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Sequence in intuitionistic fuzzy soft multi topological spaces

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Abstract: In this paper, we introduce a new type of sequence of intuitionistic fuzzy soft multi sets and their basic properties are studied. The concepts of subsequence, increasing sequence, decreasing sequence and convergence sequence of intuitionistic fuzzy soft multi sets are proposed. The concepts of cluster intuitionistic fuzzy soft multi sets of sequences are also introduced. Some basic properties regarding the above concepts are explored.

Keywords: Fuzzy set, soft set, fuzzy soft multi set, intuitionistic fuzzy soft multi set, intuitionistic fuzzy soft multi topological space.

1 Introduction

Theory of fuzzy sets, intuitionistic fuzzy sets, soft sets and soft multi sets are powerful mathematical tools for modeling various types of vagueness and uncertainty. In 1999, Molodtsov [9] initiated soft set theory as a completely generic mathematical tool for modeling uncertainty and vague concepts. Later on Maji et al. [7] presented some new definitions on soft sets and discussed in details the application of soft set in decision making problem. Based on the analysis of several operations on soft sets introduced in [9], Ali et al. [1] presented some new algebraic operations for soft sets and proved that certain De Morgan's law holds in soft set theory with respect to these new definitions. Combining soft sets [9] with fuzzy sets [20] and intuitionistic fuzzy sets [4], Maji et al. [6, 8] defined fuzzy soft sets and intuitionistic fuzzy soft sets, which are rich potential for solving decision making problems. Later Roy and others [13-16, 19] constructed the fundamental theory on fuzzy soft topological spaces. Alkhazaleh and others [2, 5, 17] as a generalization of Molodtsov's soft set, presented the definition of a soft multi set and its basic operations such as complement, union, and intersection etc., thereafter Mukherjee and others [12, 17, 18] introduced the concept of soft multi topological spaces and studied compactness and connectedness on soft multi topological spaces. In 2012, Alkhazaleh and Salleh [3] introduced the concept of fuzzy soft multi set theory and studied the application of these sets and recently, Mukherjee and Das [10, 11] studied the concepts of fuzzy soft multi topological spaces and constructed the fundamental theory on intuitionistic fuzzy soft multi topological spaces.

In this paper we introduce a new sequence of intuitionistic fuzzy soft multi sets in intuitionistic fuzzy soft multi topological spaces and their basic properties are studied. The concepts of subsequence and convergence sequence of

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intuitionistic fuzzy soft multi sets are proposed. The concepts of cluster intuitionistic fuzzy soft multi sets of sequences are also introduced. Some basic properties regarding the above concepts are explored.

2 Preliminary notes

In this section, we recall some basic notions in fuzzy set, intuitionistic fuzzy set, soft set, fuzzy soft set theory and intuitionistic fuzzy soft multi set theory.

Throughout this paper U refers to the initial universe and E be a set of parameters. P(U) denotes the power set of U, FS(U) be the set of all fuzzy subsets of U, $IFS(U)$ denotes the set of all intersection fuzzy subsets of U and A⊆ E.

Definition 2.1 [20] Let X be a non empty set. Then a fuzzy set A is a set having the form A={(x, μ_A(x)): x∈ X}, where the functions μ_A: X→ [0, 1] represents the degree of membership of each element x∈ X.

Definition 2.2 [4] Let U be a non empty set. Then an intuitionistic fuzzy set (IF set for short) A on U is a set having the form $\mathrm{A} = \{ \langle \mathrm{x}, \mu_{\mathrm{A}}(\mathrm{x}), \nu_{\mathrm{A}}(\mathrm{x}) \rangle : \mathrm{x} \in U \}$ where the functionsμ_{A}: U → [0, 1] and ν_{A}: U → [0, 1] represents the degree of membership and the degree of non-membership respectively of each element x∈ U and $0 \leq \mu_{\mathrm{A}}(\mathrm{x}) + \nu_{\mathrm{A}}(\mathrm{x}) \leq 1$, for each x∈ U.

Definition 2.3 [9] Let U be an initial universe and E be a set of parameters. Let P(U) denotes the power set of U and A⊆ E. Then the pair (F, A) is called a soft set over U, where F is a mapping given by F: A→ P(U).

Definition 2.4 [6] Let U be an initial universe and E be a set of parameters. Let FS(U) be the set of all fuzzy subsets of U and A⊆ E. Then the pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by F: A→ FS(U).

