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A Note on Essential Subsemimodules

Kishor Pawar

Department of Mathematics, School of Mathematical Sciences,
North Maharashtra University, Jalgaon – 425 001 (M.S.) India

Abstract: Let M be an R-semimodule and N non-zero subsemimodule of M. We say that N is an essential subsemimodule of M, if $N \cap K \neq (0)$ for every nonzero subsemimodule K of M. In this paper we study some useful results on essential subsemimodules and singular semimodule of semiring.

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1 Introduction

The paper is concerned with generalizing some results in ring theory and module theory. In this paper we will discuss about an extension of the notion of an essential subsemimodules of semimodules. The semiring and semimodule are important structures that have achieved an importance in recent development of theory as their usefulness to many disciplines has been discovered and exploited. Subsemimodules in semimodules are different from submodules in modules in that there are several kinds of submodules. In this paper we study some useful results on essential subsemimodules and singular semimodule of semiring. There are many different definitions of a semiring appearing in the literature. For definitions and properties of semirings, ideals, the reader is referred to [2].

Definition 1.1: A semiring is a set R together with two binary operations called addition $(+)$ and multiplication $(\cdot)$ such that $(R, +)$ is a commutative monoid with identity element 0_R; $(R, \cdot)$ is a monoid with identity element 1; multiplication distributes over addition from either side and 0 is multiplicative absorbing, that is, $a \cdot 0 = 0 \cdot a = 0$ for each $a \in R$ [2].

Definition 1.2: A semiring R is said to have a unity if there exists $1_R \in R$ such that $1_R \cdot a = a \cdot 1_R = a$ for each $a \in R$ [2].

Definition 1.3: An ideal I of a semiring R will be called subtractive (k-ideal) if for $a \in I, a + b \in I, b \in R$ imply $b \in I$ [2].

For e.g.: The set $\mathbb{N}$of non-negative integers with the usual operations of addition and multiplication of integers is a semiring with $1_{\mathbb{N}}$.

Definition 1.4: Let R be a semiring. A left R-semimodule is a commutative monoid $(M, +)$ with additive identity 0_M for which we have a function $R \times M \to M$ defined by $(r, m) \hookrightarrow r \cdot m$ and called scalar multiplication which satisfies the following conditions for all r and r' of R and all elements m and m' of M,

1. $(r \cdot r')m = r(r' \cdot m)$

*Corresponding author.
E- mail address: kfpawar@nmu.ac.in

2. $r \cdot (m + m') = r \cdot m + r \cdot m'$
3. $(r + r') \cdot m = r \cdot m + r' \cdot m$
4. $1_R \cdot m = m$ (If exists)
5. $r \cdot 0_M = 0_M = 0_R \cdot m.$

Convention: In this paper all semirings considered will be assumed to be commutative semirings with unity [2].

1 Essential Ideal

Definition 2.1: An ideal I of a semiring R is said to be an essential ideal of R if $I \cap K \neq 0$ for every nonzero ideal $K$of R [1].

Notation: We shall denote an essential ideal I of a semiring R by $I \lhd\cdot R$.
Proposition 2.2: If $0 \neq K \lhd\cdot I \lhd\cdot R$ and $\overline{K}$ is the ideal of R generated by K, then $\overline{K}$ is essential ideal of R [3].
Proof: Let L be any nonzero ideal of R. Since I is essential ideal in R, we have $I \cap L \neq 0$. Since K is essential ideal in I, we must have $0 \neq K \cap (I \cap L) \subseteq K \cap L \subseteq \overline{K} \cap L$. Thus $\overline{K}$ is essential ideal in R.
2 Essential Subsemimodules

Definition 3.1: Let M be an R-semimodule and N a non-zero subsemimodule of M. We say that N is an essential subsemimodule of M, if $N \cap K \neq (0)$ for every nonzero subsemimodule K of M.

Notation We shall denote an essential subsemimodule N of an R-semimodule M by $N \subseteq_e M$.

Clearly, that is equivalent to say $N \cap Rx \neq (0)$ for any nonzero element $x \in M$. So in particular, a nonzero left ideal I of R is an essential left ideal of R if $I \cap J \neq (0)$ for any nonzero left ideal J of R, which is equivalent to the condition $I \cap Rx \neq (0)$ for any nonzero element $r \in R$.

Proposition 3.2: Let M be a left R-semimodule. Any subsemimodule of M which contains an essential subsemimodule of M is itself essential in M.

Proposition 3.3: Let M be a left R-semimodule. If K is an essential subsemimodules of L and L is an essential subsemimodule of M then K is essential in M.

Proposition 3.4: Let M be a left R-semimodule. Let a be a non-zero element of M and let K be an essential subsemimodules of M then there is essential left ideal L of R such that $aL \neq 0$ and $aL \subseteq K$.

Proposition 3.5: Let$M$an$R$-semimodule and suppose that $N_1, N_2, \cdots, N_k$ are subsemimodules of M.then $\bigcap_{i=1}^{k} N_i \subseteq_e M$ if and only if $N \subseteq_e M$ for all i.

Proof: We only need to prove the proposition for $k = 2$. If $N_1, N_2 \subseteq_e M$, then $N_1 \subseteq_e M$ and $N_2 \subseteq_e M$ because both N_1 and N_2 contain $N_1 \bigcap N_2$.

Conversely, let P be a nonzero subsemimodule of M. Then $N_1 \cap P \neq 0$ because $N_1 \subseteq_e M$ and therefore $(N_1 \cap N_2) \cap P = N_2 \cap (N_1 \cap P) \neq 0$ because $N_2 \subseteq_e M$. Hence the proof.

3 Main Result

Definition 4.1: Let M be an R-semimodule and $x \in M$. The left **annihilator** of x in R is defined by $ann(x) = \{ r \in R \mid rx = 0\}$. Which is obviously a left ideal of R.Now, consider the set $Z(M) = \{x \in M \mid ann(x) \subseteq_e R\}$. It is easy to see that $Z(M)$ is a subsemimodule of M and we will call it the **singular subsemimodule** of M. If $Z(M) = M$, then M is called singular. If $Z(M) = 0$, then M is called **nonsingular [2]**.

Proposition 4.2: If $M = K/L$ for some R-semimodule K and some subsemimodule$L \subseteq_e K$. Then an R-semimodule M is singular. Anticipation

Proof: Suppose first that $M = K/L$ where K is an R-semimodule and$L \subseteq_e K$. Let $x = a + L \in M$ and let J be a nonzero left ideal of R. If $Ja = (0)$, then $Ja \subseteq L$ and so $ann(x) \cap J = J \neq (0)$. If $Ja \neq (0)$, then $L \cap Ja \neq (0)$ because $L \subseteq_e K$. So there exists $r \in J$ such that $0 \neq ra \in L$. That means $0 \neq r \in ann(x) \cap J$. So we have proved that $x \in Z(M)$ and hence $Z(M) = M$ i.e. M is singular.

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