\}$ is bounded sequence. Assume that $u_n \rightharpoonup u$ in E. By lemma 3.1 $u_n \to u$ in $L^t(\mathbb{R}^3)$ for any $t \in [2, 2^*)$. By relation 3.10,

$$\|u_n - u\|_E^2 = \langle I'_\lambda(u_n) - I'_\lambda(u), u_n - u\rangle + \int_{\mathbb{R}^3} |u_n|^{p-1} - |u|^{p-1}(u_n - u) dx - \int_{\mathbb{R}^3} \left(\phi_{u_n} u_n - \phi_u u\right)(u_n - u) dx.$$

By the Höder inequality, Sobolev inequality and lemma 3.3

$$\left|\int_{\mathbb{R}^3} \phi_{u_n} u_n (u_n - u) dx\right| \leq \left\|\phi_{u_n} u_n\right\|_{L^2} \|u_n - u\|_{L^2} \leq \left\|\phi_{u_n}\right\|_{L^6} \|u_n\|_{L^3} \|u_n - u\|_{L^2}$$

$$M_4 \left\|\phi_{u_n}\right\|_{D^{1,2}} \|u_n\|_{L^3} \|u_n - u\|_{L^2} \leq M_2 M_4 \|u_n\|_{L^{\frac{12}{5}}}^2 \|u_n\|_{L^3} \|u_n - u\|_{L^2},$$

where M_4 is a positive constant. Again using $u_n \to u$ in $L^t(\mathbb{R}^3)$ for any $t \in [2, 2^*)$, we obtain that

$$\int_{\mathbb{R}^3} \phi_{u_n} u_n (u_n - u) dx \to 0,$$

as $n \to \infty$. Similarly,

$$\int_{\mathbb{R}^3} \phi_{u_n} u (u_n - u) dx \to 0,$$

as $n \to \infty$. Hence,

$$\int_{\mathbb{R}^3} \left(\phi_{u_n} u_n - \phi_u u\right)(u_n - u) dx \to 0,$$

as $n \to \infty$. Thus $\|u_n - u\|_E \to 0$. Therefore, $I_\lambda(u)$ satisfies Cerami condition.

*Proof of theorem 2.3.*From proposition 4.2. $I_\lambda(u)$ satisfies Cerami condition. Next, we show that $I_\lambda(u)$ satisfies the rest conditions of theorem 2.2. First of all, we prove that $I_\lambda(u)$ satisfies I_1. Since $p \in (2,6)$, so $\lim_{|u|\to\infty} \frac{|u|^{p+1}}{|u|^2} = +\infty$. Then for any $K > 0$ there exist $\delta > 0$ such that for $|u| \geq \delta$,

$$|u|^{p+1} \geq \frac{\lambda}{4} K|u|^2. \tag{4.9}$$

Hence,

$$I_\lambda(u) \leq \frac{1}{2}\|u\|_E^2 + \frac{\lambda M_2}{4}\|u\|_E^4 - \frac{\lambda K}{4(p+1)}\|u\|_{L^2}.$$

Since, norms on finite dimension spaces W_k are equivalent,

$$I_\lambda(u) \leq \frac{1}{2}\|u\|_E^2 + \frac{\lambda M_2}{4}\|u\|_E^4 - \frac{\lambda K M_5}{4(p+1)}\|u\|_E^2,$$

where M_5 is a constant. Since

$$\frac{\lambda M_2}{4} - \frac{\lambda K M_5}{4(p+1)} < 0$$

when K is large enough, it follows that

$$a_k := \max_{u \in W_k, \|u\| = \rho_k} I_\lambda(u) \leq 0$$

for some $\rho_k > 0$ large enough. Using the lemma 3.3 and 3.1 we show that $I_\lambda(u)$ satisfies in condition I_2. By definition of I_λ,

$$I_\lambda(u) \geq \frac{1}{2}\|u\|_E^2 - \frac{1}{p+1}\int_{\mathbb{R}^3} |u|^{p+1} dx \geq \frac{1}{2}\|u\|_E^2 - \frac{1}{p+1}\|u\|_{L^p}^p \geq \frac{1}{2}\|u\|_E^2 - \frac{\beta_k^p}{p+1}\|u\|_E^p,$$

where β_k is defined in lemma 4.1. defining $r_k := \left(\frac{p\beta_k^p}{p+1}\right)^{\frac{1}{2-p}}$, implies that

$$b_k := \inf_{u \in Z_k, \|u\|_E = r_k} I_\lambda(u) \geq \inf_{u \in Z_k, \|u\|_E} \left(\frac{1}{2}\|u\|_{EW}^2 - \frac{\beta_k^p p}{p+1}\|u\|_E^p\right) \geq \left(\frac{1}{2} - \frac{1}{p}\right)\left(\frac{p\beta_k^p}{p+1}\right)^{\frac{2}{2-p}} \to +\infty$$

as $k \to \infty$. Using 2.2 completes the proof.