Definition 2.5 [11] Let $\{U_i : i \in I\}$be a collection of universes such that $\cap_{i\in I} U_i = \phi$and let $\{E_i : i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i\in I} IFS(U_i)$ where $IFS(U_i)$ denotes the set of all intersection fuzzy subsets of U_i,$E = \prod_{i\in I} E_{U_i}$ and $A \subseteq E$. A pair (F, A) is called an intersection fuzzy soft multi set over U, where F is a mapping given by F : A → U, such that $\forall e \in A$,

$$F(e) = \left(\left\{ \frac{u}{(\mu_{F(e)}(u), \nu_{F(e)}(u))} : u \in U_i \right\} : i \in I \right).$$

Definition 2.6 [11] An intuitionistic fuzzy soft multi set (F, A) over U is called an absolute intuitionistic fuzzy soft multi set, denoted by $(F,A)_U$, if $(e_{U_i,j}, F_{e_{U_i,j}}) = U_i, \forall i$.

Mukherjee and Das [11] consider an absolute intuitionistic fuzzy soft multi set $(,A)$ over U and $IFSMS_A(,A)$ denote the family of all intuitionistic fuzzy soft multi sub sets of $(,A)$ in which all the parameter set A are same.

Definition 2.7 [11] A sub family τ of $IFSMS_A(,A)$ is called intuitionistic fuzzy soft multi topology on $(,A)$, if the following axioms are satisfied:

$[O_1]$ $\Phi_A, (,A) \in \tau$, (where Φ_A has been defined in **[11]**)
$[O_2]$ $\{(F_i,A) : i \in I\} \subseteq \tau \Rightarrow \tilde{\cup}_{i\in I}(F_i,A) \in \tau$,
$[O_3]$ If $(F,A),(G,A) \in \tau$,then $(F,A)\tilde{\cap}(G,A) \in \tau$.

The pair $((,A),\tau)$ is called intuitionistic fuzzy soft multi topological space and the members of τ are called intuitionistic fuzzy soft multi open sets.

Definition 2.8 [11] The complement of an intuitionistic fuzzy soft multi set (F, A) over U is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c,A)$, where $F^c : A \to U$ is a mapping given by

$$F^c(e) = \left(\left\{\frac{u}{(\nu_{F(e)}(u),\mu_{F(e)}(u))} : u \in U_i\right\} : i \in I\right), \forall e \in A.$$

Definition 2.9 [11] $(F,A) \in IFSMS_A(,A)$ is an intuitionistic fuzzy soft multi neighbourhood of $(G,A) \in IFSMS_A(,A)$ if there exists $(H,A) \in \tau$ such that $(G,A)\tilde{\subseteq}(H,A)\tilde{\subseteq}(F,A)$.

Theorem 2.10 [11] $(F,A) \in IFSMS_A(,A)$ is an intuitionistic fuzzy soft multi open set if and only if (F,A) is a neighbourhood of each intuitionistic fuzzy soft multi set (G,A) contained in (F,A).

3 Main results

In this section we introduce a new sequence of intuitionistic fuzzy soft multi sets in intuitionistic fuzzy soft multi topological spaces and study their basic properties.

Definition 3.1 Let $((,A),\tau)$ be the intuitionistic fuzzy soft multi topological space on $(,A)$ and N be the set of all natural numbers. A sequence of intuitionistic fuzzy soft multi sets in $((,A),\tau)$ is a mapping from N to $IFSMS_A(,A)$ and is denoted by $\{(F_n,A)\}\ or \{(F_n,A) : n = 1,2,3,...\}$.

Example 3.2 Let us consider two universes $U_1 = \{h_1,h_2\}$ and $U_2 = \{c_1,c_2\}$. Let $\{E_{U_1},E_{U_2}\}$ be a collection of sets of decision parameters related to the above universes, where

$$E_{U_1} = \{e_{U_1,1} = \text{expensive}, e_{U_1,2} = \text{cheap}, e_{U_1,3} = \text{wooden}\},$$

$$E_{U_2} = \{e_{U_2,1} = \text{sporty}, e_{U_2,2} = \text{cheap}, e_{U_2,3} = 2010\text{model}\}.$$

Let $U = \prod_{i=1}^{2} FS(U_i)$, $E = \prod_{i=1}^{2} E_{U_i}$and $A = \{e_1 = (e_{U_1,1},e_{U_2,1}),\ e_2 = (e_{U_1,1},e_{U_2,2})\}$.

If we chose for n=1, 2, 3,....

$$(F_n,A)=\left\{\left(e_1,\left(\left\{\frac{h_1}{(2/5n,3/5n)},\frac{h_2}{(1/n,2/4n)}\right\},\left\{\frac{c_1}{(1/2n,1/2n)},\frac{c_2}{(5/7n,1/7n)}\right\}\right)\right),\right.$$
$$\left.\left(e_2,\left(\left\{\frac{h_1}{(1/5n,3/5n)},\frac{h_2}{(2/3n,1/3n)}\right\},\left\{\frac{c_1}{(1/2n,1/2n)},\frac{c_2}{(1/7n,1/7n)}\right\}\right)\right)\right\},$$

then $\{(F_n,A):n=1,2,...\}$ forms a sequence of intuitionistic fuzzy soft multi sets.

Definition 3.3 A sequence $\{(F_n,A)\}$ of intuitionistic fuzzy soft multi sets is said to be eventually contained in an intuitionistic fuzzy soft multi set *(F, A)* if and only if there is a positive integer m such that, $n\geq m$ implies $(F_n,A)\tilde{\subseteq}(F,A)$.

Definition 3.4 A sequence $\{(F_n,A)\}$ of intuitionistic fuzzy soft multi sets in an intuitionistic fuzzy soft multi topological space $((,A),\tau)$ is said to be convergence and converge to an intuitionistic fuzzy soft multi set (F,A) if it is eventually contained in each neighbourhood of the intuitionistic fuzzy soft multi set *(F, A)* and we say that the sequence $\{(F_n,A)\}$ has the limit (F,A) and we write $\lim_{n\to\infty}(F_n,A)=\left(\lim_{n\to\infty}F_n,A\right)=(F,A)$ or $(F_n,A)\to(F,A)$ as n $\to\infty$ or simply $F_n\to F$ as n $\to\infty$.

Example 3.5 If we consider an intuitionistic fuzzy soft multi sequence $\{(F_n,A)\}$ as in example 3.2, then

$$\begin{aligned}&\lim_{n\to\infty}(F_n,A)\\&=\lim_{n\to\infty}\left\{\left(e_1,\left(\left\{\frac{h_1}{(2/5n,3/5n)},\frac{h_2}{(1/n,2/4n)}\right\},\left\{\frac{c_1}{(1/2n,1/2n)},\frac{c_2}{(5/7n,1/7n)}\right\}\right)\right),\right.\\&\left.\left(e_2,\left(\left\{\frac{h_1}{(1/5n,3/5n)},\frac{h_2}{(2/3n,1/3n)}\right\},\left\{\frac{c_1}{(1/2n,1/2n)},\frac{c_2}{(1/7n,1/7n)}\right\}\right)\right)\right\}\\&=\left\{\left(e_1,\left(\left\{\frac{h_1}{(0,0)},\frac{h_2}{(0,0)}\right\},\left\{\frac{c_1}{(0,0)},\frac{c_2}{(0,0)}\right\}\right)\right),\left(e_2,\left(\left\{\frac{h_1}{(0,0)},\frac{h_2}{(0,0)}\right\},\left\{\frac{c_1}{(0,0)},\frac{c_2}{(0,0)}\right\}\right)\right)\right\}\\&=\Phi_A\end{aligned}$$

Theorem 3.6 If the neighbourhood system of each intuitionistic fuzzy soft multi set in an intuitionistic fuzzy soft multi topological space $((,A),\tau)$ is countable, then an intuitionistic fuzzy soft multi set *(F, A)* is open if and only if each sequence $\{(F_n,A)\}$ of intuitionistic fuzzy soft multi sets which converges to an intuitionistic fuzzy soft multi set (G,A) contained in *(F, A)* is eventually contained in *(F, A)*.

Proof. Since *(F, A)* is an intuitionistic fuzzy soft multi open set in $((,A),\tau)$, *(F, A)* is a neighbourhood of *(G, A)*. Hence, $\{(F_n,A)\}$ is eventually contained in *(F, A)*.