References

[1] A. AMBROSETTI, D.RUIZ, multiple bound states for the Schrödinger-Poisson problem, Commun. Contemp. Math., 10(3) (2008), 391–404.

[2] A. AZZOLINI, P. D'AVENIA, A. POMPONIO, On the Schrödinger-Maxwell equations under effect of a general nonlinear term, Ann. Inst. H. Poincare Anal. Non Lineair., 27(2) (2010), 779–791.

[3] T. D'APRIL, D. MUGNAL, Solitary waves for nonlinear Klein-Gordon-Maxwell and Schördinger-Maxwell equations, Proc.Roy.Soc.Edinburgh. Sect. (A)., 134(5) (2004), 893–906.

[4] A. AMBROSETTI, A. MALCHIODI, Perturbation Methods and Semilinear Elliptic Problems on R^n, Progr. Math. Birkhuser Verlag, Vol. 240 (2006).

[5] A. AZZOLINI, A. POMPONIO, Ground staes solutions for the nonlinear Schördinger-Maxwell equations, J.Math. Anal. Appl., 345(1) (2008), 90–108.

[6] V.BENCI, D. FORTUNATO, An eigenvalue problem for the Schördinger-Maxwell equations ,Topol.Methods nonlinear Anal., 11(2) (1998), 283–293.

[7] TH. BARTHS, SH. PENG, Semiclassical symmetric Schrdinger equations: existence of solutions concentrating simultaneously on several spheres, Z. Angew. Math. Phys., 58(5) (2007), 778–804.

[8] D. BONHEURE, J. VAN SCHAFTINGEN, Bound state solutions for a class of nonlinear Schrdinger equations, Rev. Mat. Iberoam., 24 (2008), 297–351.

[9] V. BENCI, D. FORTUNATO, A. MASIELLO, L. PISANI, Solitons and the electromagneticfield, Math. Z., 232(1) (1999), 73–102.

[10] D. BONHEURE, J.DI COSMO, J.VAN SCHAFTINGEN, Nonlinear Schrdinger equation with unbounded or vanishing potentials: Solutions concentrating on lower dimensional spheres, J.Differential Equations., 252 (2012) , 941–968.

[11] G.M. COCLITE, A multiplicity result for the nonlinear Schrödinger -Maxwell equations, Commun. Apll.Anal., 7(2-3) (2003), 417–423.

[12] G. CERAMI, An existence criterion for the points on unbounded manifolds, Istit. Lombardo Accad. Sci. Lett. Rend. A ., 112(2) (1979), 332–336.
[13] S.J. CHEN, C.L. TANG, High energy solutions for the superlinear Schrödinger-Maxwell ewuations, Nonlinear Anal., 71(10) (2009), 4927–4934.
[14] D. GILBARG, N.S. TRUDINGER, Elliptic partial differential equations of second order, Classics in Mathematics. Springer-Verlag, Berlin, (2001).
[15] Y.JIANG, ZH. WANG, H-S ZHOU, multiple solutions for a nonhomogeneous Schrödinger-Maxwell system in R^3, arXiv: 1204.3359v1 [math.AP], 16Apr (2012).
[16] L.LI, SH-J. CHEN, Infinitely many large energy solutions of superlinear Schrödinger-maxwell equations, Electron. j. Differential Equations., Vol.(2012) No. 224 (2012), 1–9.
[17] D. RUIZ, The Schrödinger-Poisson equation under effect of nonlinear local term, J. Funct. Anal., 232(2) (2006), 655–674.
[18] F.K. ZHAO, Y.H. DING, On Hamiltonian elliptic systes with periodic and non-periodic potentials, J.Differential Equations., 249 (2010) , 2964–2985.
[19] J. ZHANG, W. QIN, F. ZHAO, Existence and multiplicity of solutions for asymptotically linear nonpreiodic Hamiltonian elliptic system, J. Math. Anal. Appl., 399 (2013), 433-441.
[20] W. M. ZOU, M. SCHECHETER, Critical point theory and its applications, Springer, New York (2006).