Conversely, for each $(G,A)\tilde{\subseteq}(F,A)$, let $(G_1,A),(G_2,A),...,(G_n,A),...$be the neighbourhood system of (G, A) and let $(H_n,A)=\tilde{\cap}_{i=1}^{n}(G_i,A)$, then $\{(H_n,A):n=1,2,...\}$ is a sequence of intuitionistic fuzzy soft multi sets which is eventually contained in each neighbourhood of *(G, A)*. Hence, there is an m such that for $n\geq m$, $(H_n,A)\tilde{\subseteq}(F,A)$. Thus (H_n,A) are neighborhood's of *(G, A)*. This implies *(F, A)* is a neighbourhood of *(G, A)* and hence *(F, A)* is an intuitionistic fuzzy soft multi open set.

Theorem 3.7 If an intuitionistic fuzzy soft multi set *(F, A)* is open, then each intuitionistic fuzzy soft multi sequence

$\{(F_n,A)\}$ which converges to a intuitionistic fuzzy soft multi set (G,A) contained in (F, A) is eventually contained in (F, A).

Proof. Since (F, A) is open and $(G,A)\tilde{\subseteq}(F,A)\tilde{\subseteq}(F,A)$, (F, A) is a neighbourhood of (G, A) and since $\{(F_n,A)\}$ converges to a fuzzy soft multi net (G,A), it is eventually contained in each neighbourhood of (G, A). Hence, $\{(F_n,A)\}$ is eventually contained in (F, A).

Definition 3.8 Let f be a mapping over the set of positive integers. Then the sequence $\{(G_n,A)\}$ is a subsequence of a sequence $\{(F_n,A)\}$ if and only if there is a map f such that $(G_i,A)=\left(F_{f(i)},A\right)$ and for each integer m, there is an integer n_o such that $f(i)\geq m$ whenever $i\geq n_o$.

Example 3.9 Let us consider two universes $U_1=\{h_1,h_2\}$ and $U_2=\{c_1,c_2\}$. Let $\{E_{U_1},E_{U_2}\}$ be a collection of sets of decision parameters related to the above universes, where $E_{U_1}=\{e_{U_1,1}=\text{expensive},e_{U_1,2}=\text{cheap},e_{U_1,3}=\text{wooden}\}$

$$E_{U_2}=\{e_{U_2,1}=\text{sporty},e_{U_2,2}=\text{cheap},e_{U_2,3}=\text{2010model}\}.$$

Let $U=\prod_{i=1}^{2}FS(U_i)$, $E=\prod_{i=1}^{2}E_{U_i}$ and $A=\{e_1=(e_{U_1,1},e_{U_2,1}),\ e_2=(e_{U_1,1},e_{U_2,2})\}$.

We chose for n=1,2,3. . . ,

$$(F_n,A)=\left\{\left(e_1,\left(\left\{\frac{h_1}{(2/5n,1/2n)},\frac{h_2}{(3/4n,1/5n)}\right\},\left\{\frac{c_1}{(1/n,3/7n)},\frac{c_2}{(5/7n,1/7n)}\right\}\right)\right),\right.$$
$$\left.\left(e_2,\left(\left\{\frac{h_1}{(1/3n,2/5n)},\frac{h_2}{(2/3n,2/3n)}\right\},\left\{\frac{c_1}{(1/2n,1/4n)},\frac{c_2}{(1/3n,2/3n)}\right\}\right)\right)\right\},$$

and

$$(G_n,A)=\left\{\left(e_1,\left(\left\{\frac{h_1}{(1/5n,1/2n)},\frac{h_2}{(1/2n,1/2n)}\right\},\left\{\frac{c_1}{(1/5n,1/2n)},\frac{c_2}{(1/2n,1/5n)}\right\}\right)\right),\right.$$
$$\left.\left(e_2,\left(\left\{\frac{h_1}{(1/4n,3/5n)},\frac{h_2}{(1/2n,1/2n)}\right\},\left\{\frac{c_1}{(1/3n,1/3n)},\frac{c_2}{(1/4n,2/3n)}\right\}\right)\right)\right\},$$

then $\{(G_n,A)\}$ is a subsequence of the sequence $\{(F_n,A)\}$.

Definition 3.10 The complement of an intuitionistic fuzzy soft multi sequence $\{(F_n,A)\}$ in an intuitionistic fuzzy soft multi topological space $((,A),\tau)$ is denoted by $\{(F_n,A)\}^C$ and is defined by $\{(F_n,A)\}^C=\left\{(F_n,A)^C\right\}=\left\{\left({F_n}^C,A\right)\right\}$.

Example 3.11 If we consider an intuitionistic fuzzy soft multi sequence $\{(F_n,A)\}$ as in example 3.2, then complement of $\{(F_n,A)\}$ is $\{(F_n,A)\}^C=\left\{(F_n,A)^C\right\}=\left\{\left({F_n}^C,A\right)\right\}$,
where for n=1, 2, 3,. . . . . .

$$\left({F_n}^C,A\right)=\left\{\left(e_1,\left(\left\{\frac{h_1}{(3/5n.2/5n)},\frac{h_2}{(2/4n,1/n)}\right\},\left\{\frac{c_1}{(1/2n,1/2n,)},\frac{c_2}{(1/7n,5/7n)}\right\}\right)\right),\right.$$
$$\left.\left(e_2,\left(\left\{\frac{h_1}{(3/5n,1/5n)},\frac{h_2}{(1/3n,2/3n)}\right\},\left\{\frac{c_1}{(1/2n,1/2n)},\frac{c_2}{(1/7n,1/7n)}\right\}\right)\right)\right\},$$

Definition 3.12 A sequence $\{(F_n,A)\}$ of intuitionistic fuzzy soft multi sets is said to be increasing sequence if and only if for each positive integer n, $(F_n,A)\tilde{\subseteq}(F_{n+1},A)$, i.e. $(F_1,A)\tilde{\subseteq}(F_2,A)\tilde{\subseteq}(F_3,A)\tilde{\subseteq}(F_4,A)\tilde{\subseteq}$........

Definition 3.13 A sequence $\{(F_n,A)\}$ of intuitionistic fuzzy soft multi sets is said to be decreasing sequence if and only if for each positive integer n, $(F_n,A)\tilde{\supseteq}(F_{n+1},A)$, i.e. $(F_1,A)\tilde{\supseteq}(F_2,A)\tilde{\supseteq}(F_3,A)\tilde{\supseteq}(F_4,A)\tilde{\supseteq}$........

Definition 3.14 A sequence $\{(F_n,A)\}$ of intuitionistic fuzzy soft multi sets is said to be monotonic if and only if the sequence is either increasing or decreasing sequence.

Example 3.15 Let us consider two universes $U_1=\{h_1,h_2\}$ and $U_2=\{c_1,c_2\}$. Let $\{E_{U_1},E_{U_2}\}$ be a collection of sets of decision parameters related to the above universes, where $E_{U_1}=\{e_{U_1,1}=\text{expensive},e_{U_1,2}=\text{cheap},e_{U_1,3}=\text{wooden}\}$

$$E_{U_2}=\{e_{U_2,1}=\text{sporty},e_{U_2,2}=\text{cheap},e_{U_2,3}=\text{2010model}\}.$$

Let $U=\prod_{i=1}^{2}FS(U_i)$, $E=\prod_{i=1}^{2}E_{U_i}$ and $A=\{e_1=(e_{U_1,1},e_{U_2,1}),\ e_2=(e_{U_1,1},e_{U_2,2})\}$.

We chose for n=1, 2, 3…,

$$\begin{aligned}(F_n,A)=\Big\{&\Big(e_1,\Big(\Big\{\tfrac{h_1}{(1-1/5n,1/2n)},\tfrac{h_2}{(1-3/4n,1/5n)}\Big\},\Big\{\tfrac{c_1}{(1/2,3/7n)},\tfrac{c_2}{(5/7,1/7n)}\Big\}\Big)\Big),\\&\Big(e_2,\Big(\Big\{\tfrac{h_1}{(1/3,2/5n)},\tfrac{h_2}{(1-2/3n,2/3n)}\Big\},\Big\{\tfrac{c_1}{(1-1/2n,1/4)},\tfrac{c_2}{(1-1/3n,2/3n)}\Big\}\Big)\Big)\Big\},\end{aligned}$$

and

$$\begin{aligned}(G_n,A)=\Big\{&\Big(e_1,\Big(\Big\{\tfrac{h_1}{(1/5n,1-1/2n)},\tfrac{h_2}{(1/2n,1/2)}\Big\},\Big\{\tfrac{c_1}{(1/5n,1-1/2n)},\tfrac{c_2}{(1/2n,1/5)}\Big\}\Big)\Big),\\&\Big(e_2,\Big(\Big\{\tfrac{h_1}{(1/4n,3/5)},\tfrac{h_2}{(1/2n,1-1/2n)}\Big\},\Big\{\tfrac{c_1}{(1/3n,1-1/3n)},\tfrac{c_2}{(1/4n,2/3)}\Big\}\Big)\Big)\Big\},\end{aligned}$$

then the sequence $\{(F_n,A)\}$ is increasing sequence and the sequence$\{(G_n,A)\}$ is decreasing sequence.

Definition 3.16 A sequence $\{(F_n,A)\}$ of intuitionistic fuzzy soft multi sets is said to be frequently contained in an intuitionistic fuzzy soft multi set *(F, A)* if and only if for each positive integer n, there is a positive integer m such that, $n\geq m$ implies $(F_n,A)\tilde{\subseteq}(F,A)$.

Definition 3.17 An intuitionistic fuzzy soft multi set (F,A) in an intuitionistic fuzzy soft multi topological space $((,A),\tau)$ is a cluster intuitionistic fuzzy soft multi set of a sequence $\{(F_n,A)\}$ if the sequence $\{(F_n,A)\}$ is frequently contained in every neighbourhood of (F, A).

Theorem 3.18 If neighbourhood system of each intuitionistic fuzzy soft multi set in an intuitionistic fuzzy soft multi topological space $((,A),\tau)$ is countable, then for (F,A) is a cluster intuitionistic fuzzy soft multi set of a sequence $\{(F_n,A)\}$ there is a subsequence converging to (F,A).

Proof. Let $(K_1,A),(K_2,A),...,(K_n,A),...$ be neighbourhood system of (F, A) and let $(L_n,A)=\tilde{\cap}_{i=1}^{n}\{(K_i,A)\}$. Then

$\{(L_n, A) : n = 1, 2, ...\}$ is a sequence of intuitionistic fuzzy soft multi sets such that $(L_{n+1}, A) \tilde{\subseteq} (L_n, A)$ for each n and is eventually contained in each neighbourhood of (F, A). For every positive integer i, choose $f(i)$ such that $f(i) \geq i$ and $(F_{f(i)}, A) \tilde{\subseteq} (L_i, A)$ and hence $\{(F_{f(i)}, A) : i = 1, 2, ...\}$ is a subsequence of the sequence $\{(F_n, A) : n = 1, 2, ...\}$, which converges to (F, A).

Theorem 3.19 Let (F, A) be a cluster intuitionistic fuzzy soft multi set of an intuitionistic fuzzy soft multi sequence $\{(F_n, A)\}$ and (F, A) contained in an intuitionistic fuzzy soft multi set (G, A). If (G, A) is open, then the intuitionistic fuzzy soft multi sequence is frequently contained in (G, A).

Proof. Since (G, A) is open and (F, A) contained in an intuitionistic fuzzy soft multi set (G, A). Hence (G, A) is a neighbourhood of (F, A). Also, since (F, A) be a cluster intuitionistic fuzzy soft multi set of an intuitionistic fuzzy soft multi sequence $\{(F_n, A)\}$ so by the definition of cluster intuitionistic fuzzy soft multi set the sequence $\{(F_n, A)\}$ is frequently contained in every neighbourhood of (F, A) and hence, $\{(F_n, A)\}$ is frequently contained in (G,A).

4 Conclusion

Fuzzy sets, soft sets, soft multi sets, fuzzy soft sets, fuzzy soft multi sets and intuitionistic fuzzy soft multi sets are all important mathematical tools for dealing with uncertainties and vagueness. In this research work, we have introduced a new sequence of intuitionistic fuzzy soft multi sets in intuitionistic fuzzy soft multi topological spaces together with some basic properties over a fixed parameter set. The concepts of subsequence, increasing sequence, decreasing sequence and convergence sequence of intuitionistic fuzzy soft multi sets are proposed. We also introduce the concepts of cluster intuitionistic fuzzy soft multi sets of sequences and study their basic properties. As far as future, we will study the work of Halis Aygün, who has paper about fuzzy soft topological spaces and some important and interesting issues to be addressed.

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